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Hacking the quantum revolution: 1925–1975

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Abstract. I argue that the quantum revolution should be seen as an Ian Hacking type of scientific revolution: a profound, *longue durée*, multidisciplinary process of transforming our understanding of physical nature, with deep-rooted social components from the start. The "revolution" exhibits a characteristic style of reasoning – the hierarchization of physical nature – and developed and uses a specific language – quantum field theory (QFT). It is by virtue of that language that the quantum theory has achieved some of its deepest insights into the description of the dynamics of the physical world. However, the meaning of what a quantum field theory is and what it describes has deeply altered, and one now speaks of "effective" quantum field theories. Interpreting all present day quantum field theories as but "effective" field theories sheds additional light on Phillip Anderson's assertion that "More is different". This important element is addressed in the last part of the paper.

Introduction

It is usual to consider the quantum revolution to have started at the beginning of the 20th century with Planck's observation that the exchanges of energy between matter and electromagnetic radiation could be interpreted as occurring in discrete transfers, and Einstein's transformation of this hypothesis by considering radiation as consisting of quanta of discrete energy (Kuhn 1978). It is also usual to consider the revolution as still ongoing (Brown et al. 1995, Kragh 1999, Staley 2013). My paper considers the period from 1925 till the mid-1970s. The reason for 1925 is that it marks the beginning of a coherent mathematical formulation of the quantum theory. I stop in the mid 1970s with the explanation of second order phase transitions, with the establishment of the standard model, and with a general acceptance by the physics community that all present day quantum field theories are but "effective" field theories valid in a limited range of energies. The standard model – the most "foundational" present day theory – is a description in terms of quarks, leptons, gluons, electroweak bosons, Higgs bosons and their interactions formulated as a relativistic quantum field theory (RQFT). It accounts – in principle – for much of the observable physical world: mesons, nucleons, nuclei, atoms and molecules, and many properties of stars (see e.g. Gottfried and Weisskopf 1984–1986, Hooft 1997b). But of equal importance is the fact that its quantum field theoretical formulation makes possible a description such

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that knowledge of the dynamics of the submicroscopic domain and beyond, is not necessary, nor relevant, to describe the atomic, molecular and higher levels, *at the level of precision* with which we probe the latter levels: the"effective" field theories which describe these sets of levels are decoupled. The effects of what happens at very, very short distances (to the extent that "distance" retains meaning) on the dynamics at the energies being probed is encapsulated in certain parameters in the Lagrangian describing the dynamics at the level being probed. The values of these parameters are obtained by measurements. The use of effective field theories thus explains the possibility of describing the world hierarchically at a certain level of precision, and not having to know everything in order to know something. "Hierarchy" has two *primary* meanings: on the one hand, as a body of persons organized or classified according to rank or authority; and on the other, as a body of entities arranged in a *graded* series. It is in this second sense that I am and shall be using the word. Field theories conceptualized as "effective" field theories also highlight a facet of Phillip Anderson's assertion that "More is Different" as we shall see in Section 5 (Anderson 1972, and especially Weinberg 1995–2000; Gross 1999b, Duncan 2012).

The solution of the phase transition problem using a *non-relativistic* field theoretical *language* explained the *universality* features of second order phase transitions: what is *common* in the description of the phase transition near the critical point of a ferromagnet and of a simple fluid; why such a phase transition depends essentially only the dimensionality and on certain symmetry aspects of the system; and most importantly, why the microscopic dynamics responsible for the structure of the system in its different phases become essentially irrelevant at the critical point. The field theoretic formulation allows the systematic integrating out of the high frequencies, short wave length modes that are not probed by the measuring instruments. It not only makes possible the formulation of a theory of second order phase transitions, but integrating out short wavelengths modes associated with atoms and molecules enables one to derive the Navier-Stokes equations, a universal long wavelength description of the dynamics of fluids, where the particular atomic or molecular composition of the fluid is manifested in just two parameters: the density and the viscosity of the fluid (see Kadanoff and Martin 1963, Nelson 1999). The field theoretic formulation also gives deep new insights into the kind of averaging that is necessary to be able to go from a microscopic description to a macroscopic thermodynamic one (see Kadanoff 2000).

The standard model by virtue of the high energy, short distance domain it describes must be formulated as a *relativistic* quantum field theory. What characterizes *relativistic* quantum field theories (RQFTs) is their ability to *simultaneously* incorporate

(a) the principal feature of a quantum description in terms of observables represented by operators that act on vectors in a Hilbert space that represent states. Their point of departure is a Lagrangian, or more precisely an action, i.e. an $\int Ldt$ – this in order to incorporate symmetries and the ensuing conservation laws. RQFTs tacitly assert that they are as context free a description as is possible. Their "context" is the space-time of special relativity, a space-time unaffected by the processes that take place in it. Given a RFT, quantum rules can variously be imposed. Most recently, this is usually formulated in the language of Feynman path integrals or functional integrals more generally. (see e.g. Negele and Orland 1988, Weinberg 1995–2000 and 1996, Zinn-Justin 2005).

And RQFTs also incorporate

(b) the requirement of locality, namely, that what happens in a very small space time region depends only on what happens in very nearby space-time regions – and not on distant ones; and in addition, the requirement of (special relativistic) causality, that observables located at space-time points that are separated by a space-like distance commute – since they cannot be connected by anything that propagates with the velocity of light or less (see Duncan 2012).

A third feature of quantum field theories that will be important for my argument is that

(c) RQFTs can satisfy (a) and (b) and furthermore that one can calculate observable consequences from RQFTs whose ontologies – the elementary entities it refers to in its Lagrangian – form a small set. Thus, the standard model and the description of nature it entails are in terms of a RQFT whose "ontology" are six quarks, eight gluons, six leptons and some Higgs Bosons.

But, when computing observable consequences of RQFTs in perturbation theory, such as S-matrix elements, i.e. scattering amplitudes, one encounters divergences. The renormalization procedure of Dyson that indicated how to extract finite answers in the perturbative treatment of RQFTs was at one time thought to give a privileged status to some theories, the renormalizable ones. However, the effective field conceptualization has radically altered the meaning of renormalizabilty, while still keeping the computational apparatus for making calculations and eliminating in a consistent manner the divergences one encounters.

The quantum theory (QT) as expressed in the language of QFT, is one of the most remarkable theories yet devised. The range of its applicability – some 20 orders of magnitude – is staggering. But its inability to incorporate general relativity also indicates its limitations: new concepts of interactions, objects, space and time, geometry, ... will have to be considered and probably introduced. It is widely believed that relativistic quantum field theory, the *foundational* language used so far in the quantum description of nature, reached the limits of its validity *as presently formulated* in the domain in which the standard model is applicable (see e.g. Susskind 2005, and the discussions in Cao 1999). Hence the date 1975 which terminates my coverage.

The role of context

Some further general remarks concerning the process of transformation of representations and concerning periodization will help clarity the scope of the paper.

I am using the appellation "representation" in a philosophically restrained and somewhat loose manner[1], attributing to it the following meaning. Concepts, theories, models,... are *representational devices*. They are used with specific purposes in mind, in order to deal with entities embedded in specific contexts. In physics representational devices are expressed by languages obeying specific linguistic conventions. Science under this view is seen as replacing individualized representations so as to be objective. This, to the extent that science then produces observations, phenomenological/experimentally based laws, theories, models, ... that can be considered constituting an accurate, reliable, useful, ... representation of 'real world' situations/states.

Furthermore, this science in general, and physics in particular, is an international enterprise. From 1925 on and throughout the 1930s physicists everywhere were concerned with extending the boundaries of the applicability of quantum mechanics to include not only atoms, simple molecules, and solids, but they were also exploring its validity at ever smaller distances, namely its applicability when describing nuclei and nuclear forces and the processes induced by high energy cosmic rays.

Though being an international, cooperative enterprise, the institutions which nurture and support the activities of scientists are affected by the political, ideological and economic context of their national setting. With the rise of National socialism, Hitler coming to power, and the enactment of racial laws determining who could hold positions at German universities, the intellectual leadership in theoretical physics moved from Germany to the United States, Great Britain and the Soviet Union. World War II increased the dominance of the United States in all areas of physics.

Wars – hot and cold – form an essential part of the history of physics during the twentieth century, particularly in the United States where they molded the development of physics[2]. Many of the technological advances produced during World War II are well known: radar, purity of semiconductor materials (germanium, silicon), nuclear reactors, computers and computing, management technologies (operations research, systems engineering,. . .). The time scale of the changes is to be noted. The time frame involved in the development of nuclear energy, from the discovery of fission (1939) to the successful operation of the first pile (1942) and of nuclear reactors of industrial size (Hanford 1944), is probably the shortest time scale for any technology up to that time.

After World War II, the plethora of new precision instruments that became off-shelf equipment in the laboratory, the success of the renormalization program in quantum electrodynamics, masers, lasers, transistors, and PDP computers[3] opened up new worlds in atomic physics. And ever more powerful accelerators did the same in high energy physics.

The large number of high energy accelerators that were built in the US from the late 1940s till the early 1970s reflected the importance of, and the power that had accrued to, the nuclear physicists who had built the atomic bomb. Many of them had become outstanding researchers in high energy physics and trained a new generation of theorists for whom quantum field theory was the natural language because of the success of quantum electrodynamics. However, it took over a decade to overcome the challenges to RQFT posed in the late 1950s by the difficulties it encountered in accounting for the empirical data in hadron physics generated by the high energy accelerators, and the challenges posed by Landau (1955, 1956) and by Geoffrey Chew and his S-matrix approach and philosophy. The phenomenological quark model and the formulation of the electroweak theory based on gauge principles, by Steven Weinberg (1967) and somewhat later by Abdus Salam (1968), were important landmarks leading to the standard model. The success of the quark model and of the electroweak theory brought about the demise of Chew's S-matrix theory and of his associated notion of nuclear democracy, since making quarks the building blocks of mesons and nucleons explicitly violates the premises of Chew's "nuclear democracy" (see Lipkin 1997, Kaiser 1999).

It can be argued that until the mid 1970s, the privileged status of high energy physics and of high energy physicists, and the "fundamental" character attributed to relativistic quantum field theories and its metaphysics of reductionism[4], were a co-construction of the Cold War (see Schweber 1997, Wang 1999, Leslie 1993). The civil rights movement, the Vietnam war, the student upheavals, and the ascendancy of neo-liberal thought changed all that. The impact of Phillip Anderson's influential and generative article, "More is Different" is indicative of the deep change in perspective these events had generated (Anderson 1972)[5]. New importance became attached to understanding how to construct, and to "reconstruct" the world, and to appreciating the limits of the knowledge based on the high energy physicists' tenets of reductionism. John Bell and his inequalities can be considered a point of departure for the transformation at the "internal" level (see Freire 2015). High precision lasers, cavity quantum electrodynamics, quantum measurements, decoherence, the control of single and many atoms or photons in cavities, nanotechnology, quantum optics, quantum computation, cryptography, and the commercial applications of these devices and of the knowledge that makes them possible, have taken center stage (see e.g. Haroche and Raymond 2006). It can be argued that the centrality of these concerns is a reflection of the impact of neo-liberalism on politics and economics since the early 1980s (see Radder 2010, Schweber 2014). The periodization of a longer *durée*, e.g. from 1925 till the end of the 20th century, would reflect more sharply the economic, political, and cultural contexts in which the advances were made (see Schweber 1997, Krige and Pestre 1997, Kragh 1999).

Were my paper an attempt at historical epistemology, i.e. a "reflection on the historical condition *under* which, and the means *with* which, things are made into objects of knowledge" (Rheinberger 2010, pp. 2-3), it would have to give an account of the *cultural* factors[6] that made possible the process of generating the scientific knowledge I focus on[7]. I have given an overview of the *political* factors that played an important role in generating the new knowledge in Schweber 1997. In the present paper I am principally concerned with those aspects of the *history* of the quantum revolution until the mid 1970s that are relevant to characterize it as a Hacking type revolution. Belfer and Schweber (2015) have called *Hacking type revolutions* large scale, *longue durée*, scientific revolutions that introduce a new style of reasoning, affect several disciplines, transform or introduce new institutions and alter the feel of the world. Ian Hacking described the probabilistic revolution of the 19th century as such a revolution with the statistical analysis of regularities of populations as its style of reasoning and the calculus of probabilities as its language (Hacking 1987). The characteristic style of reasoning (Hacking 1982) the quantum revolution exhibits is the hierarchization of inert nature, with "hierarchy" understood as a *graded* series of domains. It has also created a particular *language* to formulate its foundational theories and models: quantum field theory[8].

My paper's intent is to corroborate this historiographical viewpoint. It highlights the social: the interactions and exchanges of knowledge and techniques between various disciplines and sub-disciplines; the co-construction of the *language* of quantum field theory by various disciplines and subdisciplines; the individuals and communities responsible for advances; the biography of individuals who made crucial contributions in order to draw attention to their particular educational trajectories and to the "metaphysics" they were committed to, the latter to be seen as a component of the enabling conditions and conditions of possibility for the knowledge they produced.

Also, as my focus is the change that occurred in the 1970s in the *conceptualization* of what a quantum field theory is, I emphasize certain earlier *conceptual* transformations that were responsible for it: renormalization, BCS, spontaneously broken symmetry, scaling, renormalization group.

My presentation is *not* a fine grained analysis on how these changes came about. I have not presented important contributions, such as the development of current algebras, the treatment of anomalies, and the role these and more generally, the role that symmetries played in arriving at QCD (see Bell and Jackiw 1969; Jackiw 1995, Adler 2004, Weinberg 1995–2000, Gross 1997, Bjorken 1997, Cao 2010). Thus Murray Gell-Mann, who played a central role with his contributions to these areas is not prominent in my narratives. Similarly, I have not recounted the researches of Schwinger and Weinberg (1966) on the pion-nucleon system which were crucial in conceptualizing effective field theories. (See Weinberg 1979, DeWitt 1996, Polchinski 1999). And the same can be said regarding my account of the solution of the phase transition problem, wherein Michael Fisher (1998) played a central and crucial role, yet does not appear prominently in my story. The reason is because he has given a beautiful, factual account of his role and researches (Fisher 1999, see also 1998), and because I concentrate on those aspects that transform the meaning of quantum field theoretical descriptions, hence the focus on Benjamin Widom, Leo Kadanoff and Kenneth Wilson.

Tian Yu Cao has given an outstanding account of the development of quantum field theories in three remarkable books he has authored (Cao 1997, 1999, 2010). But his exposition of the developments of 20th century field theories is, to some extent, philosophically driven by the aim of justifying a commitment to structural realism. Also in *From Currents Algebras to QCD* he is specifically interested in the establishment of QCD, whereas my exposition stems from an attempt to understand the developments more generally, as a historian of science and as a biographer of Hans Bethe, one of the outstanding theoretical physicist of the 20th century (Schweber 2012, 2014). Additionally, my presentation is limited by my lack of *detailed* knowledge not only of many facets of the developments until the 1970s (e.g. the development of mathematical physics and its contributions to field theory and statistical mechanics; developments in "pure" mathematics that became incorporated into theoretical physics; developments in computers and computing,. . .), and as importantly, developments since the mid-1970s. It is thus a retrospective, low resolution, narration which emphasizes certain key elements that are important for my exposition. I should also add that the technical expositions that are presented are not meant to be reconstructions of what given authors did.

The paper is organized as follows. The details of what is meant by a Hacking type revolution are outlined in Section 1. The interdisciplinary character of the revolution during the 1930s is presented in Section 2. Its subsections deal with specific aspects of the developments: 2.1, with hierarchization and the lessons learned from nuclear physics regarding symmetry; 2.2, with quantized fields; 2.3, with interdisciplinarity; 2.4, with quantum mechanical based experimental physics during the 1930s; 2.5, with the relation of mathematics and physics; and 2.6. with computing. After a Pause Section 3 does the same for the post World War II developments. Again my intent is to highlight interdisciplinary and inter-subdisciplinary aspects – and in particular, the cross-fertilization between condensed matter and high energy physics, (Jona-Lasinio 2002, Kadanoff 2000, 2009, 2013, Butterfield and Bouatta 2011), the evermore important role of mathematics, its co-development and its co-construction (see e.g. Ge 1989, Deligne et al. 1999, Atiyah 2002, Shroer 2010, Krieger 2013), and the ever greater impact of computing (see Kadanoff 1986, and e.g. Negele 2001, Boyle 2003, Troyer 2010, Ren et al. 2012, Schweber and BenPorat 2015). Even though the separateness and the entanglement of theory and experiment are crucial aspects of Hacking's conceptualization of science[9], due to the enormity of the post WWII technological, experimental and instrumental innovations and practices and my limited knowledge of them, I will say very little about the interaction between experiments and theory, though fully aware of their profound entanglement and their crucial, cooperative, mutually beneficial, indispensable relationship. Bjorken has stated the matter succinctly:

> It is my credo that technological advances drive the progress in experimental physics and that experimental physics in turn drive the theory. Without these ingredients, the most brilliant theoretical constructs languish worthlessly. There is in my opinion no greater calling for a theorist than to help advance the experiments. It is not an easy thing to do (Bjorken 1997, p. 596).

Specifically, the subsections dealing with the post WWII developments are as follows: 3.1, QED and Renormalization; 3.2, QFT and Solid State Physics; 3.2.1 Spontaneous Symmetry Breaking; 3.3, The BCS Theory; 3.4 Goldstone Bosons; 3.5, Some rigorous results in equilibrium statistical mechanics. Section 4 deals with the reconceptualization of QFT: 4.1 Scaling; 4.2, Renormalization Groups; 4.3, Benjamin Widom; 4.4, Leo Kadanoff; 4.5 Kenneth Wilson; 4.6, Peter Lepage and NRQFT; a Pause leads to section 5 which is concerned with some reflections on "More is Different"; 5.1 QCD once again; 5.2 From nuclear physics to nuclear science. A Coda is the final section of the paper.

1 Hacking-type revolutions[10]

Hacking type revolutions transform a wide range of scientific practices and are multidisciplinary, with new institutions being formed that epitomize the new directions. The time scale of Hacking-type revolutions is the *longue durée*, but the *durées* have become shorter as the scientific community has increased.

Belfer and I associate with a Hacking-type revolution a new style of scientific reasoning. Styles of reasoning are the constructs that specify what counts as scientific knowledge and constitute the cognitive conditions of possibility of science. They are made concrete through the specification of ontological and explanatory models (see Hacking 1982–2012). Hacking stressed that a style of reasoning must introduce new types of "objects, evidence, sentences (new ways of being a candidate for truth or falsehood), laws, or any rate modalities, (and most importantly), possibilities" (Hacking 2012).

Hacking (1992) gave the following examples of styles of reasoning[11]: postulation in the mathematical sciences, the deployment of experiment to control postulation and to explore by observation and measurement, the hypothetical construction of analogical models, ordering by comparison of variety and taxonomy, the statistical analysis of populations, and the historical derivation of genetic development. Note that different styles of reasoning can co-exist and are bounded in scope with definite limits of applicability[12].

Hacking type revolutions amalgamate pure and applied concerns. Hacking type revolutions are emplacement-revolutions, rather than replacement-revolutions. They change the way a science is practiced without necessarily jettisoning all the previous concepts, transforming it from within by a shift of the questions being asked and the criteria for acceptable answers, this being a characteristic of an "emplacement revolution" (Humphreys 2011). A Hacking-type revolution is characterized by a new style of scientific reasoning and conversely, the genesis of a new style of reasoning is indicative that a Hacking-type revolution is in the process of evolving, with self-authentication and self-stabilization characteristic features of the evolutionary process.

The style of reasoning I associate with the quantum revolution is characterized by the hierarchization of the inanimate microscopic world, with "hierarchy" understood as a *graded* series of domains. In my characterization of the quantum revolution as a Hacking type scientific revolution language is a crucial component. I have designated quantum field theory as the language constructed by the quantum theory. I do so because it is in this language that many of the most useful idealizations and models have been formulated. In particular, in the case of RQFT it calls attention to the presence of the vacuum, and of the fluctuations and correlations ever present there, with the consequence that no "particle" is never isolated from the vacuum. QFT, as a system with an infinite number of degrees of freedom, also points to the vacuum as making possible what is called spontaneous symmetry breaking, a crucial feature of the BCS theory of superconductivity and of the Higgs mechanism and of the standard model. And most importantly, relativistic quantum field theory, the theory which yields descriptions of nature at ever smaller distances has singled out a particular set of theories – gauge theories – and has indicated why among these only a small subset are able to account for phenomena at the distances probed by the high energy accelerators presently in operation. Furthermore, the standard model can do so in terms of a small number of foundational entities, the quarks, gluons, leptons and Higgs particle. Were there too many quarks with different flavors, asymptotic freedom, a key explanatory feature of the theory would be invalidated. Finally, RQFTs give insights into the limits of quantum mechanical representations.

The quantum revolution helped clarify the extent to which the physical world can be "hierarchisized" and its amazing diversity made plausible. The hierarchization I speak of is such that each level has associated with it its ontology and its "effective" dynamics. At the atomic level the basic ontology are nuclei and electrons. At the molecular level, ions and electrons. The dynamics is described by an "effective" non relativistic quantum field theory, wherein the electrons are described by a matter field obeying the free Schrödinger equation when non-interacting. The great simplification which occurs is that within that energy range, particle numbers are conserved. At the submicroscopic levels the descriptions are similarly in terms of an "effective" field theory and an associated ontology. An "effective" field theory is a description of phenomena in a certain energy regime bounded by some energy less than some energy cut-off Λ, in terms of wavelengths longer than some length scale h/Λ. An effective field theory assumes that the physics in the domain in which it is valid can be given in terms of "elementary" entities – the basic field excitations – out of which the composite entities that populate that domain are built. These "elementary" entities constitute the effective degrees of freedom appropriate to that scale. They depend on a more "fundamental" theory only through a small set of "running" parameters that enter in the description of the dynamics of these entities (See Weinberg 1995–2000; Banks 2008, Duncan 2012)[13].

The use of effective field theories makes the following important feature possible: as long as one does not probe beyond the energy and length scales in which they are deemed applicable, the description of the physics in their domain is essentially decoupled from the higher energy modes and thus not invalidated by discoveries at smaller distances. (Symanzik 1973, Applequist and Carazzone 1975). An ensuing issue becomes answering the question: "To what extent can one reconstruct the world knowing the most 'fundamental' effective theory now known, namely, the standard model?" This was the issue that was addressed by Anderson (1972) in "More is Different". I argue that the smallness of the number of 'kinds' of elementary entities that act as the "elementary particles" at the atomic, nuclear, and subnuclear levels is a prerequisite for making possible a hierarchical, field theoretical description of that part of the physical world. This small number relates to how many parameters enter in the Lagrangian describing the effective field theory such as masses and coupling constants. Furthermore, certain special features of the standard model make this description possible:

(1) the fact that quantum chromodynamics is a renormalizable, asymptotically free and infrared confining gauge theory defined in terms of a small number of quarks and gluons, which can bind and make a finite number of stable compound, observable, entities such as nucleons and mesons (in appropriate physical environments); and in terms of which the nuclear domain can be described and explained. And
(2) the fact that, similarly, electroweak theory is defined in terms of a finite number of leptons, and a finite number of spin 1 bosons that are responsible for the interaction between leptons and quarks, and that the theory is renormalizable. Among the leptons the electron plays a unique role because it is the lightest electrically charged particle and thus absolutely stable by virtue of conservation of electric charge. And among the bosons, the photon has the special property of remaining massless, and thus giving rise to QED.

The smallness of the number of this "foundational" ontology is responsible not only for the asymptotic freedom of quantum chromodynamics, but also for the practicality of the renormalizability of the standard model, an essential component in its calculational aspects. In all these descriptions the conservation laws play a crucial role. "More" becomes very "different" at the solid state and at the chemical level, because the basic entities of the solid state and chemical world, ions and electrons, can combine in the terrestrial context to form an almost limitless number of new compounds with startlingly different properties and mode of combinations[14]. I will return to the issue in the coda laws.

2 Quantum mechanics and QFT in the 1930s

The initial, seemingly different, formulations of quantum mechanics (QM) by Heisenberg and by Schrödinger were both motivated by the problem that Bohr had addressed in his historic 1913 papers: the explanation of the stability of atoms and molecules[15]. Schrödinger's wave mechanics, because of the simplicity of its formulation, the familiarity of its mathematical formalism, the straightforwardness of its explanation of the stability of the hydrogen atom, the intuitiveness of the interpretation of its wave functions, its visualizability, and in particular, the ease with which it could incorporate the Pauli principle – made it the widely used and prevalent approach[16] (see for example Mehra and Rechenberg 1987, Darrigol 1993, Renn 2013). Its fructiferousness made it the standard approach to elucidating and solving the problems that had proven intractable in the old quantum theory, in particular, proving the stability of atoms and molecules and calculating their properties[17].

The wave mechanical explanation of the periodic table pointed out one of the distinctive features of quantum mechanics: it mandates that for a system of particles which interact with one another through forces that make it possible for them to become a bound system, – i.e. to combine and be in state with less energy than when they are widely separated and not moving, – only discrete values of the energy of the bound system are possible. Furthermore, if the energy difference between the ground state – the state of lowest energy – of such a bound system and its first excited state is much greater than the energy it can acquire in any interaction with its environment, the system can be considered to be endowed with fixed, unchanging properties when in that context.

The success of non-relativistic quantum mechanics (NRQM) in explaining the structure of atoms and simple molecules is due to the fact that these entities can be considered – in their usual terrestrial context – to be composed of point-like electrons and of (essentially) immutable nuclei, "immutable" because the characteristic energy level separation between the ground state and first excited states in nuclei is of the order of kilovolts and higher[18]. Initially the description of the atomic world was in terms of the stable nuclei and electrons. All these nuclei were treated the same way: they were differentiated only by their mass and electric charge, parameters determined experimentally and which entered the Schrödinger equation the same way. This is what can be called a *universality* feature of the non-relativistic Schrödinger equation:

$$H\,|\Psi\rangle = E\,|\Psi\rangle \tag{2.1}$$

$$\begin{aligned} H = &-\sum_{j}^{N_e} \frac{\hbar^2}{2m}\nabla_j^2 - \sum_{\alpha}^{N_n} \frac{\hbar^2}{2M_\alpha}\nabla_\alpha^2 \\ &- \sum_{j}^{N_e}\sum_{\alpha}^{N_n} \frac{Z_\alpha e^2}{|r_j - R_\alpha|} + \sum_{j\prec k}^{N_e} \frac{e^2}{|r_j - r_k|} + \sum_{\alpha\prec\beta}^{N_j} \frac{Z_\alpha Z_\beta e^2}{|R_\alpha - R_\beta|} \end{aligned} \tag{2.2}$$

which determines the structure of atoms and molecules (Z_α and M_α are the atomic number and mass of the αth nucleus, $\boldsymbol{R}_\alpha$ is the location of this nucleus, e and m are the electron charge and mass, $\boldsymbol{r}_j$ is the location of the jth electron, and $\hbar$ is Planck's constant).

NRQM attributes a hierarchical order to the microscopic world. It can do so by virtue of the specificity of the energy levels of bound systems, by virtue of the energy scales involved and the validity of certain conservation laws. Bethe (1993) put it thus:

> Quantum theory tells you that an atom has quantum states... It says that a carbon atom is a carbon atom is a carbon atom and not something else... Only because of quantum theory are chemical compounds definite. Only because of quantum theory can we tell that carbon has four valences, that there is CH_4.

Energy scales translate into momentum scales and the latter into distance scales by virtue of Heisenberg's uncertainty principle. The non-relativistic quantum mechanical description of the structure of atoms in terms of electrons and nuclei is a low resolution, long wave length one. It implicitly assumes that one is not to probe very small distances.

2.1 Hierarchization: the 1930s

In the second half of the 19th century the macroscopic representations of the physical world became partitioned into various branches – celestial mechanics, mechanics of continuous media, electromagnetism, thermodynamics, ... with each of these areas based on a wide ranging set of principles. For example, the dynamics of fluids was described by the Navier-Stokes equations, in which entered two macroscopic parameters: the density and viscosity of the particular fluid being considered. These parameters were to be empirically determined (Darrigol 2005). The same was true for thermodynamics and the equation of state which entered into the description of the behavior of particular systems, or in the case of electrodynamics and Maxwell's equations, in which appeared three macroscopic parameters: the conductivity of the material, σ, its dielectric constant, ε, and its magnetic permeability μ, all three to be experimentally determined[19].

Clausius, Maxwell, Boltzmann, Gibbs, Lorentz, Drude and others investigated the interrelation of representations of the atomic realm with its inherent discreteness with those of the macroscopic realm with their continuum descriptions (see e.g. Buchwald 1985). Although they could formulate relationships between the macroscopic parameters and dynamical descriptions at the micro level, the attempt to obtain the observed value of the macroscopic parameters failed in most cases. This because until the end of the nineteenth century it was believed that classical physics[20] governed the dynamics of entities *at all scales* and that its "fundamental" laws were immutable. In retrospect, this commitment would have led to the recognition – by virtue of chaos and other factors – that macroscopic physics would be unpredictable when based on the details of microscopic physics. One could argue that a good deal of the success it had depended on the central limit theorem of probability theory[21].

Lorentz's 1906 Columbia lectures (Lorentz 1909), – the account of his researches that related the *macroscopic* Maxwell equations to a *microscopic* representation of the dynamics of the electromagnetic field and of charged particles after the "discovery" of the electron in 1897 – pointed out the difficulties encountered in the classical approach. But he also formulated an essentially modern exposition of a microscopically based model of electric and thermal conduction in metals and insulators. It embodied what became the paradigmatic approach to relate macro parameters to the dynamics of micro entities[22]. But in Lorentz's formulation the properties of the charged entities which appeared therein – such as their mass, their charge, their charge distribution, – were assumed to be unchanging – a characteristic feature of classical representations. Being unchanging and immutable their history could be described by the change in time of certain quantifiable *properties* attributed to them: in the case of particles their position and their momenta.

Fontana and Buss (1996) succinctly characterized the classical description and its extensions[23]:

> In a dynamical system, it is *not* the interacting entities that participate *as objects* in the formal constitution of "the system", but rather their quantitative properties and couplings. As a consequence, interaction is understood as the temporal or spatial change in the numerical value of variables. This change is captured by a set of (deterministic or stochastic) differential (or difference) equations.

What was fundamentally new in Schrödinger's wave mechanics is that the interacting entities participate as *objects*, whose structure, couplings and other attributes could change as a result of the interactions, and more particularly, that new *objects* could be formed. As important as had been the calculations of Pauli and Schrödinger in obtaining the level structure of the hydrogen atom, – a new *object* formed by the interaction of an electron with a proton, – or that of Heisenberg in explaining the level structure of the (two electron) He atom, and the subsequent calculations to explain the periodic table, a further *crucial* calculation was that of Walter Heitler and Fritz London that explained the formation of the H_2 molecule[24]. The calculation gave a new quantitative perspective on bonding and saturation. In addition, the directional characteristics of orbitals when electrons were not in *s* states were used to indicate how quantum mechanics could explain the bonding properties of the carbon atom so important to understand the structure of organic compounds. A morphic element was thus introduced into quantum mechanical explanations (Gottfried and Weisskopf 1984).

