An Experimental Test of the Trajectory Predictions of Bohmian Quantum Mechanics

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Abstract

We propose an experimental time-of-flight test of the photon "trajectory" predictions of the Bohmian version of quantum mechanics. The photon trajectories in free space, as predicted by Bohmian mechanics, deviate from expected straight-line paths, bending and turning corners in the presence of quantum interference. We propose an experiment using two such corner-turns that should, in principle, allow these hypothetical Bohmian photons to arrive at a timing detector *earlier* than conventional photons traveling on the straight-line paths predicted by standard quantum mechanics.

I. Introduction

Bohmian quantum mechanics[1] was introduced by the late David Bohm (1917-1992) in an attempt to reject the indefiniteness of Heisenberg's uncertainty principle while accounting for EPR nonlocality, thereby providing a more classically acceptable alternative to the standard quantum mechanics of Heisenberg, Schrödinger, Born, and Dirac.

It is normally impossible to test rival interpretations of quantum mechanics in the laboratory because they describe the same predictive quantum formalism. However, Bohmian mechanics represents a revision of the standard quantum formalism and in particular introduces procedures like trajectory tracing that are not a part of the standard formalism. This opens the door for experimental tests.

Among its other properties, Bohmian mechanics assumes that the particles (e.g., photons, electrons, etc.) described by the quantum formalism *simultaneously* have definite positions and definite velocities and momenta. This allows their trajectories, guided by the wave function, to be traced through space-time. Fig. 1, taken from Bohm's posthumously published book[1], shows the predicted Bohmian trajectories of coherent non-interacting massive particles passing through a Gaussian two-slit system.
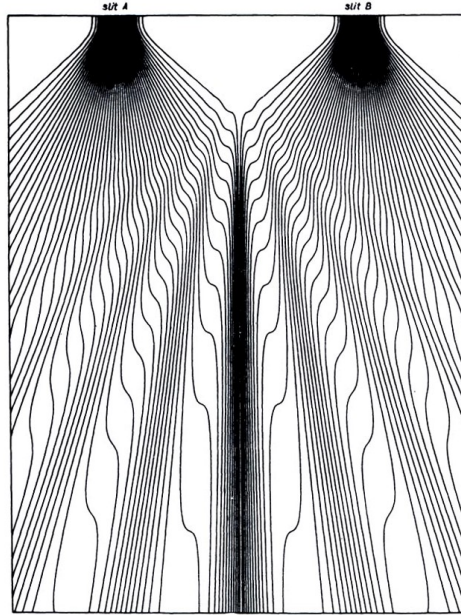
Fig. 1 Trajectories of coherent particles passing through a Gaussian two-slit system (from Bohm & Hiley[1]).

As can be seen in the lower part of Fig. 1, the expected two-slit interference pattern of maxima and minima is formed by particle trajectories that bunch together at the interference maxima while avoiding or making fast transitions across the interference minima.   It should be noted in this diagram that (a) although there are assumed to be no external forces, the Bohmian particles *do not* travel in straight lines in free space, but instead execute rather sharp turns at many locations along their paths, (b) like Faraday's electric and magnetic lines of force, Bohmian particle trajectories never intersect or cross, and (c) Bohmian particles passing through left slit *A* remain on the left side of the system and Bohmian particles passing through right slit *B* remain on the right side of the system, never crossing the vertical axis of symmetry. In Bohmian mechanics this "trajectory steering" behavior is attributed to the presence of an additional nonlocal "quantum potential" produced by quantum interference that modifies the trajectories of massive particles and photons to give results like those shown.

We are of the opinion that Bohm's assertion that such particle trajectories exist in the real world and constitute an accurate description of particle behavior is unphysical. It constitutes a Popper-falsifiable[2] theoretical prediction that can be subjected to experimental testing in the laboratory. In the present paper we propose such an experimental test.

2. Calculating Bohmian Trajectories

In order to analyze the experimental test proposed here, it is necessary, for a given quantum wave function, to calculate the corresponding particle trajectories that are predicted by Bohmian mechanics. The basic idea behind such trajectory calculations is very simple: in direct defiance of Heisenberg's uncertainty principle,

at every space point along the path of the hypothetical particle, Bohmian mechanics extracts a local vector momentum from the quantum wave function and asserts that it is the true and well-defined local momentum of the particle at that point.

