arXiv:gr-qc/0006061 v3 23 Jan 2001

Notes for a brief history of quantum gravity

Presented at the 9th Marcel Grossmann Meeting in Roma, July 2000

Carlo Rovelli
Centre de Physique Théorique, CNRS Luminy, 13288 Marseille, France
Physics Department, University of Pittsburgh, Pittsburgh, PA 15260, USA.
rovelli@cpt.univ-mrs.fr

April 20, 2001

Abstract

I sketch the main lines of development of the research in quantum gravity, from the first explorations in the early thirties to nowadays.

1 Introduction

When John Stachel asked me to prepare a brief history of the research in quantum gravity for the 9th Marcel Grossmann Meeting, I trembled at the size of the task, worried of repeating only information already known to everybody, and feared to displease my colleagues. John managed to convince me to try anyway, and here is the result. I have much enjoyed spending time in the "old archives" section of my library, and I have been surprised by some of the things I have found. I am enormously indebted with the many friends that, after the archive posting of the first draft of this work, have pointed our errors, omissions and imperfections. With their invaluable help, this history is a bit less biased and a bit less incomplete.

I have focused on quantum gravity in the strict sense: the search for a theory that could describes the quantum behavior of the full gravitational field. Thus, I do not cover important related subjects such as quantum fields in curved spacetime, applications such as cosmology related research, work on the structure of quantum constrained systems, black hole thermodynamics or extensions of quantum mechanics to general covariant theories. For lack of energy, I have also decided not to cover the numerical and lattice-like approaches to the theory – which is a serious absence.

I have no ambition of presenting complete references to all the important works; some of the references are to original works, others to reviews where reference can be found. Errors and omissions are unfortunately unavoidable and I apologize for these. I have made my better effort to be balanced, but

in a field that has not yet succeeded in finding consensus, my perspective is obviously subjective. Trying to write history in the middle of the developments is hard. Time will go along, dust will settle, and it will slowly become clear if we are right, if some of us are right, or –a possibility never to disregard– if we all are wrong.

2 Three main directions

An evident peculiarity of the research in quantum gravity is that all along its development it can be separated into three main lines of research. The relative weight of these lines has changed, there have been important intersections and connections between the three, and there has been research that does not fit into any of the three lines. Nevertheless, the three lines have maintained a distinct individuality across 70 years of research. The three main lines are often denoted "covariant", "canonical", and "sum over histories", even if these names can be misleading and are often confused. They cannot be characterized by a precise definition, but within each line there is a remarkable methodological unity, and a remarkable consistency in the logic of the development of the research.

The covariant line of research is the attempt to build the theory as a quantum field theory of the fluctuations of the metric over a flat Minkowski space, or some other background metric space. The program was started by Rosenfeld, Fierz and Pauli in the thirties. The Feynman rules of general relativity (GR, from now on) were laboriously found by DeWitt and Feynman in the sixties. t'Hooft and Veltman, Deser and Van Nieuwenhuizen, and others, found firm evidence of non-renormalizability at the beginning of the seventies. Then, a search for an extension of GR giving a renormalizable or finite perturbation expansion started. Through high derivative theory and supergravity, the search converged successfully to string theory in the late eighties.

The canonical line of research is the attempt to construct a quantum theory in which the Hilbert space carries a representation of the operators corresponding to the full metric, or some functions of the metric, without background metric to be fixed. The program was set by Bergmann and Dirac in the fifties. Unraveling the canonical structure of GR turned out to be laborious. Bergmann and his group, Dirac, Peres, Arnowit Deser and Misner completed the task in the late fifties and early sixties. The formal equations of the quantum theory were then written down by Wheeler and DeWitt in the middle sixties, but turned out to be too ill-defined. A well defined version of the same equations was successfully found only in the late eighties, with loop quantum gravity.

The sum over histories line of research is the attempt to use some version of Feynman's functional integral quantization to define the theory.

Hawking's Euclidean quantum gravity, introduced in the seventies, most of the the discrete (lattice-like, posets ...) approaches and the spin foam models, recently introduced, belong to this line.

Others. There are of course other ideas that have been explored:

- Twistor theory has been more fruitful on the mathematical side than on the strictly physical side, but it is still actively developing.
- Noncommutative geometry has been proposed as a key mathematical tool for describing Planck scale geometry, and has recently obtained very surprising results, particularly with the work of Connes and collaborators.
- Finkelstein, Sorkin, and others, pursue courageous and intriguing independent paths.
- Penrose idea of a gravity induced quantum state reduction have recently found new life with the perspective of a possible experimental test.
- ...

So far, however, none of these alternatives has been developed into a large scale research program.

3 Five periods

Historically, the evolution of the research in quantum gravity can roughly be divided in five periods.

The Prehistory: 1930-1959. The basic ideas of all three lines of research appear very early, already in the thirties. By the end of the fifties the three research programs are clearly formulated.

The Classical Age: 1960-1969. The sixties see the strong development of two of the three programs, the covariant and the canonical. At the end of the decade, the two programs have both achieved the basic construction of their theory: the Feynman rules for the gravitational field on one side and the Wheeler-DeWitt equation on the other. To get to these beautiful results, an impressive amount of technical labour and ingenuity has proven necessary. The sixties close –as they did in many other regards– with the promise of a shining new world.

The Middle Ages: 1970-1983. The seventies soon disappoint the hopes of the sixties. It becomes increasingly clear that the Wheeler-DeWitt equation is too ill defined for genuinely field theoretical calculations. And evidence for the non-renormalizability of GR piles up. Both lines of attach have found their stumbling block.

