Quantum nonlocality based on finite-speed causal influences leads to superluminal signaling

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The experimental violation of Bell inequalities using spacelike separated measurements precludes the explanation of quantum correlations through causal influences propagating at subluminal speed. Yet, it is always possible, in principle, to explain such experimental violations through models based on hidden influences propagating at a finite speed $v > c$, provided v is large enough. Here, we show that for *any* finite speed $v > c$, such models predict correlations that can be exploited for faster-than-light communication. This superluminal communication does not require access to any hidden physical quantities, but only the manipulation of measurement devices at the level of our present-day description of quantum experiments. Hence, assuming the impossibility of using quantum non-locality for superluminal communication, we exclude any possible explanation of quantum correlations in term of finite-speed influences.

Correlations cry out for explanation [1]. Our pre-quantum understanding of correlations relies on a combination of two basic mechanisms. Either the correlated events share a common cause — such as seeing a flash and hearing the thunder when a lightning strikes — or one event influences the other — such as the position of the moon causing the tides. In both cases, we expect the chain of events to satisfy a principle of continuity: that is, the idea that the physical carriers of causal influences propagate continuously through space. In addition, we expect them — given the theory of relativity — to propagate no faster than the speed of light. The correlations observed in certain quantum experiments call into question this viewpoint.

When measurements are performed on two entangled quantum particles separated far apart from one another, such as in the experiment envisioned by Einstein, Podolosky, and Rosen (EPR) [2], the measurement results of one particle are found to be correlated to the measurement results of the other particle. Bell showed that if these correlated values were due to local common causes, then they would satisfy a series of inequalities [1]. But theory predicts and experiments confirm that these inequalities are violated [3], thus excluding any local common cause type of explanation. Moreover, since the measurement events can be spacelike separated, any influence-type explanation must be based on influences propagating faster than light.

This non-local connection between distant particles presents profound interpretative difficulties and is a source of tension between quantum theory and relativity [4, 5]. However, it does not put the two theories in direct conflict thanks to the no-signaling property of quantum correlations. This property guarantees that spatially separated observers in an EPR-type experiment cannot use their measurement choices and outcomes to communicate with one another.

But quantum non-locality is not only puzzling because of its apparent conflict with relativity, it also seems to invalidate the more fundamental idea that correlations can be explained by causal influences propagating continuously in space. Indeed, according to the standard textbook description, quantum correlations are achieved through the collapse of the wavefunction, a process that is instantaneous and independent of the spatial separation between particles. Any explanation of quantum correlations via hypothetical influences would therefore require that they "propagate" at speed $v = \infty$. Clearly, one may ask whether *infinite speed* is a necessary ingredient to account for the correlations observed in Nature or whether a finite speed v, recovering a principle of continuity, is sufficient. At first, this question seems unanswerable. Indeed, provided that v is large enough, any model reproducing the non-local correlations of quantum theory through (hidden) influences propagating at a finite speed $v > c$ can always be made compatible with all experimental results observed so far. It thus seems like the best that one could hope for is to put lower-bounds on v by testing the violation of Bell inequalities with systems that are further apart and better synchronized [6, 7].

Here we show that there is a fundamental reason why influences propagating at a finite speed may not account for the non-locality of quantum theory. We demonstrate, following an original suggestion of [8, 9], that all models for quantum correlations where non-local influences propagate at a given finite speed $v > c$ give, for *any* v, predictions that can be used for faster-than-light communication. More precisely, consider any such model that correctly reproduces the quantum prediction within the range of its causal influences and ceases to violate Bell inequalities beyond this range. Then we show that it will also necessarily predict, in certain configurations, mea-

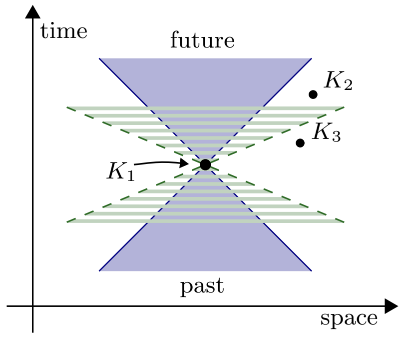

FIG. 1. Space-time diagram in the privileged reference frame. In the (shaded) light cone delimited by solid lines, causal influences propagate up to the speed of light c, whereas in the v-cone (hatched region), causal influences travel up to the speed v. An event K_1 can causally influence a spacelike separated event K_2 contained in its future v-cone, but cannot directly influence an event K_3 outside its v-cone.

