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The transactional interpretation, counterfactuals, and weak values in quantum theory

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ABSTRACT

In recent years, a number of authors have studied pre- and post-selected quantum systems and associated time symmetry considerations. In this context, numerous paradoxes have arisen which raise questions about what can be inferred about such a system based on theoretically calculated quantities such as the Aharonov–Bergmann–Lebowitz (ABL) probabilities; and, more recently, "weak values"—time-symmetric quantities applicable to pre- and post-selected systems. This paper applies to some of these problems a time-symmetric interpretation of quantum theory: the "transactional interpretation" (TI) of Cramer, first proposed in 1980. The TI picture supports the conclusion that weak values are properly interpreted as multiple-time amplitudes rather than as generalized expectation values. It also prompts a stricter constraint on the counterfactual usage of the ABL rule than the consistency of the associated family of histories, which has previously been regarded as sufficient.

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1. Introduction and background

Cramer (1980, 1986, 1988) presented his transactional interpretation (TI) in the 1980s. TI proposes that the usual quantum-mechanical state $|\psi\rangle$ characterizes an "offer wave" (OW) emitted in the usual

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forward time direction from a source, and adds to this picture the idea that absorbers in the future light cone of such a source emit advanced or backward time-directed "confirmation waves" (CW) back to the source, upon receiving all or part of such an OW. The overlap of such offer and CW may set up a four-vector standing wave whose amplitude at the source point reflects the Born probability. While many "incipient transactions" consisting of an overlap of offer and CW may occur, only one will result in an actual transaction—giving rise to an observable event—with the probability corresponding to the "final" amplitude at the source point. (For further details, see Cramer, 1986.)

It should be noted that TI has historically received little notice or acceptance. This is because its solutions to various puzzles come at a price: one must admit the counterintuitive notion of time-reversed "advanced waves" and the causal perplexities such notions inevitably raise. Thus, it undoubtedly struck many thinkers as creating at least as many conceptual problems as it solved. But this was before the onslaught of new conceptual puzzles recently raised in the context of pre- and post-selection, counterfactuals, and "strange" weak values (cf. Aharonov, Botero, Popescu, Reznik, & Tollaksen, 2002; Kastner, 2003, 2004; Vaidman, 1996). It is proposed here that the conceptual price to be paid for the solutions offered by the TI may now be seen as more equitable—and moreover, as an opportunity to break out of a constraining paradigm to a more fruitful way of understanding quantum puzzles. In addition, the growing acceptance of time-symmetric postulates in interpretational studies is evidenced by the work of Price (1996) as well as Aharonov and Vaidman (1990, 1991), Vaidman (1996, 1997, 1998a, b, 1999a, b), Miller (1997), Chiatti (1995). In particular, Price's advocacy of "advanced action" is very much along the lines of Cramer's TI. Cramer has continued to promote TI, in Cramer (1997, 2001).

Now to some further details of the "transaction" in the TI picture (for a detailed description, see Cramer, 1986). In what follows, a "pseudotime sequence" of events is described,[1] but it must be remembered that the process is an atemporal one and that the various component events do not precede or follow each other in any particular time sequence. To an observer who could discern the individual steps, they would all appear to happen "at once".

In the TI, a quantum system is produced by a source S which plays the role of an "emitter" in the Wheeler–Feynman theory. However, what the source emits is *both* a quantum mechanical wave Ψ and its time-reversed counterpart, Ψ^* (or, in Dirac notation, $|\Psi\rangle$ and $\langle\Psi|$, respectively). These waves are considered to be physically real. (However, as Cramer is using the term, "real" means something significantly less substantial than what most of us would consider physically real. For example, an electron's Ψ-wave would merely carry the possibility for an electron to be detected; the electron is not identified with its Ψ-wave.)

Cramer refers to the future-directed $|\Psi\rangle$-wave as an "OW", as mentioned above (see Fig. 1). This wave continues on (possibly being attenuated by the medium through which it passes or through spatial divergence) until it interacts with an absorber (D), which absorbs the wave and in response emits a "CW", also having two components, both advanced and retarded. What we are interested in is the worldline connecting the source S and the absorber D, where a retarded OW $|\Psi\rangle$ and an advanced CW $\langle\Psi|$ "overlap."[2] At the point where the advanced CW reaches S, there is a possibility for a transaction to occur, which consists in S reinforcing the CW and giving rise to an actual, observable event (such as the detection of a particle).

2. A specific example of the TI

To the above rough sketch we now fill in some details using a specific example, the famous (or perhaps infamous) three-state or three-box experiment.

[1] This "pseudotime" account of how a transaction occurs is subject to a serious challenge by Maudlin (2002). In Kastner (2006) I argue that TI can survive the consistency part of the Maudlin challenge but that the pseudotime account cannot be taken too seriously as describing the real ontology of transactions. It is included here only as a pedagogical/heuristic aid.

[2] According to the Wheeler–Feynman theory, the advanced wave emitted by the source and the retarded wave emitted by the absorber are exactly out of phase with the CW and OW extending beyond the source and the absorber, respectively, and thus the only nonzero field is a four-vector standing wave between source and absorber (cf. Cramer, 1986, pp. 660–661).