In 1974, Steven Hawking derives black hole radiation. Trying to deal with the Wheeler-DeWitt equation, he develops a version of the sum over history as a sum over "Euclidean" (Riemannian) geometries. There is excitement with the idea of the wave function of the universe and the approach opens the way for thinking and computing topology change. But for field theoretical quantities the euclidean functional integral will prove as weak a calculation tool as the Wheeler-DeWitt equation.

On the covariant side, the main reaction to non-renormalizability of GR is to modify the theory. Strong hopes, then disappointed, motivate extensive investigations of supergravity and higher derivative actions for GR. The landscape of quantum gravity is gloomy.

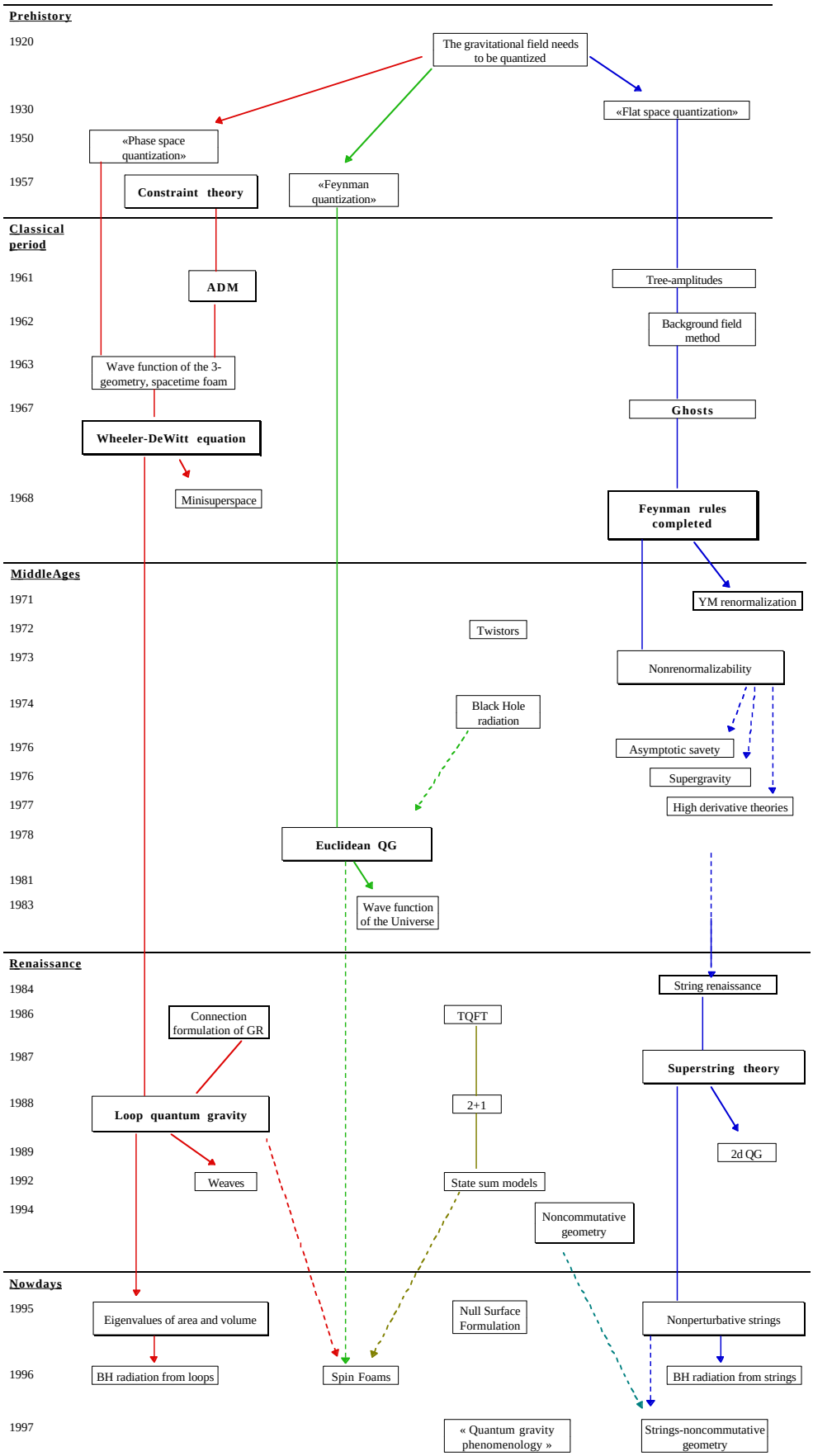
The Renaissance: 1984-1994. Light comes back in the middle of the eighties. In the covariant camp, the various attempts to modify GR to get rid of the infinities merge into string theory. Perturbative string theory finally delivers on the long search for a computable perturbative theory for quantum gravitational scattering amplitudes. To be sure, there are prices to pay, such as the wrong dimensionality of spacetime, and the introduction of supersymmetric particles, which, year after year, are expected to be discovered but, so far, are not. But the result of a finite perturbation expansion, long sought after, is to good to be discarded just because the world insists in looking different from our theories.

Light returns to shine on the canonical side as well. Twenty years after the Wheeler-DeWitt equation, loop quantum gravity finally provides a version of the theory sufficiently well defined for performing explicit computations. Here as well, we are far from a complete and realistic theory, and scattering amplitudes, for the moment, can't be computed at all, but the excitement for having a rigorously defined, nonperturbative, general covariant and background independent quantum field theory, in which physical expectation values can be computed, is strong.