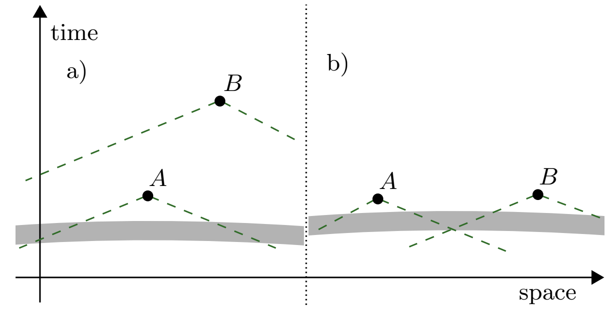

FIG. 2. Bipartite Bell experiments in a v-causal model. a) A is in the past v-cone of B. The variable λ, with probability distribution $q(\lambda)$, denote the joint state of the particles, or more generally a complete specification of any initial information in the shaded spacetime region that is relevant to make predictions about a and b [15]. In this situation, we can write $P_{A<B}(ab|xy) = \sum_\lambda q(\lambda)P(ab|xy,\lambda) = \sum_\lambda q(\lambda)P(a|x,y\lambda)P(b|y,ax\lambda) = \sum_\lambda q(\lambda)P(a|x,\lambda)P(b|y,ax\lambda)$, where we used Bayes' rule in the second equality and the fact that the measurement setting y is a free variable, i.e., uncorrelated to a, x, λ, in the last equality. b) A and B are outside each other's v-cones. As above, the variable λ represents a complete (as far as predictions about a and b are concerned) specification of the shaded spacetime region. Note that this region screens-off the intersection of the past v-cones of A and B [16]. It thus follows that $P(a|x, by\lambda) = P(a|x,\lambda)$ since any information about B is irrelevant to make predictions about a once λ is specified (see [17] for a more detailed discussion of this condition). Similarly $P(b|y, ax\lambda) = P(b|y,\lambda)$. We can therefore write $P_{A\sim B}(ab|xy) = \sum_\lambda q(\lambda)P(ab|xy,\lambda) = \sum_\lambda q(\lambda)P(a|x,y\lambda)P(b|y,ax\lambda) = \sum_\lambda q(\lambda)P(a|x,\lambda)P(b|y,\lambda)$.

surement outcome correlations that could be used by distant observers to communicate at superluminal speeds. This does not require access to any of the (hidden) quantities specified by the underlying model, but only the manipulation of measurement settings and the observation of measurement results at the level of our present-day description of quantum experiments.

For definiteness, we derive our results assuming that the speed v is defined with respect to a privileged reference frame [10]. It should be stressed that whilst the assumption of such a frame is not in line with the spirit of relativity, there is also no empirical evidence suggesting their absence [11]. In fact, there exist physical theories that assume a priviledged reference frame and are compatible with all observed data, such as Bohmian mechanics [12] and the collapse theory of Ghirardi, Rimini, and Weber [13, 14]. While both of these theories reproduce all tested quantum predictions, they violate the principle of continuity mentioned above (otherwise they would not be compatible with no-signalling as our results imply).

Models with a finite limit $v > c$ on the speed of superluminal influences.

The models that we consider assume that to each spacetime point K, we can associate in the privileged frame a past and a future "v-cone" generalizing the notion of past and future light-cones, see Figure 1. We write $K_1 < K_2$ ($K_1 > K_2$) if K_2 is in the future (past) v-cone of K_1 and $K_1 \sim K_2$ if K_1 and K_2 are outside each other's v-cones. An event at K_1 can have a causal influence on points $K_2 > K_1$ in its future v-cone and can be influenced by points $K_2 < K_1$ in its past v-cone. But there cannot be any direct causal relation between two events $K_1 \sim K_2$ that are outside each other's v-cones: any correlation between them must originate from common causes in the intersection of their past v-cones.