Here is the procedure for doing the trajectory tracing operation. We start by using conventional quantum wave mechanics to solve a wave equation and to provide solutions that are the quantum mechanical wave functions describing the particles of interest. Then, at the starting point of a trajectory to be traced, we extract the local vector momentum of the particle. This is done using the standard quantum mechanical momentum operator, which involves spatial differentiation of the wave function. Bohmian mechanics assumes that the local momentum vector points in the direction that the particle is moving, so we take a small spatial step in that direction, we apply the momentum operator again to re-evaluate the vector momentum at the new location, we take another step and do another evaluation, and the process repeats, tracing out the complete trajectory of the particle from the selected starting point.

Here is the Bohmian trajectory-tracing procedure in more mathematical detail:

(1) For a particle in a given physical environment, use a standard quantum mechanical wave equation (e.g., the Schrödinger equation or the electromagnetic wave equation) to calculate the particle's spatial wave function $\psi(x,y,z)$.

(2) Apply the momentum operators $\mathbb{P}_x \equiv (-ih/2\pi) \, \partial/\partial x$, $\mathbb{P}_y \equiv (-ih/2\pi) \, \partial/\partial y$, and $\mathbb{P}_z \equiv (-ih/2\pi) \, \partial/\partial z$ to this wave function ψ.

Example: $\mathbb{P}_x\psi = (-ih/2\pi) \, \partial\psi/\partial x = p_x\psi$, where p_x is taken to be the local momentum component in the x direction.

(3) Extract the value of the local momentum Cartesian components p_x, p_y, p_z from this operator result.

Example: $p_x(x,y,z) = [\psi^*\mathbb{P}_x\psi]/\psi^*\psi = [\mathbb{P}_x\psi(x,y,z)]/\psi(x,y,z)$
$= [(-ih/2\pi) \, \partial\psi(x,y,z)/\partial x]/\psi(x,y,z)$

(4a) *For massive particles:* Divide this p_x by the particle mass m to get the local velocity v_x, and similarly obtain v_y and v_z. Use these vector velocity components, computed at each new location, to step through the trajectory construction.

(4b) *For photons:* Divide p_x by the total photon momentum $p = h/\lambda$ to get its relativistic velocity component β_x, and similarly obtain β_y and β_z. Use these vector components computed at each new spatial location to step through the trajectory construction. [Note that: $\beta_x^2 + \beta_y^2 + \beta_z^2 = 1$]

3. *Example: Photon trajectories for crossed coherent Gaussian laser beams.*

As an example of the application of this procedure, we consider the interference effects and Bohmian photon trajectories that are predicted to be present at the crossing of two coherent Gaussian laser beams of width parameter σ that have an initial phase difference of π at the sources at t=0. The geometry of the beam crossing is shown in Fig. 2.
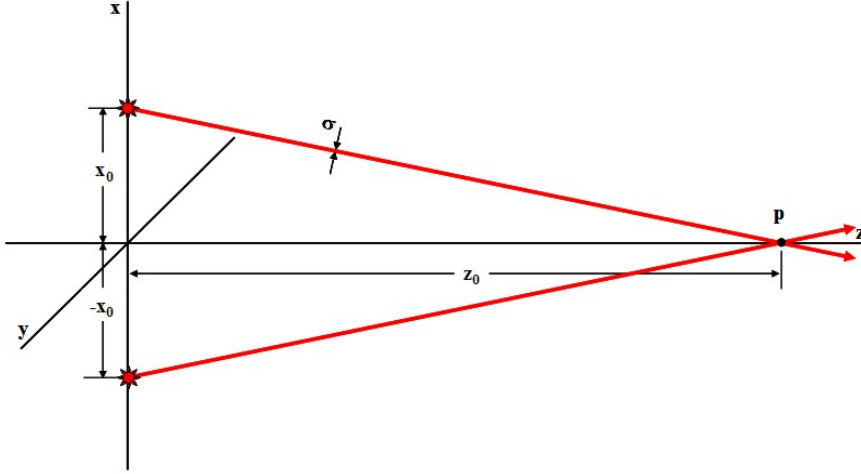
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Fig. 2. Geometrical arrangement of crossed coherent Gaussian laser beams. For the case considered, $\sigma = 1$ mm, $x_0 = 1$ cm, $z_0 = 20$ m, and the two source phases differ by π.