The quantum mechanical modeling of the atomic and nuclear world had two further attributes that were recognized early and shaped the approach to understanding phenomena at the micro and macro levels:

(1) A quantum description gives a measure of certainty to our knowledge of the world: it asserts that all hydrogen atoms in their ground state when isolated are identical, in fact indistinguishable; idem for ^{23}Na atoms in their ground state; similarly, that all lead ^{206}Pb$_{82}$ nuclei in their ground state are identical... In fact, Bethe, in his acceptance speech upon receiving the 1993 Orsted medal for his contributions to the teaching of physics, claimed "that there is a *certainty principle* (emphasis added) in quantum theory (concerning the energies of bound states and the identity of systems in such states) and that the certainty principle is far more important for the world and us than the uncertainty principle. That doesn't say that the uncertainty principle is wrong. It says that the uncertainty principle just tells you that the concepts of classical physics, position, and velocity, are not applicable to atomic structure" (Bethe 1993).

(2) As noted earlier, when computing the properties of atoms, molecules and solids the value of the parameters that enter into the Schrödinger equation describing the dynamics of the system characterizing the electron and the nuclei – such as their mass, spin, magnetic moment, electric quadrupole moments – the values of these parameters are empirically determined[25]. After the discovery of the neutron in 1932, and after models of nuclear structures had been advanced, and it had been recognized that quantum mechanics seemed to be a valid description of the dynamics of neutrons and protons, these nuclear parameters were to be explained and their value quantitatively determined by the "lower level" theory that was to account for the structure and stability of nuclei, i.e, by a description of nuclear dynamics in terms of neutrons and protons and the nuclear forces by which they interact[26].

By 1932 the general features of the quantum theory of systems composed of a finite number of particles had been characterized by Dirac and by von Neumann. Already in the first edition of his *Principles of Quantum Mechanics* Dirac (1930) noted that quantum mechanics attributes a hierarchical order to the physical world by virtue of Planck's constant,h, and the atomic length scale it introduces, h/me^2, where m is the mass of electron, and e its electric charge[27]. Furthermore, systems whose characteristic time T, mass M, and length L, are such that $ML^2/T \gg h$ are "macroscopic" and described by classical mechanics, those for which $ML^2/T \approx h$ are "microscopic" and described by quantum mechanics[28]. In 1932, John von Neumann expounded in detail his axiomatic formulation of quantum mechanics in his *Mathematische Grundlagen der Quantenmechanik*, thereby basing quantum mechanics on a seemingly rigorous mathematical foundation. In it states were represented by vectors in Hilbert space, observables were associated with self-adjoint operators, and the dynamics of the theory was implemented by a unitary operator acting on the state vector representing the system. This unitary operator, in turn, was related to the Hamiltonian, H, of the system by Stone's theorem (Mackey 1949):

$$U(t-t') = \exp[(i/\hbar)H(t-t')].$$

2.2 Quantized fields

The initial formulations of quantum mechanics considered problems involving a finite number of particles. But already in the 1925 *dreimänner Arbeit* of Born, Heisenberg and Jordan, in which the full range of matrix mechanics was presented, Jordan addressed the problem of the quantum mechanical description of a one-dimensional string, a system he described in terms of a denumerable infinity of degrees of freedom. His formulation anticipated the quantization of the free electromagnetic field. Dirac's quantum theory of the electromagnetic field – in which the creation (emission) and the annihilation (absorption) of photons could be described, and therefore the emission and absorption of photons by atoms – was the crucial initial step in the formulation of what became known as quantum electrodynamics. It was also the first quantum mechanical investigation of a system with an infinite number of degrees of freedom which could be related to empirical evidence.

It was Jordan who, with Klein and with Wigner, formulated the quantization of the non-relativistic Schrödinger equation conceived as describing the dynamics of a classical matter *field*. Just as photons were conceived as the quanta of the quantized electromagnetic field, for Jordan, electrons were the quanta of the quantized Schrödinger matter field. Furthermore, he indicated the commutation relations the field operators had to satisfy to describe indistinguishable particles obeying either Fermi-Dirac or Bose-Einstein statistics. The non-relativistic field theories Jordan considered conserved the number of particles and were completely equivalent to the description by an N-particle Schrödinger equation, with the N-particle wave function satisfying the appropriate symmetry conditions under particle exchange (see Schweber 1994, Weinberg 1995–2000, Chap. 1, Duncan 2012, Schroer 2010).

After Dirac had introduced his relativistic equation for a spin 1/2 particle Heisenberg and Pauli generalized Jordan's notion of quantum fields. They detailed a formulation of QED which was a generalization of the time dependent Schrödinger equation applied to field systems. In it, electrons or protons were to be understood as manifestations of an underlying Dirac spin 1/2 quantum field, but with only the electric current, $j_\mu(x) = e\overline{\psi(x)}\gamma_\mu\psi(x)$, and the total electric charge being conserved. Their QED was a local quantum field theory, the interaction term being of the form $j_\mu(x)A^\mu(x) = e\overline{\psi(x)}\gamma_\mu\psi(x)A^\mu(x)$ that generalized the interaction with the electromagnetic field of point like particles that have no spatial structure. However, it was Fermi's (1932) version of quantum electrodynamics that introduced the community at large, and in particular graduate students, to the use of QED and became the model for the extension of field theoretic methods in the 1930s. As Bethe asserted at the Enrico Fermi memorial symposium at the Washington meeting of the APS on April 29, 1955 shortly after Fermi's death:

> Many of you probably, like myself, have learned their first field theory from Fermi's wonderful article in the *Reviews of Modern Physics* of 1932. It is an example of simplicity in a difficult field which I think is unsurpassed. It came after a number of quite complicated papers and before another set of quite complicated papers on the subject, and without Fermi's enlightening simplicity I think many of us would never have been able to follow into the depths of field theory. I think I am one of them.

Both in earlier articles as well as in his *Reviews of Modern Physics* presentation Fermi pointed to an important new feature that QED introduced: a single charged particle when moving with constant velocity has an attached Coulomb field and an infinite number of (virtual transverse) photons attached to it that give rise to its Biot-Savart (magnetic) field. The notion of localization which seemingly was straight forward in non-relativistic quantum mechanics[29] assumed a new degree of complexity by virtue of the attached fields. Mathematicians such as von Neumann, recognized that there was something very different about the quantum mechanics of a finite number of particles and that of an infinite number of degrees of freedom, such as those Jordan had dealt with in the *dreimännerarbeit* in his quantization of a continuous string; or the Fock space associated with the quantization of a non-relativistic matter field that has an interaction term in its Hamiltonian of the form $\int d^3x d^3x' \psi^*(x')\psi(x')v(x - x')\psi^*(x)\psi(x)$; or the infinite number of zero energy and zero momentum "photons" introduced by Dirac in his initial version of QED; or the infinite number of attached photons in Fermi's QED.

Fermi's formulation of QED became the standard way to address problems involving photon emission and absorption field theoretically. Though physically transparent[30], it had one great disadvantage: *it imposed a particular gauge, the radiation gauge, which seemingly destroyed both the relativistic invariance of the formulation and locality.*

The investigations by Oppenheimer and by Waller of the self-energy of the electron in QED revealed the divergence difficulties that all *local, relativistically invariant* quantum field theories displayed (at least in perturbation theory). And these divergence difficulties proved to be a major obstacle in extending quantum mechanics to field systems. And whereas in the non-relativistic domain it was easy to introduce description of two, three and even many body interactions, and maintain Galilean invariance – this whether the description is in terms of particles or in terms of a matter field and appropriate quantization rules are imposed–this was not the case in the relativistic domain. The formulation of a Poincaré invariant description of a system of interacting *particles* proved to be an exceedingly difficult undertaking and to the best of my knowledge has not been successful.

During the 1930s the growing corpus of nuclear and cosmic ray data (see Brown and Rechenberg 1991) together with the quantum field theoretical demonstration that the electromagnetic interactions between charged particles could be explained as due to photon exchanges (Fermi 1932), Fermi's formulation of a field theory of β-decay[31], Yukawa's suggestion that in analogy to electromagnetic forces the short range nuclear forces between nucleons could be generated by the exchanges between them of a hitherto unobserved massive particle (Brown and Rechenberg 1994, 1996), a novel conceptualization of physics began assuming an ever greater importance. It consisted in recognizing – at the level of accuracy of possible physical measurements and the corresponding theoretical representations – that the physical world could be considered *hierarchically ordered* into fairly well delineated domains and concerns: the macroscopic – consisting of solids, liquids, gases, their structure, and their properties[32]; the molecular and atomic realm; the nuclear; and the sub-nuclear ones[33]. Furthermore, the physical interactions and processes by which the different levels "emerged" from one another were suggested[34]. Once again, I wish to emphasize that the term "hierarchical" is to be understood as implying that the "elementary entities" and "representations" of the various domains can be arranged in a *graded* series. It does not imply that the "bottom" domain is more "fundamental" than the others.

The atomic, nuclear, and subnuclear realms were thought to be describable by separate (foundational) ontologies and corresponding quantum dynamics. The ontologies were associated with a given level – electrons, nuclei and photons for the atomic and molecular realm, neutrons and protons for the nuclear level, with the latter's interactions at first described phenomenologically by nuclear potentials, and later assumed to be derivable from a quantum field theory of nucleons and mesons once mesons were included in the basic ontology. Additionally, neutrinos, and once again electrons (and positrons) and Fermi's theory of beta-decay – when limited to the lowest order of perturbation theory– were accepted as explaining the radioactivity of nuclei. The entities which comprised the foundational ontology were considered the building blocks of the composite objects that populated that level. It should be noted that each of these levels had a "foundational" ontology consisting of a small number of "kinds" of entities.

This novel conceptualization of physics which emerged with QFT theory in the 1930s bolstered the view that the aim of physics was to identify, classify and characterize the various realms, and their interrelations. It was the task of the theories representing the lower levels to quantitatively account for the empirically determined parameters which described the "elementary" building blocks of the higher levels. This despite the fact that there was little confidence that QFT was adequate to explain the nuclear domain in terms of subnuclear constituents (Oppenheimer 1941). Nonetheless, the microscopic and sub-microscopic levels became considered more "fundamental" since it was believed that one would eventually be able to reconstruct the higher levels in terms of the knowledge of the properties and dynamics of the entities that populated the lower levels. The success of NRQM in explaining the low energy properties of atoms and molecules in terms of electrons, nuclei, and photons; that of simple molecules in terms of their atomic constituents; and of nuclear theory in accounting for some of the properties of nuclei – such as their masses and magnetic moments in terms of the interaction potentials between nucleons as determined from neutron and proton scattering experiments,– gave support to a commitment to reductionism, the belief that a knowledge of the entities and theory at the lowest level would allow one to *derive* the properties of the higher domains (see Butterfield 2011).

But it should be remembered that the vast majority of physicists during the 1930s who were working on problems where quantum mechanics was relevant, were addressing problems where, for the most part, the non-relativistic Schrödinger equation was an adequate description of the dynamics of the matter component.

To the best of my knowledge no solid state physicist used quantum field theory to describe the electrons and ions of solids during the 1930s[35]. There is an obvious reason for this: solid state physics (and condensed matter physics more generally) deals principally with *non-relativistic systems* in which there is no creation or annihilation of massive particles. The Hamiltonians that determine the structure of atoms, molecules or solids or the dynamics of processes such as electronic conduction in metals, *conserve particle numbers*[36]. Processes involving the emission and absorption of massless photons or phonons were described using a formalism arrived at by imposing quantum commutation rules on the variables that described the electromagnetic field and the lattice vibrations at the classical level. As these entities do not carry conserved charges, superpositions of states with different numbers of photons or phonons present no problems.

The same can be said about nuclear theory during the 1930s. Problems in nuclear structure and nuclear reactions were all addressed using the Schrödinger equation with particles – neutrons and protons – as the basic observables. Only when dealing with β-decay phenomena, and only after Yukawa's seminal paper introducing meson exchanges to explain nuclear forces did field theory enter into nuclear physics. Nuclear electromagnetic effect were treated in the same way using quantum electrodynamics as in atomic and molecular phenomena.

It should also be remembered that in comparison to the number of physicists working in solid state physics and other areas of physics, the size of the community of "elementary particle" physicists using quantum field theoretical tools to address problems such as pair production by photons, Bremsstrahlung, the problem of vacuum polarization, or the derivation of nuclear forces from meson theory, was relatively small.

Broadly speaking, theorists were divided into two camps. One group considered particles – including photons – as basic *observables*, – and had to put in the Pauli principle by hand. They did consider the electromagnetic field as a classical field to be "quantized", thus yielding zero mass photons. Conversely, they could also give plausible ways of recovering the classical description in terms of photons. The other group considered fields as the basic *observables*. The field theorists viewed material particles as the quanta of quantum fields, for example free electrons as the quanta of a spin 1/2 matter field which obeyed the Schrödinger or the Dirac equation, thus explaining the identity of all electrons. The anti-commutation rules imposed on the field operators automatically yielded the antisymmetry of the wave functions describing n-particle states. Ironically, most theorists dealing with relativistic phenomena involving positrons viewed electrons as being described quantum mechanically by the *one particle* Dirac equation interpreted hole theoretically, i.e., with all the negative energy states assumed occupied. This accommodated the possibility of the creation of the positron as the "hole" created when a negative energy electron is given sufficient energy to make a transition to a positive energy state[37]. Dirac's initial confrontation with vacuum polarization was formulated hole theoretically and only later given its field theoretic formulation (see Schweber 1994).

Vacuum polarization introduced a new deep problem regarding localization to those who wanted to maintain particles as basic observable. How does one define a position operator to a particle which is *always* accompanied by a cloud of electron positron pairs. In fact, what is observed and what is to be described is the dressed particle, but the description introduces divergences when asking what is the observed charge. The concept of charge renormalization allowed the divergence to be "swept under the rug", but did not alleviate the localization problem for "particle" inclined theorists.

A further comment can be made about QM and QFT during the 1930s and this concerns symmetry considerations. The use of group theory became important in deducing the consequences of rotational invariance, of permutations symmetry due to the Pauli principle in atomic systems; and in deducing symmetry properties of electronic wave functions in molecules and crystalline solids. In the study of low energy proton-proton and neutron-proton scattering it was found that to a good approximation the nuclear forces which phenomenologically could account for the observed scattering cross-sections were independent of the electric charge carried by the nucleons: the nuclear forces could be assumed invariant under the transformation which interchanges proton and neutron. This led to a description of nuclear forces as having an SU(2) isospin symmetry in which the proton and neutron are considered to be an isospin doublet, with the group structure of isospin symmetry identical to that of the usual spin (See Mlađenović 1998). But since neutron and proton do have different electric charges, electromagnetic interactions do not respect isospin invariance, and isospin symmetry cannot be an exact symmetry. Were the symmetry an exact one the total Hamiltonian describing the nuclear interactions of a system of nucleons would commute with the generators of the isospin symmetry. All members of an isomultiplet would then have the same mass. Thus the mass differences within an isomultiplet are a good measure of the symmetry breaking. For the proton and neutron $(M_{\mathrm{n}} - M_{\mathrm{p}})/(M_{\mathrm{n}} + M_{\mathrm{p}}) \approx 0.7 \times 10^{-3}$. The concept of isospin invariance would play an important role in "high energy" physics after World War II.

The constraints that Galilean invariance imposes on nuclear potentials when describing nuclei as a non-relativistic systems composed of nucleons whose dynamics is governed by the Schrödinger equation, were investigated by Wigner, Eisenbud and others (Eisenbud and Wigner 1958). However, the constraints of Lorentz invariance for relativistic systems when *computing* their properties were not intensively considered. Thus Weisskopf in 1939 would prove that the self energy of an electron in QED diverged logarithmically to all orders of perturbation theory using non-covariant calculational methods, thereby raising questions about the reliability of his calculation (see Schweber 1994).

2.3 Interdisciplinarity: the 1930s

I have thus far been primarily concerned with conceptual issues. Let me briefly draw attention to some other aspects of the initial phase of the quantum revolution. I shall focus primarily on what happened in the US and Great Britain from 1933 until World War II. One can point there to certain general features brought about by quantum mechanics:

1. being a representation by mathematical models of the atomic, molecular and subatomic world, quantum mechanics is broadly interdisciplinary. It addresses problems in physics, in chemistry, in metallurgy, in biology and medicine[38],... And the formulation of quantum mechanical models becomes primarily the province of theorists.

Theoretical physics became a subdiscipline at the beginning of the 20th century (Jungnickel and McCormmach 1986). But theorists and experimenters interacted strongly, – symbiotically –, with one another[39]. The outstanding young theorists who came of age after the advent of quantum mechanics were masters of all of physics: Bethe can be taken as the paradigmatic example because he was so productive and was the synthesizer and expositor of the new knowledge with his *Handbuch der Physik* articles on atomic, molecular and solid state physics and his articles on nuclear physics in the *Reviews of Modern Physics*. The same is true as far as mastery of all of physics is concerned of Bloch, Condon, Heitler, Landau, London, Oppenheimer, Peierls, Slater, Wigner, ... They tackled problems in all areas of physics, though after 1932 with the discovery of the neutron and the burgeoning of the field of nuclear physics, – by virtue of the number of accelerators being built –, specialization becomes apparent. There is also specialization into the areas that became delineated as solid state physics and as chemical physics. In the Richtmyer lecture of 1951 John Slater (1951) narrated the development of solid state physics. He observed that the original steps taken by Heisenberg to explain atomic structure, Heitler and London to explain covalent bonding in molecular hydrogen, Sommerfeld, Bethe and Bloch to explain the electronic structure of metallic structures, were broadened during the 1930s to a complete investigation of all the "mechanical and chemical and thermal properties" of atoms, molecules and solids. Regarding the appropriation of chemical problems by physicists during the 1930s I will only mention Slater's 1939 textbook on *chemical physics* (Slater 1939) and refer the reader to Gavroglu and Simões's (2012) masterful exposition of the history of quantum chemistry.

There is another feature of the initial phase of the quantum revolution that should be pointed out: for many of the young theorists who came of age with the advent of quantum mechanics in 1925–1926, wave mechanics did not appear revolutionary[40]. Sommerfeld in his 1927 exposition of QM, emphasized the partial differential formulation of the Schrödinger approach and stressed the continuity of wave mechanics with older parts of physics formulated in terms of wave equations. And his students were introduced to the subject with this viewpoint[41]. It is by virtue of the Fifth Solvay Congress held in October 1927, and in particular, the discussions therein between Bohr and Einstein, and subsequently, due to the texts by Dirac, Heisenberg and others, that the revolutionary character of quantum mechanics became emphasized.

2.4 QM based experimental physics during the 1930s

I clearly cannot do justice in the compass of the present article to the new experimental practices that emerged as result of quantum mechanics in atomic and molecular physics, or in nuclear physics by virtue of the discovery of the neutron, or in "high energy" physics by virtue of the new capabilities of cloud chambers, the discovery of the positron, the investigations of cosmic ray showers and the building of ever higher energy cyclotrons.

I shall merely point to two fields of inquiries where quantum mechanics was an essential component:

(1) molecular beams as developed by Stern, and later refined by Rabi and his students
(2) the passage of charged particles through matter.

But before doing so let me indicate why I focus on these two fields.

In a perceptive article Howard Stein (1994) pointed to a general method, which he called the exact-approximate duality, by which some of the mathematically describable physical sciences have proceeded since Newton's time. Observations are first described by a *model* which simulates the observations. The model in turn gives rise to a first approximate *theory*. This theory in turn confronts the data that in the meantime may have been become more accurate, which in turn gives rise to refinements to the first approximate theory. The discrepancy between theory and the observational or experimental data thus may become smaller through successive approximation – and its confrontation with the empirical data tests not only its domain of validity but also its correctness. Think of Kepler modeling planetary motions by ellipses. These in turn gave rise to Newton's gravitational theory. But it is important to note that while Newton assumed that his laws of motion were valid, in the *Principia* he did not derive the inverse square law from the fact that observed trajectories were very nearly ellipses. Rather, as stressed by George Smith (2001), he proceeded under the constraint that conclusions hold *"quam proxime" (very nearly*) when the premises hold *quam proxime*[42]. The subsequent refinement of the theory of planetary motions (to take into account the perturbations of the planets on one another, finite size effects, ...) and the parallel instrumental and observational refinements ultimately led to the conclusion at the end of the nineteenth century that the $1/r^2$ law of universal gravitation could not account for the 43 arc-second per century advance of the perihelion of mercury[43]. This in turn eventually led to Einstein and general relativity. The advance of the perihelion of Mercury became one of the tests corroborating the validity of general relativity, and is a classic example of the *turning of data into evidence* (Smith 2002)[44].

In a series of lectures delivered at Stanford in 2007 on the use of seismology to determine the internal structure and constitution of the earth Smith (2007a,b,c) succinctly characterized his approach. He there stressed that he is concerned with "the nature, scope, and limits of the knowledge attained in individual sciences when, by their own standards, they are most successful". For him "science, ... viewed from the inside, is an endeavor to turn data into evidence, something that is difficult to do and for which theory is invariably needed". And most importantly, "the knowledge pursued in the sciences consists not merely of theory, but also, indeed even more so, of details that make a difference and the differences they make"[45]. If one accepts these tenets, and adopts the stance Smith illustrated in his lectures, then, as he emphasized, an all important element when trying to answer questions about the nature, scope, and limits of the knowledge achieved in an individual science is the determination of "how, if at all, the theory that has been used in turning data into evidence in that science has itself been tested in the process".

Furthermore for Smith, the most important evidence for a given science lies not in individual comparisons of calculations with observation, but in the *history* of the development of the field in response to such comparisons. This, as Smith emphasizes, requires a combination of historical and philosophical research: historical, because "it is a question about the evidential practices in specialized fields over many generations, and philosophical because scientists themselves are usually too preoccupied with learning more about the world than to spend their time analyzing how established theory is being tested in the process".

Rabi's molecular beam apparatus and the first experiments carried out with it (Rabi et al. 1939. See also *Molecular beams* 1965 and Ramsey 1956), the experiments and theory concerned with the passage of charged particles and radiation through matter (Bethe and Ashkin 1953), and the Davis neutrino experiments testing the assumptions and applications of the theory nuclear reactions and of beta-decay to stellar structure – and in particular the structure of the sun – are in the process of being analyzed along the lines that Smith had charted[46].

2.5 Mathematics and physics

There is a further feature of the initial phase of the quantum revolution that stands out: the entanglement of physics and mathematics. It has been repeatedly commented upon that since the 19th century there has been a deep interconnection between the development of physics and mathematics (see e.g. Gray 2008, 2013). Göttingen played a crucial role in this, and in particular the schools of pure and applied mathematics that Felix Klein, David Hilbert, Hermann Minkowski created there at the end of the nineteenth century (see Sigurdsson 1991; Rowe 1989, 2004). In fact, it can be argued that a good part of the co-construction[47] of theoretical physics and mathematics was effected there, by

(a) Felix Klein and his students, and in particular by his *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*;
(b) by Hilbert and Minkowski and their students, in particular, Emmy Noether, and the physics seminar that had been initiated by Minkowski; and
(c) by the embodiment of all of these Kleinian and Hilbertian elements in Sommerfeld and the transplantation of them into his students in his seminar in Munich (Eckert 2004, 2013, Seth 2010).

The investigation of the properties of infinite dimensional linear vector spaces (Riesz 1913), the further development of Riemannian geometry connected with general relativity (see for example Weyl 1919, Cartan 1928), and the exposition of mathematical physics in Courant and Hilbert (1924)[48] are examples of this co-construction. After the advent of quantum mechanics, the investigations of the properties of Hilbert spaces and of linear operators therein produced important works such as Stone's *Linear Transformations in Hilbert Space and their Applications to Analysis.* Einar Hille in a perceptive review of it in 1934 in the *Bulletin of the AMS* noted that

> The attitude of mathematicians towards the applications varies. Some feel a need of rationalizing their personal interests by pointing out their importance for the applications, and the applications know them not. Others assert with pride that their results cannot possibly be applied, and fate whimsically harnesses their purest dreams to the chariot of industry. Whether we want our results to be applied or not, we probably all agree that the applications have a way of posing stimulating questions which spur the progress of our science. Thus we probably owe more of our advance in analysis to the prying curiosity of the physicist than to any other agent.

This is certainly true of ergodic and probability theory during the 1930s (see e.g. Mackey 1988), of investigations of new kinds of algebraic structures such as Jordan algebras (see Jordan 1932, 1933; Jordan et al. 1934), of advances in the theory of unitary representations of non-compact groups (see Wigner 1939). All these attest to the co-construction of physics and mathematics. This will turn out to be the case in an even more pronounced fashion after World War II, and in particular after the 1980s, after the elaboration of string theory.

Here I want only to stress that in the 1930s it was clear to perceptive mathematicians that *quantum field theory* required an extension and a deeper understanding of the Hilbert spaces that quantum mechanics had introduced. It led to the investigations of rings of operators (see e.g. Murray and von Neumann 1937, von Neumann 1940), and of infinite sums and products of Hilbert spaces (see von Neumann contribution to Pauli 1935-6-6 and von Neumann 1939). These mathematical structures would become crucial elements of the language of quantum field theory after World War II.

2.6 Computing

Numerical computations have always been part and parcel of theoretical physics. The researches of Lorentz, Planck, Sommerfeld, Debye, Born, Ehrenfest, Darwin, Fowler, Schrödinger, Kemble, Slater, ... give ample evidence of this. What is new after the advent of quantum mechanics is the use of computers – analog at first, and after WWII, digital ones. They represent powerful new tools with which to try to answer the question: "How far can the world be reconstructed when one knows the foundational theory of a given level?" This is because quantum mechanics allows the computation of the structure of complex atomic structures and the comparison of the ever more precise empirical data with the predicted values of ever more complex quantum mechanical calculations demanded the help of mechanical, and subsequently electronic computers (see Wise 1995).

Douglas Hartree is the outstanding representative of physicists charting this new direction during the 1930s.

Until Hartree became aware of Vannevar Bush's differential analyzer all his work on the energy level structure of atoms using Hartree and Hartree-Fock models had been done with pencil and paper and Marchand hand desk calculators. In 1934, having learned of Bush's analyzer, he went to MIT to see it and to learn how it operated. On returning to Manchester he first built one using Meccano pieces and later had a much bigger and sturdier machine built. During and after WWII he became a central figure in the development of electronic computers both in Great Britain and in the USA (Darwin 1958).

Pause I

My aim in the coarse-grained bird's eye view I have presented of the growth of the quantum theory during the 1930s was to emphasize its interdisciplinary nature. That the quantum theory changed chemical understanding is clear. But conversely, it should be recalled that the insights that had been gleaned in chemistry regarding the saturation of the homopolar bond played an important role in formulating phenomenological nuclear forces that would yield a nuclear binding energy proportional to the number of nucleons in the nucleus (Carson 1996a,b) And the same was true of physics and mathematics.

I did not comment on the restructuring of departments of physics and chemistry, nor on the restructuring of industrial laboratories as result of the advent of quantum mechanics, nor on the economic and political context during which all this took place, as these matters have been addressed in depth in the literature (e.g. see Kragh 1999). Nor did I address the questions Jochen Büttner, Jürgen Renn, and Matthias Schemmel raised in their paper examining the break in knowledge that stemmed from the precision measurements of the frequency distribution of black body radiation (Büttner et al. 2003), namely

(1) What accounts for the breaks in the development of scientific knowledge which can be described as scientific revolutions, whether conceived as Kuhnian or otherwise?
(2) Despite such breaks is there nonetheless a continuous growth of knowledge? and
(3) Where and when do scientific revolutions occur?

These questions will be addressed *briefly* in the coda to my paper.

3 The Post WWII developments

Lee DuBridge, who had directed the Radiation Laboratory during the war, delivered the Richtmyer lecture in 1949. The title of his talk was "The Effects of World War II on the Science of Physics". The first thing he pointed to in it was the dramatic increase in the membership of the American Physical Society: from 3341 in 1938 to 8100 in 1949, and the increased prestige which the war and the post-war developments had brought to physics and physicists. But he considered the major and most permanent contribution of the war years "the vast collection of new experimental techniques and tools to which the war gave birth" (DuBridge 1949).

3.1 QED and renormalization

These instrumental advances had immediate repercussions in theoretical physics – the best known and one of the most consequential was the response to Lamb and Retherford's experiment on the fine structure of hydrogen and to Rabi, Nafe and Nelson accurate measurements of the hyperfine structure of hydrogen announced at the Shelter Island Conference in June 1947 (see Schweber 1994, 2014). These experiments stimulated crucial calculations[49] by Bethe and by Julian Schwinger that were the starting point of the modern renormalization program, and gave renewed faith to quantum field theory. The "modern era" of quantum field theory was initiated by that conference. Steven Weinberg assessed its importance concisely: "It was not so much that it forced us to change our physical theories, as it forced us to take them seriously" (Weinberg 1977).

"Taking them seriously", and in particular taking QED seriously, implied making sure that the relativistic invariance and the gauge invariance of the theory were maintained in the calculations. The ideas that Kramers and Bethe had advanced regarding mass renormalization to circumvent the self energy divergences, and those of Dirac and Weisskopf on how to circumvent the divergences that stemmed from vacuum polarization by charge renormalization, allowed Tomonaga, Schwinger and Feynman to formulate a Lorentz and gauge invariant QED so that it yielded finite values to all observable quantities – such as cross-sections, energies and lifetimes of excited states of H, ... – to fourth order of perturbation theory[50]. Dyson crowned the achievement by proving that the S-matrix of QED is in fact renormalizable to all orders of perturbation theory.

Note that the emphasis was on observables, echoing Heisenberg's philosophical commitments that led him to formulate matrix mechanics in 1925. Even though what Schwinger and Tomonaga had accomplished was seen as a triumph for field theory, the *local fields* that represented the electromagnetically interacting electrons and positrons were "too far away" from observables to be regarded as descriptive elements per se[51]. Hence the focus on the S-matrix (what Heisenberg later called the "roof" of the theory), on cross-sections, and bound state energy level structure.

Tomonaga, Schwinger, Feynman, Dyson all defined QED in terms of a Lagrangian wherein appears the parameters m_o and e_o, assumed to be the bare mass and bare charge of the electron, i.e. its mass and charge in the absence of interactions.

One can translate what Dyson had accomplished as follows[52]: To make the *perturbative* calculations well defined – and it should be stressed that QED was only understood perturbatively, and the use of perturbation theory believed justified by virtue of the smallness of the coupling constant $e^2/4\pi\hbar c \approx 1/137$, – one introduces into the theory a large cut-off momentum Λ, which modifies the theory at short distances h/Λ. One then calculates two physical processes which, if performed experimentally, determine the value of the observable mass, m, and that of the charge, e, of the electron[53]. An analysis of two such processes – e.g. electron-electron scattering and Compton scattering – yield the result that to lowest order of perturbation theory

$$\begin{aligned} e^2 &= e_o^2 - \beta e_o^2 \ln(\Lambda/m_0) \\ m &= m_0 - \gamma m_o (e_o^2/\hbar c) \ln(\Lambda/m_0) \end{aligned}$$

where β and γ are numerical constants. By inverting these relations, one can express the bare constants in terms of the physical ones:

$$\begin{aligned} e_0^2 &= e^2 \ + \ \beta_2 e^2 \ln(\Lambda/m) \\ m_0 &= m + \gamma_1 m (e^2/\hbar c) \ln(\Lambda/m). \end{aligned}$$

The startling fact is that if one expresses any other observable initially calculated in terms of e_0 and m_0 (to order e_0^2) in terms of e and m, it has a finite value (neglecting terms of order $1/\Lambda^2$) and remains finite in the limit the cut-off Λ goes to infinity[54].

Dyson formalized this procedure of "renormalization" in terms of graphs (Feynman diagrams) and indicated that for QED the thus "renormalized" S-matrix scattering amplitudes are finite to all orders of perturbation theory[55]. Furthermore, he indicated that only certain relativistic quantum field theories yielded finite results by this procedure. Renormalizability became an important selection principle for allowable field theories (see Weinberg 1995–2000, Chap. 12).