More specifically, consider a Bell-type experiment involving two particles A and B upon which measurements (labeled by) x and y are performed, yielding outcomes a and b with probability $P(ab|xy)$. A model in which causal influences can propagate at speed v, henceforth referred as a *v-causal* model, will predict the joint probabilities $P_{A<B}(ab|xy) = \sum_\lambda q(\lambda)P(a|x,\lambda)P(b|y,ax\lambda)$ if A is in the past v-cone of B, and $P_{B<A}(ab|xy) = \sum_\lambda q(\lambda)P(b|y,\lambda)P(a|x,by\lambda)$ if B is in the past v-cone of A (see Figure 2a). Intuitively, particle A can "communicate" to particle B in the first case and particle B to particle A in the second one. On the other hand, if A and B are outside each-other's v-cones, a v-causal model will predict $P_{A\sim B}(ab|xy) = \sum_\lambda q(\lambda)P(a|x,\lambda)P(b|y,\lambda)$, see Figure 2b; that is, the model will predict correlations that are formally "local" and satisfy Bell inequalities. Intuitively, the two systems cannot influence each other, but can nevertheless be correlated through the variables λ specifying their common past. The causal structure that we consider thus simply corresponds to Bell's notion of local causality [17] but with the speed of light c replaced by the speed $v > c$.

Quantum theory and v-causal models.

According to quantum theory, measurements on two separated systems prepared in the quantum state ρ yield joint probabilities of the form $P_Q(ab|xy) = \mathrm{tr}(\rho M_a^x \otimes M_b^y)$, regardless of the spacetime ordering between the measurements. A v-causal model for *quantum correlations* is then one that reproduces $P_Q(ab|xy)$ in the situations $A < B$ and $B < A$, i.e., a model such that $P_{A<B} = P_{B<A} = P_Q$. Note that it is always possible to find such a model as we can write $P_Q(ab|xy) = P(a|x)P(b|y, ax) = P(b|y)P(a|x, by)$ by the no-signaling property of quantum correlations.

More generally, in a multipartite setting, a v-causal model for quantum correlations is one that reproduces the quantum predictions when the spacetime ordering of the systems is such that a chain of causal influences can propagate sequentially from the first measured system to the last one. Consider for instance a four-partite experiment involving particles A, B, C, D and characterized by the quantum probabilities $P_Q(abcd|xyzw)$. If the spacetime ordering is $T = (A < D < B < C)$, then a v-causal model will predict $P_T(abcd|xyzw) = \sum_\lambda q(\lambda)P(a|x, \lambda)P(d|w, ax\lambda)$ $P(b|y, axdw\lambda)P(c|z, axdwby\lambda)$ and a quantum v-causal model will satisfy $P_T = P_Q$ (as above, it is always possible to find such a model thanks to the no-signaling property of quantum correlations).

Note that when the spacetime ordering is such that causal influences are restricted between certain pairs of events, a v-causal model will generally *not* be able to reproduce *all* quantum correlations. For instance, the correlations between A and B will never violate Bell inequalities when $A \sim B$. If we observe experimentally that Bell inequalities are nevertheless violated in this situation, we will have ruled out the possibility of explaining Nature through influences that propagate at the finite speed v. Accordingly, it seems like the best that one can hope for is to lower bound experimentally the speed of causal influences by synchronizing as well as possible the different measurements in a Bell experiment (an additional difficulty, though, is that the synchronization has to be done in the unknown privileged frame [6]). Our aim here is to show that there is a more fundamental reason why influences propagating at a finite speed v cannot account for the non-locality of quantum theory: for *any* finite v, they necessarily allow for superluminal communication.

No-signaling and v-causal models.

The superluminal influences allowed in a v-causal model, at the (hidden) microscopic level, need not *a priori* lead to any signaling at the the macroscopic level, that is at the level of the experimenters who have no access to the underlying mechanism and variables λ of the model, but can only observe the average correlations $P(ab|xy)$ (e.g., by rotating polarizers along different directions x, y and counting detector clicks a, b). For instance, when the spacetime ordering between the systems is such that the model reproduces the correlations predicted by quantum theory, then these correlations can obviously not be used for signaling.