Nowadays: 1995-2000. Both string theory and loop quantum gravity grow strongly for a decade, until, in the middle of the nineties, they begin to deliver physical results. The Bekenstein-Hawking black hole entropy formula is derived within both approaches, virtually simultaneously. Loop quantum gravity leads to the computation of the first Planck scale quantitative physical predictions: the spectra of the eigenvalues of area and volume.

The sum over histories tradition, in the meanwhile, is not dead. In spite of the difficulties of the euclidean integral, it remains as a reference idea, and guides the development of several lines of research, from the discrete lattice-like approaches, to the "state sum" formulation of topological theories. Eventually, the last motivate the spin foam formulation, a translation of loop quantum gravity into a Feynman sum over histories form.

The search for a quantum theory of the gravitational field



Other ideas develop in the meanwhile, most notably noncommutative geometry, which finds intriguing points of contact with string theory towards the end of the decade.

The century closes with two well developed contenders for a quantum theory of gravity: string theory and loop quantum gravity, as well as a set of intriguing novel new ideas that go from noncommutative geometry to the null surfaces formulation of GR, to the attempt to merge strings and loops. And even on a very optimistic note: the birth of a new line of research, the self-styled "quantum gravity phenomenology" which investigates the possibility –perhaps not so far fetched– that Planck scale type measurements might be within reach. And thus that we could finally perhaps know which of the theoretical hypotheses, if any, make sense.

Let me now describe the various periods and their main steps in more detail.

4 The Prehistory: 1930-1957

General relativity was found in 1915. Quantum mechanics in 1926. A few years later, around 1930, Born, Jordan and Dirac are already capable of formalizing the quantum properties of the electromagnetic field. How long did it take to realize that the gravitational field should –most presumably– behave quantum mechanically as well? Almost no time: already in 1916 Einstein points out that quantum effects must lead to modifications in the theory of general relativity [1]. In 1927 Oskar Klein suggests that quantum gravity should ultimately modify the concepts of space and time [2]. In the early thirties Rosenfeld [3] writes the first technical papers on quantum gravity, applying Pauli method for the quantization of fields with gauge groups to the linearized Einstein field equations. The relation with a linear spin-two quantum field is soon unraveled in the works of Fierz and Pauli [4] and the spin-two quantum of the gravitational field, presumably first named "graviton" in a 1934 paper by Blokhintsev and Gal'perin [5], is already a familiar notion in the thirties. Bohr considers the idea of identifying the neutrino and the graviton. In 1938, Heisenberg [6] points out that the fact that the gravitational coupling constant is dimensional is likely to cause problems with the quantum theory of the gravitational field.

The history of these early explorations of the quantum properties of spacetime has been recently reconstructed in a wonderful and fascinating paper by John Stachel [7]. In particular, John describes in his paper the extensive, but largely neglected, work conducted in the mid thirties by a Russian physicist, Matvei Petrovich Bronstein. Bronstein, (who was nephew of Leon Trozky) rederives the Rosenfeld-Pauli quantization of the linear theory, but realizes that the unique features of gravitation require a special treatment, when the full nonlinear theory is taken into account. He realizes that field quantization techniques must be generalized in such a way as to be applicable in the absence of

a background geometry. In particular, he realizes that the limitation posed by general relativity on the mass density radically distinguishes the theory from quantum electrodynamics and would ultimate lead to the need to "reject Riemannian geometry" and perhaps also to "'reject our ordinary concepts of space and time" [8]. For a detailed discussion of Bronstein early work in quantum gravity see ref [9]. The reason Bronstein has remained unknown for so long has partly to do with the fact that he was executed by the Soviet State Security Agency (the NKVD) at the age of 32. In Russia, even today Bronstein is remembered by some as "smarter than Landau". References and many details on these pioneering times are in this fascinating paper by John, which I strongly recommend to the reader. Here, I pick up the historical evolution from after World War II. In particular, I start from 1949, a key year for the history of quantum gravity.

1949

Peter Bergmann starts its program of phase space quantization of non linear field theories [10]. He soon realizes that physical quantum observables must correspond to coordinate independent quantities only [11]. The search for these gauge independent observables is started in the group that forms around Bergmann, at Brooklyn Polytechnic and then in Syracuse. For instance, Ted Newman develops a perturbation approach for finding gauge invariant observables order by order [12]. The group studies the problems raised by systems with constraints and reaches a remarkable clarity, unfortunately often forgotten later on, on the problem of what are the observables in general relativity. The canonical approach to quantum gravity is born.

Bryce DeWitt completes his thesis. He applies Schwinger's covariant quantization to the gravitational field.

Dirac presents his method for treating constrained hamiltonian systems [13].

1952

Following the pioneering works of Rosenfeld, Fierz and Pauli, Gupta [14] develops systematically the "flat space quantization" of the gravitational field. The idea is simply to introduce a fictitious "flat space", that is, Minkowski metric $\eta_{\mu\nu}$, and quantize the small fluctuations of the metric around Minkowski $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$. The covariant approach is fully born. The first difficulty is immediately recognized, in searching the propagator, as coming from the fact that the quadratic term of the Lagrangian is singular, as for the electromagnetic field, and as a consequence of gauge invariance. Gupta's treatment uses an indefinite norm state space as for the electromagnetic field.