The wave functions of the Gaussian beams above (ψ_+) and below (ψ_-) the z axis are: $\psi_+(x,y,z) = +(A/4\pi\sigma) \, \mathrm{Exp}[-((x_\theta - x_0)^2 + y^2)/(2\sigma)^2] \, \mathrm{Exp}[i \, (2\pi \, s_+)/\lambda]$ and $\psi_-(x,y,z) = -(A/4\pi\sigma) \, \mathrm{Exp}[-((x_\theta + x_0)^2 + y^2)/(2\sigma)^2] \, \mathrm{Exp}[i \, (2\pi \, s_-)/\lambda]$, where A is the wave amplitude, σ is the Gaussian width of the beam, the crossing half-angle is $\theta \equiv \mathrm{Arctan}[x_0/z_0]$, $x_\theta \equiv x \, \mathrm{Cos}[\theta] + z \, \mathrm{Sin}[\theta]$, $s_+ \equiv z \, \mathrm{Cos}[\theta] - (x-x_0) \, \mathrm{Sin}[\theta]$, and $s_- \equiv z \, \mathrm{Cos}[\theta] - (x+x_0) \, \mathrm{Sin}[\theta]$. Here $s_\pm$ are the time-dependent longitudinal locations within the two beams as they propagate.
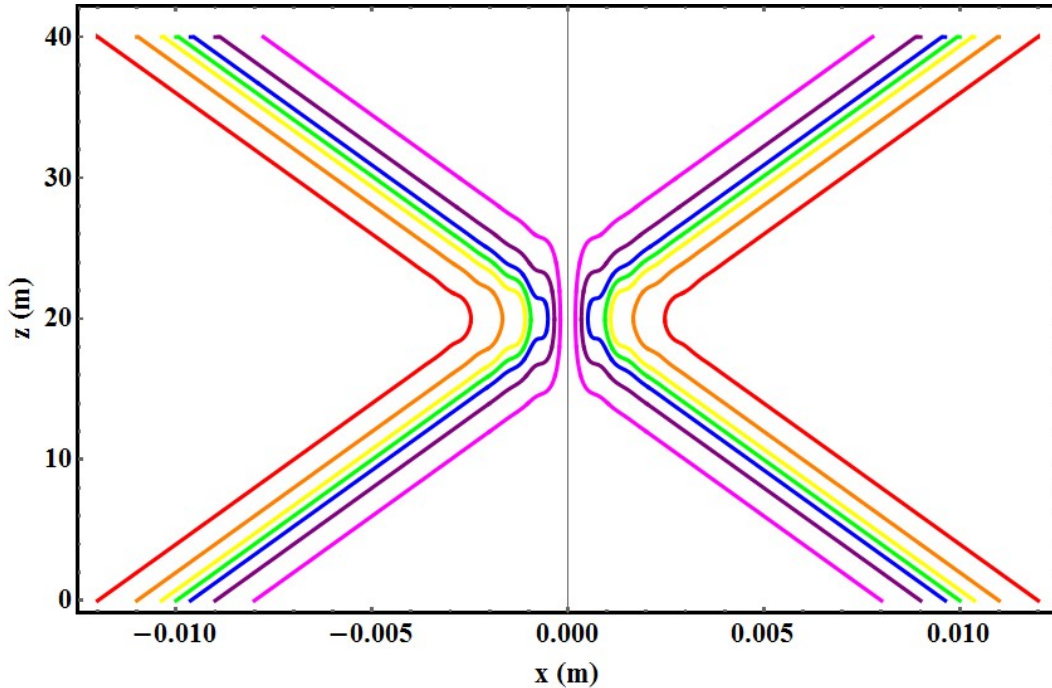
Fig. 3. Calculated Bohmian particle trajectories for a beam cross point located at $z_0 = 20$ m downstream from the two sources.

Fig. 3 shows the Bohmian trajectories of photons with initial starting points selected to be in the most intense parts of the two beams, as calculated with the net wave function, which is the sum of the two crossing Gaussian-beam wave functions given above. Fig. 4 shows these same calculated trajectories (white) superimposed on the absolute square of the net wave function near the beam crossing point at z = 20 m.