A sufficient condition for the correlations P not to be exploitable for signaling is that they satisfy a series of mathematical constraints known as the "no-signaling conditions". In the bipartite case, these are the conditions that the marginal distributions $\sum_b P(ab|xy) = P(a|x)$ and $\sum_a P(ab|xy) = P(b|y)$ for the measurement outcomes of one system be independent of the measurement made on the other system. This guarantees that the correlations P cannot be used by experimenters to signal to one another by their choice of measurement settings x or y. In the case of four parties (on which we will focus below), no-signaling is the condition that the marginal distributions for the joint system ABC be independent of the measurement performed on system D, i.e., that

$$\sum_d P(abcd|xyzw) = P(abc|xyz)\,, \qquad (1)$$

together with the analogous conditions for systems ABD, ACD, and BCD. These conditions imply that the marginal distribution for any subset of systems are independent of the measurements performed on the complementary subset.

If the conditions (1) are violated – and if it is possible for the experimenters to evaluate the marginal correlations ABC at a spacetime point outside of D's future light-cone – then the correlations P can be exploited to communicate faster-than-light from D to the combined system ABC. Such superluminal communication does not require any control of the mechanism underlying the v-causal model, but only makes use of the correlations between the macroscopic variables a, b, c and x, y, z, w. We now show that a v-causal model for quantum correlations will necessarily lead to such superluminal communication.

v-causal models for quantum correlations imply the possibility of superluminal communication.

Our result is based on the following Lemma.

Lemma. *Let $P(abcd|xyzw)$ be a joint probability distribution with $a, b, c, d \in \{0, 1\}$ and $x, y, z, w \in \{0, 1\}$ satisfying the following two conditions.*

i) The conditional bipartite correlations $BC|AD$ are local, i.e., the joint probabilities $P(bc|yz, axdw)$ for systems BC conditioned on the measurements settings and results of systems AD admits a decomposition of the form $P(bc|yz, axdw) = \sum_\lambda q(\lambda|axdw)P(b|y, \lambda)P(c|z, \lambda)$ for every a, x, d, w.

ii) P satisfies the no-signaling conditions (1).

Then the following inequality is satisfied

$$
\begin{aligned}
S = &-3\langle A_0\rangle - \langle B_0\rangle - \langle B_1\rangle - \langle C_0\rangle - 3\langle D_0\rangle \\
&- \langle A_1B_0\rangle - \langle A_1B_1\rangle + \langle A_0C_0\rangle \\
&+ 2\langle A_1C_0\rangle + \langle A_0D_0\rangle + \langle B_0D_1\rangle \\
&- \langle B_1D_1\rangle - \langle C_0D_0\rangle - 2\langle C_1D_1\rangle \\
&+ \langle A_0B_0D_0\rangle + \langle A_0B_0D_1\rangle + \langle A_0B_1D_0\rangle \\
&- \langle A_0B_1D_1\rangle - \langle A_1B_0D_0\rangle - \langle A_1B_1D_0\rangle \\
&+ \langle A_0C_0D_0\rangle + 2\langle A_1C_0D_0\rangle - 2\langle A_0C_1D_1\rangle \\
&\leq 7,
\end{aligned} \tag{2}
$$

where we have introduced the correlators $\langle A_x\rangle = \sum_{a=0}^{1}(-1)^a P(a|x)$, $\langle A_xB_y\rangle = \sum_{a,b=0}^{1}(-1)^{a+b}P(ab|xy)$, $\langle A_xB_yC_z\rangle = \sum_{a,b,c=0}^{1}(-1)^{a+b+c}P(abc|xyz)$, *and so on.*