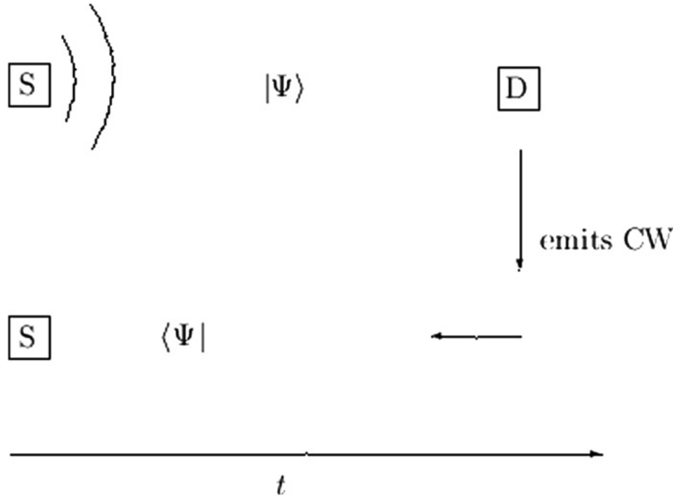

Fig. 1. Schematic diagram of the transaction process.

Fig. 2 shows the basic setup for the three-state experiment. It usually involves three boxes or shutters labeled A, B and C (essentially three possible locations at which a particle could be found). In this version, we use a "three-slit" arrangement, in which detectors might be placed at one or more of the slits. Particles are pre-selected in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|a\rangle + |b\rangle + |c\rangle) \tag{1}$$

and post-selected in the state

$$|\phi\rangle = \frac{1}{\sqrt{3}}(|a\rangle + |b\rangle - |c\rangle) \tag{2}$$

with obvious notation. In the "pseudo time sequence" referred to earlier, what takes place under TI is as follows: particles are created by a source S and then preselected at time t_0 in the state $|\psi\rangle$ (thus any particle not originally in that state will not proceed to the experiment). After t_0, an OW $|\psi\rangle$ propagates in the usual time direction through the apparatus until at time t_1 it encounters the three possible locations, each equipped with detectors **A**, **B**, and **C**, respectively, in the experiment illustrated in Fig. 3.

Let us label this nondegenerate observable (which unambiguously detects which of the three locations the particle passed through) Q. We now consider the OW component propagating through location A; analogous features hold for the other two paths. Detector **A** can be considered as an attenuating filter which admits only components in the state $|a\rangle$. Thus the OW continuing on for $t > t_1$ will have its amplitude reduced by the factor $\langle a|\psi\rangle$ and will now be in the state $|a\rangle$, as shown on the path segment just to the right of **A**. When the OW reaches the final (post-selection) detector at t_2, it is again filtered so that its new amplitude is $\langle\phi|a\rangle\langle a|\psi\rangle$ and its state is $|\phi\rangle$. This OW reaches absorber **D** and is absorbed.

Now, according to TI the absorber must emit an "advanced" (complex conjugate) CW of the same amplitude; thus the new CW is characterized by $\langle\psi|a\rangle\langle a|\phi\rangle\langle\phi|$, as shown on the upper (A) path segment to the left of **D**. This CW continues in the negative time direction (to the left on the figure) toward detector **A**, where it is filtered as before, acquiring a factor of $\langle\phi|a\rangle$ for a new state of $\langle\psi|a\rangle|\langle a|\phi\rangle|^2\langle a|$. "Finally" (in the pseudotime sequence), the CW reaches the pre-selection filter where its amplitude is further reduced by a factor of $\langle a|\psi\rangle$, resulting in the final CW state $|\langle\psi|a\rangle|^2|\langle a|\phi\rangle|^2\langle\psi|$. According to TI, the source S may be stimulated to reinforce this CW with a probability proportional to its amplitude (which is now identical with the square of the amplitude of the final OW, and which

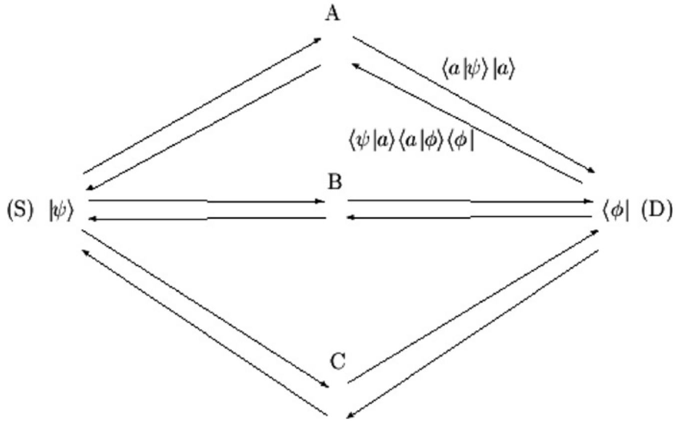

Fig. 2. Three-state experiment with detectors at all three locations.