As can be seen in Fig. 4, the Bohmian photon trajectories tend to follow interference maxima, they occasionally make rapid transitions across interference minima, and they "bounce" at the cross point so that they do not cross the line of right-left symmetry. It is our view that this predicted behavior of photon trajectories is unphysical and moreover is testable because the modified nonlinear photon path lengths have time-of-flight implications.
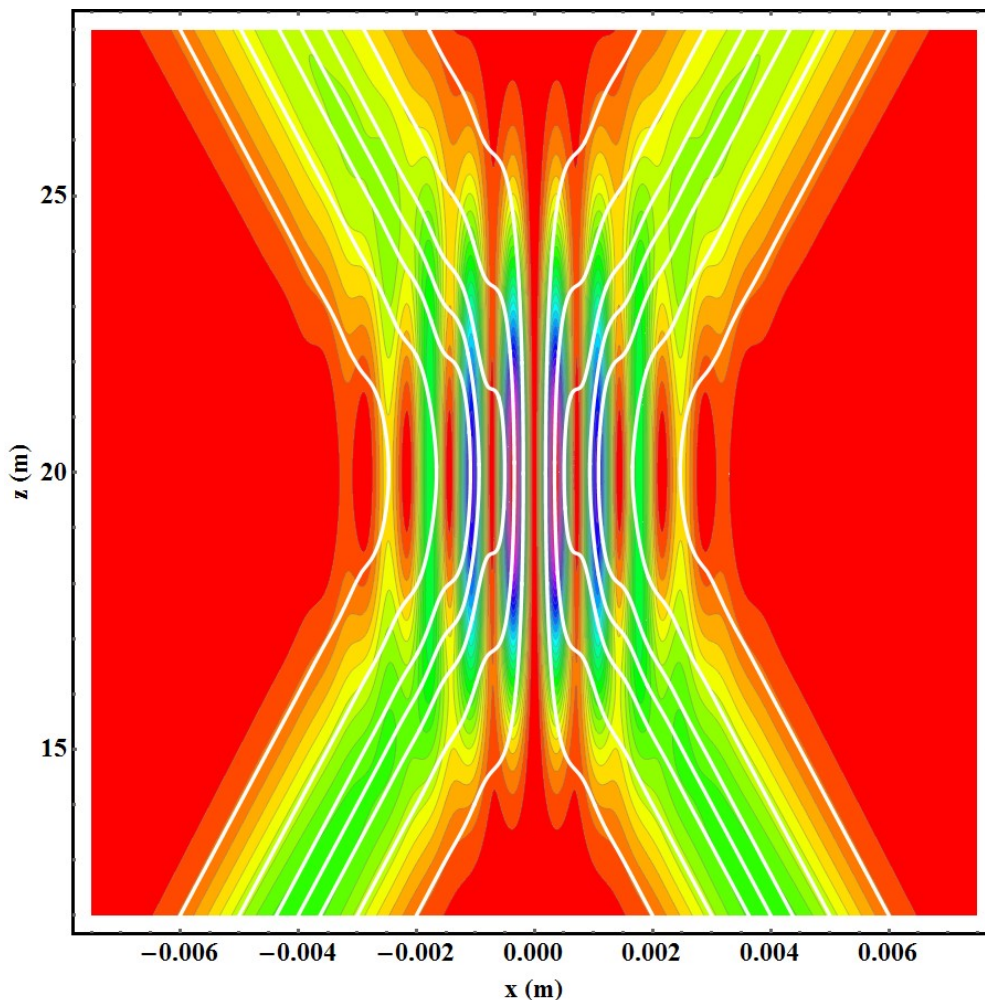

*Fig. 4. Wave function intensity ψ*ψ of crossed coherent Gaussian light beams, with calculated Bohmian trajectories (white) superimposed. In the contour plot, violet represents maximum wave function intensity and red represents minimum or zero wave function intensity.*