Proof. Let $P_{AD}(00|00)$ denote the AD marginal probabilities $P(a=0,d=0|x=0,w=0)$ and let $P_{B|AD}(b|y)$ denote the $B|AD$ probabilities $P(b|y,a=0,x=0,d=0,w=0)$, and define similarly $P_{C|AD}(c|z)$ and $P_{BC|AD}(bc|yz)$. Consider the following inequality

$$
\begin{aligned}
I = &P(1000|0000) + P(0001|0010) + P(0011|0011) \\
&+ P(0100|0011) + P(1000|0100) + P(0011|0110) \\
&+ P(0000|0111) + P(0111|0111) + P(0010|1000) \\
&+ P(1100|1000) + P(0010|1100) + P(1100|1100) \\
&+ P_{AD}(00|00)\big[1 - P_{B|AD}(0|0) - P_{C|AD}(0|0) \\
&\quad + P_{BC|AD}(00|00) + P_{BC|AD}(00|01) \\
&\quad + P_{BC|AD}(00|10) - P_{BC|AD}(00|11)\big] \geq 0\,.
\end{aligned} \tag{3}
$$

This inequality is satisfied by any correlations P fulfilling condition i). Indeed, the first twelve terms and the term $P_{AD}(00|00)$ are clearly positive. Moreover, the term in square brakets is positive since $1 - P_{B|AD}(0|0) - P_{C|AD}(0|0) + P_{BC|AD}(00|00) + P_{BC|AD}(00|01) + P_{BC|AD}(00|10) - P_{BC|AD}(00|11) \geq 0$ is nothing but the Clauser-Horne-Shimony-Holt (CHSH) inequality [18] for the BC correlations conditioned on $a=0, x=0, d=0, w=0$ and is thus non-negative according to condition i). Using the no-signaling conditions, it is now easy to see that S can be written as $S = 7 - 8I$, which implies $S \leq 7$ [19]. □

Note that inequality (2) is violated by quantum theory, since measuring the state

$$
\begin{aligned}
|\Psi\rangle = &\frac{17}{60}|0000\rangle + \frac{1}{3}|0011\rangle - \frac{1}{\sqrt{8}}|0101\rangle + \frac{1}{10}|0110\rangle \\
&+\frac{1}{4}|1000\rangle - \frac{1}{2}|1011\rangle - \frac{1}{3}|1101\rangle + \frac{1}{2}|1110\rangle.
\end{aligned}
$$

with the operators

$$
\begin{aligned}
\hat{A}_0 = -U\sigma_x U^\dagger, \quad \hat{A}_1 = U\sigma_z U^\dagger, \quad \hat{B}_0 = H, \\
\hat{B}_1 = -\sigma_x H \sigma_x, \hat{C}_0 = -\hat{D}_0 = \sigma_z, \quad \hat{C}_1 = \hat{D}_1 = -\sigma_x,
\end{aligned}
$$