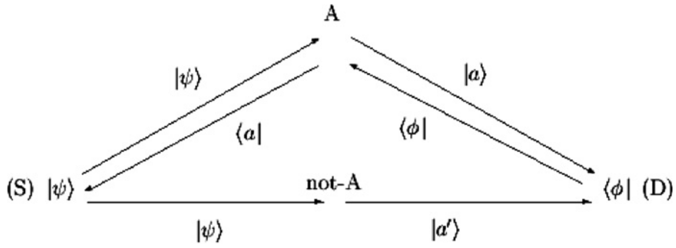

Fig. 3. No confirmation wave can be returned along the path corresponding to not-A.

therefore provides an ontological understanding of the origin of the Born probability rule). Such a reinforcement constitutes a "transaction," with the consequent realization of an actual particle transmitted between S and D and having been localized at A at the time t_1.

Similar processes occur for the other possible paths B and C, with analogous final amplitudes $|\langle\psi|b\rangle|^2|\langle b|\phi\rangle|^2$ and $|\langle\psi|c\rangle|^2|\langle c|\phi\rangle|^2$. In this case, all these final amplitudes are equal to $\frac{1}{3}$ and the particle's probability of being found at each of the three locations is therefore $\frac{1}{3}$.

A different situation occurs when a degenerate observable is measured, which only detects whether a particle went through a single particular location. Fig. 3 depicts such a measurement, of observable A corresponding to a detector **A** only at location A. In this situation, the analysis of the OW component along the path corresponding to **A** is the same as in the measurement of nondegenerate observable Q (as shown in Fig. 2—in Fig. 3 the amplitudes are omitted and only the bra/kets are shown); however, the remainder of the pre-selection state OW looks quite different as it proceeds through the apparatus. This component now corresponds to a state orthogonal to $|a\rangle$, namely the state $|a'\rangle = (1/\sqrt{2})(|b\rangle + |c\rangle)$. This state is also orthogonal to the post-selection state $|\phi\rangle$ and will therefore not be passed through the final filter $|\Phi\rangle$; thus it cannot be absorbed by D, and no CW can be returned. Thus no transaction is possible for components of the initial OW not detected by **A**. The only transaction possible for the experiment measuring observable A occurs for the OW component which passes filter **A**. An analogous situation holds for measurement of the degenerate observable B with a detector **B** placed only at location B.

Yet a different situation occurs when a detector **C** is placed only at location C; this is pictured in Fig. 4. In this case, both components of the OW—the component $\langle c|\psi\rangle|c\rangle$ detected by **C** as well as the component $\langle c'|\psi\rangle|c'\rangle$ (where $|c'\rangle = (1/\sqrt{2})(|a\rangle + |b\rangle)$) not detected by **C**—are able to reach the absorber D, since neither is orthogonal to the final state $|\phi\rangle$. Therefore two transactions are possible: (1) a transaction corresponding to the component of the OW detected by **C**, which has final amplitude

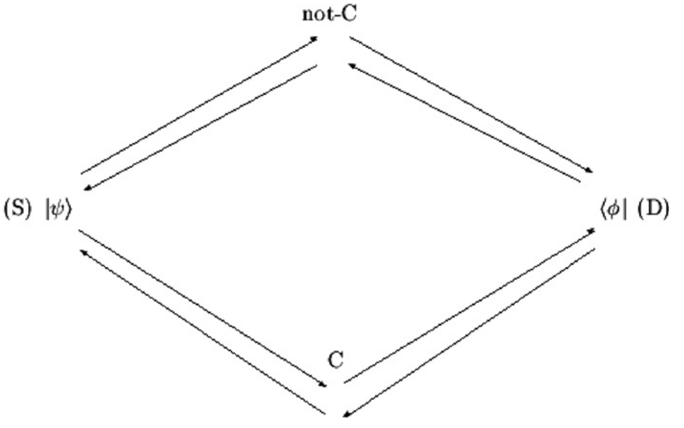

Fig. 4. Transactions are possible on either path, not-C or C.

$|\langle\psi|c\rangle|^2|\langle c|\phi\rangle|^2 = \frac{1}{9}$ and (2) a transaction corresponding to the component of the OW not detected by **C**, with final amplitude $|\langle\psi|c'\rangle|^2|\langle c'|\phi\rangle|^2 = \frac{4}{9}$.

We will return to this example when discussing weak values, which are introduced in the next section.

3. Weak values in the TI picture

"Weak values" are quantities introduced by Aharonov and Vaidman (1990) in the context of pre- and post-selection experiments (such as the three-state example discussed above).

The weak value of the operator O with respect to states $|a\rangle$ and $|b\rangle$ is defined as

$$\langle O\rangle_{\mathrm{w}} = \frac{\langle b|O|a\rangle}{\langle b|a\rangle} \tag{3}$$

Several authors have used weak values as indicators of properties in pre- and post-selected systems, and as answers to counterfactual questions about the properties of such systems between measurements. For example, in the three-box case introduced above, the degenerate operator C (represented in Fig. 4) has a weak value of -1 for the given pre- and post-selection states. Vaidman (1996) has claimed that this should be interpreted as an "element of reality" indicating that there is "-1" particle in box C in this experiment, and that this prediction could be confirmed by measuring the pressure in that box and finding it to be negative.[3]

In this section we look at the weak values of projection operators in particular (since these are the ones used to make the above sorts of inferences about the properties of pre- and post-selected systems), and consider what might be the ontology underlying weak values in the TI picture.