Within two years after the Lamb-Retherford experiment, renormalization allowed spectacular progress in QED with ever more precise calculations yielding close agreement with ever more precise measurements of the Lamb shift and the hyperfine structure of the hydrogen atom (see Yokoyama and Kubo 1990).

The success of the Schwinger-Feynman-Dyson approach in QED depended on a strict enforcement of relativistic and gauge invariance. It was then believed that gauge invariance would guarantee the vanishing of the photon mass. The usual justification given for the masslessness of the photon was that by virtue of gauge invariance, the self-energy of the photon would necessarily have the form

$$\Pi_{\mu\nu}(p) = (g_{\mu\nu}p^2 - p_\mu p_v)\,\Pi(p^2)$$

which seemed to imply that it must vanish at $p^2 = 0$, and thus guarantee a zero mass for the photon. But in 1961 Schwinger (1961, 1962) gave an argument strongly suggesting that associating a gauge transformation with a local conservation law does not necessarily require a zero-mass vector boson. He pointed out that if the interactions were strong enough $\Pi(p^2)$ could acquire a pole at $p^2 = 0$, thus making the photon massive (Schwinger 1962). These insights had actually been noted earlier in condensed matter physics after solid state theorists had adopted field theoretical tools (Anderson 1963).

These observations reinforced the prevalent view: QED is a successful theory after renormalization because the coupling constant $e^2/\hbar c$ is very small, justifying the perturbative approach to the removal of the divergences encountered when computing scattering amplitudes, level shifts and lifetimes of excited states of atoms.

3.2 QFT and solid state physics

The application of the formalism that had been so successful in QED to the strong interactions and to the weak interactions immediately ran into difficulties. The meson-nucleon interaction, though describable by a renormalizable theory, required the coupling constant to be large to explain the strength of the nucleon-nucleon interaction, which invalidated perturbation theoretic calculations. And the weak interactions, as modeled by the Fermi theory were unrenormalizable.

This was not the case for atomic many body systems. The field theoretic formalism and methods that had been developed for quantum electrodynamics by Schwinger, Feynman and Dyson became incorporated into solid state theory after their articles appeared in print[56], their calculational methods expounded in lecture courses and summer schools, and the summer school notes disseminated and their content incorporated in the first textbooks on the subject[57]. Feynman diagrams in many-body theory were first used as calculational aids[58]. Diagrammatic techniques were introduced in the zero-temperature quantum many body problem in 1954 by Keith Brückner and co-workers; by Jeffrey Goldstone and Bethe in the theory of nuclear matter in 1955; and by John Hubbard and by Gell-Mann and Brückner (1957) in the theory of the electron gas in 1957. Takeo Matsubara, already in 1955, began applying Green's function methods in the calculation of the partition function of an interacting many body system. In 1959 Martin and Schwinger introduced a powerful formalism, based on Schwinger's functional methods for generating Green's functions, which allowed the formulation of a variety of *non-perturbation* theoretic approximations. Detailed expositions of these Feynman and Schwinger based advances were given in 1961 by Abrikosov, Dzyaloshinskii and Gorkov (1961,1963) in the Russian edition of their *Methods of Quantum Field Theory in Statistical Physics* and by Kadanoff and Baym (1962) in their *Quantum Statistical Mechanics. Green's Function Methods in Equilibrium and Nonequilibrium Problems.* But it should be noted that the underlying metaphysics of solid state physicists and nuclear theorists using these methods still considered particles as the basic observables.

Actually, the use of quantum field theory in condensed matter physics had started somewhat earlier. The field theoretic formulation of the many body problem in terms of creation and annihilation operators (and the associated Fock space) makes the indistinguishable particles of the system anonymous, a valuable translation that simplifies of the mathematical description of the dynamics, and which in turn allows new possible approximations.

In 1947 Bogoliubov used a field theoretic formalism to explain the properties of liquid He[4][59], and to give a microscopic foundation to the phenomenological two fluid model that Landau and Tizsa had proposed to account for the superfluidity of liquid He^4. Bogoliubov formulated his theory and his approximations in terms of the equations of motions the creation and annihilation obeyed. The corresponding Hamiltonian formulation is the following

$$H = \int d^3r \left[\psi^*(r)\frac{-\hbar^2}{2m}\nabla^2\psi(r) + \int d^3r \int d^3r' \psi^*(r)\psi(r)v(r-r')\psi^*(r')\psi(r')\right] \tag{3.1}$$

where $\psi^*(r)$ and $\psi(r)$ are respectively, a creation and annihilation operator for a helium atom at r, and $v(r-r')$ represents the pairwise interaction potential between two helium atoms, including the hard core repulsion at short inter particle distance. As shown by Einstein, Bose-Einstein condensation – i.e. the macroscopic occupation of a single particle level, – can occur in a system of non interacting bosons, i.e. when $v=0$. But such a system would not display superfluidity. Superfluidity requires interactions between the bosonic particles that play the role of the helium atoms in the model Hamiltonian for the system.

The use of creation and annihilation operators was a key element in the approximation method Bogoliubov introduced. He used the fact that in zeroth approximation the ground state of liquid ^{4}He is macroscopically occupied by zero momentum particles and showed that when the hard core interaction between two helium atoms is taken into account, a justifiable approximation is obtained by replacing the creation and annihilation operator for the zero momentum one particle state by classical c-numbers. The resulting theory did not conserve particle numbers and broke the gauge symmetry $\psi \to e^{i\alpha}\psi$; $\psi^* \to e^{-i\alpha}\psi^*$ of the original formulation, where ψ^* and ψ represent creation and annihilation operators for a helium atom. Crucial in the formulation of the approximation was the fact that the thermodynamic limit $N \to \infty$, $V \to \infty$ but N/V remaining finite was taken: phase transitions can only occur in systems with an infinite number of particles[60]. Bogoliubov's theory was one of the first derivations of a *mean field theory* starting from a microscopic theory. However, its approximations were such that it could not exhibit the richness of the field theory exhibited when $T \neq 0$.

3.2.1 Spontaneous symmetry breaking[61]

Bogoliubov's theory was important because it was later recognized as one of the first examples of what became known as spontaneous symmetry breaking of a continuous symmetry. The symmetry in question in Bogoliubov's model is the invariance of the Hamiltonian under the global U(1) gauge symmetry

$$\psi \to e^{i\alpha}\psi;\ \psi^* \to e^{-i\alpha}\psi^*$$

which, by Noether's theorem, yields a conserved current, from which the conservation of the total particle number

$$[\mathrm{H}, \mathrm{N}] = 0$$

follows. Conversely when expressed in terms of the many-particle wave functions, the freedom of choice of the global phase of the many particle wave function is responsible for the conservation of total particle number, which in turn implies a global U(1) gauge symmetry. Later developments indicated that it is more fruitful to associate Bose-Einstein condensation and He^4 superfluidity with spontaneous symmetry breaking than with macroscopic occupation of a single particle state. But spontaneous symmetry breaking cannot occur when the normalized ground state of the many particle Hamiltonian is *non degenerate*, that is, is unique. This because under this circumstance the transformation law for the ground state under any symmetry of the Hamiltonian must be multiplication by a phase factor[62]. Hence the ground state when unique must transform according to the trivial, one dimensional representation of the symmetry group. There is thus no possibility of spontaneous symmetry breaking by which the ground state transforms *non-trivially* under a continuous symmetry of the Hamiltonian[63].

Evading this constrain requires allowing $N \to \infty$, $V \to \infty$ but particle density N/V remaining finite, i.e. taking the thermodynamic limit; or alternatively requiring a fixed chemical potential and a fixed external pressure. When using the latter approach the Hamiltonian 3.2.1. becomes

$$H_\mu = \int d^3r \left[\psi^*(r)\left(\frac{-\hbar^2}{2m}\nabla^2 - \mu\right)\psi(r) \right.$$
$$\left. + \int d^3r \int d^3r' \psi^*(r)\psi(r)v(r-r')\psi^*(r')\psi(r')\right]$$

where μ is the chemical potential.

Consider the non-interacting case[64]. In momentum space it becomes

$$H_{(0)\mu} = \sum_n (\varepsilon_n - \mu)a_n^* a_n; \quad [a_n,\ a_{n'}^*] = \delta_{nn'}; \quad [a_n,\ a_{n'}] = 0$$

and differs from $H_{(0)}$ in that all the energy eigenvalues have been shifted by μ. But behind the seeming simplicity of the change there has occurred a dramatic transformation: the dimensionalities of the eigenstates of $H_{(0)\mu}$ can change drastically. Thus the choice $\mu = \varepsilon_0$ make the ground state energy vanish. Furthermore, all the states

$$|N_0,\ 0,\ 0,\ldots\rangle = \frac{(a_0^*)^{N_0}}{\sqrt{N_0!}}|0\rangle$$

are orthogonal eigenstates of $H_{(0)\mu}$ with the same energy eigenvalue 0. Any linear combination of such states remains an eigenstate of $H_{(0)\mu}$. From such linear combinations one can construct *coherent* states,

$$\left|\sqrt{V}\phi, 0, 0, \ldots\right\rangle_{cs} = e^{\sqrt{V}\phi a_0^*}|0\rangle$$

and in particular, *ground state* eigenfunctions

$$|\phi\rangle_{gs} = e^{\sqrt{V}(\phi a_0^* - \phi^* a_0)}\,|0\rangle$$

with

$${}_{gs}\langle\phi|\ \phi'\rangle_{gs} = e^{-V\frac{|\phi-\phi'|^2}{2}}$$

which are all eigenstates of $H_{(0)\mu}$ with zero energy, but not of the number operator. In fact,

$${}_{gs}\langle\phi|\ \psi(r,t)|\ \phi'\rangle_{gs} = \phi$$
$${}_{gs}\langle\phi|\,\psi^*(r,t)\ \psi(r,t)|\ \phi'\rangle_{gs} = |\phi|^2\,.$$

In the case of interacting particles the chemical potential is chosen by requiring that at zero temperature

$$\langle 0|\, \psi^*(r,t)\, \psi(r,t)\, |0\rangle = \frac{N}{V}.$$

Spontaneous symmetry breaking is said to occur when the ground state of a system is degenerate and is not a singlet under the action of a continuous symmetry group that leaves the Hamiltonian determining the dynamics of the system invariant. A quantity like $\langle 0|\, \psi(r,t)\, |0\rangle$ which vanishes when the ground state is non-degenerate but becomes non-vanishing with spontaneous symmetry breaking is called an order parameter.

3.3 The BCS theory[65]

John Bardeen, Leon Cooper, and John Schrieffer's (BCS) field theoretically formulated theory of superconductivity was a momentous development (Bardeen, Cooper, Schrieffer 1957; see also Cooper and Feldman 2011)[66]. Bardeen at the time was 49 years old, and had already won a Nobel prize for his contribution to the creation of the transistor. Cooper was a post-doctoral fellow working with Bardeen, and Schrieffer a graduate student with Bardeen as his supervisor. All three were in the department of physics at the University of Illinois in Champaign-Urbana, Illinois.

Until the BCS theory, superconductivity was considered the one outstanding problem that quantum mechanics had not been able to explain. A key insight of the BCS theory was the recognition of the consequences of the existence of an attractive force between electrons near the Fermi surface stemming from their interactions with the lattice vibrations, i.e. with phonons. That interaction makes possible the formation just below the Fermi surface of bound pairs of electrons of opposite momentum and opposite spin, called Cooper (1956) pairs, which destabilizes the Fermi surface. The fact that all the energy levels up to the Fermi surface are filled is an essential feature. Thus the importance of the Pauli principle in determining the structural features and the dynamics of the valence electrons in a metal.

The simplified model BCS adopted is defined by the Hamiltonian

$$H = \sum_{\boldsymbol{k}\sigma} (\varepsilon_{\boldsymbol{k}} - \mu) c^*_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} - \frac{g}{V} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} c^*_{\boldsymbol{k}+\boldsymbol{q}\uparrow} c^*_{-\boldsymbol{k}\downarrow} c_{-\boldsymbol{k}'+\boldsymbol{q}\downarrow} c_{\boldsymbol{k}'\uparrow} \tag{3.1}$$

which is to describe the physics of the set of states centered around the Fermi surface with energies between $\epsilon_F + (\omega_D/2)$ and $\epsilon_F - (\omega_D/2)$. A net attraction operates when g is a positive constant. The $c_{\boldsymbol{k}\sigma}{}^*$ and $c_{\boldsymbol{k}\sigma}$ operators are the usual creation and annihilation operators for a Fermion of momentum $\boldsymbol{k}$ and spin σ, and satisfy the anticommutation rules

$$\begin{aligned} \left\{c^*_{\boldsymbol{k}\sigma},\ c_{\boldsymbol{k}'\sigma'}\right\} &= \delta_{\boldsymbol{k}\boldsymbol{k}'}\, \delta_{\sigma\sigma'} \\ \left\{c_{\boldsymbol{k}\sigma}, c_{\boldsymbol{k}'\sigma'}\right\} &= \left\{c^*_{\boldsymbol{k}\sigma},\ c^*_{\boldsymbol{k}'\sigma'}\right\} = 0. \end{aligned} \tag{3.2}$$

In the BCS theory the ground state becomes characterized by the assumption that the operator $\sum_{\boldsymbol{k}} c_{-\boldsymbol{k}\downarrow} c_{\boldsymbol{k}\uparrow}$ has a non vanishing expectation value in it:

$$\begin{aligned} \frac{g}{V} \sum_{\boldsymbol{k}} \langle \Omega_S|\, c_{-\boldsymbol{k}\downarrow} c_{\boldsymbol{k}\uparrow}\, |\Omega_S\rangle &= \Delta \\ \frac{g}{V} \sum_{\boldsymbol{k}} \langle \Omega_S|\, c^*_{\boldsymbol{k}\uparrow} c^*_{-\boldsymbol{k}\downarrow}\, |\Omega_S\rangle &= \overline{\Delta}. \end{aligned} \tag{3.3}$$

The ground state $|\Omega_S\rangle$ thus clearly cannot contain a definite number of particles and does not respect the gauge invariance of the Hamiltonian: it exemplifies *spontaneous symmetry breaking.* Bogoliubov (1958) and Valatin (1958) showed that introducing the operators

$$\begin{aligned}\alpha_{\boldsymbol{k}\uparrow} &= \cos\theta_{\boldsymbol{k}} c_{\boldsymbol{k}\uparrow} + \sin\theta_{\boldsymbol{k}} c^*_{-\boldsymbol{k}\downarrow}\\ \alpha^*_{-\boldsymbol{k}\downarrow} &= \sin\theta_{\boldsymbol{k}} c_{\boldsymbol{k}\uparrow} - \cos\theta_{\boldsymbol{k}}\, c^*_{-\boldsymbol{k}\downarrow}\end{aligned} \tag{3.4}$$

brings the Hamiltonian to diagonal form:

$$\sum_{\boldsymbol{k}\sigma} \lambda_{\boldsymbol{k}} \alpha^*_{\boldsymbol{k}\sigma}\alpha_{\boldsymbol{k}\sigma} + \sum_{\boldsymbol{k}} (\varepsilon_{\boldsymbol{k}} - \lambda_{\boldsymbol{k}}) \tag{3.5}$$

with

$$\begin{aligned}\lambda_{\mathbf{k}} &= \sqrt{(\Delta^2 + \varepsilon^2_{\boldsymbol{k}})^2}\\ \cos 2\theta_{\boldsymbol{k}} &= \varepsilon_{\boldsymbol{k}}/\lambda_{\boldsymbol{k}}; \quad \sin 2\theta_{\boldsymbol{k}} = -\Delta/\lambda_{\boldsymbol{k}}.\end{aligned} \tag{3.6}$$

The ground state of the diagonalized Hamiltonian (5), $|\Omega_S\rangle$, is the vector annihilated by all the annihilation operators $\alpha_{\boldsymbol{k}\sigma}$. This condition is met by the vector

$$\begin{aligned}|\Omega_S\rangle &= \prod_{\boldsymbol{k}} \alpha_{-\boldsymbol{k}\downarrow}\alpha_{\boldsymbol{k}\uparrow}\,|\Omega\rangle\\ &: \prod_{\boldsymbol{k}} (\cos\theta_{\boldsymbol{k}} - \sin^2\theta_{\boldsymbol{k}} c^*_{-\boldsymbol{k}\downarrow} c^*_{\boldsymbol{k}\uparrow}\,|\Omega\rangle\end{aligned} \tag{3.7}$$

where $|\Omega\rangle$ is the vacuum state of the $\{c^*_{\boldsymbol{k}\sigma},\, c_{\boldsymbol{k}'\sigma'}\}$ algebra.

Upon substituting this last equation into equation (3.3) yields an equation which determines Δ

$$\frac{g}{V}\sum_{\boldsymbol{k}} \langle\Omega_S|c_{-\boldsymbol{k}\downarrow}c_{\boldsymbol{k}\uparrow}\,|\Omega_S\rangle = \Delta. \tag{3.8}$$

The excited states of a BCS superconductor, the quasiparticle states $\alpha^*_{\boldsymbol{k}\sigma}\,|\Omega\rangle$, are separated from the ground state by a finite energy gap: the energy $\lambda_{\boldsymbol{k}}$ of a quasiparticle of momentum $\boldsymbol{k}$ does not go to zero as $\boldsymbol{k}$ goes to zero, but to Δ[67]. The energy gap is a further indication that the gauge symmetry of the original Hamiltonian is spontaneously broken.

The conditions under which spontaneous symmetry breaking can occur in a quantum field theory formulated in terms of a Lagrangian are follows. Noether's theorem asserts that if the action, $\int \mathcal{L}\, d^4\mathrm{x}$, of the quantum field theory is invariant under a *continuous* group of symmetry transformations there exists a conserved current, $j_\mu(x)$, that satisfies the continuity equation $\partial^\mu j_\mu(x) = 0$. The charge $Q = \int j_0(\mathbf{x},t)d^3x$ serves as a generator of the symmetry transformations. The latter are implemented on the Hilbert space of states by unitary operators. Given the charge density $\mathrm{j}_0(\mathrm{x})$, introduce the charge operator

$$\mathcal{Q}_\Omega(t) = \int_\Omega j^0(x,t)d^3x$$

the integration being over a finite space domain of volume Ω. The symmetry breaking condition can be defined as the existence of a (not necessarily local) operator Φ such that

$$\lim_{\Omega\to\infty} \langle 0|[Q_\Omega, \Phi]|0\rangle \neq 0$$

where $|0\rangle$ is a translationally invariant ground state. Furthermore, the spontaneous breakdown of a *continuous* symmetry also implies the existence of a massless spinless particle. (See Guralnik, Hagen and Kibble 1964; see also Kibble 2009a,b).

The point of departure for these developments was an influential paper by Nambu published in 1959, in which he suggested that the masses of elementary particles might arise in a similar way as in BCS: the vacuum state might likewise not respect the symmetries of the theory, and like the quasiparticles of the BCS theory elementary particles might acquire mass. Nambu and Jona-Lasinio (1961) went on to construct a model that of a massless fermion field $\psi(x)$ that exhibited these characteristics. It was based on the Lagrangian density

$$\mathcal{L} = i\overline{\psi}(x)\gamma^\mu\partial_\mu\psi(x) \, + \, g\left[(\overline{\psi}(x)\psi(x))^2 \, - \, (\overline{\psi}(x)\gamma_5\psi(x))^2\right]$$

which is invariant under the phase changes

$$\psi(x) \to e^{i\alpha}\psi(x) \quad \text{and} \quad \psi(x) \, \to \, e^{i\beta\gamma_5}\,\psi(x).$$

By Noether's theorem, there would therefore be a vector current, $j^\mu = \overline{\psi}(x)\gamma^\mu\psi(x)$, that is conserved and an axial vector current, $j_5^\mu = \overline{\psi}(x)\gamma^\mu\gamma_5\psi(x)$, that is conserved. Nambu and Joan-Lasinio then stipulated that the ground state, i.e. the vacuum state, does not respect the chiral symmetry and that the symmetry is broken spontaneously by a non-zero expectation value: $\langle 0|\,\overline{\psi}(x)\psi(x)\,|0\rangle \neq 0$. They then showed that this would imply a non-zero mass for the "quasiparticle" of that theory, which they identified with the nucleon.

However, their model also predicted the existence of a pseudoscalar, mass zero, spin zero particle – the "Goldstone" (1961) boson of that model[68].

3.4 Goldstone Bosons

In 1961 Goldstone in an important paper exhibited a simple *relativistic* field theory that manifested spontaneous symmetry breaking. It consisted of a complex scalar field ϕ with the Lagrangian density

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - V(\phi)$$
$$V(\phi) = m^2\phi^*\phi + \frac{1}{2}\lambda(\phi^*\phi)^2$$

where m and λ are the mass and self-interaction coupling constant of the scalar field. The model is invariant under a global change of phase

$$\phi(x) \to e^{i\alpha}\,\phi(x)$$

which transformations define the Abelian symmetry group U(1). When $m^2 > 0$ the model is that of a self-interacting scalar field, whose quanta are particles and antiparticles of mass m. However when $m^2 < 0$ the potential V

$$V(\phi) = -\left|m^2\right|\phi^*\phi + \frac{1}{2}\lambda(\phi^*\phi)^2$$

has minima on the circle $|\phi|^2 = \left|m^2\right|/\lambda$. Therefore in the ground state, the vacuum state, the value of ϕ will be non-zero, with a magnitude close to $|\phi| = \sqrt{|m^2|/\lambda}$, but with arbitrary phase α. There will be a *degenerate* family of vacuum states $|0_\alpha\rangle$, labeled by the phase angle α, each $|0_\alpha\rangle$ being the vacuum state of a distinct Hilbert space constructed by applying the field operators to it[69].

Massless Goldstone bosons also appear in this model. This can be seen by choosing a particular minimum, e.g. the one where ϕ is real and positive, and expanding about that point. Upon defining the shifted real fields φ_1, φ_2 by

$$\phi = (v + \varphi_1 + i\varphi_2)$$

and inserting them in the Lagrangian density one finds that a term in φ_2^2 does not appear. The model thus describes two kinds of particles: massive φ_1 quanta and massless φ_2 quanta. The massless φ_2 quanta are *Goldstone* bosons. That their presence is required is a consequence of a theorem that Goldstone, Salam and Weinberg (1962) proved: massless Goldstone bosons are present in any *manifestly Lorentz covariant* theory in which a *continuous* symmetry is spontaneously broken.

Although Nambu and Jona-Lasinio (1961) and Anderson (1963) had indicated that in the *non-relativistic* context the Goldstone boson could have a mass, most field theorists believed that in a relativistic theory the Goldstone-Salam-Weinberg theorem required that the Goldstone bosons be massless.

The story of how that limitation was overcome has been repeatedly told (see e.g. Close 2011). Yang and Mills in 1954, and independently a student of Salam, Shaw, in his PhD dissertation submitted to Cambridge University in 1955, formulated the first non-Abelian gauge theory by promoting the global SU(2) isospin symmetry to a local symmetry, thereby introducing an isospin triplet of massless vector gauge fields. Although initially intended as a theory of the strong interactions, it was soon recognized that it might serve as a model for the weak interactions. The fact that parity was not conserved in the weak interactions, and that the four fermion Fermi theory of weak interactions – when considered as a theory wherein the interaction term is of the form of a current-current coupling $J^\mu J_\mu$, with currents of the form $\overline{\psi}_1\gamma^\mu(1 - i\gamma_5)\psi_2$, i.e. involving interactions between vector and axial vector currents, accounted for the experimental data to lowest order of perturbation theory, suggested that the weak interactions might be mediated by charged vector bosons. And in direct analogy with QED where the interaction between charged particles $j^\mu(x)D(x - x')j_\mu(x')$ is the result of exchanges of vector bosons stemming from QED interpreted as a gauge theory, it suggested the possibility of a gauge theory of the weak interactions. However its feasibility had to face an immediate difficulty: the weak interactions were short ranged, and hence the intermediate vector bosons mediating them had to have a large mass, whereas it was believed that gauge bosons described by a Yang-Mills theory were necessarily massless, and therefore gave rise to long range forces. To transform such a theory into a model for the weak interactions demanded that the spin 1 bosons associated with the gauge fields have mass. Like in superconductivity, spontaneous symmetry breaking was seen as the mechanism whereby the bosons acquired mass, but the difficulty of the associated scalar Goldstone mesons had to be overcome. Higgs (1964), Englert and Brout (1964), and Guralnik, Hagen and Kibble (1964) solved the puzzle on how to do so in relativistically invariant gauge theories. They showed that when a gauge theory is combined with an additional field that spontaneously breaks the symmetry group, the gauge bosons indeed consistently acquire a nonzero mass. And Higgs, in particular, pointed out that in a gauge theory formulated in the radiation gauge, the fixing of the gauge breaks the relativistic invariance and renders the Goldstone-Salam-Weinberg theorem inapplicable (Higgs 1997; see Kibble 2009a).

These developments coincided with the developments of methods to quantize non-Abelian gauge theories (Fadeev and Popov 1967, DeWitt 1967) and the proof of their renormalizability even in the broken symmetry phase (‘t Hooft 1971, Lee and Zinn-Justin 1972a,b). This led to the acceptance of the Weinberg-Salam quantum field theoretical model of electroweak interactions (Weinberg 1967, Salam 1968).

The formulation of the theory of strong interactions proved to be much more involved, and required the elucidation of the dependence of the cross section of deep inelastic electron-proton scattering on energy. These 1968 experiments at SLAC introduced the notion of scaling by Bjorken, and of essentially freely moving sub-nuclear constituents of hadrons by Feynman which he called partons. Their connection with the quarks that had been introduced by Gell-Mann and Zweig as fictitious entities to elucidate the $SU(3)$ symmetries of the hadron spectrum, and the incorporation of the color properties attributed to them by Greenberg, eventually led to the present day theory of the strong interactions, quantum chromodynamics, a non-abelian gauge theory. Its crucial property of being asymptotically free, as proven by Gross and Wilczek (1973a,b 1974) and by Politzer (1973) explained the weakness of the interactions between quarks at short distances, and thus the success of the parton model in explaining scaling. Furthermore, the analysis of renormalizable relativistic quantum field theories by the renormalization group approach that Curtis Callan and David Gross (Callan 1968, 1970, 1972, 1973) and Kenneth Wilson (1973, 1984) had developed from 1968 to 1973 led to the deeply significant conclusion that only gauge theories of the Yang-Mills type could have the property of being asymptotically free[70].

Internal and space-time symmetry considerations played a crucial role in the development of quantum field theory. Symmetry breaking likewise played a critical role[71]. As noted, quantum field theory was enlarged and greatly extended by the BCS theory. The subsequent clarification of the notion of spontaneous symmetry breaking (SBS) by Nambu, Anderson, Goldstone, Salam, Weinberg, Higgs, Kibble, Lee and others was an important step in accepting Yang-Mills gauge theories and the eventual formulation by Bjorken, Feynman, Gross and Wilczek, Politzer, 't Hooft, Gell-Mann, and others, of quantum chromodynamics (see Cao 2010) and of the standard model (see Wilczek 2000). BCS was one of the points of departure for establishing gauge quantum field theories as the representations of the foundational theories describing the strong and the electroweak theories, i.e. "explaining" the subnuclear world down to distances of the order of 10^{-17} cm. (see Cao 1997, 1999).

Mathematical physicists played an important role in this achievement by extending and clarifying the mathematical aspects of quantum field theory. The mathematical physics tradition that had existed at Göttingen until 1933, became transported to the United States with the appointment in the early 1930s of von Neumann, Wigner, and Hermann Weyl at Princeton (See Heims 1980, Mehra 2001, Sigurdsson 1996). Hitler coming to power in 1933 and the subsequent dismissal of university professors because of their non-Aryan descent or their political views, resulted in a substantial migration of German mathematicians to the US, e.g. Richard Courant and Kurt Friedrichs to NYU, Hans Radamacher to the University of Pennsylvania, Solomon Bochner to Princeton University, ... others elsewhere (see Siegmund-Schultze 2009). Their presence contributed importantly to the efflorescence of mathematical physics in the US.

The strong Russian mathematics tradition since the end of the nineteenth century produced a very talented group of researchers in the Soviet Union in the post World War II years, – Bogoliubov, Gelfand, Minlos, Dobrushin, Sinai, Arnold, Fadeev, ... – who addressed and solved important physical problems with methods that meet the highest standard of mathematical rigor. (See Fadeev 1995). Parallel activities, but in a more specialized area took place after the war in Great Britain (James Lighthill, Cyril Domb, ...), in Sweden (Lars Gärding, ...), in Japan (Kato, Kodaira, ...), and in France, where mathematics held a unique status among the disciplines ever since the French revolution and the founding of the École Normale and the Polytechnique (Laurent Schwartz, Lichnerowitz, Kastler, Louis Michel, ...).

In 1947 Léon Van Hove and Res Jost were visiting members at the Institute of Advanced Study in Princeton and Arthur Wightman was an instructor at Princeton University. All three attended the lectures that von Neumann was giving at the Institute on C* algebras. All three had studied mathematics as undergraduates. All three of them made crucial contributions to the development of relativistic quantum field theory and all three became the founders of important schools of mathematical physics and the mentors of several generations of outstanding students. Rudolf Haag is another member of that generation of mathematical physicists. He was responsible for novel, important new approaches to the synthesis of special relativity and quantum theory and likewise was the mentor of several generations of mathematical physicists. Cyril Domb, of that same generation, created a school of mathematical physics at King's College London. He and his school made crucial contributions to understanding how the symmetry of particular structures manifests itself in the critical exponents characterizing a phase transitions[72]. I refer the reader to the obituary of Jost by Pais (2000); that of Van Hove by Casimir (1992) (see also Giovannini 2000); that of Domb by Fisher (1995) (see also Domb 1990); those of Wightman by his students[73]; and to Kastler's encomium of Haag on the occasion of his 80th birthday (Kastler 2003; see also Haag 2010), for detailed assessments of their contributions.

The importance of mathematical physics is attested by the fact that in the early 1960s the Boulder, the Brandeis, and the Cargèse summer schools each devoted lecture courses on various topics in mathematical physics. The entire 1968 International School of Physics "Enrico Fermi" which was directed by Res Jost, was devoted to "Local Quantum Theory" with several of the lecture courses given by mathematicians: Dixmier on C* algebras, Malgrange on partial differential equations, and Glimm on functional integration and models for quantum field theory (Jost 1969). Some 80 students attended the school. See also the courses of the 1970 Les Houches summer school where the interaction between mathematicians, mathematical physicists and theoretical physicists is on display (DeWitt and Stora 1971)[74].

In 1960 the American Institute of Physics started publishing a journal devoted exclusively to mathematical physics, the *Journal of Mathematical Physics.* In 1965, under the aegis of Springer-Verlag, another publication, *Communications in mathematical physics,* committed to even more rigorous expositions than the *Journal of Mathematical Physics* made its appearance. In 1974 some 200 mathematical physicists attended the International Symposium on mathematical physics and physical mathematics organized by the Polish Academy of Sciences (Maurin and Raczka 1976). Andrew Lenard, himself an important contributor to the discipline, commented at the 1971 Battelle Seattle *Rencontres*

> The maturing and growth of modern mathematical physics is one of the striking intellectual developments of the last two decades. Even more importantly mathematical physics fosters a cooperative and unifying spirit between practitioners of different areas of expertise (Lenard 1973).