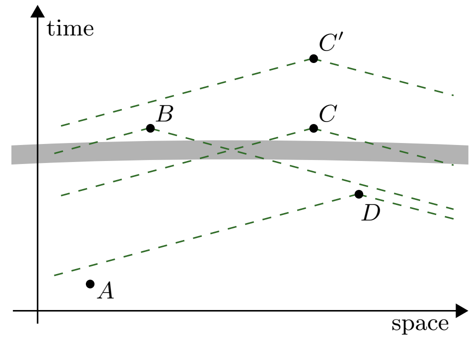

FIG. 3. Four-partite Bell-type experiment characterized by the spacetime ordering $R = (A < D < B \sim C)$. Let λ describe any relevant information from the past of A, B, C, D and in addition let μ be a (sufficiently complete) specification of the shaded region, which screens-off the intersection of the past v-cones of B and C. Note that μ may depend on the value of the past variables a, x, d, w, λ and is thus characterized by a probability distribution $q(\mu|axdw\lambda)$. Since $B \sim C$, we have as in Figure 2b $P(b|y, cz\mu) = P(b|y,\mu)$ and $P(c|z, by\mu) = P(c|z,\mu)$. We can thus write $P_R(abcd|xyzw) = \sum_\lambda q(\lambda)P(a|x,\lambda)P(d|w,ax\lambda)\sum_\mu q(\mu|axdw\lambda)P(b|y,\mu)P(c|z,\mu)$. This implies that the correlations BC conditioned on AD are local since $P_R(bc|yz,axdw) = \sum_\mu \tilde{q}(\mu|axdw)P(b|y,\mu)P(c|z,\mu)$ where $\tilde{q}(\mu|axdw)=\sum_\lambda q(\lambda)\times P(a|x,\lambda)P(d|w,ax\lambda)q(\mu|axdw\lambda)/\sum_\lambda q(\lambda)P(a|x,\lambda)P(d|w,ax\lambda)$. Let us now show that the marginal correlations ABD are quantum. For this, suppose that a choice is made after the shaded spacetime region (and in a way that is independent of the variables μ) to delay the measurement on particle C, up to the point C'. Thus, B now lies in the past v-cone of C and the spacetime ordering is $T = (A < D < B < C')$. We can then write $P_T(abcd|xyzw) = \sum_\lambda q(\lambda)P(a|x,\lambda)P(d|w,ax\lambda)\sum_\mu q(\mu|axdw\lambda)P(b|y,\mu)P(c|z,by\mu)$, from which it follows that $P_T(abd|xyw) = P_R(abd|xyw) = \sum_\lambda q(\lambda)P(a|x,\lambda)P(d|w,ax\lambda)\sum_\mu q(\mu|axdw\lambda)P(b|y,\mu)$. But since by assumption $P_T = P_Q$, we deduce that $P_R(abd|xyw) = P_Q(abd|xyw)$. Similarly, it can be shown that $P_R(acd|xzw) = P_Q(acd|xzw)$.

where $U = \cos(\frac{4\pi}{5})\sigma_z - \sin(\frac{4\pi}{5})\sigma_x$ and H is the Hadamard matrix, yields $S = 7.2014 > 7$.

Assume now that there exists a v-causal model for the above quantum correlations, that is, that there exists a model such that $P_T = P_Q$ if T is a spacetime ordering that does not constrain causal influences between any pair of systems. Consider the predictions of such a model in a configuration where the quantum predictions need not be reproduced *completely* such as in the configuration of Figure 3, characterized by the ordering $R = (A < D < B \sim C)$, i.e., superluminal influences can propagate from system A to all the other ones and from system D to systems B and C, but cannot propagate between systems B and C. By definition of the model, it then follows that the correlations $BC|AD$ are local, i.e., condition i) of the Lemma is satisfied, see Figure 3 for details. Since $B \sim C$, as noted above, we should not expect, in general, that $P_R = P_Q$. However, inequality (2) is completely determined by the value of the tripartite terms $P(abd|xyw)$ and $P(acd|xzw)$, where the model does give the same predictions as quantum theory for the spacetime ordering R, that is, any v-causal model for these quantum correlations satisfies $P_R(abd|xyw) = P_Q(abd|xyw)$ and $P_R(acd|xzw) = P_Q(acd|xzw)$, see caption of Figure 3. Intuitively, this is because the marginal correlations ABD (ACD) are well-defined, that is independent of the measurements performed on C (B), and in particular independent of whether this measurement is delayed, in which case the correlations should reproduce the quantum ones. Therefore, any quantum v-causal model for the above quantum correlations will also yield $S = 7.2014 > 7$ in the configuration of Figure 3. Inequality (2) is thus violated and therefore one of the two conditions of the Lemma must be violated. But since the model satisfies condition i), it must necessarily violate condition ii), i.e., the correlations P_R must violate the non-signaling conditions (1).

This implies that the correlations P_R can be used for superluminal communications. Indeed, since they violate the no-signaling conditions, at least one of the tripartite correlations ABC, ABD, ACD, or BCD must depend on the measurement setting of the fourth party. This is not the case for the marginal ABD (ACD) as it is defined independently of C (B) (and equal to the quantum marginal). It thus follows that either the marginal ABC must depend on the measurement setting w of system D or that the marginal BCD must depend on the measurement setting x of system A. In both cases, these marginals can be evaluated outside the fourth party's future light-cone and can thus be explicitly used for superluminal communication, see Figure 4.