4. A simple test of Bohmian mechanics involving time-of-flight modification

As can be seen from Fig. 4, Bohmian mechanics predicts that photons are deflected at the crossings of interfering coherent beams and that the photon trajectories never cross. This characteristic behavior can be used to create a photon "shortcut" that influences a time-of-flight measurement. A simple prototype of such a time-of-flight measurement is shown in Fig. 5.
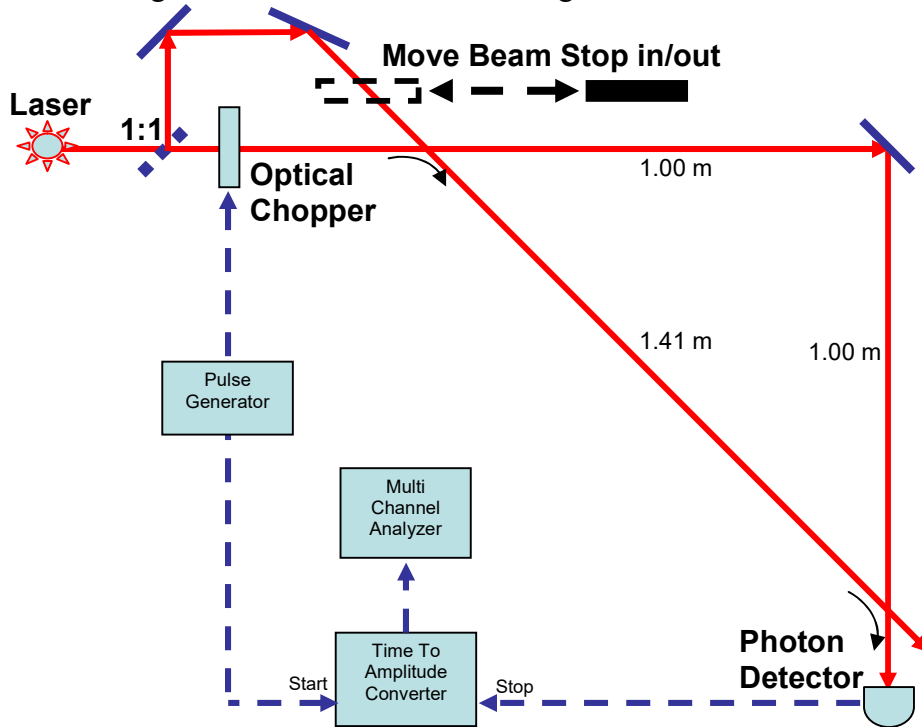
Fig. 5   *Simple prototype time-of-flight test of Bohmian mechanics. Because of deflections at the two cross points, Bohmian photons should "cut the corner" (arrows) and arrive at the photon detector <u>before</u> photons taking the expected right-angle path. The movable beam stop should shift the observed flight time by about 2 ns if the Bohmian trajectory predictions are valid.*

Here a polarization-independent beam splitter produces two coherent beams that are arranged to cross and interfere at two spatial locations. The diagonal beam is continuous, while the right-angle beam is chopped by an optical chopper cell and its time of flight is monitored by a time-to-amplitude converter (TAC) unit. The spatial coherence length of the laser is assumed to be large enough (several meters) that the two beams interfere coherently at both crossing regions.

According to Bohmian mechanics, the photons from the optical chopper should bounce at each of the two crossings, cut the corner of the beam path (see arrows), and arrive at the detector about 2 ns *before* the photons taking the right angle path predicted by standard quantum mechanics. This is a Popper-falsifiable prediction.

5. Improvements to the simple time-of-flight test of Bohmian mechanics

The problem with this simple prototype design is that the two coherent beams cross at 45°. This means that for visible light there would be around 10^5 microscopic interference fringes present in the crossing region. Random air currents and thermal gradients could easily wash out the interference effects. Therefore, the physical setup must be modified to employ beams that cross at a small angle, preferably a fraction of a degree. Fig. 6 shows how one can use a misaligned Mach-Zehnder interferometer to produce small-angle crossings.
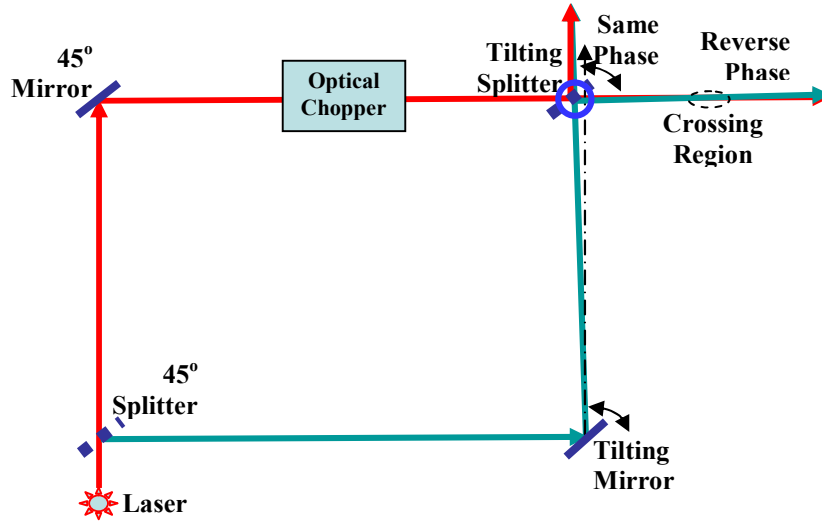