Although I have not indicated the fields of "pure mathematics" which were seeded, fertilized or extended by problems stemming from physics, and in particular, from quantum field theory, I do want to call attention to the co-construction of physics and mathematics (see e.g. Michel 1998). Indicative of how deep this interaction has been more generally is the fact that both Arthur Wightman and Arthur Jaffe became chair of their department of *mathematics,* at Princeton University and at Harvard, respectively[75]. Arthur Jaffe in fact became the president of the American Mathematical Society in 1997–1998. I can also point to Elliott Lieb. Lieb is a professor of physics and of mathematics at Princeton University. He has made outstanding contributions to the explanation of the stability of matter (see Lieb and Seiringer 2010). In 1999 with Michael Loss he wrote a textbook on *analysis* that the *American Mathematical Society* has published and that underwent an expanded second edition in 2001 (Lieb and Loss 2001).

One should not infer from the preceding remarks that mathematics and theoretical physics are "*co-produced*": a great deal of mathematics is the free creation by imaginative, "pure" mathematicians without any connection to physics[76]. Rather an analogy with human languages is useful. Human languages developed originally as a recursive means of communication among cooperating individuals that gave the species immeasurable *adaptive* advantages. It eventually evolved to produce myths, poetry and plays, novels, ... objective means of communicating deep insights into the dynamics of individuals, societies, political systems, ... Similarly, mathematics developed in conjunction with recording the seasons for agricultural and religious purposes, recording the results of measuring lengths and areas of agricultural fields, levels of water, recording commercial transactions, *predicting* the motion of the planets, ... It developed into the amazing language and discipline we call modern mathematics with wide and deep applications in all human activities and enterprises. In his 1954 *Origin of Geometry* Husserl writes of the historical intervention of writing and its significance for the development of sciences

> "The importance of the written, documenting linguistic expression is that it makes communications possible without immediate or mediate personal address; it is so to speak communication become virtual" (quoted in Rheinberger 2010, p. 76).

Mathematics is a language that makes possible *unambiguous* writing and communication, and thus of objective assessments by the community. Rheinberger in his beautiful exposition of the historization of epistemology when commenting on Derrida's connection to Husserl explicates Derrida's term *historiality* as an "iterative-recursive production of meaning in the irrevocable extoriorization of a generalized writing", one that leaves no trace of an origin. Mathematics has this attribute of *historiality*.

It is the case that many developments in mathematics were stimulated by physics – e.g. the development of the calculus. But the subsequent formulation of measure theory and Lebesgue integration was an internal development within mathematics, later made use of by physics in coming to terms with the subtleties of Hilbert spaces and operators therein, and in probability theory.

One can reject Platonism, but one would be hard put to deny that mathematics has not captured some elements of reality (see Jackiw 1999). But the conception of reality I speak of is Hacking's. A reality that is entrained by our representations, not the other way around. As Hacking stresses: "the first peculiarly human invention is representation. Once there is a practice of representing, a second order concept follows in train. This is the concept of reality, a concept which has content only when there are first-order representations" (Hacking 1983, p. 7; quoted in Rheinberger 2010, p. 81).

What Jost, Wightman, Haag, Domb, ... were able to accomplish in the 1950s was to transform what had been individual activities into a flourishing collective activity, a discipline. They and their students worked on problems in statistical mechanics (thermodynamic limit, ...); quantum mechanics (rigorous definition of the operators appearing in its formulation, scattering theory, existence of bound states, quantum logic, ...); in quantum field theory (axiomatic and later constructive field theory[77], investigations of the analyticity of Wightman functions, the CPT theorem, the connection between spin and statistics, the formulation of relativistic field theories as Euclidean ones, the justification of the LSZ formalism, the putting of the renormalization program on a firmer mathematical footing, the Haag-Kastler algebraic C* approach to QFT, ...).

It can be argued that a great deal of "co-construction" took place after World War II because quantum field theory required new mathematics (e.g. the proper definition of the field operators in QFT as distributions). Streater and Wightman's *PCT, Spin and Statistics, and All That* (1964), Jost's *The General Theory of Quantized Fields* (1965), and David Ruelle's *Statistical Mechanics: Rigorous Results* (1969) are the paradigmatic examples of the approach. Ruelle's book – which has become known as

"The Book" – gives a thorough overview of the ways rigorous mathematical analyses had secured some of the foundations of statistical mechanics and had established what kinds of systems it could describe.

One of the aims of the *history* of science is to answer such questions as: "What made it possible for Wightman, Jost, Haag, Kastler, Domb, … to create schools of mathematical physics, and for the students that they trained to form a new discipline with all the accoutrements that go with it, such as professional journals, prizes, endowed chairs?". In the United States, the answer has to do with passage of the GI bill that allowed World War II veterans to obtain a college education at essentially no expense to them. Universities thereafter became restructured with great emphasis placed on research in the physical sciences and in applied mathematics, this research being lavishly supported by the government as part of its pursuance of the Cold War. The accompanying overhead payments allowed universities to support activities and functions not directly supported by the government such as appointments and scholarships in the arts and the humanities. Mathematical physics was surely one of the beneficiaries of this expansion (see Kerr 1991, and e.g. Leslie 1993, Schweber 1997, Wang 1999, Soo and Carson 2004). During the Cold War national prestige, and concomitant factors, were similarly at play in the Soviet Union and elsewhere[78].

The above sketch, at best, adumbrates the situation until the 1970s. Both mathematics and mathematical physics have undergone deep changes. Indicative of the profound transformation mathematics has and is undergoing is the impact of computers on mathematics requiring the mathematical community to once again consider what constitute a proof; and more generally, the demands of computer science. Similarly, the symbiotic relation between some parts of mathematics and string theory has raised the possibility of having "theoretical mathematics" (Jaffe and Quinn 1993).

But with quantum field theories becoming seen as "effective", the mathematicians' requirement of consistency in their mathematical formulation lost some of its force (Schnitzer 1999). Nonetheless, a dark cloud hangs over relativistic quantum field theories. The perturbative expansions used in calculating observable effects are at best asymptotic, and very probably diverge. In fact, it has been shown that in $s+1$ dimensions (with s equal or greater than 3) in the φ^4 theory treated perturbatively the renormalized coupling constant vanishes when the cut-off goes to infinity, and the theory is thus trivial (Fernandez et al. 1992). Only in two dimensional space-time has it been shown non-perturbatively that non trivial field theories exist.

3.5 Equilibrium statistical mechanics

Rigorous results in Statistical Mechanics played an important role in clarifying the approaches to be taken in the theoretical explanations of phase transitions.

As stated in Ruelle's "Book" the main problem of equilibrium statistical mechanics is the study of equilibrium states of *infinite* systems and the relation of such states to the interactions that give rise to them; infinite because only under those conditions can the sharp discontinuities exhibited by phase transitions be explained in a rigorous fashion. The study of intensive properties of systems with an *infinite* number of particles stems from the recognition that many of the phenomena exhibited by macroscopic systems – such as phase transitions – can be accounted for rigorously only in the thermodynamic limit, i.e., in the limit as the number of particles, N, and the volume, V, become infinite but the ratio N/V tends to a definite limit. It therefore becomes important to ascertain under what circumstances the thermodynamic limit exists[79].

The systems considered are assumed to be enclosed in a volume V and describable by a (non-relativistic) Hamiltonian

$$H_N = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \Phi_N(r_1, r_2, \ldots, r_N).$$

The partition function of the system is defined as

$$\begin{aligned} Z(\beta, N, V) &= \exp[-\beta F(\beta,\ N, V)] \\ &= \frac{1}{N!}\left(\frac{m}{\beta h}\right)^{3N} \int\limits_V \ldots \int\limits_V dr_1 \ldots dr_N \exp(-\beta \Phi_N) \end{aligned}$$

with $\beta = 1/k_B T$, T the temperature, and k_B Boltzmann's constant. One is interested in the free energy F per particle, which quantity becomes precise only in the thermodynamic limit, i.e. as $V \to \infty$, with $V/N \to a$ finite constant.

The existence of a thermodynamic limit was proven for a large class of systems whose interaction potential satisfy the condition: $\Phi_N(r_1, r_2, \ldots, r_N) \geqslant -NB$ in the classical case (with B a finite constant independent of $N$for all r's) and thus excludes the possibility of the system collapsing (as would be the case for purely gravitational interactions). A second requirement is that the potential should not become too positive at large separations to insure against explosions. Systems of point particles interacting through Coulomb forces do not satisfy the first criterion. Dyson and Lenard showed that in the quantum case $E_o(N) \geqslant -NB$ will hold provided that the negatively charged particles (and/or the positive ones) obey the Pauli principle (Dyson 1968, Dyson and Lenard 1967,1968, Krieger 1996).

Some further questions that must be answered affirmatively are the following: (i) does the free energy per unit volume in the limit V $\to \infty$, have the property that it does not depend on the shape of container in which the system is enclosed in the thermodynamic limit? This in order to be able to consider physical situations in which surface effects and particular shapes are irrelevant. (ii) is F/N, when it exists, a convex function of the density and a concave function of β, the inverse temperature, in order to guarantee the thermodynamic stability of the system? (see Lebowitz 1976; also Krieger 2012).

The close linkage between statistical mechanics (SM), condensed matter physics and quantum field theory (QFT) during the 1960s and thereafter owes much to the researches of Yang, T.D. Lee, Martin, Schwinger, Ginibre, Dyson, Eckmann, Lieb, Ruelle, Fröhlich, Lebowitz, Wightman, Symanzik... mathematically inclined theoretical physicists or "mathematical physicists".

The effective incorporation of Feynman's path integral and functional integration methods into statistical mechanics and condensed matter physics played a key role in the exchanges between these two fields. The initial insights that were responsible for the linkage between SM and QFT were due to Feynman and Schwinger. Already in 1940, Feynman, while working on his thesis, had noted the close parallel between his path integral method of calculating transition amplitudes and the quantum statistical mechanics method of computing thermal averages of observables as traces of the observables multiplied by the density matrix describing the system. In the latter, when expressed as a path integral the inverse temperature assumes the role of a complex time. Similarly, Schwinger had made an analysis of the (analyticity) properties of the Green's functions he had introduced into quantum field theory (Schwinger 1951) in terms of which he formulated special relativistic quantum field theories and worked out their relation to the Green's functions for the corresponding Euclidean field theories (Schwinger 1958). To obtain the latter from the former, the time coordinate of the relativistic case can be thought of as having been analytically continued to purely imaginary values[80]. The analytic continuation to complex values of the time coordinate makes connection with the statistical mechanical case. These connections were given rigorous formulations by Osterwalder and Schrader (1973)[81] for the relativistic-Euclidian connection, and by Symanzik (1966, 1969) for the statistical mechanics-field theory connection.

These transfers and interactions form but a small part of the history of the connections between condensed matter physics and quantum field theory during the 1950s and later on. Paul Martin (1979) in his exposition of the history of the transfer of Schwinger's field theoretic techniques to statistical mechanics, and in particular, in their application to the description of phase transitions and the derivations of the properties of superfluids, has told part of the story and indicated where to obtain further information[82].

4 Reconceptualizing QFT

My paper is about breaks, discontinuities and continuities. Such characterizations only emerge when looking at changes of the *collective* resources and conceptualizations of the community. They are attributions to the "collective thought" of the community (Fleck 1935, 1979).

Breaks, discontinuities and continuities have a temporal dimension. They cannot be seen on too small a time scale. Thus attributing a deep change to a revolution implies that something about it has a long temporal stability. An individual may disagree with what has become the collective assessment, and by virtue of his/her disagreement may contribute to the abandonment of a previously held "collective thought". This was the case of John Bell and the abandonment by the physics community of some features of the Copenhagen interpretation of quantum mechanics (see Freire 2015). Individuals indeed make key contributions to the process, but they do so in response to issues and problems raised by other members. Furthermore, individuals make their crucial contributions usually while working on a specific problem related to empirical data, making use of collective knowledge.

In the next few sections I look at the contributions of several individuals who contributed significantly to the solution of the phase transition problem: Benjamin Widom, Leo Kadanoff and Kenneth Wilson. The solution to that problem was an important component in the reconceptualization of quantum field theory. Needless to say, others also made very important contributions to the phase transition problem, e.g., Ralph H. Fowler, Lars Onsager, Bruria Kaufman, Chen-Ning Yang, Cyril Domb, Alexander Patashinskii, Valery Pokrovskii, Robert Griffith, James Langer, Tai T. Wu, Eugene Stanley, Giovanni Jona-Lasinio… and especially Michael Fisher. I focus on the above three individuals because the end point of the story I want tell is the reconceptualization of quantum field theory that Wilson's work brought about.

4.1 Critical exponents and Landau's mean field theory

The magnetization of a ferromagnet, the density in a liquid-gas system, the concentration in a binary mixture all exhibit a sharp discontinuity when the system undergoes a phase transition. These phase transitions can be made continuous by adjusting some parameters, such as the temperature, that controls the environment that the system is placed in. The point at which the discontinuity vanishes is called the critical point. Thus at temperatures above the critical point the magnetization of a ferromagnet vanishes, the difference between the density of the liquid and that of the vapor phase vanishes; in fact, the liquid and vapor phase become indistinguishable.

Ever since the time of van der Waals and Maxwell, physicists and chemists have studied the discontinuities in the physical behavior that occur when systems in thermodynamic equilibrium undergo phase transitions[83]. Phenomenological explanations of transitions for real gases that seemed to account for the discontinuities semi-quantitatively were advanced by many people – most notably by Landau in 1937 (see Landau 1937, Tisza 1966, Callen 1960, Landau and Lifshitz 1969). Yet the notion of a phase transition remained somewhat vague.

Attempts to explain the properties of phase transition in liquids were begun in 1870s by van der Waals, and of those in ferromagnets at the beginning of the 20th century by Pierre Curie, Paul Langevin and others. Already in 1873 van der Waals proposed an equation of state of a fluid that attempted to take into account the finite size of and the interactions between the atoms or molecules making up the gas-liquid system. For the case of one mole of the substance it takes the simple form:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where a and b are constants depending on the particular substance being described. Maxwell formulated a rule by which one could obtain from van der Waals equation a curve which has a horizontal piece in the P-V plane corresponding to the liquid-gas coexistence curve. At the critical point, T_c, V_c, P_c, the horizontal line becomes a point. From this one deduces that

$$V_c = 3b \quad P_c = a/27b^2 \qquad T_c = 8a/27bR$$

van der Waals also noted that by introducing the reduced volume $v = V/V_c$, reduced pressure $p = P/P_c$. and reduced temperature $t = T/T_c$ his equation took the reduced form

$$\left(p + \frac{3}{v^2}\right)(3v - 1) = 8t$$

implying that the measured values for different substances should be described by this same equation (see van der Waals 1910).

The derivation of an equation of state for a gas-liquid system starting from interatomic or intermolecular forces between the constituent microscopic entities is a very difficult statistical mechanics problem; explaining phase transitions for such systems an even more difficult task. Attention therefore switched to phase transitions in ferromagnets, where modeling these substances proved much more successful and rewarding. Pierre Curie and Paul Langevin initiated this program. In the early 1920s Wilhelm Lenz and Ernst Ising introduced a simple model to study the ferromagnetic to paramagnetic phase transition. It consisted of a regular lattice, where at each lattice site is located a spin which can point either up or down. When two nearest neighbor spins are parallel there is an interaction energy $-J$, when antiparallel $+J$. Each spin can also interact with an external magnetic field B (measured in units such that the value of Bohr magneton is in it). The Hamiltonian for the system was postulated to be

$$H = -J \sum_{(i,j)} \sigma_i \sigma_j - B \sum_i \sigma_i$$

where the summation in the first term is over all pairs of nearest neighbor spins. The partition function of the system, from which all thermodynamic functions can be obtained is given by

$$Z_N = e^{-F/k_B T} = \sum_{(\sigma_1, \sigma_2 \ldots, \sigma_N)} \exp(-H/k_B T)$$

where F is the free energy, k_b the Boltzmann constant, and the summation being over the 2^N spin configurations.

The general features of the Ising model are readily exhibited. The magnetization of the system is given by

$$M = \langle \sigma \rangle = \frac{1}{Z} \sum_i \sigma_i e^{-H/k_B T}.$$

At zero magnetic field, when $B = 0$, and high temperature the spins point randomly in the + and − direction and M vanishes. At very low temperature the spin-spin interaction tends to line up the spins and produce a non vanishing M. As the temperature is raised M gradually decreases until the critical (Curie) temperature, T_c, is reached, where M vanishes.

Ising proved that there can be no phase transition in the one-dimensional model. In 1936 Peierls advanced a seminal argument to establish the existence of a phase transition and spontaneous magnetization at low temperature in the two dimensional Ising model, the simplest atomic model of a ferromagnet (Peierls 1936). And in 1944 Lars Onsager[84], making use of results of Kramers and Wannier (1941), succeeded in obtaining an exact solution of the two dimensional Ising model in the sense that he explicitly calculated the free energy of this model (Onsager 1944) for a square lattice. His exact solution exhibited a logarithmic singularity of the specific heat near the critical point indicating that the quantitative predictions of Landau's "mean field" theories were not trustworthy. Onsager's solution provided one of the stimuli that led to the intense reinvestigation of the behavior of thermodynamic quantities near discontinuities. Ten years after Onsager's solution had appeared, Yang obtained an exact expression for the spontaneous magnetization of the two dimensional Ising model on a square lattice[85].

Let me briefly recall Landau's theory as formulated for ferromagnets (Landau 1937. See Fisher 1967, Wilson and Kogut 1974, Wilson 1974). As indicated above, the critical temperature T_c, the Curie temperature, is the temperature below which the ferromagnet spontaneously magnetizes. Above T_c the magnetization is zero. For temperatures close to, but below T_c, both in simple models – such as Langevin's – and empirically, it is found that the magnetization varies as $(T - T_c)^\beta$. Experiments indicate that for many ferromagnets β has the value $\sim$0.35. The exponent β is an example of a critical exponent. In Landau's mean field $\beta = 1/2$. That Landau's mean field theory was inadequate in describing ferromagnetic phase transitions was made clear by Onsager's solution of the two dimensional Ising model (Onsager 1944). He found $\beta = 1/8$.

A microscopic description of a ferromagnet, such as the Ising model, introduces the crucial notion of a correlation length. For temperatures above T_c the correlation length can be defined as follows: If one imagines aligning a spin at a given position in a certain direction, the correlation length ξ measures the distance from that spin that other spins will be aligned in the same direction. Below T_c, ξ is infinite due to the spontaneous magnetization. Above T_c, ξ is finite, but as $T \to T_c$, ξ must $\to \infty$. Near T_c, ξ behaves as $(T - T_c)^{-\nu}$, where ν is a second critical exponent. Experimentally, for a three dimensional ferromagnet ν is found to have the value $\sim$0.6–0.7. Landau's theory predicts the value 0.5 for it. Onsager's solution for the 2 dimensional Ising model yields $\xi = 1.0$.

The great difficulty in explaining quantitatively the behavior of the ferromagnet at the critical point is the non analyticity in T of the magnetization and of the thermodynamic functions. Thus M is 0 below T_c, and varies as $(T - T_c)^\beta$ above T_c

with β non integer. Since the partition function

$$Z = e^{-F/kT} = \sum_i e^{-E_i/kT}$$

for any finite sum over the configurations i of the magnet, with E_i the energy of the ith configuration, is analytic in T (except at $T = 0$) the non-analyticity near T_c of thermodynamic functions such as F, the free energy, cannot be derived when i is finite. Only for systems with an infinite number of configurations can Z diverge. Finite quantities like the magnetization then have to be computed by a limiting procedure, usually taken to be the thermodynamic limit: $N \to \infty$, $V \to \infty$ but N/V remaining finite. Under those circumstance analyticity is no longer guaranteed.

In Landau's mean field theory the discrete lattice of the ferromagnet is discarded, and a local magnetization $M(x)$ is introduced. $M(x)$ is understood to be the average magnetization over a region of radius L surrounding the point x, with $a \ll L \ll \xi$ where a is the lattice spacing and ξ, the correlation length. Landau assumed that the free energy of a ferromagnet could be expressed in term of $M(x)$ as follows:

$$F = \int d^3x \left\{ [\nabla M(x)]^2 + R(T)M^2(x) + U(T)M^4(x) - B(x)M(x) \right\}$$

where $R(T)$ and $U(T)$ are temperature dependent constants, assumed to be analytic in T at T_c, and $B(x)$ is the external magnetic field. One readily determines that in Landau's theory the correlation length above T_c is $\xi(T) = R(T)^{-1/2}$. Thus $R(T_c) = 0$ in order for $\xi = \infty$ at T_c. Near T_c, the analyticity of R means that

$$R(T) = R'(T_c)(T - T_c)$$

and hence, assuming that $R'(T_c)$ is not zero $\xi \propto (T - T_c)^{-1/2}$. Similarly, assuming that U is positive on both sides of T_c, when $B = 0$ the minimum of F for $T < T_c$ occurs at

$$M = [-R/2U]^{1/2} \propto (T_c - T)^{1/2}$$

i.e. $\beta = 1/2$.

After World War II, due to theoretical advances and because the experimental data on the critical exponents for the liquid gas phase transition in simple systems had become much more accurate and reliable, it became clear that Landau theory made wrong predictions.

In 1965 Ben Widom, a physical chemist at Cornell, indicated that the equation of state of a simple substance, which in general is a function of the two variables, e.g. $\rho = f(T, p)$, *scales* near the critical point. He found that the free energy function F as a function of pressure and temperature near the phase transition point depended only on the ratio h/t^{Δ}, where $t = (T - T_c)/T_c$ and $h = p - p_c$ and Δ is a constant. That it depended only on the ratio h/t^{Δ} is what is meant by *scaling*. But Widom could not explain the reason for the scaling (Widom 1965, see also Widom 1974). Fischer (1964, 1965) had made some parallel observations regarding scaling.

Accounting for the scaling became the point of departure of Leo Kadanoff for his model of block scaling in the Ising model. He asked how, near the transition point, the description of the model might change if one replaced a block of spins by a single spin, thereby changing the length scale and thereby also having reduced the number of degrees of freedom. He found that with new effective values of $(T - T_c)/T_c = t$, and of J/k_BT (J is the spin-spin coupling and k_B is Boltzmann's constant) the system is describable by the same model, but with new parameters. The decimation[86] reduces the number of degrees and requires new couplings, but the description of the physics does not change. The result thus manifests a notion of scale-invariance. Stated slightly differently, Kadanoff noted that in the vicinity of a critical point the *diverging* correlation length sets the length scale, allows the size L which replaces the original lattice length a of the original model to be *arbitrary*, as long as $a \ll L \ll \xi$, and therefore requires observables not to depend on L. If an observable O has the dimensionality of $[\text{length}]^{D_O}$ then near the critical point it should obey the *scaling* form

$$O \sim \kappa_O \xi^{D_O}$$

with κ_O a dimensional less function.

Ken Wilson being at Cornell became aware of Widom's, Fisher's, and Kadanoff's work and in 1971 formulated a complete theory of second order phase transitions. He did so by recognizing the limitations in Kadanoff's formulation, and considered a free energy functional that included *all possible couplings allowed by symmetry*, i.e. all possible Hamiltonians for the given system. He observed that when "integrating out" successively degrees of freedom that the corresponding scale changes and renormalizations can produce a closed algebra of couplings. Call $\boldsymbol{K}$ the (infinite) set of coupling constants. Call the operator which transforms the set with each thinning $\boldsymbol{R}$. There exist a special set of couplings $\boldsymbol{K}^*$ which are invariant under $\boldsymbol{R}$:

$$\boldsymbol{K}^* = \boldsymbol{R}(\boldsymbol{K}^*).$$

A set of couplings $\boldsymbol{K}^*$ which satisfy this equation are said to be a fixed point of the renormalization group transformation $\boldsymbol{R}$. The system at a critical point has a scale-invariant dynamics which is independent of the length scale. The representation of the model near each fixed point can be considered to be the representation of a distinct, separate physical theory. Wilson's work deeply and dramatically extended the earlier formulation of renormalization group methods by Stueckelberg and Petermann (1953), by Gell-Mann and Low (1954), and by Boguliubov and Shirkov (1959).

4.2 Scaling more generally

As just noted, in the mid 1960s theorists in statistical mechanics were confronted with new empirical data that challenged their understanding of key phenomena in their discipline.

A somewhat similar development took place in high energy physics in the analysis of high energy electron-proton scattering.

At high energies the cross-sections for inelastic electron-proton scattering described by the process $\mathrm{e} + p \to \mathrm{e}' + \mathrm{X}$, where X is any (final) state that has the same hadronic quantum number as a proton, can, to a very good approximation, be expressed in terms of two functions, W_1 and W_2, that are defined in terms of the matrix element of the electromagnetic current operator, $\langle Xp'|\, j^{\mu}_{em}\, |p\rangle$, between the initial state of a proton of 4 momentum p, and the final hadronic state X of momentum p'. Figure 4.2.1 indicates the approximation involved in the calculation of the transition amplitude; l_1 and l_2 are the initial and final 4-momenta of the electron, and X represents a possible hadronic state. For high energy electrons the single photon exchange can be justified.

W_1 and W_2 are functions of $Q^2 = (p - p')^2$ and of $\nu = p \cdot Q/M$ where M is the mass of the proton.

A function describing a property of hadrons that can be measured in high-energy scattering experiments is said to *scale* when its value is not determined by the energy at which the measurement is being carried out, but is determined by dimensionless

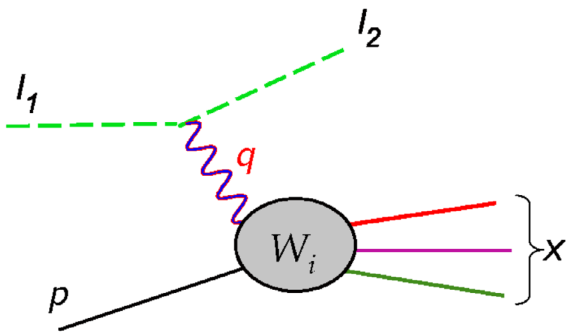


Fig. 4.2.1.

kinematic quantities such as the ratio of the energy to a momentum transfer. Increasing the energy of the probing particle implies decreasing its wave length, and thus being able to resolve more precisely the spatial features of what is being probed. Thus the property of "scaling" for a cross-section implies that the resolution scale of the scatterer *does not depend on length*, and hence that effectively what ever is responsible for the scattering has a point-like substructure. Scaling behavior for the structure functions of deep inelastic scattering of electrons on nucleons was first proposed by James Bjorken in 1968.

The deep inelastic electron-nucleon scattering experiments of Friedman, Kendall and Taylor at SLAC (see Kendall 1991, Friedman 1997) corroborated Bjorken's scaling hypothesis and his, and Feynman's, conjectures regarding the point like structure of the elementary constituents of hadrons. Feynman called them partons, and they became identified with Gell-Mann's "hypothetical" quarks (see Cao 2010, Chaps. 5 and 6, for the detailed story).

Since deviations from strict scaling are present in any quantum field theoretical description and interpretation of the experiments, Bjorken scaling cannot be exact. In fact, summing the effects of these deviations to all order of perturbation theory in any of the field theories describing the strong interactions that were known in the late 1960s resulted in the proposed scaling behavior of the amplitude becoming invalidated. Only in theories whose effective couplings vanishes as the resolution scale increases indefinitely, i.e. vanishes at very small distances or equivalently at very high energy, would scaling survive. However, no such theory was known at the time. The effort to find an example of such a theory culminated with the discovery of asymptotic freedom in non-abelian gauge theories (t'Hooft 1971, Gross and Wilczek 1973, Politzer 1973).

This made possible the formulation of quantum chromodynamics (QCD), the quantum field theory of quarks and gluons with a fundamental SU(3) color symmetry that had been proposed previously by Fritzsch, Gell-Mann and Leutwyller (Fritzsch 1971, 1972, 2012) as a possible quantum field theory describing the strong interactions (see Cao 2010). And the predictions of QCD have been fully confirmed by modern high energy experiments. QCD has become the foundational theory of strong interactions and accounts for all the observed phenomena up to an energy of 1 Tev with an accuracy better than 5% (see Gross 1999a).

QCD predicts the detailed form of violations of the scaling behavior at high energy of the relevant physical quantities through the distinctive quantum field theoretic effect of dimensional transmutation. Very high energy means that massless theories can be used to describe the phenomena. However, renormalization in such theories brings in new, *arbitrary,* mass scales. The arbitrariness in the choice of the these mass scales, means, once again, that observables cannot depend on these length scales (see e.g. Peskin and Schroeder 1995; Weinberg 1996; Tung 2009)

The relationship of Kadanoff scaling and Bjorken scaling can be made more explicit. Accounting for the Bjorken scaling laws and for their possible violation led to the investigation of the relationship of scale invariance and physics at very high energy where the masses of particles could be neglected. This led to a re-examination by Callan (1970) and by Symanzik (1970, 1971) of Gell-Mann and Low's renormalization group method by which they had addressed the behavior of QED at ultra high energy where the mass of the electron could be neglected. It resulted in the formulation of a generalization of their approach to arbitrary field theories, and to new insights into non-pertubative aspects of quantum field theory. The Callen-Symanzik renormalization group equations are a consequence of the requirement that the renormalized amplitudes be independent of the arbitrary masses introduces by renormalization. Dimensional transmutation is described by the detailed analysis of this process (see Weinberg 1983, and Vol. II of Weinberg 1995–2000).

The renormalization group equation that Kadanoff derived in his analysis of the Ising model was likewise a consequence of the fact that near the critical point the equation of state of a magnet could not depend on the (scale) length of the larger cell he introduced to decimate degrees of freedom. Callan-Symanzik's context is the high energy, short wave length domain, Kadanoff's the low energy, long wave length domain.

Callan's, Symanzik's formulations of renormalization group methods and that of Wilson which generalized and put on a firm footing Kadanoff's approach, deeply altered the understanding of what is implied by a quantum field theoretic representation[87].

4.3 Renormalization groups. A coarse-grained historical overview

To recapitulate. During the 1950s and early 1960s the much more precise characterization of the critical point by experiments (Fairbank et al. for liquid ^{4}He, Voronel for the liquid-vapor transition, Benedek and Heller for magnetic systems, ...) and by high temperature series expansions (Domb and coworkers, Essam, Sykes, Fisher,...) were important factors in making the problem of phase transitions of central concern to the theoretical physics community (see Fisher 1967).

The first phase transition that was believed understood when described at the atomic level was the Bose-Einstein condensation of non-interacting spin zero particles i.e. of ideal, monatomic (Bose) quantum gases. Understanding the dynamics at the atomic level of phase transitions in liquid helium and of superconductivity in lead were turning points for solid state physics. Identifying what were the universal features of materials undergoing phase transitions became a central and challenging problem. The BCS theory, which accounted for many of the phase change properties of certain superconductors became the focus of many experimental and theoretical investigations with the above in mind. It turned out, as we have noted, that the formulation of BCS theory was also a transition point for quantum field theory and high energy physics.

Two separate research lines can be discerned as stemming from BCS. One, principally located in high energy physics, leads from spontaneously broken symmetries and Yang-Mills gauge theories, to Higgs, to Guralnick, Hagen and Kibble and to Englert and Brout, to Weinberg's and Salam's electroweak theory, to Gell-Mann-Low, Callan-Symamzik renormalization group methods, to asymptotic freedom and the standard model, and the award of the Nobel prize in Physics[88] in 1979 to Sheldon Glashow, Abdus Salam and Steven Weinberg *"for their contributions to the theory* *of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current";* in 1999 to Gerardus 't Hooft and Martinus J.G. Veltman *"for elucidating the quantum structure of electroweak interactions in physics";* in 2004 to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction";* and in 2013 to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*[89].