1957

Charles Misner introduces the "Feynman quantization of general relativity" [15]. He quotes John Wheeler for suggesting the expression

$$\int exp\{(i/\hbar)(\text{Einstein action})\}\ d(\text{field histories})), \tag{1}$$

and studies how to have a well defined version of this idea. Misner's paper [15] is very remarkable in many respects. It explains with complete clarity notions such as why the quantum hamiltonian must be zero, why the individual spacetime points are not defined in the quantum theory and the need of dealing with gauge invariance in the integral. Even more remarkably, the paper opens with a discussion of the possible directions for quantizing gravity, and lists the three lines of directions, covariant, canonical, and sum over histories, describing them almost precisely with the same words we would today![1]

At the end of the fifties, all the basic ideas and the research programs are clear. It is only a matter of implementing them, and seeing if they work. The implementation, however, turns out to be a rather herculean task, that requires the ingenuity of people of the caliber of Feynman and DeWitt on the covariant side, and of Dirac and DeWitt, on the canonical side.

5 The Classical Age: 1958-1969

1950

The Bergmann group, and Dirac [16], work out the general hamiltonian theory of constrained system. For a historical reconstruction of this achievement, see [17]. At the beginning, Dirac and the Bergmann group work independently. The present double classification in primary and secondary constraints and in first and second class constraints, still reflects this original separation.

1959

By 1959, Dirac has completely unraveled the canonical structure of GR.

1961

Arnowit, Deser and Misner complete what we now call the ADM formulation of GR, namely its hamiltonian version in appropriate variables, which greatly simplify the hamiltonian formulation and make its geometrical reading transparent [18].

In relation to the quantization, Arnowit, Deser and Misner present an influential argument for the finiteness of the self energy of a point particle in classical GR and use it to argue that nonperturbative quantum gravity should be finite.

1962

Feynman attacks the task of computing transition amplitudes in quantum gravity. He shows that tree-amplitudes lead to the physics one expects from the classical theory [19].

[1] To be sure, Misner lists a 4th approach as well, based on the Schwinger equations for the variation of the propagator, but notices that "this method has not been applied independently to general relativity", a situation that, as far as I know, has not changed since.

DeWitt starts developing his background field methods for the computation of perturbative transition amplitudes [20].

Bergmann and Komar clarify what one should expect from a Hilbert space formulation of GR [21].

Following the ADM methods, Peres writes the Hamilton-Jacobi formulation of GR [22]

$$G^2(q_{ab}q_{cd} - \frac{1}{2}q_{ac}q_{bd})\ \frac{\delta S(q)}{\delta q_{ac}}\frac{\delta S(q)}{\delta q_{bd}} + \det q\, R[q] = 0,$$

which will lead to the Wheeler-DeWitt equation. q_{ab} is the ADM 3-metric and G the Newton constant.

1963

John Wheeler realizes that the quantum fluctuations of the gravitational field must be short scale fluctuations of the geometry and introduces the physical idea of spacetime foam [23]. Wheeler's Les Houches lecture note are remarkable in many respects, and are the source of many of the ideas still current in the field. Just to mention two others: "Problem 56" suggests that gravity in 2+1 dimensions may not be so trivial after all, and indicates it may be an interesting model to explore. "Problem 57" suggests to study quantum gravity by means of a Feynman integral over a spacetime lattice.

1964

Penrose introduces the idea of spin networks, and of a discrete structure of space controlled by $SU(2)$ representation theory. The construction exists only in the form of a handwritten manuscript. It gets published only in 1971 [24]. The idea will surprisingly re-emerge 25 years later, when spin networks will be found to label the states of loop quantum gravity [25].

1964

Beginning to study loop corrections to GR amplitudes, Feynman observed that unitarity was lost for naive diagrammatic rules. DeWitt [26] develops the combinatorial means to correct the quantisation (requiring independence of diagrams from the longitudinal parts of propagators). These correction terms can be put in the form the form of loops of fictitious fermionic particles, the Faddeev-Popov ghosts [27]. The key role of DeWitt in this context was emphasized by Veltman in 1974 [29]:

> ...Essentially due to this, and some deficiencies in his combinatorial methods, Feynman was not able to go beyond one closed loop. DeWitt in his 1964 Letter and in his subsequent monumental work derived most of the things that we know of now. That is, he consider the question of a choice of gauge and the associated ghost particle. Indeed, he writes the ghost contribution in the form of a local Lagrangian containing a complex scalar field obeying Fermi statistics.

Somewhat illogically this ghost is now called the Faddeev-Popov ghost.

On the other hand, however, in comparison with the complicated combinatorics of DeWitt, the Faddeev-Popov approach has the merit of a greater technical simplicity and of a transparent geometrical interpretation, which are probably the reason for its popularity. It is in the work of Faddeev that the key role played by the gauge orbits (and not fields at a given point) as true dynamical variables, is fully elucidated [28].

1967

Bryce DeWitt publishes the "Einstein-Schrödinger equation" [30].

$$\left((\hbar G)^2 (q_{ab} q_{cd} - \frac{1}{2} q_{ac} q_{bd}) \frac{\delta}{\delta q_{ac}} \frac{\delta}{\delta q_{bd}} - \det q \, R[q] \right) \Psi(q) = 0,$$

Bryce will long denote this equation as the "Einstein-Schrödinger equation", attributing it to Wheeler –while John Wheeler denoted it as the DeWitt equation– until, finally, in 1988, at a Osgood Hill conference, DeWitt gives up and calls it the way everybody else had been calling it since the beginning: the "Wheeler-DeWitt equation".

The story of the birth of the Wheeler-DeWitt equation is worth telling. In 1965, during an air trip, John had to stop for a short time at the Rahley-Durham airport in Noth Carolina. Bryce lived nearby. John phoned Bryce and proposed to meet at the airport during the wait between two planes. Bryce showed up with the Hamilton-Jacobi equation for GR, published by Peres in 1962 and mumbled the idea of doing precisely what Shrödinger did for the hydrogen atom: replace the square of the derivative with a second derivative. Surprising Bryce, John was enthusiastic (John is often enthusiastic, of course), and declared immediately that *the* equation of quantum gravity had been found. The paper with the equation, the first of Bryce's celebrated 1967 quantum gravity trilogy [30, 31], was submitted in the spring of 66, but its publication was delayed until 1967. Apparently, also because of difficulties with publication charges ...