First, we note that the weak value of an operator A when the pre- and post-selection states are the same (say $|\psi\rangle$) is identical to the usual expectation value $\mathrm{Exp}_{\psi}(A) = \langle\psi|A|\psi\rangle$. It is of course well known that if A is a projection operator corresponding to some outcome a then

$$\mathrm{Exp}_{\psi}(A) = \mathrm{Pr}_{\psi}(a) \tag{4}$$

that is, the expectation value of a projection operator gives the Born probability of the associated outcome. Should we then think of weak values as analogous to some kind of generalized probability, or expectation value, for the pre- and post-selection case?

[3] It is not clear how this could be done, since one must throw out runs not fulfilling the post-selection criterion. Thus the qualifying particles can only be collected in one place *after* post-selection; whereas measuring pressure would seem to require the existence of a collection of appropriately pre- and post-selected particles in box C at the same time.

The TI provides an interesting account of the similarity in form between weak and expectation values of projection operators which shows that these two quantities have very different ontological interpretations and that the answer to the above question should be negative. First, Fig. 5 shows how the Born Rule arises under TI (one can think of the states in this figure as the ones with the same labels in the three-state example).

An OW is emitted by a source *S* and passes through a filter (such as a polarizing filter) preparing the system in state $|\psi\rangle$. It then passes through another filter which allows only the component $|a\rangle$ corresponding to the projection operator *A*. Finally it is absorbed at detector *D*. The detector, according to TI, emits an advanced CW whose amplitude is the complex conjugate of the OW as it arrives as *D*. The CW undergoes an analogous filtering process—essentially the time reverse of that described above—and arrives back at *S*. Its amplitude at *S* reflects the Born Rule corresponding to the probability of the system's being found in state $|a\rangle$ given that it was prepared in state $|\psi\rangle$.

Next, Fig. 6a shows the situation corresponding to the weak value $\langle A\rangle_w = \langle\phi|A|\psi\rangle/\langle\phi|\psi\rangle$.

The experiment begins in the same manner as in Fig. 5, but an additional ϕ (post-selection) filter is added after the *A* filter. The quantity $\langle\phi|A|\psi\rangle$, which is just the unnormalized weak value, now describes the amplitude of the *OW only* just before it reaches the detector *D*. A CW will of course be returned as before, but the point is that the (unnormalized) weak value of *A* characterizes only the

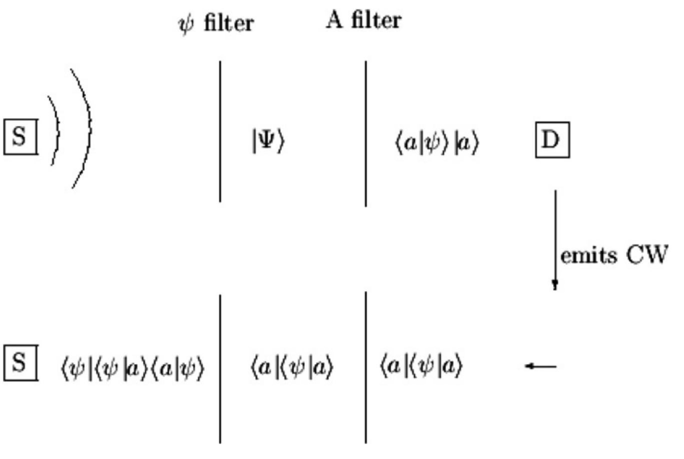

Fig. 5. The TI picture of the Born probability.

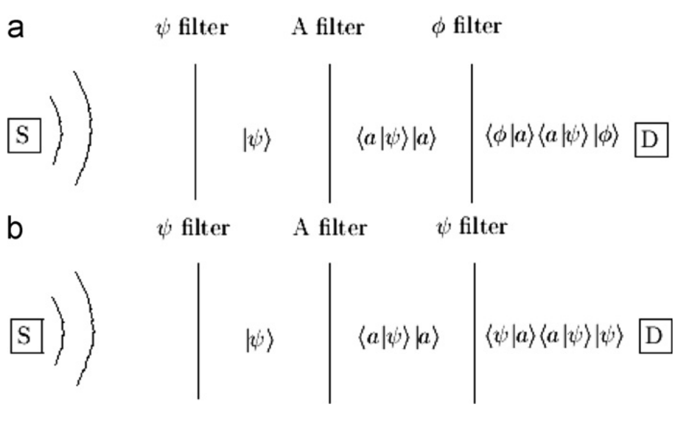

Fig. 6. The TI picture of the weak value.

OW portion of the experiment. (Note that, with the states identified with those in the three-box example, the weak value of A is unity.)

Now, if we replace the final ϕ filter with a ψ filter (Fig. 6b), we see that the weak value looks formally identical to the expectation value $\text{Exp}_\psi(A)$, equivalently the Born probability, $\Pr_\psi(a)$, shown in Fig. 5. Yet the weak value of the projection operator A and its expectation value (understood as the Born probability) correspond to two very different physical situations under TI: one characterizes only an OW component, and the other characterizes an incipient transaction in which offer and CW have overlapped. The Born probably is the amplitude of a comfirmation wave while the weak value of A in the pre- and post-selection state ψ is the amplitude of an offer wave.