Sidney Coleman's name would undoubtedly appear in the above list of Nobel laureates were it not for certain contingencies – such as the fact that the Nobel prize can be shared by only three people. As Howard Georgi (2011) forcefully stated in his *Biographical Memoir* of Coleman

> He was as one of the leading quantum field theorists in the world [and] ... an indispensible player in the resurgence of quantum field theory in the 1960s and 1970s, and indeed he taught particle physicists, established and aspiring, a new way of thinking about quantum field theory. ... Symmetry had been an important tool in physics from the beginning. (Already in his Ph.D. thesis) ... Coleman understood that he was bringing to the forefront a set of algebraic techniques in group representation theory that had usually played a secondary role to analysis in theoretical particle physics. The important objects were not the symmetry transformations themselves but their "generators" – associated with infinitesimal transformations. He understood the amazing power of these algebraic tools in working with quantum field theory. He knew that he had to explain them carefully to his elders (and eventually to his students). and he knew that he had the requisite skills to do it.

3

The other research line stemming from BCS leads from it back to Onsager and Ising models, to phase transitions more generally, to universality in phase transitions, correlation lengths, critical exponents, and running coupling constants, to Wilson renormalization group methods, and to the formulation by Wilson and Fisher of an expansion about four dimensions (Wilson and Fisher 1972) thereby linking statistical mechanics calculations with efficient, diagram-based perturbative methods of quantum field theory[90].

Fisher's lecture (Fisher 1999), Ken Wilson's Nobel lecture (Wilson 1983) and his review article with Kogut (Wilson and Kogut 1974) and Domb (1996) narrate these developments.

In the mid 1970s the two research lines converged and gave rise to a new understanding of quantum field theory based on renormalization group methods. The origin of these approaches was in work of Gell-Mann and Low (1954). Their results were based on the observation that the renormalization procedure introduced an unexpected property to the perturbative expansion of QED. If the bare electron is assumed massive, the renormalized charge of the electron can be defined by the Coulomb interaction of two electrons at rest at a large distance from one another. For massless bare electrons this process is not feasible as massless particles always travel with the velocity of light. One then has to introduce an arbitrary mass scale μ to define the renormalized charge $\mathrm{e}(\mu)$, defined in terms of some process wherein electrons interact electromagnetically in collisions at momenta of order μ. The so deduced renormalized charge, $\mathrm{e}(\mu)$ is called the effective charge at scale μ. Since that mass scale is arbitrary one can introduce other pairs $\{e', \mu'\}$ which give rise to the same physical results. The

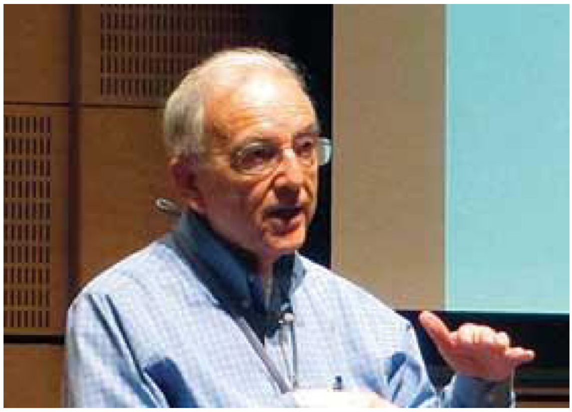


Fig. 4.4.1. Benjamin Widom.

set of transformations $\{e, \mu\} \to \{e', \mu'\}$ that leave the physics the same is called the renormalization group. The same can be done for the case of massive bare electrons. In both cases for an infinitesimal change of scale the variation in the renormalized charge with change of scale can be expressed as a "flow" equation

$$\mu \frac{de^2(\mu)}{d\mu} = \beta\left(e^2(\mu)\right)$$

where the function β can be calculated perturbatively as a series expansion in powers of $e^2(\mu)$. For large distances $e^2(\mu)$ would be the charge as determined by the Coulomb potential, for distances of the order or smaller than the Compton wave length $\hbar/mc$, $e^2(\mu)$ embodies the screening effects of the pair fluctuations in the vacuum[91].

Kenneth Wilson had deep insights into these methods and their use. Kadanoff's attempts to explain the scaling behavior observed in phase transitions was an important stepping stone in Wilson's formulation of his renormalization group method.

In the next two subsections I present some biographical materials of two of the persons whose work deeply influenced Wilson in arriving at his formulation of renormalization group methods: Benjamin Widom and Leo Kadanoff. A third person who, perhaps, was even more important, Michael Fisher, has told his part in the story in great detail. I refer the reader to his article (Fisher 1999). The stress on the first part of these biographical materials is on the educational institutions that molded Widom, Kadanoff and Wilson. This in order to see them as an important component of the enabling conditions that made it possible for them to accomplish what they did.

4.4 Benjamin Widom

Benjamin Widom was born in Newark, New Jersey on October 13, 1927. He grew up in Brooklyn, New York and attended lower Manhattan's Stuyvesant High School, from which he graduated in 1945. The high school had outstanding science teachers and attracted students interested in science. Already then Widom was recognized as an exceptional student. He thereafter went to Columbia as a chemistry major. Upon completing the requirements for a BA degree at midyear, he went to Cornell in February 1949 for graduate studies in physical chemistry. He had planned to work with John Bragg, a young assistant professor who worked on statistical mechanics. But Bragg left Cornell for a job at the General Electric research laboratory in Schenectady in June 1949. As the department felt that it had to allow Widom to do a theoretical thesis, the possibilities open to him were working with Paul Flory, a statistical mechanician, who worked on the properties of polymers in solutions, or working with Simon Bauer on chemical kinetics, and in particular, on the quantum mechanical description of inelastic collisions of molecules. Widom recalled"... what did I know? I thought, well, quantum mechanics is a very jazzy subject, and that sounded great, and statistical mechanics sounded dull, so I chose to do quantum mechanics with Simon Bauer"[92]. To prepare himself he took all the graduate courses a student intending to become a theoretical physicist would take: classical mechanics and electrodynamics, with Philip Morrison; mathematical methods of physics, with Mark Kac; quantum mechanics, with Ed Salpeter; advanced quantum mechanics and statistical mechanics, with Hans Bethe. He even audited a good part of Freeman Dyson's course on quantum field theory.

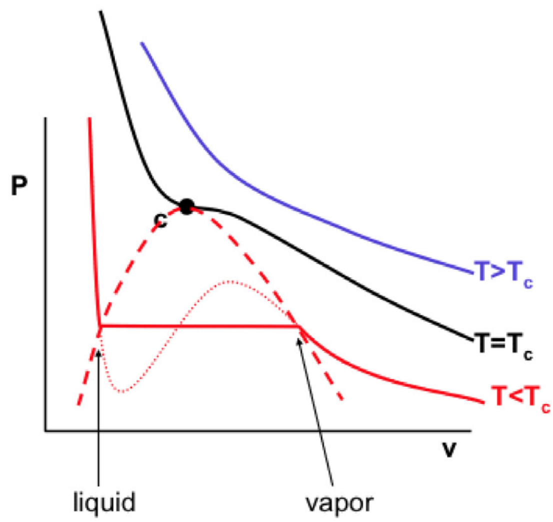
It was standard practice for a graduate student who intended to become a physical chemist to take graduate physics courses because at Cornell the examination for admission to the PhD required the coverage of two minor subjects in addition to the major subject, with the minors having to be outside the field of the major subject[93]. Physical chemistry graduate students normally had one minor in physics and one in mathematics. What was unusual was the number of advanced courses in physics and mathematics that Widom took. Interestingly, as a physical chemistry major he did not have to take any laboratory courses, and in fact didn't take any.

In 1952 after finishing an extensive semi-classical calculation of the de-excitation of vibrationally excited CO_2 molecules in collisions with H_2O molecules in the gas phase, Widom went to the University of North Carolina as a postdoctoral fellow to work with Oscar K. Rice (see Rice 2008). He knew of Rice's early work in energy transfer theory, and went to work with him thinking that he would be continuing along the lines of his thesis. But when he came to Chapel Hill, Rice told him that he was free to work on any topic he chose, but that if he wanted to talk to him, he would have to learn something about phase-equilibrium and critical points, "because that's what he was working on at the time". It was with Rice that Widom "really learned phase transition theory and got to appreciate thermodynamics and statistical mechanics". Rice was then trying to resolve the discrepancy between the theoretical prediction of the shape of the coexistence curve of most models of phase transitions, what is now called the critical point exponent beta, and the experimentally determined value of beta.

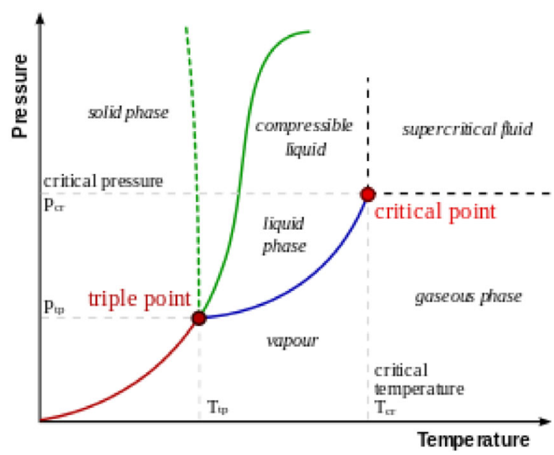
That discrepancy had been known since Van der Waals had proposed his equation of state and his corresponding principle. However, it was only in 1945 after E.A. Guggenheim published *his* paper on corresponding states that the community took notice of the problem. Guggenheim had put the coexistence curves of the rare gases and those of small molecules such as methane and nitrogen on a common scale and showed that their coexistence curves all coincided, but with an exponent for beta much closer to 1/3 than to the 1/2 that simple models predicted.

Besides theoretical investigations Rice was also doing experiments on binary liquid mixtures. It was then known through the work of Lee and Yang (1952) that a lattice gas description of a binary liquid mixture with its phase separation could be transcribed into an Ising model with permanent magnetization. In fact, soon after Widom arrived in Chapel Hill, Rice assigned him the task of reporting on Yang and Lee's paper in a seminar. This was also his introduction to the papers of Kramers and Wannier (1941) and of Onsager on phase transitions in two dimensional Ising models. He mastered the contents of these difficult papers and they became the point of departure for his own investigations of the statistical physics description of phase transitions.

"Trying to understand the deviations of the critical point exponents from their mean field values and trying to construct some kind of an equation of state that would incorporate these" is the problem Widom worked on in Chapel Hill. He remembers



The red line (between the liquid and vapor markings) is the coexistence curve on a pv diagram.



The blue line is the coexistence curve on a pT diagram.

Fig. 4.4.2.

working on what became known as the parametric model in critical phenomena, a way of expressing the equation of state in the neighborhood of a critical point that incorporates non-classical critical point exponents[94]. "But I never got that far. I never succeeded".

In 1954 Widom left Chapel Hill to return to the Cornell Chemistry Department as an instructor. He remained at Cornell since then, and became one of its most distinguished faculty members.

Widom spent the academic year 1961/62 on a sabbatical leave at the university of Amsterdam working in de Boer's group in statistical mechanics. Both Ezechiel G.D. "Eddie" Cohen and Hans Van Leeuwen were there working on the derivations of equations of state in quantum statistical mechanics using diagrammatic expansions. In discussions with them and the other members of de Boer's group Widom learned about the work of Michael Fisher on correlation functions and deviations from Ornstein-Zernike theory, and of the researches of Cyril Domb on the values of critical exponents obtained by series expansions and Padé approximations near the critical point. This is also where he learned all that was being done experimentally in liquid helium and where he got to understand Onsager's work on the two dimensional Ising model "more deeply... I knew about it long before, but I got a greater appreciation of it then. And I was still very excited about non-classical coexistence curves, and I saw them everywhere".

He spent the academic year 1965/66 in the Chemistry Department of the University of Reading, at the invitation of E.A. Guggenheim. While there he worked on how to correct the classical equations of state so as to incorporate the experimentally known critical-point singularities. A careful analysis of experimental data, suggested that the behavior of a fluid in the neighborhood of its critical point could be characterized by the property of a certain function $\Phi(x, y)$ that approximated the equation of state near its critical point and replicated the experimental data. The function that Widom had constructed had the astonishing property that the constant-volume heat capacity C_v computed from it diverged as $\ln(1/|T - T_c|)$ as T approached T_c both from above or below, with the density ρ fixed at ρ_c (the critical isochore). The same kind of logarithmic divergence of C_v had been found by Onsager in his analytic solution of the two-dimensional Ising model. Widom went on to establish that it was a certain homogeneity feature of the function he had conjectured for the equation of state in the neighborhood of the critical point that was responsible for the logarithmic divergence of C_v.

The equation of state that Widom was considering related the fluid's chemical potential to its density, ρ, and its temperature, T. In $\Phi(x, y)$, $x = (T - T_c)/T_c$, and $y = a|\rho - \rho_c|^\alpha$, with a and α constants. Widom had conjectured that near the critical point $\Phi(x, y)$ is a *homogeneous function of its variables,* with $\Phi(x, y)$ said to be homogeneous[95] of degree f-1 if

$$\begin{aligned}\Phi(x, y) &= y^{f-1}\,\Phi(x/y, 1)\\ &= x^{f-1}\,\Phi(1, y/x) \qquad \text{if } \; x > 0\\ &= (-x)^{f-1}\,\Phi(-1, y/-x) \qquad \text{if } \; x < 0.\end{aligned}$$

Expressed in term of the free energy function F of the system Widom had found that near the phase transition point, F when written in the "scaled" form

$$F(t = (T - T_c)/T_c,\; h = p - p_c) = Vt^{\beta+\Delta} f^*(h/t^\Delta) + \text{non-singular terms}$$

with V the volume of system, and β, Δ constants, F yielded an equation of state that represented the transition fairly accurately. Explaining the scaling became the point of departure of Leo Kadanoff.

4.5 Leo Kadanoff[96]

Leo P. Kadanoff was born New York City in 1937. His mother was a schoolteacher and his father an attorney. Both his undergraduate degree and his PhD in physics are from Harvard University which he attended from 1954 to 1960. He clearly stood as an exceptional student as he was part of the Schwinger "in" group by the time he was beginning his graduate studies. Being a member of the "in group" meant that he had lunch three times a week at Chez Dreyfus[97] with Julian Schwinger, his post-docs and young faculty associates. Being the most junior person in the group he would sit at the

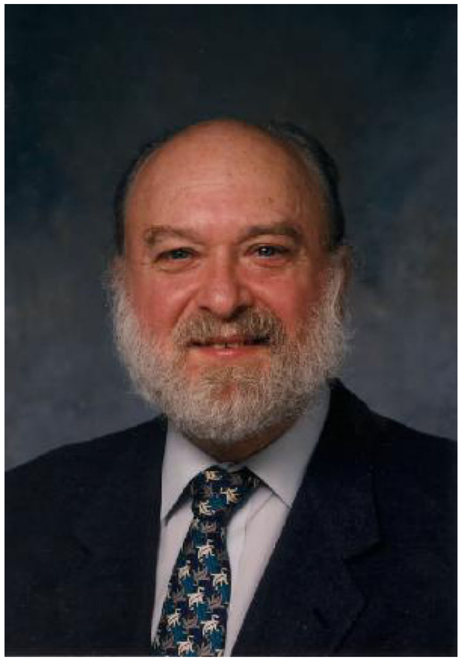


Fig. 4.5.1. Leo Kadanoff.

very end of the table. But he also noticed that there were "a tremendous number of people... chasing after a very limited amount of Julian's time" and therefore decided to work with Roy Glauber and Paul Martin and asked them to be his thesis advisers (Kadanoff 1960)[98]. They were "more easily available", and it also seemed "natural" for him to be involved with them.

After obtaining his PhD Kadanoff spent two years as a postdoctoral fellow at the Niels Bohr Institute in Copenhagen. During that time, with Gordon Baym, who had been a fellow graduate student at Harvard, he wrote a book on Schwinger's Green's function methods applied to condensed matter physics (Kadanoff and Baym 1962). Their book became a classic and is still being widely used as a textbook. While in Copenhagen in the winter of 1961-1962 he was offered – and accepted – an assistant professorship at the University of Illinois in Urbana[99]. The University of Illinois physics department was, and has remained being, an outstanding research center in solid state physics. Fred Seitz was then the chair of the department, and Bardeen, Pines, Schrieffer and Slichter, were the senior people working in condensed matter physics. In 1963 Gordon Baym joined the department (see Kadanoff 2014).

In December 1963 the Urbana solid state group, joined by Paul Martin, made a trip to the Soviet Union (see Hoddeson and Daitch 2002). Kadanoff there met Migdal, Polyakov, Patashinskii, Pokrovskii and other theorists and experimentalists actively working on the problem of phase transitions. It was probably there that he first heard of Voronel's experiments on the liquid-gas phase transition that established that his measurements of the approach to the critical point did not agree with Landau's mean field predictions. Kadanoff had known of Fairbank's experiments on liquid ^{3}He and ^{4}He, and in particular, of Fairbank's experiments exhibiting the divergent character of the specific heat of liquid ^{4}He at the lambda point.

The next big landmark in Kadanoff's professional life was accepting an invitation by Neville Mott, then the head of the Cavendish, to spend the academic year 1965/6 at Cambridge University. He arrived there being totally absorbed with "critical phenomena" and found himself "with plenty of time on [his] hands".

Kadanoff was then hard at work learning Onsager's solution of the two dimensional Ising model, studying Bruria Kaufman's spinor version of it (Kaufman 1949, Kaufman and Onsager 1949), and *meticulously* going through Schultz, Mattis and Lieb's field theoretical formulation of the solution (Schultz et al. 1964). Their approach made connections with Kadanoff's previous extensive investigations of superconductivity (see Kadanoff 1960 to Kadanoff 1966). Inspired by Schultz, Mattis and Lieb's paper, and using its formulation, he spent the fall doing "a hideous, seat-of-the-pants, hard-working type calculation of spin correlations in the two-dimensional Ising model". One of the results of his labors was an analytic expression for the asymptotic form of the two-spin correlation function for large separation of the two spins (Kadanoff 1966a,b) just above and below the critical point. The correlation length, ξ, is defined as the decay length of the correlation function in the limit of large separations of the spin:

$$C(r_1 - r_2) = \langle \sigma(r_1)\sigma(r_2)\rangle - \langle \sigma(r_1)\rangle \langle \sigma(r_2)\rangle$$
$$\sim \exp\left[-\frac{|r_1 - r_2|}{\xi}\right].$$

The expressions for the correlation length just above $\mathrm{T_c}$ that he obtained from what he accurately called his "long and difficult" calculation (Kadanoff 1999, p. 158) were that the $C(r_1 - r_2)$ – in the case of a square lattice, with nearest neighbor interactions and all the coupling constants identical,– were of the form $C_{\gtrless} \propto \varepsilon^{1/4} f_{\gtrless}(\varepsilon R)$ where ε is a measure of the distance from the critical point and $R = |r_1 - r_2|$. The result is valid for $\varepsilon \to 0$ but with εR remaining finite. Explicitly for large x

$$f_{>}(x) = e^{-x}(\pi x)^{1/2} 2^{-3/8}$$

and

$$f_{<}(x) = e^{-2x}(x)^{-2}\pi^{-1} 2^{-21/8}.$$

The calculations were important for Kadanoff. They gave him a technical mastery of the two dimensional Ising model that but very few people had. He had rephrased Onsager's calculation into the language of Green's functions and of correlations, making use of Mattis, Schultz and Lieb's field theoretical formulation. Furthermore, "It contained the germs of much of what was to prove to be the correct theory of second order phase transitions, all worked out in a particular example". He lectured on his calculations at King's College, and there met Michael Fisher for the first time. However, the paper's publication was delayed for over a year as it was twice rejected by the *Journal of Mathematical Physics,* whose editor and reviewers did not recognize its importance. Kadanoff thereafter submitted it to *Nuovo Cimento* where it was published in August 1966.

Shortly after coming to Cambridge Kadanoff had a conversation with Paul Martin – who was passing through Cambridge – during which Martin related to him what he had heard at the April 1965 Bureau of Standards conference on phase transitions[100], and told him of Ben Widom's lecture which had impressed him. At the conference, Widom had indicated that if the equation of state of a substance near its critical point exhibited certain homogeneity features, certain relations among the critical exponents could be derived which were corroborated experimentally in some systems. Martin had called them "magic relations". Kadanoff's spin correlation work also involved these critical indices. Given the great admiration and respect that Kadanoff had for Paul Martin, the fact that Martin had been so deeply impressed by Widom's work suggests that the challenge of how to justify Widom's homogenity assumption and derive Widom's "magic relations" must have been on Kadanoff's mind thereafter.

For the model of a ferromagnet that Kadanoff was considering, that of an Ising model in an external magnetic field, the free energy per spin, is a function $f = f(B,T)$. Widom's scaling hypothesis then stated that asymptotically close to the critical temperature the singular part of the free energy per spin $f_s = f_s(B,\epsilon)$ is a homogeneous function, i.e., there exist two numbers, a_B and a_T such that for all positive λ

$$f_s\left(\lambda^{a_B} B, \lambda^{a_T} T\right) = \lambda \; f_s(B,T)$$

or equivalently

$$f_s(b,T) = \left(\frac{1}{B}\right)^{-1/a_s} f_s\left(1, \left(\frac{1}{B}\right)^{a_T/a_s} T\right)$$

i.e. that the function $f_s(b,T)\,\left(\frac{1}{B}\right)^{+1/a_s}$ is a function of the scaled variable $(\frac{1}{B})^{a_T/a_s}T$.

The challenge for Kadanoff thus was how to translate the insights obtained from his calculations of properties of the Ising model – such as the spin-spin correlation length as a function of temperature – so that the properties of the free energy near the critical temperature that Widom had conjectured could be proven.

The reason for the singularities of the thermodynamic functions near a critical point was understood qualitatively. Consider the case B = 0. Near the critical point the system is trying to choose between the two possible directions of the magnetization. The interaction between spins is such it will favor having up spins near an up spin and to have down spins near other down spins. There therefore tend to be large regions in which there is a net excess of up spins and other large regions in which fluctuations tend to produce a net excess of down spins. As the temperature gets closer and closer to the Curie temperature, the size of these regions will get larger and larger. In the two dimensional Ising model when $B = 0$, the coherence length, which measures the size of these fluctuating regions, grows as $(T - T_c)^{-\nu}$ with $\nu = 1$. For the three dimensional model $\nu = 0.64$ as determined by a Padé approximation when expanding the free energy function as determined from the partition function. In the two dimensional case the large scale fluctuations in the regions are described by the $C(r_1 - r_2)$ function that Kadanoff had calculated. Their characteristic range is given by the correlation length, and thus diverge as the Curie point is reached. The reason for the singularities in the derivatives of thermodynamic functions can then be simply stated: statistical mechanics relates thermodynamic derivatives to correlation functions. In particular, the magnetic susceptibility $\chi = [\partial M/\partial B]_T$ is proportional to $\int C(r)dr$. If the range of $C(r)$ diverges, $\int C(r)dr$ will diverge. It is the long range fluctuations that produce the divergences in such quantities as the specific heat and the susceptibility. Though Kadanoff could derive the divergent nature of these observable quantities in the two dimensional Ising model, the Onsager solution he used did not yield a physical picture of what was happening. And the same was true for the analytical methods Domb, Fisher and others had produced in the three dimensional case.

Kadanoff recalls that during the Christmas week he had a "sudden vision… A gift from the gods… a simple view of how these magnitudes relations might be true in general. In modern terms, I had developed a scaling analysis of the critical behavior of Ising models based on the idea of running coupling constants, i.e. couplings which depended on the distance scale… The awfully complex and convoluted extension of the Onsager solution which I had previously done could now be explained in terms of a few simple and appealing ideas" (Kadanoff 1999, p. 158).

What Kadanoff called his “sudden vision” became the basis of the paper he published in *Physics*. Although it did not produce an unambiguous method for deriving the critical exponents from first principles, it allowed him to formulate convincing arguments for his derivation of relations among the critical exponents. Furthermore, his plausible arguments became the basis for understanding scaling and for the rigorous formulation of renormalization group methods by Wilson.

What Kadanoff did was to divide the lattice – assumed to be a square one with spacing a, – into cells of size L, measured in lattice constants, which contained many spins but the length of whose sides were small compared to the coherence length ξ: $1 \ll L \ll \xi$. The important variable in each cell, labeled by α, was taken to be its total spin λ_α:

$$\lambda_\alpha = \sum_{r\, in\, cell} \sigma_r .$$

Kadanoff then assumed that near the phase transition there was enough correlation so that within each cell most of the spins were pointing either up or down, and therefore that λ_α takes on but two values $\lambda_\alpha = c\mu_\alpha$ with $\mu_\alpha = \pm 1$ and c a constant, a function of the length L, which Kadanoff took to be

$$c = L^x$$

with x not known. λ_α is a collective variable describing the property of cell α, which is similar to the σ_{i} which described the property of the spin located at site i in the original lattice. Thus Kadanoff assumed that he could describe the system by an effective Hamiltonian

$$H_{\mathit{eff}} = -J_{\mathit{eff}} \sum_{\substack{\alpha\alpha' \\ nearest\, neighbors}} \mu_\alpha\, \mu_{\alpha'} - Bc \sum_\alpha \mu_\alpha$$

and that the new effective Hamiltonian contains once again only nearest neighbor interactions, and no terms interpretable as three spin, four spin, ... interactions of the form

$$-J^{(2)}_{\mathit{eff}} \sum_{\alpha\alpha'\alpha''} \mu_\alpha\, \mu_{\alpha'} \mu_{\alpha''} - J^{(3)}_{\mathit{eff}} \sum_{\alpha\alpha'\alpha''\alpha'''} \mu_\alpha\, \mu_{\alpha'} \mu_{\alpha''} \mu_{\alpha'''} + \ldots$$

This is the crucial assumption in Kadanoff’s work. The effective Hamiltonian is thus again an Ising model problem but with a new lattice constant La, a new magnetic field $cB = L^x B$, and a new coupling constant, J_{eff}. Changing J to J_{eff} implies a different distance from $\mathrm{T_c}$ and Kadanoff implemented this by assuming that $J \to J_{\mathit{eff}}$ is equivalent to

$$\frac{T - T_c}{T_c} \to L^y \frac{T - T_c}{T_c} .$$

The three transformations are then claimed to represent an invariance of the Ising model near T_c under the simultaneous change of length scale, magnetic field and $T - T_c$. But near the critical point, by virtue of the size of the correlation length, the choice of L is arbitrary – it can always be chosen so that $a \ll L \ll \xi$, – and therefore all physically measurable quantities must be independent of the artificial length L. That the consequence of this are scaling laws can be seen from the following. Consider the magnetization M and therefore of $\sum_i \langle \sigma_i \rangle$. $\langle \sigma_{\mathrm{i}} \rangle$ is a function of $T - T_c$ and of B:

$$\langle \sigma_i \rangle = g(T - T_c, B).$$

But

$$\sum_{r\,in\,cell} \langle \sigma_r \rangle = L^x \langle \mu_\alpha \rangle = L^x g\left(L^y(T-T_c), L^x B\right)$$

since $\langle \mu_\alpha \rangle$ also comes from the solution of an Ising problem but with a modified value of $T - T_c$ and of B. Since summation over all cells gives a factor L^d, where d is the dimensionality of the lattice,

$$\begin{aligned} M = \sum_i \langle \sigma_i \rangle &= F\left((T-T_c), B\right) \\ &= L^{x-d} F\left(L^y(T-T_c), L^x B\right). \end{aligned} \tag{4.1}$$

The result cannot depend on L which is only possible if

$$M = F\left((T-T_c), B\right) = (T-T_c)^{(d-x)/y}\, \mathrm{f}\left(\frac{T-T_c}{B^{y/x}}\right). \tag{4.2}$$

Although the function f is unknown and thus does not yield the value of the critical exponent, it nonetheless allows the determination of the critical exponent in terms of x and y.

Kadanoff made a similar analysis for the structure of the spin-spin correlation function and determined it must be a function of the form

$$\langle \sigma_r \sigma_{r'} \rangle = (T-T_c)^{2\beta} G\left((r-r')(T-T_c)^{1/y}, \frac{T-T_c}{B^{y/x}}\right) \tag{4.3}$$

in order to be independent of L. Since, as he had determined in the two dimensional case, this correlation function must contain $(r-r')/\xi$ it follows that

$$\xi \sim (T-T_c)^{-1/y}$$

or $y = 1/\nu$.

Onsager's, Domb's, and Fisher's work indicated that *at* the criticality, the spin-spin correlation function $C(r)$ has a power law decay for $r \to \infty$:

$$C_c(r) \approx \text{constant}/r^{d-2+\eta} \quad \text{as} \quad r \to \infty \tag{4.4}$$

where d is the dimensionality of the model, and η defines a new critical exponent $d-2+\eta$. The power law dependence implies scale invariance, and is thus explained by Kadanoff. To see this rescale length by a factor b, and simultaneously rescale σ by some factor b^ω. Under the rescaling $C_c(r) \to b^{2\omega}$ constant$/b^{d-2+\eta} r^{d-2+\eta}$ as $r \to \infty$. If $\omega = 1/2\ (d-2+\eta)$ the dependence on b drops out and $C_c(r)$ is scale invariant.

From the various relations Kadanoff had derived for various critical exponents he could obtain the various equalities that had been gotten thermodynamically between the critical exponents.

Under Kadanoff's assumptions, the first "decimation" yielded an Ising model whose effective coupling J' was not the same as the J in the original Ising model that specified the interaction between the spin σ_i and nearest neighbor σ_{i+1}, nor did the new $G' = \kappa B'/k_B T$ have the same value as the old one, $G = \kappa B/k_B T$. Note that the model is assumed to consist of N spins:

$$H = -J \sum_{i,j}^{N} \sigma_i \sigma_j + \kappa B \sum_{i}^{N} \sigma_i.$$

The requirement that the physics when expressed in terms of the new variables be the same as the old, can be translated into the requirement that the partition function have the same value

$$Z_N(K, G) = Z_{N/L}(K', G')$$

with $K' = J'/k_B T$, $G' = \kappa B'/k_B T$ and $K = J/k_b T$, $G = \kappa B/k_B T.K$, G describe the interaction at the microscopic scale a, correlation length ξ, whereas K', G' do so at the scale La with a correlation length ξ/L since its length is measured in units La in length.

The process can be iterated. Call K_n the effective coupling constant describing the theory at length scale $L^n a$ over which microscopic fluctuations have been averaged out, and with corresponding correlation length $\xi/L^n a$. When $\xi/L^n a$ becomes of order 1, long range correlations cease to play any role: the effective Hamiltonian which determines the corresponding partition function, with K_n satisfying the initial condition $K_1 = K$ determines the dynamics at that scale. The sequence of points generated by an iteration of the map will be different for different values of the "initial condition" K. The singular case is when the system is initially at its critical point so that ξ is infinite, in which case ξ' is also infinite.

The equation relating K_n to K can be thought off as an equation describing the "running" of the coupling constant with length. Kadanoff realized that he had figured that one could have such a running coupling constant.

> I had never heard of the work of Stückelberg and Gell-Mann and Low ... I'm not so sure about the Boguliubov-Shirkov volume, but I had never heard of the Western stuff. So I was completely naive of renormalization group concepts, but knew that here was a way of getting an effective coupling constant which was changing with scale. I presented it to my colleagues in Urbana soon after and they loved it, and it was presented soon after at Cornell, and they loved it, and I loved it.

Incidentally, when writing up his results for publication, Kadanoff recalled Martin's comments about Widom's lecture at the Washington NBS conference and discovered that Widom had anticipated some of his results and had published them in *Journal of the Chemical Physics* (Widom 1965). As he had had discussions with Bardeen about the obligations of authors to represent correctly the earlier literature he "did the right thing", and rewrote the introduction to the paper he was writing saying that it was based upon the work of Widom.