1967

John Wheeler discusses the idea of wave function $\Psi(q)$ of the "3-geometry" q, and the notion of superspace, the space of the 3-geometries in [32].

Penrose starts twistor theory [33].

The project of DeWitt and Feynman is concluded. A complete and consistent set of Feynman rules for GR are written down [31, 27].

1969

Developing an idea in Bryce's paper on canonical quantum gravity, Charles Misner starts quantum cosmology: the game of truncating the Wheeler-DeWitt equation to a finite number of degrees of freedom [34]. The idea is beautiful, but it will develop into a long lasting industry from which, after a while, little new will be understood.

The decade closes with the main lines of the covariant and the canonical theory clearly defined. It will soon become clear that neither theory works.

6 The Middle Ages: 1970-1983

1970

The decade of the seventies opens with a world of caution. Reviving a point made by Pauli, a paper by Zumino [35], suggests that the quantization of GR may be problematic and might make sense only by viewing GR as the low energy limit of a more general theory.

1971

Using the technology developed by DeWitt and Feynman for gravity, t'Hooft and Veltman decide to study the renormalizability of GR. Almost as a warm up exercise, they consider the renormalization of Yang-Mills theory, and find that the theory is renormalizable – result that has won them this year Nobel prize [36]. In a sense, one can say that the first physical result of the research in quantum gravity is the proof that Yang-Mills theory is renormalizable.

1971

David Finkelstein writes his inspiring "spacetime code" series of papers [37] (which, among others ideas, discuss quantum groups).

1973

Following the program, t'Hooft finds evidence of un-renormalizable divergences in GR with matter fields. Shortly after, t'Hooft and Veltman, as well as Deser and Van Nieuwenhuizen, confirm the evidence [38].

1974

Hawking announces the derivation of black hole radiation [39]. A (macroscopically) Schwarzshild black hole of mass M emits thermal radiation at the temperature

$$T = \frac{\hbar c^3}{8\pi k G M}$$

The result comes as a surprise, anticipated only by the observation by Bekenstein, a year earlier, that entropy is naturally associated to black holes, and thus they could be thought, in some obscure sense, as "hot" [40], and by the Bardeen-Carter-Hawking analysis of the analogy between laws of thermodynamics and dynamical behavior of black holes. Hawking's result is not directly connected to quantum gravity –it is a skillful application of quantum field theory in curved spacetime– but has a very strong impact on the field. It fosters an intense activity in quantum field theory in curved spacetime, it opens a new field of research in "black hole thermodynamics" (for a review of the two, see [43]), and it opens the quantum-gravitational problems of understanding the statistical

origin of the black hole (the Bekenstein-Hawking) entropy. For a Schwarzshild black hole, this is

$$S = \frac{1}{4}\,\frac{c^3}{\hbar G}\,A \tag{2}$$

where A is the area of the black hole surface. An influential, clarifying and at the same time intriguing paper is written two years later by Bill Unruh. The paper points out the existence of a general relation between accelerated observers, quantum theory, gravity and thermodynamics [42]. Something deep about nature should be hidden in this tangle of problems, but we do not yet know what.

1975

It becomes generally accepted that GR coupled to matter is not renormalizable. The research program started with Rosenfeld, Fierz and Pauli is dead.

1976

A first attempt to save the covariant program is made by Steven Weinberg, who explore the idea of asymptotic safety [44], developing earlier ideas from Giorgio Parisi [45], Kenneth Wilson and others, suggesting that non-renormalizable theories could nevertheless be meaningful.

1976

To resuscitate the covariant theory, even if in modified form, the path has already been indicated: find a high energy modification of GR. Preserving general covariance, there is not much one can do to modify GR. An idea that attracts much enthusiasm is supergravity [46]: it seems that by simply coupling a spin 3/2 particle to GR, namely with the action (in first order form)

$$S[g,\Gamma,\psi] = \int d^4x\,\sqrt{-g}\,\left(\frac{1}{2G}R - \frac{i}{2}\,\epsilon^{\mu\nu\rho\sigma}\,\psi_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma\right),$$

one can get a theory finite even at two loops.

1977

Another, independent, idea is to keep the same kinematics and change the action. The obvious thing to do is to add terms proportional to the divergences. Stelle proves that an action with terms quadratic in the curvature

$$S = \int d^4x\,\sqrt{-g}\,\left(\alpha R + \beta R^2 + \gamma R^{\mu\nu}R_{\mu\nu}.\right),$$

is renormalizable for appropriate values of the coupling constants [47]. Unfortunately, precisely for these values of the constants the theory is bad. It has negative energy modes that make it unstable around the Minkowski vacuum and not unitary in the quantum regime. The problem becomes to find a theory renormalizable and unitary at the same time, or to circumvent non-unitarity.

1978

The Hawking radiation is soon re-derived in a number of ways, strongly reinforcing its credibility. Several of these derivations point to thermal techniques [48], thus motivating Hawking [49] to revive the Wheeler-Misner "Feynman quantization of general relativity" [15] in the form of a "Euclidean" integral over *Riemannian* 4-geometries g

$$Z = \int Dg\ e^{-\int \sqrt{g}R}.$$

Time ordering and the concept of positive frequency are incorporated into the "analytic continuation" to the Euclidean sector. The hope is double: to deal with topology change, and that the Euclidean functional integral will prove to be a better calculation tool than the Wheeler-DeWitt equation.