Therefore the TI picture reinforces the conclusion in Kastner (2004) that weak values should be thought of as corresponding to amplitudes in pre- and post-selection experiments rather than as generalized expectation values. (The above conclusion was arrived at in that paper in part by noting that standard quantum mechanics admits the kind of analysis accompanying Fig. 6—that the amplitude of the state vector of the system just prior to detection is equal to the unnormalized weak value.) Under TI, weak values characterize the amplitudes of OW (or CW) having passed through several measurement events (such as the filters in the above discussion). They do not correspond to the amplitudes of incipient transactions in which offer and CW have overlapped.

While some authors, such as Vaidman (1996) and Aharonov et al. (2002) have made weak values the basis of ontological claims,[4] according to the TI picture this would be unwarranted since weak values correspond only to amplitudes, rather than to expectation values understood as sums of eigenvalues weighted by Born probabilities. In connection with the three-box example discussed in the previous section, this means that the weak value of unity for A (or B) should not be interpreted as indicating the possession of a property such as being located in box A (or B); nor should the weak value of -1 for C be taken as indicating "-1 particle in box C."

However, we can now see that the "multiple time probability" naturally applicable in the context of an experiment like that in Fig. 6 would be obtained by following the resulting CW back through the apparatus until it reaches the source—as in the discussion accompanying Fig. 2 of Section 2. The final amplitude would be the absolute square of the OW amplitude; that is,

$$\Pr(a\&\phi|\psi) = |\langle\phi|A|\psi\rangle|^2 \tag{5}$$

This is also, of course, a standard result in quantum theory for the probability of a sequence of events (cf. Bub, 1997, p. 232). We can obtain from this what looks like the conditional probability of outcome a given the pre- and post-selection by using the usual rules of probability (that is, dividing the right-hand side of (5) by the probability of state ϕ given ψ); the result is

$$\Pr(a|\psi\&\phi) = \frac{|\langle\phi|A|\psi\rangle|^2}{|\langle\phi|\psi\rangle|^2} = |\langle A\rangle_{\text{w}}|^2 \tag{6}$$

The above result (i.e., the right-hand side) was obtained in Kastner (2004) but, as was pointed out there, (6) is not a well-behaved probability[5] unless certain additional "consistency" conditions apply. We will therefore call it a "pseudoprobability." In the following sections we consider what significance the pseudoprobability has when it is rendered well behaved through appropriate additional constraints.

4. TI and the possession of properties

In the TI, the possession of properties corresponding to values of observables is underdetermined by such theoretical quantities as probabilities or weak values. Instead, the precise nature of the particular experiment must be specified, as the latter will determine what types of transactions are

[4] Aharonov et al. (2002) also find a weak value of -1 arising in the Hardy (1992) experiment and also interpret it as indicating "-1 particles" present at a certain location.

[5] For example, it yields values larger than 1.

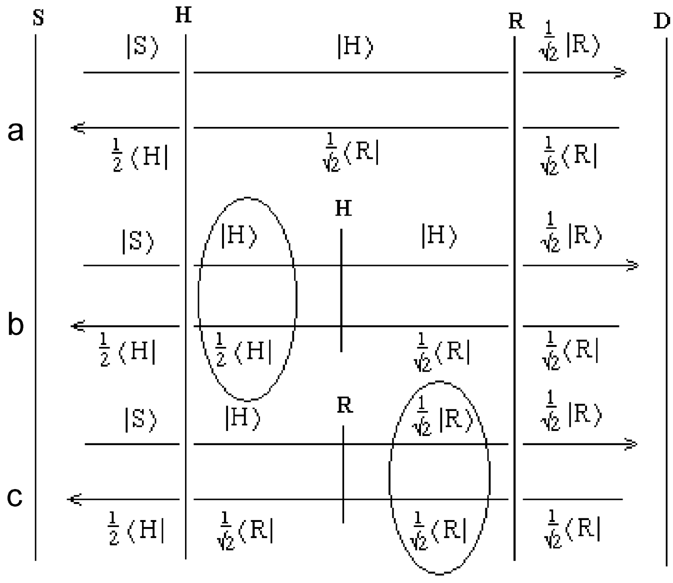

Fig. 7. Cramer's example of a photon/polarizer experiment.

possible; and under the TI, property possession corresponds to a special type of transaction (to be described below).

For example, under TI it may the case that a system seems to be characterised by a weak value of unity (which means, according to Eq. (6), that it also has a pseudoprobability and possibly even a "proper probability" of unity) but at no time will the system actually possess the property corresponding to that weak value. A simple illustration is the example presented in Cramer (1986, p. 690), Fig. 14, (reproduced here as Fig. 7) which schematically presents a pre- and post-selection setup for a polarized photon (isomorphic to a spin-$\frac{1}{2}$ particle). The photon is pre-selected in a horizontal state of polarization (H) (which we can think of as analogous to a state of "spin up along z" for a spin-$\frac{1}{2}$ particle; the state of vertical polarization (V) would be orthogonal and would correspond to "spin down along z") and post-selected in a right-circular state of polarization (R) (which we can think of as analogous to a state of "spin up along x"; the orthogonal state is left-circular polarization (L)).