> So the work was reported and well-received, and then I did something of which I've always been rather proud. I decided that the right way of making this known to the world was to make contact with the experiments. I'd been involved with a thesis which was done a tiny bit slowly, and there were previous publications by other people, so the student had a problem. But the student's thesis was well-received in the end, because he had referred to all the experimentalists who worked in the field. And I absorbed that information. And I decided that I would run a seminar which would include all the experimental work I could find.
>
> The materials research lab had just been set up at Urbana, and my colleagues in the lab kindly, or maybe occasionally not-so-kindly, allowed me to commandeer their graduate students and post-docs, to give lectures in the seminar, and in the end to put together a review paper[101]. This is something we did over a course of six or eight months. We reviewed, I believe, every experiment that we could reasonably find involving critical phenomena. And managed to fit them all into some picture which included this new scaling point of view, based, of course, on the phenomenology that had been developed by Widom.

Based upon the phenomenology which had also in parallel, and earlier than my work, I believe, been developed by Patashinskii and Pokrovskii[102].

Kenneth Wilson learned what Kadanoff had done from various sources and transformed Kadanoff's running coupling constants into a precise operational scheme that was free of the assumption that the decimated Hamiltonian (or equivalently stated, that the Hamiltonian obtained by integrating out short length degrees of freedom) was of the same form as the original Hamiltonian. His approach was relevant to both phase transitions and QFT, unifying the concept of renormalization group.

For the simple 2 dimensional Ising model outlined above Wilson looked upon the initial Kadanoff thinning as a transformation from the interaction $\mathcal{H}_a(S)$ to a new effective interaction $\mathcal{H}_{2a}(S)$ at scale $2a$:

$$\mathcal{H}_{2a}(S) = \mathcal{T}[\mathcal{H}_a(S)]$$

with $K_2 = t(K)$. $\mathcal{H}_{2a}(S)$ can be thought as having been obtained by summing, $\sum$, the contribution to the partition function of all the spins but one in cells of size a.

$\mathcal{H}_{2a}(S)$ is then defined by

$$e^{-\mathcal{H}_{2a}(s)/kT} = \sum e^{-\mathcal{H}_a(s)/kT}.$$

But in contrast to Kadanoff Wilson took $\mathcal{H}_a(S)$ to describe not merely nearest neighbor interactions but to include to include all possible couplings generated by the transformation, with K standing for the set of all these coupling constants.

The transformation can be recursively iterated

$$\mathcal{H}_{2^n a}(S) = \mathcal{T}\left[\mathcal{H}_{2^{n-1}a}(S)\right]$$

with $K_n = t(K_{n-1}) = t(t(K_{n-2})) = \ldots$ If the repeated application of $\mathcal{T}$ produces an effective interaction whose asymptotic form is essentially independent of the initial interaction one has produced a mechanism that explains the universality of the model. And such asymptotic interactions will be fixed points of the transformation $\mathcal{T}$:

$$\lim_{n\to\infty} \mathcal{H}_{2^n a}(S) = \mathcal{H}^*(S)$$
$$\mathcal{H}^*(S) = \mathcal{T}\left[\mathcal{H}^*(S)\right]$$

where $K^* = t(K^*)$. The important point is that the analysis of fixed points is independent of the assumed initial Hamiltonian and is determined by the equation $K^* = t(K^*)$[103]. In the two dimensional Ising model, there are three fixed points. At one, at the largest size, the temperature goes to zero and becomes unimportant corresponding to the magnetized state. At the second, $T \to \infty$ and the system is disordered and unmagnetized. At the third point, the correlation length is infinite and corresponds to the critical point.

Fisher has given a detailed exposition of the technical aspects of the above in Fisher (1999) and I refer the reader to it.

4.6 Kenneth Wilson

Kenneth Wilson was born in 1936, and became the oldest of six siblings. Kenneth's mother had been trained as a physicist and had taken graduate courses in physics. Kenneth's father, E Bright Wilson, was a distinguished professor of chemistry at Harvard University. In 1935 he had written a textbook with Linus Pauling from

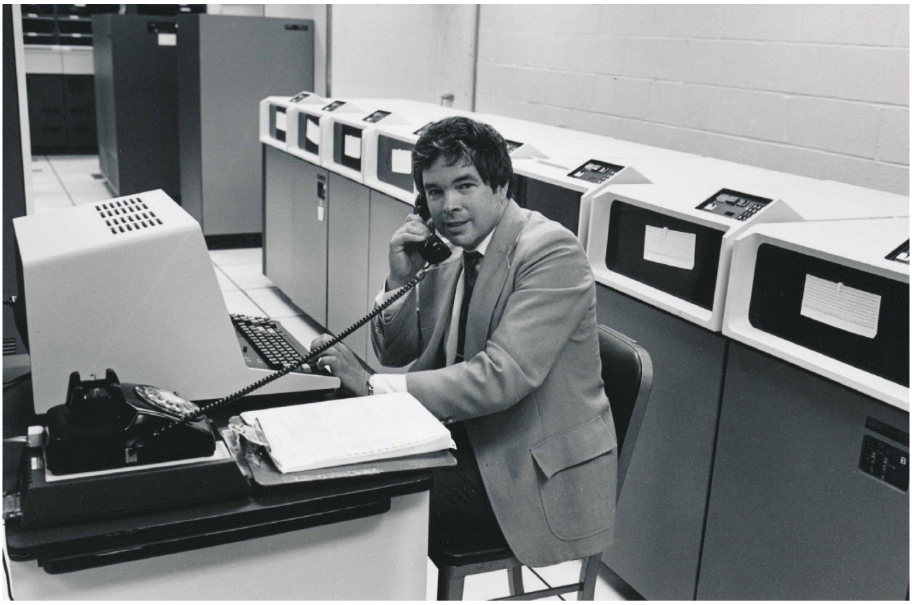


Fig. 4.6.1. Kenneth Wilson (1936–2013). Photograph by Sol Goldberg, Cornell University Photography. Division of Rare and Manuscript Collections, Cornell University Library.

which several generations of physics and chemistry graduate students learned quantum mechanics[104]. In 1952 he wrote an equally influential book on the scientific method[105].

Kenneth ("Ken") Wilson did his undergraduate studies at Harvard majoring in mathematics. He recalled that already then he "was fascinated with the question of how you approximate mathematical equations in order to solve them". He went to Caltech in 1956 as a graduate student in physics. Although he had not realized it at the time he had essentially completed a PhD working with Arnold Arons during his junior and senior year at Harvard[106].

Very early on when Ken was at Cal Tech, Jon Mathews[107], one of the young high energy theorists he had gotten close to, took him to the Jet Propulsion Lab and introduced him to the Datatron Burroughs computer[108] with its punched cards input that was being developed there. Mathews showed him how to use it and Ken did some programming on it. "I was fascinated with computers starting from that point"[109].

Sometime toward the end of his second year at Cal Tech he started working with Murray Gell-Mann. Gell-Mann suggested to him looking at K meson-nucleon scattering, treating the nucleon as a fixed source. He read extensively on fixed source theory, both strong coupling theory and Chew-Low theory. He then also studied the Gell-Mann-Low paper on the renormalization group and realized that its approach could be applied to the fixed source model. Making use of the insights of that paper he discovered that great simplifications took place when he extrapolated the fixed source equations to high energies. In fact, he could then obtain exact solutions to these equations. These results became part of his PhD dissertation and were the beginning of his interest in the renormalization group. He recalled that when he gave a seminar on his thesis someone asked "Yes, that's fine, but what good is it?" and that Feynman, who was in the audience, answered saying: "Don't look a gift horse in the mouth!"[110].

In 1959, before completing his PhD, he came back to Harvard as a Junior Fellow. He learned what was going in field theory and S-matrix theory at the time from Francis Low and Kenneth Johnson, worked on Mandelstam's bootstrap approximation and tried to program a numerical solution[111]. He used the MIT computer located in the Registrar's office, "and that went exactly no place, because I could look at the output only once a day. And you can't do computing on that basis"[112]. A stay at CERN – from January 1962 till January 1963 – where people were working on S-matrix theory convinced him that the only subject he wanted to pursue was quantum field theory applied to strong interactions.

> I rejected S-matrix theory because the equations of S-matrix theory, even if one could write them down, were too complicated and inelegant to be a theory. In contrast, the existence of a strong coupling approximation as well as a weak coupling approximation to fixed source meson theory helped me believe that quantum field theory might make sense.

In 1963 Wilson accepted an assistant professorship at Cornell. He was promoted to an associate professorship with tenure two years later. This with essentially no publication record.

> In fact, there was one or two papers on the publications list when I was granted tenure. Francis Low complained that I should have made sure there was none. Just to prove that it was possible.

After arriving at Cornell he started making models involving the elimination of degrees of freedom in field theoretical models (Wilson 1965). These researches culminated with the work he did 1969 at SLAC in which in the fixed source model he "sliced up" the momentum space continuum into shells up to some cutoff momentum, and took the Hamiltonian for the largest momentum slice as the unperturbed Hamiltonian and the terms for all the lesser slices as the perturbation[113]. If one started with n momentum slices, selected the ground state of the unperturbed Hamiltonian for the nth slice, one ended up with an effective Hamiltonian for the remaining $n-1$ slices. This new Hamiltonian was identical with the original Hamiltonian but with only $n-1$ slices kept, except that the meson-nucleon coupling constant g was renormalized (i.e., modified)[114]. This work was a real breakthrough for he had found a natural basis for a renormalization-group analysis: the elimination of one momentum scale from the problem.

Wilson spent the summer of 1966 at the Aspen Center for Physics in Colorado[115]. Before going to Aspen, he had attended a theoretical chemistry seminar at Cornell in which Ben Widom had presented his scaling equation of state (Widom 1965)[116]. Already at that presentation he was puzzled by the absence of any theoretical basis for the form Widom wrote down. He was at that time ignorant of what was going on in critical phenomena, and what made Widom's work so important a development.

While in Aspen he studied Onsager's solution of the two-dimensional Ising model in the field-theoretic formulation given by Schultz, Mattis, and Lieb (1964). As he worked through the paper he realized that there were applications of his renormalization-group ideas to critical phenomena, and discussed this with some of the solid-state physicists at Aspen. He was informed by them that he had been "scooped" by Leo Kadanoff and that he should look at his preprint (Kadanoff 1966).

In his Nobel lecture[117] and in his HRST interviews Wilson detailed how he arrived as his formulation of renormalization group methods by amalgamating his knowledge of Euclidean (imaginary time) quantum field theories on a lattice with the grand partition function in statistical-mechanics. He generalized Kadanoff's approach of "integrating out" degrees of freedoms with the methods he had used in "slicing up" momentum space in the fixed source model he had investigated, by freeing himself from looking at models with a finite number of coupling constants. The immediate proximate cause for this development was being asked by Ben Widom to give a seminar to explain what Di Castro and Jona-Lasinio (1969) had done in their paper that had applied the field theoretic renormalization-group formalism to critical phenomena.

I will not repeat the story here. For informative, valuable and insightful accounts of what Wilson did and accomplished I again refer the reader to Wilson and Kogut (1973); to Wilson's Nobel prize lecture (Wilson 1983); to Fisher's very informative paper (Fisher 1999) which gives a personal account of the process by which Wilson arrived at his formulation of the solution of the phase transition problem and his own involvement in the process; to Wilson's 2004 interviews with Babak Ashrafi, Sam Schweber and George Smith available on the HRST website at Cal Tech; and to the articles written by Kadanoff, Fisher, Brezin, and Peskin for presentations at the Kenneth Wilson Memorial colloquium that the Cornell physics department organized in the fall 2014. Instead I will present the application of Wilson's approach to formulating nonrelativistic quantum field theory (NRQFT).

But before doing so I want to point to two areas in which Wilson became deeply involved in the 1970s and 1980s: computing and quantum chemistry[118].

In describing Wilson's education I have pointed to his close connection to computers and computing[119]. This coupling became ever stronger thereafter. His solution of the Kondo problem relied on that relationship and so did his formulation of lattice gauge theory (see Wilson 2005)[120]. Wilson stressed the importance of computers for him in one of his HRST interviews:

> In thinking and trying out ideas about 'what is a field theory', I found it very helpful to demand that a correctly formulated field theory be soluble by computer, the same way an ordinary differential equation can be solved on a computer, namely with arbitrary accuracy in return for sufficient computing power.

He there also commented on he why relied so heavily on computers and computations when doing his researches on phase transitions:

> Why was I interested in a very high accuracy in the case of critical phenomena? ... [It] was because I wanted to get the leading irrelevant operator, from a numerical simulation. I wanted to be able to not just calculate the exponents associated with the relevant operators, the things that take you away from the critical point, but the leading irrelevant operator, where, as you go to larger and larger sizes at the same time the effect becomes smaller and smaller, and so if you can't do very high precision you just can't see it. That was part of understanding the phenomena: to identify the leading irrelevant operator, identify its strength. All of a sudden that meant that if you're going to do this numerically, you had to have high accuracy. So, I had very good reasons why I wanted to be able to not just do a rough simulation.

Wilson's involvement in quantum chemistry was the result of discussions with his father who "used to get very wrought up about computational quantum chemists" claiming what they wetre doing was "Garbage in, garbage out". That induced him to spend time studying quantum chemistry after he and his wife moved to Ohio State[121]. He had already done so towards the end of his stay at Cornell in the mid 1980s, but took the matter up very seriously when at Ohio State (Wilson 1990, Wenzel et al. 1996). In his HRST interview he recalled that

> What I found was that the people who did the important work (in quantum chemistry) worked on algorithms. They improved the algorithms for solving quantum chemistry problems on computers. They couldn't do the calculations they wanted to do, so they worked on algorithms. And it was the algorithmic work that was absolutely essential. When the computers got better, and they could do serious things, it was the work on algorithms that made the difference

and the people that my father knew (and thought highly of) made contributions to serious algorithm developments.

Pause

This is not an appropriate setting to present the technical details in the development of renormalization group methods and their application to quantum field theory, high energy physics and condensed matter physics, nor am I competent to do so. I have focused on the contributions of Widom, of Kadanoff and of Wilson in the initial development of the approach to emphasize the mathematical and field theoretical tools they had acquired in their graduate training, and the importance of their interactions with other theorists in condensed matter physics, in chemistry, in computing.

Kadanoff, his post-docs and his graduate students made important contributions to the further elucidation of quantum field theory. Of particular importance was Kadanoff's and Wegner's analysis of the fixed points of the renormalization group equation

$$\boldsymbol{K}^* = \boldsymbol{R}(\boldsymbol{K}^*)$$

by considering the behavior of $\boldsymbol{K}$, i.e. its dependence on the parameters it embodies, near a fixed point value.

To sketch what they did recall the two dimensional Ising model. In that model, one considers $\boldsymbol{K}$ a function of the deviation from the critical temperature, $\varepsilon = (T - T_c)/T_c$, and of h, the dimensionless magnetic field: $\boldsymbol{K} = (\boldsymbol{K}_0, h, \varepsilon)$. The pair ε, h constitute the essential set of what Kadanoff and Wegner called the *fields* of the model. Their more general analysis to include all possible couplings when near criticality yielded a classification of carefully chosen "fields" (what they called "scaling fields") into being relevant, marginal and irrelevant. In the criticality investigations, this division of perturbations into relevant, irrelevant, and marginal depends on whether they grow or decay upon renormalization toward large scales. "Relevant" fields grow under the renormalization group transformation and become more and more important as larger distance scales are reached. Irrelevant fields tend to diminish with successive renormalization group transformations and effectively do not contribute at large scales. Marginal fields, when present, do not change as the scale changes (see e.g. Kadanoff 2000, pp. 257–262).

In the review of criticality that Kadanoff and his group had published in 1967 in the *Reviews of Modern Physics* use had been made of the notion of universality. The classification scheme he and Wegner introduced makes the notion precise: a universality class includes many different kinds of systems that all have the same behavior because of the *irrelevance* of microscopic detail.

Both Kadanoff and Wilson contributed importantly to what has become known as the short distance expansion of products of local operators in quantum field theory (see Duncan 2012). And both Kadanoff and Wilson were strongly coupled to computers, made heavy use of them, and often came to "think" like them[122]. In fact, Wilson made crucial contributions to physics in general, and to high energy physics in particular by virtue of his mastery of computing: His formulation of QCD on a lattice, gave a plausible explanation of the color confinement of quarks and gluons.

4.7 Lepage and NRQED

An aspect of the transformation brought about by Wilson's viewpoint is beautifully conveyed in the lectures Peter Lepage gave at the TASI summer school in Boulder,

Fig. 4.7.1. One-loop self-energy diagram.

Colorado in the summer 1989 on "What is renormalization?" and illustrated with QED (Lepage 2005). QED is defined by the Lagrangian density

$$\mathcal{L} = -\frac{1}{2}F^{\mu\nu}(x)F_{\mu\nu}(x) + \overline{\psi(x)}\left(i\gamma^{\mu}\partial_{\mu} - e_0\gamma^{\mu}\left(A_{\mu}(x) + A^{e}_{\mu}(x)\right) - m_0\right)\psi(x).$$

Given that the interaction between the fields is local – reflecting the structurelessness assumption regarding the entities described by the fields – one must first regulate the theory. This can be done by introducing a cut off, Λ_0, thus removing from the theory all states having energies greater the cut-off Λ_0. The renormalization of the theory makes use of the fact that the effects of the very high energy states of the theory on its low energy behavior can be simulated by a set of new local interactions. The effect of the removal from the theory of all states with energies between Λ_0 and some new cut off Λ ($\Lambda_0 \gg \Lambda$) for processes at energies much lower than Λ can be compensated by the addition to the Lagrangian the correction

$$\delta\mathcal{L}_0 = -e_0 c_0\left(\frac{\Lambda}{\Lambda_o}\right)\,\overline{\psi}(x)\gamma^{\mu}\psi(x)\left(A_{\mu}(x) + A^{e}_{\mu}(x)\right)$$

where $c_0\ (\Lambda/\Lambda_0)$ is dimensionless constant proportional to

$$c_0\left(\frac{\Lambda}{\Lambda_o}\right) \propto \frac{e_0^2}{\hbar c}\log\left(\frac{\Lambda}{\Lambda_o}\right).$$

These contributions stem from the one loop vertex and vacuum polarization diagrams. Simple dimensional and power counting arguments indicate that the removal of the states between Λ and Λ_0 affects all other scattering amplitudes by factors proportional to $(p/\Lambda)^2$ or higher powers of (p/Λ) where p (with $p \ll \Lambda$) is the characteristic momentum of the processes being considered. However, the mass of the electron is strongly affected by the removal of the states between Λ and Λ_0, stemming from the one-loop electron self energy diagram (Fig. 4.7.1).

Its contribution can be simulated by adding a term of the form

$$\delta\mathcal{L}_m = -m_0\widetilde{c}\,(\Lambda/\Lambda_o)\,\overline{\psi}(x)\psi(x)$$

proportional to m_0 (by chiral symmetry) and where $\widetilde{c}\,(\Lambda/\Lambda_o)$ is dimensionless and proportional to $\frac{e_0^2}{\hbar c}\log\left(\Lambda/\Lambda_o\right)$. Thus the theory with Lagrangian

$$\mathcal{L} = -\frac{1}{2}F^{\mu\nu}(x)F_{\mu\nu}(x) + \overline{\psi(x)}\left(i\gamma^{\mu}\partial_{\mu} - e_{\Lambda}\gamma^{\mu}\left(A_{\mu}(x) + A^{e}_{\mu}(x)\right) - m_{\Lambda}\right)\psi(x)$$

and coupling parameters

$$\begin{aligned} e_{\Lambda} &= e_0\left(1 + c_0(\Lambda/\Lambda_0)\right) \\ m_{\Lambda} &= m_0\left(1 + \widetilde{c_0}(\Lambda/\Lambda_0)\right) \end{aligned}$$

give the same results as the original theory with cutoff Λ_0, up to corrections of order p/Λ, where p is a momentum typical of order p/Λ.

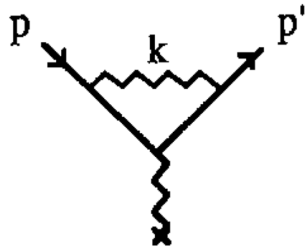


Fig. 4.7.2. One loop vertex diagram.

Using the same Wilsonian approach, Caswell and Lepage have indicated how to transform the above relativistically invariant formulation of QED into an effective field theory that corresponds to the customary non relativistic theory, but in which relativistic and radiative corrections can be systematically included. To do so they remove all relativistic states by cutting off all momentum integrations in perturbation theory at $p \sim m$, where m is the electron's mass, and add correction terms to the Lagrangian that compensate for the effect of the cutoff. The errors introduced by the cut-off are of order $p/\Lambda \sim v/c$. To remove them they consider the contribution of internal photons with momenta k or order or larger to the one loop radiative correction to the amplitude for an electron to scatter off an external field (see Fig. 4.7.2).

Given that non-relativistic external electrons have momenta $p, p' \ll \Lambda$, the difference between the correct amplitude T and the amplitude $T^{(\Lambda)}$ in the cutoff theory can be expanded in a Taylor series in p/Λ and p'/Λ:

$$\begin{aligned} T - T^{(\Lambda)} &= \frac{e(\Lambda)m(\Lambda)c_1(\Lambda)}{\Lambda^2}\overline{u(p')}\widetilde{A}^{\mu}_{ext}(q)\,\sigma_{\mu\nu}q^{\nu}u(p) \\ &+ \frac{ie(\Lambda)c_2(\Lambda)}{\Lambda^2}q^2\,\overline{u(p')}\widetilde{A}^{\mu}_{ext}(q)\gamma_{\mu}u(p) \\ &+ O\left(\frac{1}{\Lambda^3}\right) \end{aligned}$$

where $q = p' - p$ and the coefficients c_1 and c_2 are dimensionless. Since it is of order p/Λ the first term dominates the corrections. Its effect can be incorporated in the cutoff theory by adding a new local interaction to the Lagrangian

$$\delta\mathcal{L}^{(\Lambda)}(x) = \frac{e(\Lambda)m(\Lambda)c_1(\Lambda)}{2\Lambda^2}\,\overline{\psi}(x)\sigma_{\mu\nu}F^{\mu\nu}(x)\psi(x)$$

$\delta\mathcal{L}^{(\Lambda)}(x)$ introduces a "bare" anomalous magnetic moment for the electron, and though non renormalizable causes no problem since the theory is cut off at Λ. Dimensional analysis indicates that adding $\delta\mathcal{L}^{(\Lambda)}(x)$ removes all errors of order p/Λ. Cardwell and Lepage have indicated how the process can be generalized to higher order in p/Λ and the theory made as accurate as wanted without increasing the cutoff.

Having admitted "non-renormalizable" interaction in the Lagrangian, the question is not "Is the theory renormalizable?", but rather how renormalizable is it, i.e. how large are the non renormalizable interactions in the theory.

QED is a low energy approximation to some (unknown) "supertheory" valid at very short distances, or equivalently valid beyond some very high energy beyond which QED is no longer valid. Choose the cut off to be that energy. The supertheory affects low energy phenomena, but only through the values of the coupling constants that appear in the cut-off Lagrangian:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}F^{\mu\nu}(x)F_{\mu\nu}(x) + \overline{\psi(x)}\left(i\gamma^\mu\partial_\mu - e_\Lambda\gamma^\mu A_\mu(x) - m_\Lambda\right)\psi(x)\\ &+ \frac{e_\Lambda m_\Lambda \kappa_1}{\Lambda^2}\overline{\psi(x)}F^{\mu\nu}(x)\sigma_{\mu\nu}\psi(x) + \frac{e_\Lambda\kappa_2}{\Lambda^2}\overline{\psi(x)}F^{\mu\nu}(x)\sigma_{\mu\nu}\psi(x)\\ &+ \frac{\kappa_3}{\Lambda^2}\overline{\psi(x)}\gamma_\mu\psi(x)\overline{\psi(x)}\gamma^\mu\psi(x) + \ldots\end{aligned}$$

The coupling constants (the κs) in this Lagrangian will *presumably* be calculable in a future super theory. But since this super theory is at present *non-existent,* the values of the κs are to be obtained empirically, by measurements. These non renormalizable terms are certainly present. The fact that they are not needed in the description of nature at the present level of accuracy indicates that Λ is very large, as in any scattering process they contribute terms of order $(p/\Lambda)^2$ raised to some power, where p is a characteristic momentum of the process being described. It is because one can show that it is warranted to neglect the contributions of non-renormalizable terms in the Lagrangian that very low energy approximations to arbitrary high energy *dynamics* can be formulated in terms of renormalizable theories.

Note what has happened in this Wilsonian approach. One can encapsulate ignorance of short distance microphysics in a finite number of parameters but these parameters depend upon the definition of "short distance microphysics": the parameters depend upon where one puts the boundary between what one knows and what one doesn't know. The introduction of an ultraviolet cutoff Λ into QED defines "short distance microphysics" to be physics above Λ. Furthermore, when this is done the coupling "constants" in the Lagrangian are no longer constants: they are functions of the value of the cutoff; they "run".

5 Some reflections on "More is Different"

In the previous sections I have looked at models of systems which allowed taking an effective theory believed valid at the microscopic level and correlating it to observables at a level at which measurements on the system are made. The simplest of these was the Ising model, a model designed to exhibit the relation between the micro and the macro levels of description, and to indicate the assumptions made in going from one level of description to the other. Kadanoff's clustering and decimation approach explained the observed scaling at criticality in the macroscopic level of description that Widom had emphasized. Wilson then formulated a consistent renormalization group methodology that embodied Kadanoff's approach, thereby also explaining the observed universality features of phase transitions, and with Fisher formulated algorithms that enabled the calculation of critical exponents.

The Ising model has a parameter in it, the temperature T, that the experimentalist can vary. And the system it models can be *fabricated.* One can construct materials – e.g. carbene,– which behave essentially two-dimensionally. And by constructing crystals with different atomic constituents one can vary K, and much more. The same is true of "ordinary" superconductors and of high T_c superconductors, indicating the great powers of experimentalists in creating new materials and being able to differentiate between type I and type II superconductors. In fact, when in 1911 Kamerlingh-Onnes observed superconductivity in various metallic materials, he created phenomena that had never existed before, thus echoing what Nernst had pointed out a few years earlier: Chemists produce materials that had never existed before (Nernst 1896). The process of creating high T_c superconductors also made clear the limitations of the BCS theory.

With QED one enters a new domain. QED is an effective field theory, valid up to some energy determined by the level of precision at which one probes the system. This is true of QED limited to the description of the interactions of electrons and positrons with the electromagnetic field (e.g. Compton scattering), or of QED that includes the presence of an external field (as is the case when the proton in a hydrogen atom is approximated by a static Coulomb field). The eleven digits the most accurate measurements of the hyperfine structure of hydrogen have yielded cannot be explained by a QED which limits itself to electrons and positrons. Vacuum polarization effects due to muon pair productions must be taken into account as well as nucleon structure factors (See Gabrielse in Kinoshita 1990).

In its simplest version, when restricted to electrons and photons, QED is a local theory that incorporates Poincaré and gauge symmetries. By virtue of these properties its observable consequences, as calculated from its S-matrix in perturbation theory, are divergent and must be regularized and renormalized. The experimental fact that its coupling constant at low energies is small, $\mathrm{e}^2/4\pi hc \approx 1/137$ is invoked to justify the pertubative approach and gives rise to the hope that the series expansion is an asymptotic one. QED when restricted to electrons, positrons and photons has a further special attribute: the electron is the lightest charged lepton and (seemingly) absolutely stable, revealing no structure to the shortest distance probed thus far, thus allowing a point like description in all its theoretical manifestation.

This is not the case of the theories whose aim is to explain the nuclear domain. Quantum field theories that assumed nucleons and mesons to constitute its "elementary" ontology required large couplings, invalidating the use of perturbation theory. Furthermore, by the mid-1950s it was known that nucleons are extended object with a size of the order of 1 fermi, thus invalidating the assumption of no structure when trying to formulate theories valid at high energy. Nonetheless, as expounded in Walecka (1995) a very effective quantum field theoretic *hadrodynamics* (QHD) was developed that fairly accurately described nucleon-nucleon and hadron-nuclear interactions at low and intermediate energies, namely, below 350 Mev or so. After the establishment of the standard model how to relate QHD to the (QCD) and the standard model became the outstanding problem in nuclear physics.

5.1 QCD once again

One of the characteristics of the standard model is that its formulation as an effective quantum field theory is a description in terms of structureless entities that are *unobservable*. The QCD part of the standard model is a local, non-Abelian gauge theory based on the gauge group SU(3). It is formulated in terms of six types of quarks (flavor) the u(p), c(harm), t(op) with electric charge 2e/3 and the d(own), s(trange), b(ottom) quarks with electric charge –e/3. Each quark has three colors – the reason for the 3 in SU(3)[123]. There are eight massless spin 1 bosons, the gluons, that are responsible for the interaction among quarks. They also interact with one another since they carry a color charge. The theory with massive quarks can be written in the deceptively simple form

$$\mathcal{L} = \frac{1}{4} F_{\alpha}^{\mu\nu} F_{\alpha\mu\nu} - \sum_{n} \overline{\psi_n} [\not{\partial} - ig \not{A}_{\alpha} t_{\alpha} + m_n] \psi_n \qquad (5.1)$$

where the ψs are quark field operators with the subscript n labeling the quark flavors, $F_{\alpha}^{\mu\nu}$ is the gauge covariant strength tensor of the gluon field, g is the coupling constant, and the t_{α} are a complete set of generators of color SU(3) in the **3** representation.

Its formulation on a lattice (lattice gauge theory) gives strong support to the notion that quarks and gluons are always confined to the interior of hadrons and are never asymptotically free. This confinement is a dynamic property of QCD. Lattice gauge theory calculations indicate that the energy of a system composed of a static quark and static antiquark separated by a distance R grows linearly with R. This became the basis for the explanation why quarks are never seen. Only color neutral particles like baryons and mesons can appear as isolated entities. Confinement justifies why in the phenomenological quark model baryons like the proton and neutron can be considered as color neutral bound states of three quarks, totally antisymmetric in quark color charge; and that mesons could be considered color neutral bound states of a quark and an antiquark.

The second remarkable property of QCD is asymptotic freedom. Recall that it was the search for a theory that displayed asymptotic freedom that led to QCD: asymptotic freedom explains the scaling observed in deep-inelastic scattering of electron and neutrino off protons.

In QED, vacuum polarization shields a point electric charge, so that the renormalized charge as a function of the distance at which it is measured increases as the distance decreases. In the approximation where the quark masses can be considered as negligible compared to the energies being probed the β function which determines the behavior of the renormalized coupling constant with distance was computed by Gross and Wilzcek, and by Politzer

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left(11 - \frac{2}{3}n_f\right) + O(g^5). \tag{5.2}$$

The minus sign indicates that there is antishielding, with the coupling constant decreasing at shorter distances. The theory is asymptotically free as long as there are no more than 16 quarks with masses below the energy scale of interest. The factor 11 comes from the presence of the non-Abelian color gauge group. It has been shown that only non-Abelian gauge theories exhibit asymptotic freedom.

I want to stress the importance of the presence of the n_f term in the β function. *QCD yields asymptotic freedom only if n_f is a small number*. It is a “foundational” theory which must delimit the number of “elementary” entities it incorporates in its ontology in order to have the particular desired property.