1980

Within the canonical approach, the discussion focuses on understanding the disappearance of the time coordinate from the Wheeler-DeWitt theory. The problem has actually nothing to do with *quantum* gravity, since the time coordinate disappears in the *classical* Hamilton-Jacobi form of GR as well; and, in any case, physical observables are coordinate independent, and thus, in particular, independent from the time coordinate, in whatever correct formulation of GR. But in the quantum context there is no single spacetime, as there is no trajectory for a quantum particle, and the very concepts of space and time become fuzzy. This fact raises much confusion and a vast interesting discussion (whose many contributions I can not possibly summarize here) on the possibility of doing meaningful fundamental physics in the absence of a fundamental notion of time. For early references on the subject see for instance [50].

1981

Polyakov [51] shows that the cancellation of the conformal anomaly in the quantization of the string action

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g}\ g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b \eta_{ab}.$$

leads to the critical dimension.

1983

The hope is still high for supergravity, now existing in various versions, as well as for higher derivative theories, whose rescue from non-unitarity is explored using a number of ingenious ideas (large N expansions, large D expansions, Lee-Wick mechanisms...). At the 10th GRG conference in Padova in 1983, two physicists of indisputable seriousness, Gary Horowitz and Andy Strominger, summarize their contributed paper [53] with the words

> In sum, higher derivative gravity theories are a viable option for resolving the problem of quantum gravity ...

At the same conference, supergravity is advertised as a likely final solution of the quantum gravity puzzle. But very soon it becomes clear that supergravity is non-renormalizable at higher loops and that higher derivatives theories do not lead to viable perturbative expansions. The excitement and the hope fade away.

In its version in 11 dimensions, supergravity will find new importance in the late 1990s, in connection with string theory. High derivative corrections will also reappear, in the low energy limit of string theory.

1983

Hartle and Hawking [55] introduce the notion of the "wave function of the universe" and the "no-boundary" boundary condition for the Hawking integral, opening up a new intuition on quantum gravity and quantum cosmology. But the Euclidean integral does not provide a way of computing genuine field theoretical quantities in quantum gravity better than the Wheeler-DeWitt equation, and the atmosphere at the middle of the eighties is again rather gloomy. On the other hand, Jim Hartle [56] develops the idea of a sum over histories formulation of GR into a full fledged extension of quantum mechanics to the general covariant setting. The idea will later be developed and formalized by Chris Isham [57].

Sorkin introduces his poset approach to quantum gravity [54].

7 The Renaissance: 1984-1994

1984

Green and Schwarz realize that strings might describe "our universe" [58]. Excitement starts to build up around string theory, in connection with the unexpected anomaly cancellation and the discovery of the heterotic string [59].

The relation between the ten dimensional superstrings theory and four dimensional low energy physics is studied in terms of compactification on Calaby-Yau manifolds [60] and orbifolds. The dynamics of the choice of the vacuum remains unclear, but the compactification leads to 4d chiral models resembling low energy physics.

Belavin, Polyakov and Zamolodchikov publish their analysis of conformal field theory [61].

1986

Goroff an Sagnotti [62] finally compute the two loop divergences of pure GR, definitely nailing the corpse of pure GR perturbative quantum field theory into its coffer: the divergent term is

$$\Delta S = \frac{209}{737280\pi^4}\,\frac{1}{\epsilon}\int d^4x\,\sqrt{-g}\;R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\epsilon\theta}R^{\epsilon\theta}{}_{\mu\nu}.$$

1986

Penrose suggests that the wave function collapse in quantum mechanics might be of quantum gravitational origin [63]. The idea is radical and implies a re-thinking of the basis of mechanics. Remarkably, the idea may be testable: work is today in progress to study the feasibility of an experimental test.

1986

String field theory represents a genuine attempt to address the main problem of string theory: finding a fundamental, background independent, definition of the theory [68]. The string field path, however, turns out to be hard.

1986

The connection formulation of GR is developed by Abhay Ashtekar [69], on the basis of some results by Amitaba Sen [70]. At the time, this is denoted the "new variables" formulation. It is a development in classical general relativity, but it has long ranging consequences on quantum gravity, as the basis of loop quantum gravity.

1987

Fredenhagen and Haag explore the general constraint that general covariance puts on quantum field theory [71].

1987

Green, Schwarz and Witten publish their book on superstring theory. In the gauge in which the metric has no superpartner, the superstring action is

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} \left(g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b - i\psi^a \gamma^\mu \partial_\mu \psi^b \right) \eta_{ab}.$$

The interest in the theory grows very rapidly. To be sure, string theory still obtains a very small place at the 1991 Marcel Grossmann meeting [72]. But, increasingly, the research in supergravity and higher derivative theories has merged into strings, and string theory is increasingly viewed as a strong competing candidate for the quantum theory of the gravitational field. As a side product, many particle physicists begin to study general relativity, or at least some bits of it. Strings provide a consistent perturbative theory. The covariant program is fully re-born. The problem becomes understanding why the world described by the theory appears so different from ours.

1988

Ted Jacobson and Lee Smolin find loop-like solutions to the Wheeler-DeWitt equation formulated in the connection formulation [73], opening the way to loop quantum gravity.