It is well known that the time-symmetric probabilities (and therefore the weak values[6]) associated with either H or R in the intervening time between pre- and post-selection are both unity. Explicitly, for the weak values we have

$$\langle H\rangle_{\mathrm{w}} = \frac{\langle R|H|H\rangle}{\langle R|H\rangle} = 1; \quad \langle R\rangle_{\mathrm{w}} = \frac{\langle R|R|H\rangle}{\langle R|H\rangle} = 1 \tag{7}$$

However, according to TI what actually happens to the system is underdetermined by these values. In fact, several different experiments are possible in this situation, and only one experiment is applicable to each ontological claim about a particular observable. This is because under TI, a system only has a determinate property corresponding to an eigenvalue x when both the retarded OW and the advanced CW "overlap" and reinforce each other in states $|x\rangle$ and $\langle x|$, respectively.

As shown in the figure, despite the fact that the theoretical weak value of H is unity, when experiment (a) is performed, the system never achieves a determinate value for H throughout the time interval between pre- and post-selection. Only under experiment (b) can the system be said to

[6] As shown in Aharonov & Vaidman (1990), a probability of unity or zero for an observable under pre- and post-selection corresponds to a weak value of unity or zero, respectively; the converse is also true for two-valued observables.

possess H with certainty, and then only in the time interval prior to the H measurement. Similarly, under experiment (c) the system can be said to possess the property (R) with certainty, but only in the interval after the R measurement.

Thus, under TI, the value $\langle H\rangle_w = 1$ cannot be interpreted as indicating a possessed property of a given system unless H is actually measured (and, even then, only in the appropriate time interval where OW and CW overlap in the same state). This addresses the "paradox" in which a system seems to be characterized by determinate values for noncommuting observables. But according to TI, such values do not apply together if both observables cannot be simultaneously measured.

What can TI say about the controversial problem of the counterfactual usage of the time-symmetric Aharonov–Bergmann–Lebowitz "ABL Rule" for the probability of outcomes of measurements performed between pre- and post-selection[7]—that is, the question of whether the rule yields valid probabilities for outcomes corresponding to observables that have not actually been measured at time t_1? It has been argued by some authors (Cohen, 1995; Griffiths, 2002; Kastner, 1999a, b) that the ABL rule can be counterfactually applied only in cases in which the relevant histories form a consistent family. Though Vaidman (1998a) has objected that this restricts the counterfactual usage of the ABL rule to uninteresting cases, under TI some of these cases have more ontological content than has previously been supposed. On the other hand, under TI we find reason to think that consistency is not always enough to support the counterfactual usage of the ABL rule. Before dealing with this issue, we briefly review the consistent histories formulation.

5. TI and consistent histories

In the consistent histories formulation pioneered by Griffiths (cf. 1996, 1999, 2002), one can assign standard "classical" or Kolmogorov-type probabilities for different sequences of events, called 'histories," provided that the set of such histories fulfills a consistency criterion (see below) which ensures that probabilities for distinct histories are additive. A history F is a projector on the multiple-time Hilbert space $\breve{H}$ of the system corresponding to the number n of events/times considered, i.e.:

$$\breve{H} = H_0 \odot H_1 \odot H_2 \odot \cdots \odot H_n$$

In Griffiths' terms, such a set of histories corresponds to a particular decomposition of the multiple-time identity $\breve{I}$, into a set of "minimal elements" $\{F_i\}$:

$$\breve{I} = \sum_i F_i \tag{8}$$

We can consider F_i as the "atomic" or most specific histories belonging to the particular decomposition.[8] The consistency condition is (for zero Hamiltonian; this is easily generalized to cases of nonzero Hamilton; cf. Griffiths, 1996, pp. 2761–2762).

$$\mathrm{Tr}[F_i^\dagger F_j] = 0 \tag{9}$$

for all $\{i,j\}$ in the given decomposition of $\breve{I}$.

In the three-state experiment illustrated in Figs. 2, 3 and 4, it has been noted that each of the setups for measuring observables A and B (both of which have a weak value of unity) corresponds to a consistent family of histories (cf. Cohen, 1995; Kastner, 1999b). The experiment in which A is measured (call this "Experiment A") corresponds to a decomposition of the multiple-time identity $\breve{I}$ as

$$\breve{I} = \{\Psi_0 + \tilde{\Psi}_0\}\{A_1 + \tilde{A}_1\}\{\Phi_2 + \tilde{\Phi}_2\} \tag{10}$$

[7] The "ABL" rule was formulated in ABL (1964). It is a time-symmetric expression giving the conditional probability for the outcome of a measurement performed between pre- and post-selection measurements. As shown in Cohen (1995) and further discussed in Kastner (1999b), the ABL rule, which is a well-behaved probability, is a special case of the "pseudoprobability" (6) which obtains when the associated family of histories is consistent.

[8] More general histories Y can be constructed from the minimal elements according to $Y = \sum_i v_i F_i$, where v is either zero or one.

where capital letters denote the projector corresponding to the state with the same lower-case label, a subscript denotes the time index of the event, and $\tilde{\Psi} = 1 - \Psi$. This family of histories is consistent according to (9). The decomposition corresponding to the experiment in which B is measured (obtained by replacing A with B in the above expression—call this "Experiment B") is similarly consistent.