5.2 From nuclear physics to nuclear science

Even though it involves developments after the time period I am concerned with, I want to sketch what happened to “nuclear physics” after the standard model was formulated and corroborated by a wide ranging set of experiments and the detection of the 3K thermal cosmic background radiation gave support to a Big Bang model of cosmogenisis (see Bernstein and Feinberg 1986; Peebles 1993; Weinberg 2008): it transformed itself into “nuclear science”. I do so because

(a) The developments illustrate how QCD is used to explain and *reconstruct* the nuclear domain;
(b) to again stress that the *finiteness* of n_f is an essential element in the derivation of the properties of the higher levels. Use is also made of regularization and renormalization, and once again the finiteness of the set of “elementary” entities is an essential feature of the derivation of effective mean field theories that represent the dynamics of the higher levels. And because
(c) another reason for giving a bird-eye’s view of the history of the transformation of nuclear physics into nuclear science is that “nuclear science”, the discipline that evolved/emerged from “nuclear physics”, explicitly identifies itself as an *evolutionary* science that deals with history[124]. The transformation can only be understood by considering the quantum revolution as a Hacking type revolution with its continuities, practices, institutions, and wider cultural contexts. Furthermore, how

to understand the "nuclear physics" to "nuclear science" transformation is surely grist to a historian's mill.

Let me begin with a brief review of the history of nuclear physics[125]. The "traditional" way of doing nuclear physics – before the 1970s – was to approximate solutions of the many body Schrödinger equation by Hartree-Fock wave functions, i.e. by taking the many body wave function to be an anti-symmetrized product of one-particle wave functions which are then determined variationaly. A justification for this was that the shell model of nuclei provides a good first approximation to nuclear structure, and is conceptualized in terms of single particle wave functions. The interaction potentials used in the Schrödinger were static two-body potentials obtained by fitting the nucleon-nucleon scattering data and the structural properties of the deuteron. These two nucleon potentials all were strong, short ranged and repulsive at short distances.

One then calculated the properties of light nuclei using these potentials. To study the properties of heavy nuclei one first investigated the properties of nuclear matter. Nuclear matter is a hypothetical homogeneous system consisting of an equal number of neutrons and protons in which the Coulomb interaction between protons is neglected, and the volume, V, and the total number of nucleons, $B = N + P$, go to infinity but the density, B/V, remains finite. In fact, the bulk binding energy, $\frac{E}{B} - M$, and the saturation density are the two parameters that characterize the system. The material formed by nucleons at the center of a heavy nucleus like lead or uranium is well described as nuclear matter, whose bulk binding energy $\cong -15.75$ Mev.

Until the 1970s a great of effort was spent in applying Hartree-Fock methods to calculate the properties of nuclear matter using phenomenological two-body potentials. These studies yielded more realistic, *density dependent* interaction potentials which were then used to study the structure of heavy nuclei.

Despite the impressive results obtained using non-relativistic many-body techniques, the explanation of nuclear properties in terms of static two and three body potentials is clearly inadequate for understanding nuclear properties at high energy, nor was it considered "fundamental". Already in the late 1930s and in the early 1950s meson theories were used to derive nuclear potentials. By the 1960s with the enormous amount of accurate data obtained from high energy accelerators regarding hadrons, a nucleon-nucleon potential was derived arising from the exchange of a (pseudoscalar, isospin 1) π meson, a (scalar, isospin 0) σ meson, a (pseudovector, isospin 0) ω meson and a (pseudovector, isospin 1) ρ meson. This potential when introduced in a relativistic two nucleon equation accounted for the observed nucleon-nucleon scattering phase shifts up to 350 Mev.

A more satisfactory treatment of the hadronic interactions is a field theoretic one, wherein the hadrons (i.e. the nucleons and the above mesons) are all described field theoretically based on a Lorentz invariant, local Lagrangian density. The resulting formulation is what was named quantum hadrodynamics. (QHD).

Like QED, QHD is required to be renormalizable, hence characterized by a finite set of parameters (the couplings constants and the masses of the mesons and nucleon, i.e. the entities assumed to constitute the basic ontology) appearing in the Lagrangian density. Renormalizability in that context puts severe restriction on the possible local Lagrangian densities. Since renormalizable quantum field theories are finite in the limit as the cut-off introduced to regularize the theory goes to infinity, they are the least sensitive to the short distance behavior of hadronic interactions.

QHD has been very successful in explaining many novel features of nuclear structure, such as the electromagnetic exchange currents generated by the meson exchanges and the generation of three body forces by virtue of the $\rho\pi\pi$ interaction. The application of QHD to derive nuclear properties is expounded in Walecka (1995), and in various articles in the *Advances in Nuclear Physics* since the 1980s (e.g. Serot and Walecka 1986).

How to "derive" QHD from the standard model became a challenging problem after the establishment of the standard model. The challenge was taken up and what has been accomplished until the mid 1990s can be found in Walecka (1995). See also Vretenar and Weise (2003).

QCD covers an extremely wide range of phenomena, from hadrons and nuclei to matter under extreme conditions of temperature and density as in stars, in supernovae explosions, and under conditions that existed right after the Big Bang. At very short distance scales, $r \ll 0.1$ fm, QCD is a theory of weakly interacting, pointlike quarks and gluons. Their dynamics is essentially determined by the requirements of local gauge invariance under SU(3) color, and the conservation laws that follow from this. This is the domain of perturbative QCD.

How the quarks and gluons fields interact to form colorless nucleons and mesons, and how these localized clusters of confined quarks and gluons interact collectively in nuclei are the initial questions to be answered as far as nuclear physics is concerned. Nuclei in the terrestrial context have the property that their density is almost constant, approximately 0.17 nucleons/fm (approximately 10^{15} gr/cm^3). Thus the average separation between nucleons is a little over twice the size of the individual nucleon, which is of the order of .7 fm. At these distance scales, QCD is to be realized as a theory of pions coupled to nucleons (and possibly other heavy, almost static hadrons).

Lattice gauge theory calculations (and its extensive computational apparatus) proved that the color confinement to the interior of hadrons is indeed a dynamic property of QCD. In fact these calculations have been able to determine the properties of the lightest mesons and nucleons. In the nuclear domain where the quark field can be approximated by only including massless u and d quarks the theory exhibits chiral symmetry. The generation of the mass of the physical hadrons then arises from the spontaneous breaking of the chiral symmetry. The properties of QCD further simplify in the limit where the number of colors, $\mathrm{N_c}$, goes to infinity. The results of the calculations in this limit give strong support to the assertion that hadrons are the low energy degrees of freedom for QCD (see Walecka 1995, Chap. 37).

One can thus consider the process of reconstructing the nuclear domain as follows:

> As a first approximation the effective field theory that represents low-energy QCD involves only the u and d quarks, the lightest quarks, and is constructed according to the symmetries and symmetry breaking patterns of QCD. It indicates and gives strong support to models in which only the stable states of lowest mass, i.e. pions and nucleons, are present in the effective field theory that is to represent the next level in the hierarchy, i.e., essentially QHD. In it the quark-gluon substructure of pions and nucleons and the details of the short-distance dynamics, except for conservation laws, are encapsulated in the various constants appearing in the Lagrangian of the effective theory. But this implies that this effective theory is valid only in a limited domain, and implicitly assumes that the very short distance properties are not probed/resolved at nuclear scales.

The strongly-coupled dynamics of gluons and quarks gives a complex structure to the QCD vacuum, i.e., to its ground state, and its properties reflect the chiral symmetry of the starting point and its spontaneous breaking[126]. To simulate the properties of the QCD ground state as manifested in medium and heavy nuclei, *density dependent* interactions whose strengths are determined empirically are introduced in the QHD effective theory that is to apply in that context. The effective field theory thus encodes the substructures of pions and nucleons by incorporating structure constants (such as the masses of the mesons and nucleons) that are determined by experiments, and introduces contact interactions with density dependent coupling parameters that are likewise determined empirically. For what has been accomplished thus far with this approach and its limitations see Vretenar and Weise (2003).

There are other aspects of the story that need to be told. I have adumbrated (with very low resolution) theoretical developments in the immediate post WWII period and jumped to the 1980s! There are of course also the experimental and instrumental developments to be narrated, as well as all that happens in the intermediate stages (see e.g. Ericson 1992). But there is a striking development that I want to focus on (again with low resolution!).

The QCD representations of the structure of hadrons in terms of quarks and gluons are radically different from the representations of the structure of the many body systems physicists had dealt with, such as those for protons and neutrons in nuclei, electrons in atoms, atoms in molecules and solids. In those cases a Hartree-Fock many-body theory, or its density functional or relativistic generalization, can be justified if the "particles" are heavy in the energy scale of interest and the underlying field theory can be dealt with perturbatively. This is the case for atomic physics where the binding energies are of the order of electron-volts; the energy scale for pair production a Bev; photons do not interact with one another as they obey linear equations of motion; and the exchanges of (transverse) photons between electrons can be treated perturbatively. In QCD at the energy scales of interest the situation is reversed: the energy scale of the hadrons (the nucleons, ρ, π, ω mesons . . . , the bound entities that "emerge" from the interactions of the quarks and gluons) is hundreds of Mevs whereas that of the u and d quark masses is a few Mevs; the gluons interact strongly with one another, their equations of motion being non-linear; and pertubative expansions invalid. The theory has resisted the formulation of non-perturbative, analytical approaches in 3+1 space-time dimensions and until now only numerical methods have been successful in making quantitative predictions starting from first principles[127].

The observable consequences of QCD obtained from lattice QCD calculations are startlingly different from what one would expect from a perturbation theoretic treatment of the theory. But the computing power needed for precise and accurate lattice gauge calculations is huge and the cost of performing such calculations on supercomputers prohibitive. To overcome these constraints physicists actively took part in the development of high-performance parallel computers.

In the 1980s, they built the Cosmic Cube machines at Caltech, the Columbia University computers, the GF11 project at IBM, the Italian APE computers, the Fermilab ACPMAPS installation and the PACS machines in Japan. All these machines were designed and built to simulate lattice QCD (Riesselmann 2005). The performance of these early machines was measured in megaflops (one million floating point operations per second). By the early 1990s their computing power reached tens of gigaflops. By 2003 the top-performance computers built for lattice QCD calculations – the QCD-on-a-chip (QCDOC) machines – had over 10 000 microprocessors, and operated at a peak performance of 10 teraflops, but required only about 100 kilowatts of power (as compared to the 1000s of kilowatts supercomputers operating at these speeds) thereby greatly reducing the cost of operation. As a result of the performance of these machines the USQCD collaboration was founded in 1999 in order to create software and to stimulate utilization of these dedicated hardware resources for lattice gauge theory calculations[128].

The design and the building of these QCD supercomputers often took place in close cooperation with industry. Feynman spent a few months every summer in Boston in the early 1980s helping to develop the Connection computers of Thinking Machines, the company his friend Danny Hillis had co-founded with Sheryl Handler to develop the rapid parallel processing computer he had designed at MIT (Hillis 1989)[129]. The QCDOC collaboration at Columbia under the leadership of Norman Christ worked closely with computer scientists at IBM's Thomas J. Watson Research Center[130].

The exceptional price-to-performance ratio of QCDOC machines (approximately $1 per megaflop) and their low power consumption made them the foundation of a new generation of IBM supercomputers known as BlueGene/L which are widely used to understand protein folding and in big data biological research.

I conclude this side trip by noting that Norman Christ, the visionary leader of the QCD collaboration at Columbia and Brookhaven, is the Ephraim Gildor Professor of *Computational Theoretical Physics,* indicating that computational physics has now become institutionalized.

All this was happening within the discipline that used used to be called "nuclear physics". From a community which in the 1950s was committed to investigating the low energy properties of nuclear structure and running Van der Graff and other low energy accelerators, it became the operators of "meson factories" in the late 1960s, and after the 1980s of two large accelerators exclusively devoted to hadrodynamics:

(1) the Continuous Electron Beam Accelerator Facility (CEBAF) at the Jefferson Laboratory[131] and
(2) the Relativistic Heavy-Ion Collider (RHIC) at the Brookhaven National Laboratory which came on-line at the start of the new millennium[132].

The extent of the transformation can be obtained from the Nuclear Science Advisory Committee's (NSAC) April 2002 long range plan, "Opportunities in Nuclear Science", that it submitted to the Department of Energy (DOE) and the National Science Foundation (NSF). The plan outlined a framework for the continued growth of "nuclear science" in the USA. However, pressures on the Federal Budget prevented funds becoming available to implement the plan. In March 2005 both DOE and NSF, in order to decide what should be funded, requested that NSAC "examine the existing research capabilities and scientific efforts [supported by DOE and NSF], assess their role and potential for scientific advancements in the context of international efforts and determine the time and resources (the facilities, researchers, R&D and capital investments) needed to achieve the planned programs. NSAC should then identify and evaluate the scientific opportunities and options that can be pursued at different funding levels for mounting a world-class, productive national nuclear science program". In June 2005 NSAC responded, and its report gives an insight into "nuclear science".

In their report, the NSAC issued recommendations based on what they considered the primary mission of nuclear science in the coming decade, namely: "*explaining, at the most fundamental level, the origin, evolution and structure of the baryonic matter of the universe – the matter of stars, planets, and life itself*"

More specifically, the report indicated that

> Nuclear science is driven by fundamental investigations of the origin, evolution and structure of strongly interacting matter. Progress on [its] broad mission requires a balanced attack on key questions in three different, highly intertwined frontiers:
>
> (1) the strong nuclear force (quantum chromodynamics or QCD) and its implications for the origin of matter in the early universe, quark confinement, the role of gluons and the structure of the proton;
> (2) the study of nuclei and nuclear astrophysics, which addresses the origin of the elements, the structure and limits of nuclei, and the evolution of the cosmos; and
> (3) the standard model and its possible extensions as they bear on the origin of matter and the properties of neutrinos, neutrons, and other subatomic particles.

In its report, NSAC also identified questions that typified the issues that contemporary nuclear science addressed. I list some of them:

- *What is the nature of the quark-gluon matter of the early universe and what transitions led to our present world of protons and neutrons?*
- *What does QCD predict for the properties of nuclear matter?*
- *What binds protons and neutrons into stable nuclei and rare isotopes?*
- *What is the origin of simple patterns in complex nuclei?*
- *When and how did the elements from iron to uranium originate?*
- *What causes stars to explode?*
- *What are the masses of neutrinos and how have they shaped the evolution of the universe?*
- *Why is there more matter than antimatter?*
- *What are the unseen forces that disappeared from view as the universe cooled?*

The report went on to analyze in detail the theoretical investigations of these questions and the extensive experimental activities which drives them and nurtures them. It then went on to make detailed recommendations to ensure the continued support of nuclear science and to guarantee that valuable, generative new knowledge and new practices will be produced. This in the face of drastically diminished federal funds available for the support of nuclear science.

What is clear from the above is that nuclear science conceives itself to be an evolutionary science, a dramatic transformation from nuclear physics whose primary aim was to understand and explain the static properties of nuclear structure, or at best the decay properties of the unstable isotopes[133]. The detailed history of that transformation surely merits investigation, an investigation which would include the cultural and political factors that brought it about.

Incidentally, one of the critical points for the transformation of nuclear physics into an evolutionay nuclear science was a paper by Kirzhnits and Linde (1972) in which they qualitatively investigated the question whether a *global* broken symmetry of elementary particle physics would be restored if the temperature were sufficiently high. Weinberg (1974) took up the problem and investigated the case of *local* broken symmetries in *relativistic quantum field theories* – and in particular, in gauge theories – and showed how to calculate the critical temperature for general renormalizable fields using a Feynman diagrammatic approach. At Weinberg's suggestion, Jackiw also took up the problem of spontaneous symmetry breaking at finite temperature and showed how to calculate the critical temperature more generally using functional methods (Dolan and Jackiw 1974, see also by Kirzhnits and Linde 1976). It should be evident how to use these field theoretic results to transform temperature changes into temporal changes and use them to conjecture a historical process for the evolution of the universe in a big bang scenario.

I have pointed to the transformation of nuclear physics into nuclear science to illustrate how physics "reconstructs" the physical world. I have done so also to stress that what makes possible this reconstruction into a hierarchical levels, and deliver fairly precise quantitative values for the properties of the composite objects that form the basic ontology of the higher level is that a finite number of entities make up the ontology at the lower level. And the uses of field theoretic representations are crucial elements in the assignment of the dynamics.

One way of indicating why chemistry is different from physics is to recognize that organic chemistry's basic ontology are carbon atoms and molecules – and there are a huge number of the latter, and many ways that they can attach to carbon based structures and interact electromagnetically with one another and form new compounds that are quasi-stable in the terrestrial context. Furthermore these extended entities are more complex, and their interactions less constrained than the ones usually investigated in physics. More "is indeed different".

What is at issue are the "effective" theories by which the chemical world is reconstructed. And as emphasized by Butterfield (2011), reductionism is to be understood as deduction in the reconstruction of the physical world.

Only conceptualizing the quantum revolution as a Hacking type revolution – with its emphasis on the interdisciplinary aspect of the developments: experimental practices, technology, mathematics, chemistry, computing, . . . and on continuity – can do justice to a narration of the dramatic transformation that has taken place. Surely, since the 1980s doing science has a palpably different feel. One does it in new institutions with colleagues with many different disciplinary skills. The factors that determine the intellectual agenda have become much more entangled, and the knowledge being created and its products much more consequential.

Coda

I have given an overview of the developments of the quantum field theoretic description of that part of the physical world that can be isolated and to high accuracy be shielded from effects outside its boundaries. My narrative emphasized BCS – with BCS both as a theory and as standing for three theorists, Bardeen, Cooper, and Schrieffer –, phase transition, scaling, renormalization groups. . . In the case of phase transitions I presented some of its theoretical aspects and pointed to three theorists – Widom, Kadanoff and Wilson – who made crucial contributions to the solution of this problem and to the two of the three – Kadanoff and Wilson – who thereafter made key contributions to the understanding of quantum field theory in general. They individualize the collective contributions of the physics and physical chemistry community at large.

Although quantum field theory is at the focus of my narration it should not be inferred that QFTs were, are, responsible for all the major theoretical advances in condensed matter physics or high energy physics. The developments of band theory, quasiparticle theories and of nuclear physics in the 1930s, of the nuclear shell model, the phenomenology of phase transitions, the phenomenological quark model in the 1960s should dispel that notion. What I wanted/want to stress is the *generative* quality of QFTs, or as Bacon put it their "fructiferousness".

I focused on scaling and renormalization group methods because they brought about deep unifications, at both the theory level and at the communal level. Modern quantum field theoretical representations (as generated by Wilson) have unified condensed matter physics and high energy physics. I can learn what would be needed in the use of QFT in high energy physics studying Altland and Simons's *Condensed Matter Field Theory* or Zinn-Justin's *Quantum Field Theory and Critical Phenomena,* and conversely, study Steven Weinberg's *Quantum Field Theory* or Anthony Duncan's *The Conceptual Framework of Quantum Field Theory* to be a condensed matter field theorist. The unification of course runs much deeper than pedagogy.

To get an assessment of the magnitude of what has been accomplished recall that after the inception of quantum field theory as the language with which to describe nature at the microphysics level the divergences encountered presented an insurmountable problem. These divergences were seen to be the consequence of the very structure of quantum mechanics, namely unitarity and locality. It was seemingly impossible to analyze microphysics without at the same time analyzing what happens at arbitrarily small length scales. There seemed to be no way seen of disentangling the two domains. The problem was aggravated by the fact that it was believed that theories such as QED and meson theories were ultimate, fundamental theories. As emphasized by Jean Zinn-Justin (2007), – an important contributor to the elucidation of the field theoretic approach to statistical mechanics and to the detailed field theoretic calculations of critical exponents, – the phase transition problem in fact raised a further problem. Since some details of the microphysics dynamics survived at the macrophysics level – indicating that the physics at the micro length scale does not decouple from macro length scale physics in the case of phase transitions, – it was possible that the usual assumption that the physics at different length scale decouple was invalid and therefore that the usually unquestioned predictability of macrophysics was questionable[134].

A way of resolving the divergences difficulty was given in the late 1940s. It was realized that in perturbation theory one could hide the divergences of QED by not asking how the microphysics at arbitrarily small length scales accounts for the mass or charge of an electron but rather by expressing the bare charge and mass in terms of their measured value as determined by two physical processes. However, this procedure, renormalization, seemed to work only for only a limited number of theories, the so-called renormalizable field theories.

This in turn raised the question: "What is special about perturbatively renormalizable theories?", this especially after the establishment of the standard model which has met all experimental tests up to the present, except for some small modifications to incorporate the non-vanishing of the neutrino masses. The standard model is a corroboration of ideas based on renormalizable quantum field theory. One could therefore come to believe that renormalizability is a law of nature.

Wilson's approach to QFT with its formulation of the renormalization group equations and the notion of scale-dependent effective interactions, stemming from his solution of the phase transition problem by integrating out degrees of freedom has given an insightful, plausible, explanation for the success of the renormalizability criterion. The "final theory" – if one such exists! – when its ultrahigh energy or ultra short distance degrees of freedom – what ever these might mean or correspond to – are "integrated out", generates at the microphysics level a large-distance physics with effective local interactions between the fields whose quanta are the light particles of the standard model (the quarks and gluons of quantum chromodynamics and the quarks, leptons and bosons of electroweak theory). This field theory comes with a cut-off and contains all the local interactions between the gauge fields allowed by space-time and other symmetries. These interactions can be classified according to the dimensions of their coupling constants. Non-renormalizable interactions are automatically suppressed by powers of the cut-off. Renormalizable interactions have dimensionless coupling constants which "run" logarithmically with the scale, survive at large distances and determine the (relatively) low energy physics. The super-renormalizable interactions will presumably be absent[135].

This implies that QFTs are somewhat temporary, but robust and very effective, constructions. The history of the quantum revolution interpreted as a Hacking type revolution is the preliminary, ongoing account of these temporary, constructions.

It remains for me to address briefly the questions mentioned in the Introduction, that Büttner, Renn, and Schemmel (2003) had raised. I offer the following comments as a tentative exploration of possible answers to their first question: "What accounts for the breaks in the development of scientific knowledge which can be described as scientific revolutions, whether conceived as Kuhnian or otherwise?".

I would like to suggest that the appellation "revolution" requires either a change in the metaphysics of the contemporaneous scientific knowledge or an instrumental or technological advance that dramatically alters the practice of the science. New styles of reasoning indicate revolutions in which changes in the underlying metaphysics have affected and restructured epistemologically several areas of scientific knowledge. All the revolutions associated Hacking's styles of reasoning have this character[136]. The ever growing size of the scientific community, the ever growing interdependence of the various disciplines, the ever greater symbiosis of science, applied science and technology, and the ever deeper entanglement of science with national and international politics, with governing and government, and with religion imply that what "accounts for the breaks in the development of scientific knowledge which can be described as scientific revolutions" will depend on which revolution one is looking at and on the level of description. And when analyzed at a fine enough temporal level of resolution there will be a continuous growth of knowledge.

This incidentally answers Büttner, Renn, and Schemmel's second question, "Despite such breaks is there nonetheless a continuous growth of knowledge?" Individuals may seem to offer contributions which at one level of resolution seem to introduce breaks. A finer analysis always indicates the resources that the community had provided the individual, through education and training, and the works of previous researchers.

Crick and Watson introduced the possibility of a new style of reasoning and a new language with their double helix, and revolutionalized biology. The continuity of their findings and conceptualization with previous work has been extensively investigated[137]. Similarly, the introduction of PCR altered the practice of molecular biology to such an extent that one may call it a revolution. But it is only the amalgam of the double helix, of the genetic code, of DNA and RNA in all their manifestation, of PCR, of genomic sequencing . . . , and of all the other remarkable advances in genetics together with all this has done to medicine and its practice, to the pharmaceutical industry, together with the new research institutions that have created, that I would characterize what has happened in molecular biology as a Hacking type revolution, though I am unable to delineate its "language". Only when all of these are taken into account has a new feel to the world ensued. PCR created a revolution, a larger revolution than a "Kuhnian" one, but by itself not a Hacking-type one. The ever growing process of specialization concomitant with the ever greater role that science, applied science and technology play in the postmodern world seem to make Kuhnian notions (without commensurability) applicable only to the practices of small subdisciplines.

I have reserved the appellation Hacking type revolutions to truly "big" scientific revolutions. The relativity revolution and the quantum revolution were such big revolutions. They were responses to the crisis the physical sciences faced at the end of the nineteenth century in trying relate microscopic descriptions to macroscopic ones. The quantum revolution resolved that issue for many systems and phenomena[138] and in addition did something that no prior scientific revolution had done. Absent a foundational theory describing the dynamics and ontology of the microscopic world – as had been the case in classical physics – past justification for the belief in the correctness of scientific knowledge derived from its ability to generate new technologies. Poincaré, Bachelard and many other historians and philosophers of science of the first third of the 20th century were committed to that view. But starting from the standard model quantum field theory allows a partial reconstruction of the ontology of the physical world – and this in turn allows a partial a coherent, consistent reconstruction of the past history of the physical world. The relativity revolution understood as encompassing general relativity with its black holes and other singularities revolutionized our understanding of cosmology. Our conceptualization of the physical universe (universes?) has been transformed by it. Modern cosmology makes the big bang a credible theory. In fact we can conceive of interpreting some astronomical data as revealing the collision of two galaxies with giant black holes at their center! The justification of belief in the validity of scientific knowledge has been transformed by these two revolutions, partly because of the process of self-authentification they generate – but also because they are fairly precise in stating their limitations. The present relativistic quantum field theoretical representation is surely not applicable to describe phenomena at the Planck scale where gravitational effects become dominant – and probably before that. Conversely, general relativity is an effective field theory, limited because it does not incorporate quantum attributes.

The answer to Büttner, Renn, and Schemmel's last question: "Where and when do scientific revolutions occur?" depends of course on whether we understand it as posing a question regarding the past or regarding the future. The common denominator of all past scientific revolutions was the existence of an educational system that provided the tools and supportive environment for some students to grow, for some competent teachers to be innovative and independent. The cultural, political, and economic contexts clearly have determinative roles in this. And conversely. Each of the past scientific revolutions needs to be analyzed with this perspective in mind. There is little question – given the pace of scientific developments – that one of the enabling conditions for future scientific revolutions will continue to be the existence of educational systems that will nurture curiosity and independence and provide the necessary support for some off-scale individuals to achieve their potential to the fullest. Beyond that, the dramatic technological advances that we are witnessing make it difficult – if not impossible – to predict where and when new scientific revolutions will occur.

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Endnotes

[1]What is meant by 'representation' is the focus of many controversies in philosophy.

[2]I have in mind not only WW I and WW II and the extensive Cold War (the latter both in time and in space), but in the case of the USA the Korean War, Vietnam, and Iraq.

[3]The PDP (Programmed Data Processor) (*PDP*) minicomputers were developed at the Lincoln Laboratory of MIT in the early 1950s.They were first manufactured commercially by the Digital Equipment Corporation (DEC) in 1957. The name 'PDP' avoided the use of the term "computer" which at the time were thought of as being large, complicated, and expensive machines. Georges Doriot, who helped DEC get started with funds from his venture capital enterprise, the American Research and Development Corporation, insisted that the word "computer" not be part of the name of the machine in order to aim it at users who could not afford the larger computers.

[4]What is meant by reductionism has been the subject of an extensive literature within the philosophy of science, and the philosophy of physics in particular. See Batterman 2012, Fayerabend 1962, Castellani 2002, Butterfield 2011, Butterfield and Bouatta 2011.

[5]The events which led to the writing by Anderson of "More is Different" are narrated in Schweber 1997.

[6]See in this connection Gottfried and Wilson 1997.

[7]Re historical epistemology and epistemic objects see the stimulating article by Hasok Chang (2011). My paper is indeed about epistemic objects in the sense that Chang advocates: "entities that we identify as constituent parts of reality. The designation 'epistemic' [is used] as relating to the human process of seeking knowledge, as an indication that I wish to discuss objects as we conceive them in our interaction with them, without a presumption that our conceptions correspond in some intractable sense to the shape of an 'external' world that is entirely divorced from ourselves". Chang 2011, 413. His exposition is particularly relevant when the "epistemic *objects*" are unobservable, as in the case of quarks. See also Chang 2012.

[8]The stress on language is my own. Not every Hacking type revolution may have a particular language associated with it. The biological revolution of the 20th century initiated by Crick and Watson is certainly a Hacking type revolution, but it would be difficult to associate a *particular* language to it.

[9]This in contrast to Kuhn for whom theory played a dominant role.

[10]The following material is taken from R. Belfer and S.S. Schweber "Hacking Scientific Revolutions". To be published.

[11]These are Alistair Crombie's "revolutions". Crombie associated with each of them a new *style of thinking*. Crombie's revolutions and styles of *thinking* were the origin of Hacking's styles of *reasoning*. See Crombie 1995, Hacking 1985, 1992.

[12]See also Hacking (1992, 11) where he discusses the new elements introduced by a new style of reasoning.

[13]See also Cao and Schweber (1993) where this approach was first analyzed.

[14]See in this connection Kadanoff's Intoduction to Section B. Scaling and Phase Transitions in Kadanoff (1999).

[15]Helge Kragh has noted that Bohr's 1913 papers are usually understood to deal with one-electron atoms. This, however, is a false impression. Bohr originally thought his theory "would lead to a new understanding of the constitution of all matter, whether the physicist's atoms or the chemist's molecules... [T]he very title of his publication, "On the constitution of atoms and molecules", indicates that it was addressed as much to chemists as to his colleagues in physics" (Kragh 2013).

[16]Thus the fact that electrons are indistinguishable and obey the Pauli principle translated into the requirement that the Schrödinger wave functions describing an n electron system

change sign under the exchange of any two electrons. This in turn, translates into the readily visualizable insight that no two electrons with the same spin orientation can be located at the same point in space.

[17]The inability of classical physics to account for the stability of atoms in terms of trajectories of point-like charged particles was one of its outstanding problems. Recall that any model to describe the dynamics of entities interacting through electromagnetic based on Newton's laws of motion cannot explain the stability of the systems. Ibid for the old quantum theory. See in particular Chapters 6 and 7 of Kragh 2012.

[18]By virtue of Planck's constant these energy can be translated into lengths. In the atomic case the characteristic length is $\mathrm{h/me}^2 = 0.53\times10^{-8}$ cm. In the nuclear case the characteristic length is of the order 10^{-13} cm. Furthermore, the fact that the size of nuclei is very small compared to atomic dimensions and that their mass is very large compared to that of an electron justifies the approximation that in atoms and molecules the nuclei interact with the electrons only through electrostatic Coulomb forces.

[19]The conventional Maxwell equations are phenomenological, macroscopic, equations describing the interaction between electromagnetic fields and matter that do not reflect the atomicity of matter or electric charges. They are expressed in terms of four field quantities: the electric field intensity **E**, the magnetic induction **B**, the electric displacement **D**, the magnetic field **B** and the matter current density **j** and its charge density ρ with the latter two quantities satisfying the charge conservation law. The constitutive equations which describe the behavior of isotropic matter in the presence of the electromagnetic field

$$\begin{aligned} \mathbf{j} &= \sigma\mathbf{E} \\ \mathbf{D} &= \varepsilon\mathbf{E} \\ \mathbf{B} &= \mu\ \mathbf{H} \end{aligned}$$

introduce three new experimentally determined parameters that depend on temperature, on the density, chemical composition of the material and on the frequency and intensities of the fields.

[20]I am using the term "classical physics" is the same way as Büttner, Renn, and Schemmel (2003). It is there used as an epistemological term. "More specifically, it is not used as "reflecting contemporary parlance, nor does it refer to the precise extension of knowledge in the given historical period (namely, that of "classical physics"), nor to its contemporary state of acceptance but rather to the organization of this knowledge around a conceptual canon".