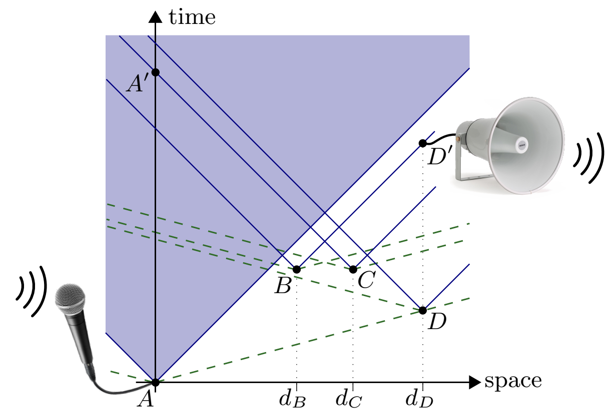

FIG. 4. Let the four systems of Figure 3 lie along some spatial direction at, respectively, a distance $d_B = \frac{1}{4}(1+\frac{1}{r}) + \frac{1}{1+r}$, $d_C = \frac{1}{4}(1+\frac{1}{r}) - \frac{1}{1+r}$, $d_D = 1$ form A, where $r = v/c > 1$, and let them be measured at times $t_A = 0$, $t_B = t_C = \frac{2}{c+v}$, $t_D = 1/r$. Suppose that the correlations P_R produced by a v-causal model are such that the BCD marginal correlations depend on the measurement x made on the first system A. If parties B and C broadcast (at light-speed) their measurement results, it will be possible to evaluate the marginal correlations BCD, at the point D'. Since this point lies outside the future light-cone of A (shaded area), this scheme can be used for superluminal communication from A to D'. Similarly, if the ABC marginal correlations depend on the measurement w made on D, they can be used for superluminal communication from D to the point A'.

Conclusion

We have shown that if the non-local effects that we observe in Bell experiments were due to hidden influences propagating at *any* finite speed, then non-locality could be exploited for superluminal communication. Our results therefore uncover a new aspect of the complex relationship between multipartite quantum non-locality and the impossibility of signaling [20–22].

Our results answer a question first raised in [8, 9]. Partial progress on this problem was made in [23], where a conclusion similar to ours was obtained for a particular set of *non-quantum* correlations. The approach of [23], based on the tripartite configuration considered in [8, 9], does not seem, however, to generalize in a straightforward way to the physically relevant case of quantum correlations. Our approach is based instead on the introduction of a general formulation of the concept of v-causality, which allowed us to go beyond the configuration originally considered in [8, 9]. It would be interesting to understand if the conceptual framework presented here can be used to rule out finite-speed influences for any conceivable non-local theory or if there exist non-local theories compatible with finite speed influences.

This work illustrates the difficulty to modify quantum physics while maintaining no-signaling. If we want to keep no-signalling, it shows that quantum non-locality must necessarily relate discontinuously parts of the universe that are arbitrarily distant. This gives further weight to the idea that quantum correlations somehow arise from outside spacetime, in the sense that no story in space and time can describe how they occur.

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[10] Or a particular foliation of spacetime into spacelike hyperplanes. One reason to assume a privileged frame is that it is not easy to formulate a fully relativistic model (i.e. with the Lorentzian metric as the only spacetime structure) containing superluminal causal influences that are limited in spacetime and that does not lead to causal paradoxes [4]. The results presented here, however, can probably be adapted to any model of this sort. Also, our results apply to models with many preferred frames defined by the measuring devices, in this case even for $v = \infty$ [8].
[11] In fact, even in a perfectly Lorentz-invariant theory, there can be natural preferred frame due to the non-Lorentz-invariant distribution of matter – a well-known example of this is the reference frame in which the cosmic microwave background radiation appears to be isotropic. See C. Lineweaver, L. Tenorio, G. F. Smoot, P. Keegstra, A. J. Banday, P. Lubin, Astrophys. J. **470**, 38 (1996).
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[15] Strictly, for a v-causal model, only the shaded region that is in the past v-cone of A can have a causal influence on A; likewise for the other figures. However, all our arguments still follow through even if we consider spacetime regions of the kind depicted.
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