Fig. 6 Misaligned Mach-Zehnder interferometer produces a small-angle beam crossing. (Note that all beams have the same wavelength, and the colors are only used to indicate the two paths.)

Here a Mach-Zehnder interferometer is slightly misaligned to produce a small-angle downstream beam crossing. The apparent red-blue beam crossing shown in the blue circle is avoided by arranging for the beams to be slightly displaced from each other in the direction perpendicular to the diagram so that they do not interfere in this region.

Fig. 7 shows an improvement of the setup shown in Fig. 5 using this technique to arrange for very small beam-cross angles.
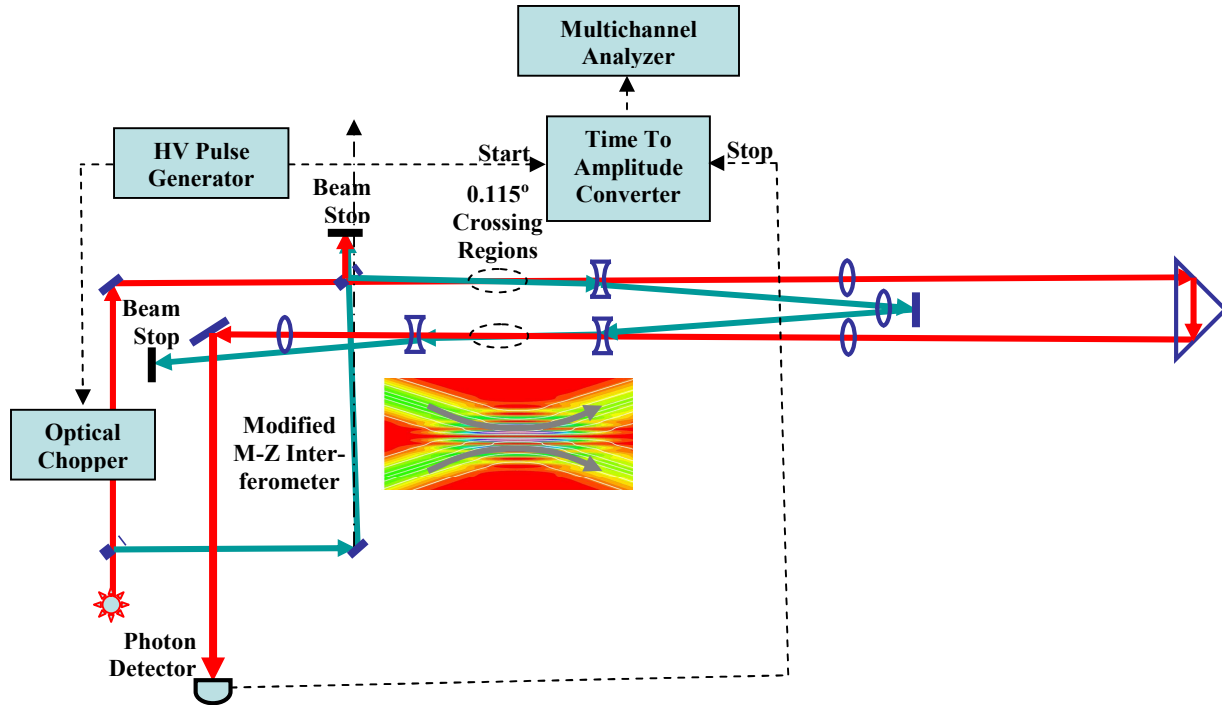
Fig. 7   Initial misaligned Mach-Zehnder interferometer is used to produce two small-angle beam crossings. Chopped Bohmian photon path is red-blue-red. (Note that all beams have the same wavelength, and the colors are only used to indicate the two paths.)