Note that neither the experiment in which detectors are placed at all three locations, nor the experiment in which a detector is placed only at location C, satisfy the consistency criterion. The feature these experiments have in common is that there is more than one OW component that can reach the final detector. In contrast, the consistent families obtaining in the Experiments A and B are characterised by the availability of only a single path for the OW from the source to the detector. Trivially, then, any detected particle had to have gone through location A (or B, if Experiment B is being performed).

As noted above, the issue of consistency in pre- and post-selected systems was initially raised in Cohen (1995) in the context of his critique of the counterfactual usage of the Aharnov–Bergmann–-Lebowitz (ABL) rule. Cohen argued that the counterfactual usage was generally not valid, but that it was valid when the sequence of events, including the counterfactual measurement, satisfied the Griffiths consistency condition.

As an illustration of how such counterfactuals fare under TI, consider again the example of a polarized photon discussed by Cramer (our Fig. 7). Suppose that in the actual world no measurement was made at time t_1 (no polarizing filter was inserted between the initial and final filters, as depicted in process (a) in the figure). Then, according to TI, the photon had no determinate value for either its linear or circular polarization throughout the time interval $[t_0, t_2]$. But the set of histories corresponding to the partition of the multiple-time identity including a measurement of, say, the linear polarization at time t_1 is consistent (as is well known; cf. Griffiths, 1996, p. 2769) for the (isomorphic case of a spin-$\frac{1}{2}$ particle). Specifically, the relevant decomposition is

$$\check{I} = \{H_0 + V_0\}\{H_1 + V_1\}\{R_2 + L_2\} \tag{11}$$

Suppose we can indeed use the ABL rule counterfactually for such consistent families. Then the ABL rule can be used to obtain the counterfactual probability of outcome H at the time t_1, which in this case is unity. Assuming that perfect certainty of an outcome at a particular time corresponds to property possession at that time, this result tells us that, *had* an H-type filter been in place at time t_1, the particle would definitely have been horizontally polarized at that time. Under TI, this means that its OW and CW would have overlapped in states $|h\rangle$ and $\langle h|$, respectively. This conclusion now differs ontologically from what was the actual case when no such measurement occurred at t_1. For in the actual case, the particle had no determinate value for its polarization: that is, it would be correct neither to say that the particle possessed property h, nor that it possessed property r, throughout the time interval in question (between the H and R measurements).

On the other hand (as noted at the end of Section 4), TI provides reason to think that consistency, while a necessary condition for validity of the counterfactual usage of the ABL rule, is not sufficient. To see why, look at the three-box example and consider the counterfactual claim (based on the given pre- and post-selection):

> CFA: "Given that I actually made no measurement of position A, B, or C at time t_1, *if* I had opened box A in the intervening time, the particle would definitely (with ABL probably 1) have been there."

Were consistency sufficient, CFA would be valid, since adding the measurement of A to the family of histories defined by the given pre- and post-selection results in a consistent family. (Griffiths refers to this procedure as a "refinement"; cf. Griffiths, 1996, pp. 2763–2764.)

But now consider also

> CFB: "Given that I actually made no measurement of position A, B, or C at time t_1, *if* I had opened box B in the intervening time, the particle would definitely (with ABL probably 1) have been there."

Since we can alternatively make a consistent "refinement" of the pre- and post-selection event set through the addition of B and its complement at t_1 to the partition of $\check{I}$, CFB has to be just as valid as A. Though the consistency of the event sets underlying each of these claims separately assures that they are based on a well-behaved probability expression, we are nevertheless faced with contrary[9] dispositional conclusions[10] regarding the same particle, and which have no ontological support from either the pre- or post-selection state (in contrast to the example of Fig. 7).

Griffiths deals with the above difficulty by saying that one cannot "combine" the two contrary inferences (as does Vaidman). Yet the problem that remains is that each statement, "separately," is claimed to be true, and each applies to the *same* particle. It is important to keep in mind that each counterfactual above is a claim about what the result of an intermediate A or B measurement would definitely have been when no such measurement was actually performed, so the fact that they both apply to the same particle (whether or not we explicitly "combine" them in a linguistic sense) constitutes a strong contrary dispositional claim. Such a claim is substantively different from the well-known fact that quantum particles can be in superpositions of classically distinct states independent of specific measurement claims. To argue that a particle subject to CFA and CFB is not in a contradictory property state[11] because of a linguistic maneuver (i.e., applying Bell's "unspeakable" rule to one or the other of a set of statements about a given particle, each of which is claimed to be true) is not, it seems to this author, to solve the fundamental difficulty raised by the two contrary counterfactuals CFA and CFB.

TI provides a solution to the above condundrum on the ontological rather than the semantic level. It is based on a distinction between these two cases that only appears under the TI picture and which shows why consistency assures a valid counterfactual interpretation for the first (polarization) case but not the second (three-box) case.