[21]Recall Khinchin's observation that the Boltzmann distribution of the energy of a single molecule in systems of weakly correlated molecules is universal, i.e. it is independent of the form of the interaction, provided it is of short range – this by virtue of the central limit theorem (Khinchin 1949).

[22]Lorentz's researches consisted in deducing the macroscopic Maxwell equations on the basis of the motion *in vacuo* (which for him still meant in the luminiferous ether) of the microscopic charged particles that make up the atoms and molecules of matter. He proposed that what are now called the Maxwell-Lorentz equations which represent the interaction of the electromagnetic field with matter described in terms of its atomic constituents. Lorentz assumed the Maxwell-Lorentz field equations to hold at every point of space and the problem he set out to solve was to demonstrate that the fields **E, D, B, H** that appear in Maxwell's equations could be understood as the superposition of microscopic field quantities averaged over a volume the size of which depended on the "fineness" with which one probed experimentally small regions of space and time. Similarly for the charge density ρ and current density **j**: they were the result of an averaging procedure over the volumes of the charged entities making up the atoms, molecules or ions making up the matter. Thus depending on the system under investigation and the kinds of experimental probes used the charge density under certain circumstances could be assumed to get contributions from all the atoms and

molecules of the matter. If only measurements probing regions of space large compared to atomic and molecular sizes are made the level of the theoretical description should mirror this fact and only averages over such regions should enter the representation.

By the early 1930s Lorentz's formulation of the relationship between micro and macro descriptions had become foundational. It became widely disseminated by van Vleck's exposition of how to calculate electric and magnetic susceptibilities using quantum mechanics (Van Vleck 1932). See Rosenfeld 1951, Duncan and Janssen, 2006.

[23]This when attempting to formulate a computer language appropriate to describe chemical reactions!

[24]By indicating how the charge density of the two electrons when in a singlet spin state lowered the energy by being locatable between the two protons, thus increasing the attractive forces between electrons and protons and shielding the repulsive force between the two protons, Heitler and London formulated the quantum mechanical basis for the covalent bond (see Pauling and Wilson 1935).

[25]I note here that laboratory life is an integral aspect of a Hacking type revolution, but irrelevant for Kuhn in his characterization of scientific revolutions.

[26]It did so first in term of phenomenological internucleonic potentials (see Bethe 1986).

[27]Another important number regarding the relation between the micro and macro realms had been introduced by chemistry in the 19th century: a macroscopic volume of liquid, solid or gaseous matter contains of the order of Avogadro's number of atoms, Avogadro's constant, $N_A = 6.022 \times 10^{23}$/mole, being the number of atoms in a mole of a pure substance. By definition, the mass of a mole of ^{12}C is 12 gr. Reflecting the magnitude of Avogadro's constant, Boltzmann's constant, $k_B = 1.38 \times 10^{-16}$ erg/K, connects the atomism of the micro level with the continuum macro realm through the equation $R/k_B = N_A$, where R is the universal gas constant, 8.3×10^7 erg/K. For a concise history of atomism in the 19th century see Chapter 5, "The reality of molecules", in Pais 1982. Specifically, the equation $R/k_B = N_A$ links the atomistic dynamics with thermodynamics through a statistical mechanical derivation of the equation of state of a very dilute gas. The micro-macro linkage is also connected through the equation $F/N_A = e$, wherein e is the charge of an electron, and F, Faraday's constant, is the amount of charge necessary to deposit one mole of a monovalent substance in electrolysis.

[28]This length has the magnitude 0.53×10^{-8} cm, in contrast to centimeters and meters for macroscopic distances. Nuclear forces introduced a further new length scale: the fermi, 10^{-13} cm

[29]The position operator of a particle, $\boldsymbol{q}$, had delta functions as its eigenfunctions: $\boldsymbol{q}|q'\rangle = q'|q'\rangle$; $\langle q''|q'\rangle = \delta(q" - q')$.

[30]Thus the Hamiltonian for a system of non-relativistic relativistic particles interacting with the electromagnetic field took the form which exhibited the Coulomb interaction:

$$H = \frac{1}{8\pi}\int d^3x \left[\frac{1}{c^2}\left(\frac{\partial \boldsymbol{A}}{\partial t}\right)^2 + (\nabla \times \boldsymbol{A})^2\right] + \sum_i \frac{\left(\boldsymbol{p_i} - \frac{e}{c}\boldsymbol{A}(\boldsymbol{q}_i)\right)^2}{2m_i}$$
$$+ \frac{1}{2}\sum_{i,j} \frac{e^2}{|\mathbf{q}_i - \mathbf{q_j}|} \quad \text{with} \quad \nabla \cdot A = 0.$$

[31]Though similar to quantum electrodynamics (QED) in asserting that the electron and neutrino were not present in nuclei prior to the processes $n \to p + e^- + \nu$, $p \to n + e^+ + \bar{\nu}$ occuring, the interaction was not mediated by the exchange of a particle, but was a four fermion interaction of the form $\psi_n(x)\,\psi_p(x)\psi_e(x)\,\psi_\nu(x)$.

[32]The macroscopic domain initially became represented by two approaches. In one, quantitative explanations of the structure and properties of crystalline solids, such as metals, insulators and semi-conductors were to be given by quantum mechanics in the same way as

it accounted for the structure and properties of simple molecules. The second approach – the domain of statistical mechanics – was concerned with the quantitative account of the thermal properties of matter. In it additional probabilistic assumptions were made, justified by virtue of the huge number of particles involved.

[33]One might add to these the cosmological – consisting of galaxies and their constituents, their evolution and dynamics. These hierarchies were not considered to be independent: accurate measurements of atomic energy levels reveal nuclear and subnuclear properties. Similarly, the recent startling discovery of the necessity of the presence of cold dark matter – consisting of as yet undiscovered subnuclear entities – in order to make sense of new cosmological observational data is proof of the linkage between the various levels. But it must also be noted that these observations have not destabilized our amazingly accurate representations of the atomic world. And needless to say, the linkage of the levels is made explicit as soon as one tries to answer evolutionary questions.

[34]This hierarchical vision of the physical world and a delineation of the four kinds of forces believed operating in nature would be stated explicitly in 1946 by John Archibald Wheeler (1946).

[35]Peierls (1932) did not in his 1932 review article on the electron theory of metals; Bethe did not in his 1933 masterful review of the field in his *Handbuch der Physik* article with Sommerfeld; nor did Mott and Jones (1936); nor did Seitz (1940). They did use the QED analogy in describing phonons and their interactions.

[36]In fact the mass superselection rule of Galilean invariant theories forbids massive particle creation or annihilation. Moreover, the models when formulated in the language of non-relativistic quantum field theories that conserve particle number are equivalent and readily transcribed to the Schrödinger equation formulation.

[37]See Heitler (1936). How to fill the negative energy states when dealing with multi-electron systems was not addressed in hole theory. See for example, Dirac, Fock, Podolsky (1932). The issue is automatically resolved in the field theoretical formulation. See Wentzel (1943).

[38]e.g. the effects of X-rays.

[39]For a presentation of many aspects of the interaction see Kragh 1999, Katzir et al. 2013, Katzir 2013.

[40]See for example the historical introduction to Kemble (1929).

[41]Eckert (2004) stresses this point in the biography of Sommerfeld. See Sommerfeld 1929.

[42]It was Laplace who deduced the inverse square law from the elliptical motion of a planet – and made it into the law of universal gravitation.

[43]This number is what remains of the observed 574 arc-second per century of precession after the Newtonian gravitational effects of Venus, Jupiter, earth, are subtracted.

[44]In a lecture in 2005 on Simon Newcomb and celestial mechanics George Smith extended his previous analysis of the Newtonian *quam proxime* stance by comparing it to Duhem's position, for whom "A physical theory (such as celestial mechanics) is a system of mathematical propositions, deduced from a small number of principles which aim to present as simply, as completely, and as exactly as possible a set of experimental laws... A true theory is a theory which represents in a satisfactory manner a group of experimental laws... Agreement with experiment is the sole criterion of truth for a physical theory. ... Physics ... is a symbolic painting in which continual retouching gives greater comprehensiveness and unity, and the whole of which gives a picture resembling more and more the whole of the experimental facts (Cohen and Smith 2002).

[45]This has happened for non-relativistic quantum mechanics, quantum electrodynamics, electroweak theory, the standard model. Like Newton's theory of universal gravitation quantum electrodynamics, for example, is an idealization, an approximate theory that is *assumed to hold exactly* in certain specifiable experimental circumstances, such as in the case of an

electron in a Penning trap, or a muon in storage ring, and experimenters and theorists investigate "details that make a difference and the differences they make".

[46]Wolf Beiglboeck, the editor of *EPJH* indicated that "It would be interesting to know what precisely Stein and Smith mean when one applies this to particle physics and RQFT with complicated huge machines, once used and quickly dismantled, where observations are made by even more tricky electronic and computer based tools, far from direct human perceptions, and an equally tricky theoretical machinery on the other side, consisting not only of particle theory but also, to understand the measurement's apparatus, of solid state physics etc. – in other words of the whole hierarchy as discussed in the article".

[47]I have avoided the use of the term "co-production" as I want to make clear that a good deal of mathematics is self-generated by mathematicians, and have no initial connection with physics. See Jasanoff 2006.

[48]This classic "yellow peril" as the Springer books were called, became an important component of the toolkit of theorists tackling wave mechanical problems.

[49]I owe the notion of a "crucial calculation" to Howard Schnitzer. See Schweber 1994 where the idea is applied to Bethe's calculation of the Lamb shift in H and Schwinger's calculation of the anomalous magnetic moment of the electron using renormalization concepts. Other examples come readily to mind: Einstein's calculation of the advance of the perihelion of Mercury using his formulation of general relativity, Pauli's calculation of the spectrum of hydrogen using Born, Jordan and Heisenberg's matrix mechanics, the BCS calculation of the energy gap in a superconductor, and many other instances in modern particle and condensed matter physics.

[50]It should be noted that although Schwinger's and Feynman's approach to quantum electrodynamics yielded identical answers to the calculated values of experimentally measured properties such as the magnetic moment of an electron or positron and the cross-section for Compton scattering as a function of the photon's energy, or the Lamb shift, the metaphysics underlying their approach was radically different. For Schwinger fields were the basic observables; for Feynman particles. Feynman's propagators referred to trajectories of particles, and his diagrams were their (perturbative) visual representations. Furthermore, after 1949 Schwinger emphasized non-perturbutive formulations. See especially Schwinger 1951, 1996.

[51]Bjorken made this statement regarding local fields associated with strongly interacting particles – hence the initial focus on current algebras (Bjorken 1997, p. 589).

[52]I am here following Lepage (2005). Although listed as (2005) it is a "Preprint" of the lectures Lepage delivered at the TASI Boulder summer school in 1989 and wasn't Arxiv'ed at the time. It should be noted that the clarity and seeming simplicity of Lepage's presentation hides the difficulties encountered in justifying the general conclusions, This already in quantum electrodynamics – where for example the handling of the wave function renormalization is not explicitly addressed, nor the handling of higher loop divergences – and especially so in quantum chromodynamics where gauge fixing and renormalization get entangled. See volume II of Weinberg's Quantum Theory of Fields for all the difficulties encountered in non-abelian gauge theories and the hard and challenging work necessary to overcome them.

[53]Or equivalently, one can calculate the energy of two widely separated electrons at rest, and require it to be equal to e^2/R (where R is the distance between them) and similarly demand that the energy of free electron of momentum p be equal to $E = c\sqrt{p^2 + (mc)^2}$ where m is the observable, experimental mass of an electron.

[54]Wave function renormalizations complicate the matter somewhat.

[55]See Weinberg 1995–2000 for the detailed proof of the renormalization of the S-matrix in QED.

[56]In particular Feynman's 1949 *Physical Review* articles and Schwinger's 1951 *PNAS* papers. See Schwinger 2000, Feynman 2001.

[57]Akhiezer and Berestetsky 1949, Jauch and Rohrlich 1955. Martin and Schwinger in their 1959 paper, The Theory of Many Particle Systems I., give a partial list of papers using field theoretic methods. Pines (1961) in his *The Many-Body Problem* gives a historical overview of field theoretical approaches to the many body problem in the first two chapters of the book, as well as a list of references. See also Martin's informative 1979 article on the impact of Schwinger on the development of condensed matter physics. Those prominent in the early post World War II developments include among others Van Hove, Hugenholtz, Bardeen, Matsubara, Migdal, Beliaev, Goldstone, Pines, Schwinger, Martin, Nozière, de Dominici, Claude Bloch, …

[58]Mattuck (1967) in his widely used textbook emphasizes this aspect of the use of Feynman diagrams in solid state physics.

[59]Helium 4 exists in a liquid form only at the extremely low temperature of about 4 K (–269 °C). Its critical temperature is 5.2 K.

[60]For a highly informative exposition of phase transitions see Kadanoff 2009.

[61]The following is adapted from Mudry 2008.

[62]There is a mathematical theorem, the Stone-von Neumann theorem, concerning a system described quantum mechanically by variables $q_1,\ q_2,\ \ldots q_n;\ p_1, p_2, \ldots p_n)$ that satisfy the commutation rules

$$[q_l,\ p_m] = i\hbar\delta_{lm} \qquad [q_l,\ q_m] = 0 \quad [p_l,\ p_m] = 0$$

and whose dynamics is determined by a Hamiltonian

$$H = \sum_{j=1}^{n} \frac{p_j^2}{2m_j} + U\left(q_1,\ q_2, \ldots q_n\right)$$

that satisfies certain properties such as having a lower bound, and certain growth properties reflecting properties of the potentials between particles so that one can prove that H is self-adjoint. The theorem states that there is essentially a *unique* (irreducible) realization of the theory in which the states are represented by square integrable functions, and form a Hilbert space where the qs act by multiplication on the coordinate functions and $p_l = i\hbar\,\frac{\partial}{\partial q_l}$. Furthermore, it can be shown that the ground state does not change sign and is *unique*. Any symmetry transforms the eigenstates of H according to its irreducible unitary representation. The ground state thus transforms into itself and there cannot be any symmetry breaking. See Mackey 1949.

[63]Note also the following. Consider a Hamiltonian that is invariant under a symmetry group. Let U be an element of the symmetry group, then $\mathrm{U}^{-1}\mathrm{HU} = \mathrm{H}$. If $|a\rangle$, $|b\rangle$ are two eigenstates of H that belong to an irreducible representation of the symmetry group, there will be a U that connects them: $\mathrm{U}|a\rangle = |b\rangle$, and from $\mathrm{U}^{-1}\mathrm{HU} = \mathrm{H}$ it follows that

$$\begin{aligned}\mathrm{H}|a\rangle &= \mathrm{E_a}|a\rangle = \mathrm{U}^{-1}\mathrm{HU}|a\rangle \\ &= \mathrm{U}^{-1}\mathrm{E_b}|b\rangle = \mathrm{E_b}|a\rangle\end{aligned}$$

from which we deduce that $\mathrm{E_a} = \mathrm{E_b}$. The symmetry of the Hamiltonian manifests itself in the degeneracy of the energy eigenstates that belong to the same irreducible representation of the symmetry group. But implicit in the statement $\mathrm{U}|a\rangle = |b\rangle$ is the invariance of the ground state under the symmetry: $\mathrm{U}|0\rangle = |0\rangle$. The states $|a\rangle$ and $|b\rangle$ can be obtained from the ground state by the action of some appropriate creation operators

$$|a\rangle = \Psi_\mathrm{a}^*|0\rangle \quad \text{and} \quad |b\rangle = \Psi_\mathrm{b}^*|0\rangle$$

and therefore

$$|b\rangle = \Psi_\mathrm{b}^*|0\rangle = \mathrm{U}|a\rangle = \mathrm{U}\Psi_\mathrm{a}^*|0\rangle = \mathrm{U}\Psi_\mathrm{a}^*\mathrm{U}^{-1}\mathrm{U}|0\rangle.$$

But

$$\mathrm{U}\Psi_\mathrm{a}^*\mathrm{U}^{-1} = \Psi_\mathrm{b}^*$$

follows only if $\mathrm{U}|0\rangle = |0\rangle$.

[64]Incidentally, without the μN term, the ground state would be the state with no particle present.

[65]Most of the article mentioned in the following sections can be found in Taylor 2001.

[66]Superconductivity is characterized by the vanishing of the electrical resistance below some critical temperature, T_c, and by the Meissner effect, the exclusion of magnetic fields from the inside of the superconductor below T_c. See Schrieffer's (1964) *The Theory of Superconductivity.*

[67]Furthermore, these quasiparticles do not have a definite charge, the operator creating a quasiparticle of momentum $\mathbf{p}$ being a linear combination of an spin up electron of momentum $\boldsymbol{p}$ and a spin down hole of momentum $-\boldsymbol{p}$. Bogoliubov 1958, Valatin 1958, Nambu 1960, Schrieffer 1964.

[68]Interestingly, Nambu and Jona-Lasinio noted that in the BCS theory the particle that plays the role of the Goldstone boson is not massless because of the existence of the long range Coulomb interaction.

[69]Furthermore, the representation of the canonical commutation relations on this Hilbert space is inequivalent to the usual Fock-space representation.

[70]See the lectures by David Gross (1975) on the "Applications of the renomalization group to high energy physics" that he gave at the 1974 Les Houches Summer School in Theoretical Physics.

[71]In *Inward bound* Pais (1982) has given a lucid, insightful overview of symmetry considerations in particle physics.

[72]Arthur Wightman (1922–2013); Res Jost (1918–1990); Leon van Hove (1924–1990); Rudolf Haag (1922); Cyril Domb (1920–2012).

[73]https://www.princeton.edu/physics/arthur-wightman/;

[74]Lanford's lectures on functional analysis take up one fifth of the volume *Statistical Mechanics and Quantum Field Theory.* See also Glimm and Jaffee's lectures, and those by Ruelle, Lieb, Ginibre, Hepp and Epstein and Glaser.

[75]Since the advent of string theory, the interaction, cross-fertilization and co-production has been pronounced, evident, and unambiguous.

[76]However, to support the notion of "co-production" see for example the beautiful lecture by Shlomo Sternberg (2005) on the relation between differential geometry, geodesics, general invariance and the equation of motion of a single particle in general relativity; and more generally the books he has written on these subjects. But also consult the interview by Marcel Berger (2000) of Mikhael Gromov, and the review by Grove (2001) of Gromov's book on Riemannian and non-Riemannian spaces for a somewhat different position from that of Sternberg.

[77]See Wightman's contributions to Brown, Dresden and Hoddeson (1989) and to Cao (1999) for a history of the developments in axiomatic and constructive field theory. See also Velo and Wightman (1973).

[78]It would be interesting to compare the factors that operated in the various national settings (US, France, Soviet Union, Germany, Switzerland,...) that made possible the establishment of the discipline of mathematical physics in them, To compare the different kinds of problems addressed in the various settings and to see what these reflected; to compare the status of the discipline in the differing settings; to determine whether the practitioners became members of physics or mathematics departments...

[79]The first proof of the existence of the thermodynamic limit were published by van Hove (1949, 1950), Yang and Lee (1952), Ruelle (1963), Fisher (1964). For a review see Lebowitz (1968).

[80]A Wick rotation converts a quantum field theory in d –1 dimensions of space and one dimension of time into a d-dimensional problem in statistical mechanics to a problem. A full exposition can be found in Jean Zinn-Justin (2002).

[81]See Glimm and Jaffee (1987) for an exposition of Osterwalder and Schrader's axioms and theorems.

[82]My aim is somewhat different as I concentrate on the events from roughly 1960 till 1973 with a focus on renormalization group methods.

[83]See e.g. the articles in *Physica* 73(1), 1974, especially Martin Klein's.

[84]Lars Onsager is a central figure in the story I tell. For biographical material on Onsager and an exposition of his mathematical and scientific work see Longuet-Higgins and Fisher 1991. Onsager won the Nobel Prize in Chemistry in 1968.

[85]Onsager had written down the result earlier at a conference but had not indicated its derivation.

[86]Leo Kadanoff pointed out to me that the word "decimation" had a technical meaning in 1965–1975. It meant summing over all but one spin in the block and using the unsummed spin as a collective variable. However, this approach did not give good answers at higher orders. The word "block spin" and "collective variable" were in common use then.

[87]I am not competent to present the details of this history. For a detailed, incisive account of the developments I refer the reader to Cao (1997, 1999, 2010). For highly informative, perceptive accounts of the technical aspects and an indication of the extent to which condensed matter physics and high energy physics continue to fructify each other see Weinberg (1995–2000), Zinn-Justin (2002), Duncan (2013) and Altland and Simons (2010).

[88]Sidney Coleman's name should have appeared in the following list. His contributions to and influence on the developments of quantum field in the period 1960-1985 was enormous. He was deeply involved in the calculations proving asymptotic freedom in gauge theories (see Georgi 2011). Similarly, Robert Brout and Thomas Kibble played a crucial role in formulating the "Higgs" mechanism (see Brout's remarkable article (1997)).

[89]I refer the reader to David Gross's 1974 Les Houches lectures on "Applications of the Renormalization Group to High-Energy Physics" and to Frank Wilczek's article on "Quantum Field Theory" in the March 1999 Centenary issue of the *Reviews of Modern Physics*, in which the impressive successes of quantum field theory and that of the standard model are presented.

[90]In their crucial paper, Wilson and Fisher (1972) determined a set of fixed points associated with a large class of phase transitions (ferromagnets, liquid-vapour, ...) by using a method that extends to non-integer values the space dimension d of the perturbative expansion. They established that near $d = 4$ ($d = 4 - \varepsilon$ with $\varepsilon \to 0$), universal quantities could be calculated as an expansion in ε. They thereby provided the first examples of analytic estimates of critical exponents that differed from their mean-field values. Quantum field theory methods then made possible a general derivation of scaling properties and provided efficient methods for calculating universal quantities. See Zinn-Justin 2002, p. 207.

[91]For an insightful and helpful presentation of these ideas see Delamotte (2004).

[92]The biographical material in the present section is taken from the interview by Babak Ashrafi and Sam Schweber with Benjamin Widom, carried out as part of the History of Recent Science and Technology, Physics of Scale project, that the Dibner Institute sponsored from 1998 to 2004. It was directed by Babak Ashrafi. The Physics of Scale project was one of the four undertaken. See http://authors.library.caltech.edu/5456/1/hrst.mit.edu/hrs/renormalization/public/index.html. See http://authors.library.caltech.edu/5456/1/hrst.mit.edu/hrs/renormalization/public/index.html for the Physics of scale project and http://authors.library.caltech.edu/5456/1/hrst.mit.edu/groups/renormalization/interview/q-and-a.tcl_topic_id=40&topic=Widom.html for the interview with Benjamin Widom.

[93]The members of Widom's thesis committee were Bauer, Bethe and Kac. They examined him for the admission to candidacy examination, and approved his dissertation after his thesis examination.

[94]The model was later successfully developed by people like Peter Schofield.

[95]Note that even though the homogeneity of $\Phi(x, y)$ is only asserted to hold asymptotically as the critical point is approached, and though the entire theory is limited to the immediate neighborhood of the critical point, nevertheless it is clear from the equations defining homogeneity that infinite ranges of the arguments of all $\Phi(x, y)$ are relevant.

[96]See HRST interview with Leo Kadanoff. http://authors.library.caltech.edu/5456/1/hrst.mit.edu/hrs/renormalization/Kadanoff/index.htm.

[97]"Chez Dreyfus" was a French restaurant on Church Street near Harvard Square that specialized in such foods as venison and bear meat. At the time the Harvard Faculty served horse meat steaks every Monday. Kadanoff remembered that Schwinger would have a $2 steak for lunch, whereas everyone else would make do with 40 cents hamburgers.

[98]The superconductivity part of the dissertation was done under Martin's supervision. The part of the thesis that dealt with the acceleration of charged particles by the electromagnetic field found in lasers (described field theoretically) was carried out under Glauber's direction.

[99]He became a professor of physics there in 1965.

[100]In April 1965 two hundred scientists from eleven countries met in Washington for what Cyril Domb later called "the founding conference of critical phenomena". In the Introduction to the published proceedings, conference organizer Melville Green pointed out four recent developments that motivated this ambitious undertaking: (1) Plausible theoretical predictions of non-classical phase transitions were finally extended successfully from two dimensions to three using painstaking series expansions. (2) The experimental demonstration that even classical fluids can exhibit sharp singularities in the specific heat at the critical point. (3) The use of NMR techniques to show strong similarities between ferromagnetic and liquid-vapor transition phenomena. (4) The experimentally confirmed breakdown of the classical Ornstein-Zernike theory of critical opalescence. Green emphasized the diversity of the research agendas represented at the conference, which brought together scientists who seldom had occasion to interact with each other. Central to their discussions was the debate over the true nature of critical singularities, and the central premise of the conference itself: "in what senses are the various critical phenomena truly *like* each other?" *Critical Phenomena.* Proceedings of a Conference held in Washington, D.C., April 1965. National Bureau of Standards Miscellaneous Document 273, December 1966.

[101]Kadanoff et al. (1967).

[102]Kadanoff (2000).

[103]For a very accessible exposition of fixed points see Leo Kadanoff "Relating Theories via Renormalization", `arXiv:1102.3705v1` (2011).

[104]Pauling and Wilson (1935).

[105]Wilson (1952). The book was reprinted by Dover Press in 1991 and in 2013! The Introduction of the 1991 edition states: "After discussing such basics as the choice and statement of a research problem and elementary scientific method, Professor Wilson offers lucid and helpful discussions of the design of experiments and apparatus, execution of experiments, analysis of experimental data, errors of measurement, numerical computation and other topics. A final chapter treats the publication of research results".

[106]He had done a theoretical study of the propagation of sound in an ocean that had an upper layer of constant temperature and constant sound velocity and a thermocline underneath it in which the sound velocity was variable, thus giving rise to a shadow zone. He worked out an elaborate mathematical theory of how the sound penetrated into that shadow zone.

[107]Jon Mathews (1932–1979) was lost at sea in December 1979 during a sailing trip around the world with his wife. He was then professor of theoretical physics at Cal Tech. See http://calteches.library.caltech.edu/582/2/Mathews.pdf.

[108]See http://archive.computerhistory.org/resources/text/Burroughs/ElectroData.204.1956.102646116.pdf.

[109]HRST. Interview with Kenneth G. Wilson, 6 July 2002. Interview recorded in Gray, Maine. Interview conducted by Physics of Scale collaborators: Babak Ashrafi, Karl Hall, and Sam Schweber. Edited by A. Martínez and S.S. Schweber.

[110]Ibid.

[111]The work on the Mendelstam representation became part of his PhD thesis. Wilson 1961.

[112]HRST. Interview with Kenneth G. Wilson, 6 July 2002.

[113]In each slice the Hamiltonian contained both a free-meson energy term and an interaction term, so this new perturbation method was neither a weak coupling nor a strong coupling perturbation.

[114]The modification was a factor involving a nontrivial matrix element of the ground state of the nth-slice Hamiltonian.

[115]The center was founded in 1962 and Bethe was influential in getting it started and supported.

[116]The seminar was a joint effort of Fisher and Widom. It had been initiated by Fisher when he joined the chemistry department in the summer of 1966. See Fisher 1999.

[117]Wilson was awarded the Nobel Prize by himself in 1982. When he was called on the telephone to receive the news he insisted that he share the prize with Michael Fisher. Upon being informed that he would be forfeiting the prize if he insisted on this demand he accepted receiving it by himself.

[118]The entanglement of quantum physics with both computing and the computational aspects of quantum chemistry, the latter as manifested by the use of density functional methods and the incorporation of renormalization group ideas therein, is an important component of my argument to validate my claim that the quantum revolution should be interpreted as a Hacking type revolution. See Schweber and BenPorath (2014) and the references therein.

[119]Fisher in Fisher (1999) describes the help he and his graduate student Howard Tarko obtained from a computer program that Wilson had developed to learn and understand the Domb-Fisher techniques of large scale series expansions to derive critical exponents.

[120]And this is not to say anything about his contributions and involvement in establishing national supercomputing centers and his directorship of the Cornell Center for Theory and Simulation in Science and Engineering from 1985 till 1988.

[121]The Wilsons moved to Ohio in 1988 when Ken's wife, Alison Brown, accepted the directorship of the Ohio State University's Computing Center. She was then the associate director of the Cornell Computer Center. Wilson had met her at a folk dance when doing a Swedish dance called the hambo. They married in 1982.

[122]See Kadanoff's General Introduction to *From Order to Chaos II*.

[123]Flavor actually comes in families of pairs that match quarks with leptons. The (u,d) family is paired with the electron and the electron neutrino. In each family of quarks and leptons, the sum of the electric charges must add up to zero. Since the u electric charge is (2/3)e and the d electric charge is (–1/3)e, in order for the quark charges to cancel the total electronic charge of the leptons, there must be three color charges.

[124]The point of departure.

[125]I here rely heavily on Serot and Walecka 1986 and Walecka 1995. See also Mottelson 1991, Mlađenović, 1998.

[126]The chiral symmetry exhibited by the QCD theory of massless quarks is broken by the condensation of quark/anti-quark pairs in the vacuum. Recall BCS.

[127]Recall that it was by "discretizing" "euclidianized" QCD on a four dimensional lattice, that Wilson made very plausible the confinement of both quarks and gluons and therefore that these entities cannot be observed as free particles; or stated field theoretically that neither quarks nor gluons could be represented as excitations of the vacuum.

[128]USQCD is funded by the Department of Energy through its "Scientific Discovery through Advanced Computation" (SciDAC) program, and through the Office of Science's High Energy Physics and Nuclear Physics programs. A majority of the 200 or so United States lattice gauge theorists are members of USQCD. For the activities and resources of USQCD see http://www.usqcd.org/collaboration.html.

[129]For the work of MIT lattice gauge theorists and their collaboration with Thinking Machines see Negele 2001.

[130]See http://phys.columbia.edu/~cqft/.

[131]The stated mission of Jefferson Laboratory is "to provide forefront scientific facilities, opportunities and leadership essential for discovering the fundamental structure of nuclear matter; to partner in industry to apply its advanced technology; and to serve the nation and its communities through education and public outreach".

[132]RHIC is an accelerator capable of accelerating heavy ions, such as gold ions (gold atoms which have had their outer cloud of electrons removed)to relativistic velocities and have them collide with one another. In 2006 the Nuclear Science Advisory Committee (NSAC) to the Department of Energy (DOE) and the National Science Foundation (NSF) recommended closing the ion collider at Brookhaven National Laboratory in Upton, N.Y., rather than eliminating other costly facilities; this in order to allow the construction of a Rare Isotope Accelerator (RIA). RHIC survived the threat and is still operating.

[133]It is the case that many of the nuclear physics laboratories established after World War II identified themselves as "Laboratory of Nuclear Science" or "Institute of Nuclear Science", e.g. at MIT, Chicago, Iowa, ... The appellation "nuclear *science*" was to differentiate the activities therein from "nuclear *engineering*", the appellation given to the work that physicists did at the Met Lab and at Los Alamos during the war.

[134]Zinn-Justin raises this possibility in the Introduction to his 2007 OUP *Phase transitions and renormalization group.*

[135]See Zinn-Justin 2007 for an elaboration of this viewpoint.

[136]The same is true of Alistair Crombie's revolutions. See Crombie 1995, Hacking 1985, 1992.

[137]See for example Judson 1979, Lwoff and Ullmann 1980, Müller-Wille and Rheinberger 2012.

[138]There are of course many macroscopic phenomena – e.g. hydrodynamics, chaos, turbulence – where microscopic composition is only marginally relevant See Kadanoff's essays (1999) where these matters are discussed.