Here the M-Z interferometer produces the first crossing and a downstream arrangement of lenses and mirrors produces a second crossing. Because of the small crossing angles, there are only a few wide interference fringes produced at the cross points. If desired, the lenses could be eliminated by simply using a longer flight path. The Bohmian photons that are initially in the red chopped beam are predicted to bounce at the cross points, following the blue beam path from the first cross point to the second and then rejoining the red beam. This would produce an early arrival of the photons and a significant time-of-flight shift in the measurements. Blocking the blue beam would cause the Bohmian photons to follow the red path only and would increase their arrival time by a measurable time increment. Failure to observe a significant time-of-flight shift with the blue beam blocked vs. unblocked would constitute a falsification of Bohmian mechanics.

6. Beam chopping and Fourier effects on wavelength and interference

There is one complication with this TOF measurement that should be mentioned. A continuous light beam from a single-mode laser is assumed to have a fairly definite angular frequency ω and wave number k. Chopping or modulating such a beam introduces sidebands, causing the resulting beam to include a broader spectrum of angular frequencies and wave numbers. This results in modification of interference effects between the chopped beam and a coherent continuous beam, as

in the proposed experiment, and may also have an undesirable effect on the predicted Bohmian photon trajectories.

We have investigated this problem is some detail with Fourier transforms and have calculated its effect on Bohmian trajectory predictions. We find that in the interference between a chopped and a continuous beam, the Bohmian trajectories become "noisy", and there is a significant probability that some fraction of the Bohmian photons will not bounce at a cross point.

The result of this on the proposed experiment under the Bohmian scenario would be a reduction in the fraction of photons arriving early in the TOF measurement. This should not be a problem in falsifying the Bohmian predictions, because in the standard quantum mechanics predicts that *none* of the photons should arrive early. In sharp contrast, in the Bohmian scenario, even in the presence of chopping and frequency broadening, a significant fraction of the Bohmian photons should have early arrival.

7. Time-of-flight resolution and precision

Let us briefly consider whether there are any experimental limitations or quantum effects that might compromise the proposed measurement. In the configuration shown in Fig. 7, it should be relatively easy to make the path-length difference between the red and blue beams around 10 m. Such a path-length difference would result in a time-of-arrival shift of about 33 ns.

It might be argued that Bohmian mechanics only predicts trajectories, not flight times. However, the momentum extraction procedure described above, which is an integral part of trajectory construction, implies a velocity and flight-time increment at each step of the process, and these increments can be summed. In the case of photons, that velocity is the speed of light. Therefore, time-of-flight predictions are implicit in trajectory predictions.

The time resolution of the experiment depends on the chopped-beam rise time produced by the chopper. In the proposed experiment, a fast Pockels-cell chopper can produce a leading-edge rise time of better than 0.5 ns. An off-the-shelf time-to-amplitude converter unit can provide a time-resolution uncertainty of better than 0.01 ns. If a red 700 nm laser is used in the experiment, its intrinsic quantum-uncertainty in time[3] is about 4×10^{-7} ns. An off-the-shelf laser of modest intensity should provide millions of counts in the final time-of-flight spectrum and a coherence length of many meters.

Thus, there should be no difficulty in obtaining an experimental time-of-flight measurement having a statistical precision of tens of standard deviations. Such a measurement should be able to confirm or falsify the early-arrival Bohmian trajectory prediction, even assuming the presence of a contaminating fraction of non-bouncing photons, as described above. The TOF test should therefore be definitive and unambiguous.

We have also found that by pulsing *both* beams and adjusting the path lengths, crossing locations, and pulse rates to insure their joint presence of pulse maxima at both cross point, we can create a condition in which such trajectory noise is eliminated and *all* of the Bohmian photons bounce at the cross points and arrive

early. However, this refinement seems unnecessary for a definitive test, since standard quantum mechanics predicts that there should be no early arrivals at all.

8. Conclusion

We have presented an experimental design that offers the opportunity to falsify, in the Popper sense of that term, either the predictions of Bohmian quantum mechanics of the behavior and trajectories of photons in coherent Gaussian light beams or the predictions of standard quantum mechanics. This experiment is relatively simple, as compared to many experiments currently being performed in quantum optics laboratories around the world. We encourage quantum optics experimentalists to take on this test as a project. Its results should be of fundamental importance to our understanding and evaluation of the rival interpretations of standard quantum mechanics and it variations.

References:

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