The insight offered into this situation by TI is that, in the three-box experiment, the offer and CW never overlap in the same state even when the A (or B) filter is present (see Fig. 3). Thus the particle *never has a determinate value of the property corresponding either to location A or B in this experiment*. This certainly seems counterintuitive—one might say, "if I observed the particle to be in the box, how can it not be determinately "in the box"? Yet it must be remembered that if we actually did open a particular box and find the particle there, it would be detected and a CW would be generated. Then an overlap of offer and CW in the same box state would result, and we could say that the particle possessed that property. But in the absence of such an actual detection in a particular box, and *in the context of the impending post-selection measurement*, which does not commute with the box location observable, "measuring" the box location state does not result in unambiguous possession of that property in the sense of an overlap of offer and CW in the same state. (This is because the kind of "measurement" assumed in the three-box experiment just corresponds to placing a filter which only allows the $|a\rangle$ or $|b\rangle$ component to pass—the particle has not actually been detected at the intermediate time unless it is stopped by the filter, in which case it will never be post-selected.)

An alternative argument is based on the following intuition: for a counterfactual claim like CFA (or CFB) to be true, it must be the case that the counterfactual measurement being contemplated does not "disturb" the post-selection of the particle; it "would have been post-selected anyway" regardless of the measurement.[12] In the Cramer polarization example, the intervening H or R measurement is benign in that it either (i) simply verifies the preselection or (ii) moves the post-selection up to an earlier time.[13] Since an ontological trace of the post-selection result (R) "already" exists in the form of the CW traveling backward in time from the post-selection, the particle in

[9] We use "contrary" in the same sense as Kent, who raises a similar objection in Kent (1997).

[10] Assuming that dispositions can be represented by counterfactuals in the usual way.

[11] Here, "contradictory property state" is taken to mean that the particle in question is predicted to be found with certainty upon measurement in two different boxes depending on which measurement happens to be performed.

[12] This consideration addresses the requirement that background conditions used to evaluate counterfactuals must be *cotenable* with the antecedent; i.e., invoking the antecedent does not affect the truth the background conditions. For a detailed discussion, cf. Kastner (1999a) or (2003).

[13] For sticklers for time symmetry, the same argument can be made in the reverse time direction, changing the roles of the pre- and post-selection measurements.

question is "fated" to be found in the state *R*—so it doesn't matter *when* that measurement takes place.

In contrast, a contemplated measurement of *A* envisions the pre-selected particle having to "decide" between states $|a\rangle$ and states $|a'\rangle$ (the latter corresponding to "not-a."). This decision was never actually made, however, in the performed experiment; so there is therefore no basis for assuming that counterfactually, the particle would choose $|a\rangle$. (The same observation of course applies to a contemplated measurement of *B*.) The only "choice" the particle actually made was in going from the pre-selection state $|\psi\rangle$ to the post-selection state $|\phi\rangle$ as opposed to some other eigenstate of the post-selection observable. The fact that the particle could be considered as having been "fated" to finally choose $|\phi\rangle$ from its initial preparation in state $|\psi\rangle$ provides no basis for a preference either for $|a\rangle$ over $|a'\rangle$ or $|b\rangle$ over $|b'\rangle$ at an intermediate time. This feature is reflected in the lack of determinateness of the property *a* (or *b*) under TI, when we look at the nature of the offer and CW arising in the experiment (see Fig. 3 and note that overlapping bras and kets differ).

In terms of cotenability, what this means is that one cannot claim either of the following statements about background conditions necessary for validity of the counterfactuals CFA and CFB (see footnote 11):

(a) if I had measured *A*, the particle would still have been post-selected or
(b) if I had measured *B*, the particle would still have been post-selected.

This is because the conditions of the actual world (i.e., the actual offer and CW), in which neither *A* nor *B* was measured, contain no information about a transition between *A* (or *B*) eigenstates to the post-selection state, in contrast to the polarization example of Fig. 7.[14]

The underlying ontology of the two cases is very different, even though the ABL probability of unity applies in both cases and they each satisfy the consistency criterion. The difference is evident only when one examines the nature of the offer and CW arising in each experiment. In one, there is an ontological basis for applying the ABL rule counterfactually; in the other, there is not.

Thus, provided one is willing to entertain the nonstandard ontology of TI, it has told us something new about the attribution of properties in the context of pre- and post-selection experiments. A consistent family, it turns out, is a necessary but not sufficient condition for a valid counterfactual usage of the ABL rule. What is also required is that offer and CW would "concur" as to the system's state in the time interval immediately prior to, or subsequent to, the time index of the counterfactual measurement.

6. Conclusion

Cramer's transactional interpretation has been applied to several commonly discussed pre- and post-selection experiments. It has been argued that TI provides insight into the nature of time-symmetric weak values: namely, that they should be interpreted as multiple-time amplitudes, rather than as generalized expectation values. Weak values of projection operators—even when the pre- and post-selection states are the same—do not reflect Born probabilities, which under TI arise from the overlap of offer and confirmation waves, but instead characterize the amplitude of an offer wave component only.

It has also been argued that TI provides reason to think that property inferences based on a counterfactual usage of the ABL rule require more than satisfaction of the Griffiths consistency criterion; it is also required that the system's offer and confirmation waves overlap in the same state corresponding to the claimed possessed property, in the time interval directly adjacent (either before or after) the counterfactual measurement.

[14] Once again, in keeping with time symmetry one may interchange the roles of the pre- and post-selection state in this discussion.

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