1988

The "loop representation of quantum general relativity" is introduced in [74]. For an early review, see [75]. It is based on the new connection formulation of GR [69], on the Jacobson-Smolin solutions [73], and on Chris Isham's ideas on

the need of non-Gaussian, or non-Fock representations in quantum gravity [76]. Loop quantization had been previously and independently developed by Rodolfo Gambini and his collaborators for Yang Mills theories [77]. In the gravitational context, the loop representation leads immediately to two surprising results: an infinite family of exact solutions of the Wheeler-DeWitt equation is found, and knot theory controls the physical quantum states of the gravitational field. Classical knot theory, with its extensions, becomes a branch of mathematics relevant to describe the diff invariant states of quantum spacetime [78].

The theory transforms the old Wheeler-DeWitt theory in a formalism that can be concretely used to compute physical quantities in quantum gravity. The canonical program is fully re-born. Today, the theory is called "loop quantum gravity".[2] For a review, complete references, and an account of the development of the theory, see [79].

1988

Ed Witten introduces the notion of topological quantum field theory (TQFT) [64]. In a celebrated paper [65], he uses a TQFT to give a field theoretical representation of the Jones polynomial, a knot theory invariant. The expression used by Witten has an interpretation in loop quantum gravity: it can be seen as the "loop transform" of quantum state given by the exponential of the Chern Simon functional [78].

Formalized by Atiyah [66], the idea of TQFT will have beautiful developments, and will strongly affect later development in quantum gravity. General topological theories in any dimensions, and in particular BF theory, are introduced by Gary Horowitz shortly afterwards [67].

1988

Witten finds an ingenious way of quantizing GR in 2+1 spacetime dimensions [80], (thus solving "problem 56" of the 1963 Wheeler's Les Houches lectures) opening up a big industry of analysis of the theory (for a review, see [81]). The quantization method is partially a sum over histories and partially canonical. Covariant perturbative quantization seemed to fail for this theory. The theory had been studied a few years earlier by Deser, Jackiw, t'Hooft, Achucarro, Townsend and others [82].

1989

Amati, Ciafaloni and Veneziano find evidence that string theory implies that distances smaller than the Planck scale cannot be probed [83].

1989

In the string world, there is excitement for some nonperturbative models of strings "in 0 dimension", equivalent to 2D quantum gravity [84]. The excitement dies fast, as many others, but the models will re-emerge in the nineties [85], and

[2] It is sometimes called also "quantum geometry", but the expression "quantum geometry" is used by a variety of other research programs as well.

will also inspire the spin foam formulation of quantum gravity [86].

1992

Turaev and Viro [87] define a state sum that on the one hand is a rigorously defined TQFT, on the other hand can be seen as a regulated and well defined version of the Ponzano-Regge [88] quantization of 2+1 gravity. Turaev, and Ooguri [89] find soon a 4d extension, which will have a remarkable impact on later developments.

1992

The notion of *weave* is introduced in loop quantum gravity [90]. It is evidence of a discrete structure of spacetime emerging from loop quantum gravity. The first example of a weave which is considered is a 3d mesh of intertwined rings. Not surprising, the intuition was already in Wheeler! See Figure 1, taken from Misner Thorne and Wheeler [91].

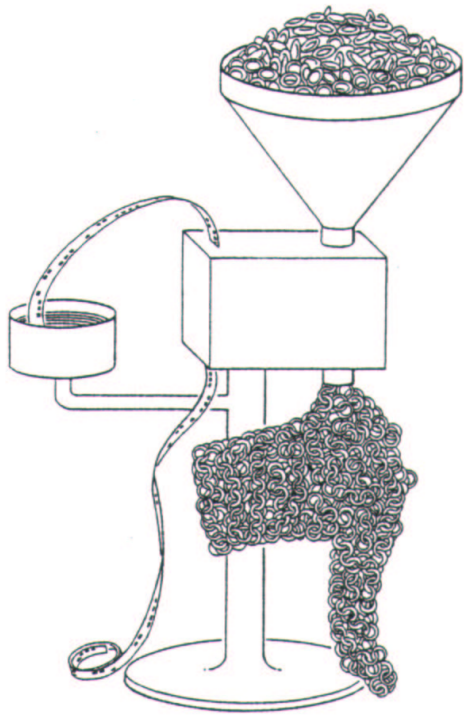


Figure 1: The weave, in Wheeler's vision.

1994

Noncommutative geometry, often indicated as a tool for describing certain aspects of Planck scale geometry, finds a strict connection to GR in the framework of Alain Connes' noncommutative geometry. Remarkably, the Connes-Chamseddine "spectral action", just the trace of a simple function of a suitably defined Dirac-like operator D

$$S = Tr[f(D^2/(\hbar G))],$$

where f is the characteristic function of the $[0, 1]$ interval, turns out to include the standard model action, as well as the Einstein-Hilbert action [92].

8 Nowadays: 1995-2000

1995

Nonperturbative aspects of string theory begin to appear: branes [93], dualities [94], the matrix model formulation of M theory [95] .... (For a review, see for instance [96]). The interest in strings booms. At the plenary conference of a meeting of the American Mathematical Society in Baltimore, Ed Witten claims that

> "The mathematics of the next millennium will be dominated by string theory"

causing a few eyebrows to raise.

The various dualities appear to relate the different versions of the theory, pointing to the existence of a unique fundamental theory. The actual construction of the fundamental background independent theory, however, is still missing, and string theory exists so far only in the form of a number of (related) expansions over assigned backgrounds.

1995

Using the spin network orthonormal basis found on the Hilbert space of loop quantum gravity, a main physical result is obtained within loop quantum gravity: the computation of the eigenvalues of area and volume [97]. The main sequence of the eigenvalues of the area is labeled by an n-tuplet of half integers $\vec{j} = \{j_1 \dots j_n\}$ and is

$$A_{\vec{j}} = 8\pi\hbar G \sum_{i=1,n} \sqrt{j_i(j_i+1)}.$$

The result is rapidly extended and derived in a number of ways [98].

A rigorous mathematical framework for loop quantum gravity is developed [99].

1995

Ted Newman and his collaborators introduce the Null Surface Formulation of GR [100].

1996

The Bekenstein-Hawking black hole entropy is computed within loop quantum gravity as well as within string theory, almost at the same time.

The loop result is obtained by computing the number of (spin-network) states which endow a 2-sphere with a given area [101], as well as by loop quantizing the classical theory of the field outside the hole and studying the boundary states [103]. These gravitational surface states [104] can be identified with the states of a Chern-Simons theory on a surface with punctures [102]. The computation is valid for various realistic black holes, and the 1/4 factor in (2) is obtained by fixing the undetermined dimensionless parameter present in loop quantum gravity (the Immirzi parameter).

In string theory, the computation exploits a strong coupling/weak coupling duality, which, in certain supersymmetric configurations, preserves the number of states: the physical black hole is in a strong coupling situation, but the number of its microstates can be computed in a weak field configuration that has the same charges at infinity. The calculation method is thus rather indirect, and works smoothly only for certain extremal black holes; remarkably, however, one obtains precisely the 1/4 factor of equation (2), as well as other key aspects of the Hawking radiation phenomenology [105].

1996

A rigorously defined, finite and anomaly free hamiltonian constraint operator is constructed by Thomas Thiemann in loop quantum gravity [106]. Some doubts are raised on whether the classical limit of this theory is in fact GR (the issue is still open), but the construction defines a consistent general covariant quantum field theory in 4d.

1996

Intriguing state sum models obtained modifying a TQFT are proposed by Barrett and Crane, Reisenberger, Iwasaki and others [86] as a tentative model for quantum GR. All these models appear as sums of "spin foams": branched surfaces carrying spins.

In the meanwhile, the loop representation is "exponentiated", à la Feynman, giving rise, again, to a spin foam model, corresponding to canonical loop quantum gravity [107]. These developments revive the sum over histories approach.

1997

There is a lively discussion on the difficulties of the lattice approaches in finding a second order phase transition [109].

Intriguing connections between non commutative geometry and string theory appear [108].

1998

Juan Maldacena shows [110] that the large N limit of certain conformal field theories includes a sector describing supergravity on the product of Anti-deSitter spacetimes and spheres. He conjectures that the compactifications of M/string theory on an Anti-deSitter spacetimes is dual to a conformal field theory on the spacetime boundary. This leads to a new proposal for defining M-theory itself in term of the boundary theory: an effort to reach background independence (for M theory) using background dependent methods (for the boundary theory).

A consequence of this "Maldacena conjecture" is an explosion of interest for an idea by Gerard t'Hooft, developed and promoted by Leonard Suskind: the "holographic principle". According to this principle (considered in a number of variants) the information on the physical state in the interior of a region can be represented on the region's boundary and is limited by the area of this boundary.

1998

Two papers in the influential journal *Nature* [112] raise the hope that seeing spacetime-foam effects, and testing quantum gravity theories might not be as forbidding as usually assumed. The idea is that there is a number of different instances (the neutral kaon system, gamma ray burst phenomenology, interferometers ...) in which presently operating measurement or observation apparata, or apparata that are going to be soon constructed, involve sensitivity scales comparable to –or not too far from– the Planck scale [113]. If this direction fails, testing quantum gravity might require the investigation of very early cosmology [114].

Today

For critical discussions of current direction of research in quantum gravity, see for instance [115].

9 Concluding remarks

The lines of research that I have summarized in Section 2 have found many points of contact in the course of their development and have often intersected each other. For instance, there is a formal way of deriving a sum over over histories formulation from a canonical theory and viceversa; the perturbative expansion can also be obtained expanding the sum over histories; string theory today faces the problem of a finding its nonperturbative formulation, and thus the typical problems of a canonical theory, while loop quantum gravity has mutated into the spin foam models, a sum over history formulation, using techniques that can be traced to a development of string theory of the early nineties. Recently, Lee Smolin has been developing an attempt to connect nonperturbative string theory and loop quantum gravity [116]. However, in spite of this continuous cross fertilization, the three main lines of development have

kept their essential separation.

As pointed out, the three direction of investigation where already clearly identified by Charles Misner in 1959 [15]. In the concluding remark of the *Conférence internationale sur les théories relativistes de la gravitation*, in 1963, Peter Bergmann noted [117]

> "In view of the great difficulties of this program, I consider it a very positive thing that so many different approaches are being brought to bear on the problem. To be sure, the approaches, we hope, will converge to one goal."

This was almost 40 years ago ...

The divide is particularly macroscopic between the covariant line of research on the one hand and the canonical/sum over histories on the other. This divide has remained through over 70 years of research in quantum gravity. The separation cannot be stronger. Here is a typical comparison, arbitrarily chosen among many. On the covariant side, at the First Marcel Grossmann Meeting, Peter van Neuwenhuizen writes [118]

> "... gravitons are treated on exactly the same basis as other particles such as photons and electrons. In particular, particles (including gravitons) are always in flat Minkowski space and move *as if* they followed their geodesics in curved spacetime because of the dynamics of multiple graviton exchange. [...] Pure relativists often become somewhat uneasy at this point because of the the following two aspects entirely peculiar to gravitation: 1) [...]. One must decide before quantization which points are spacelike separated, but it is only after quantization that that the fully quantized metric field can tell us this spacetime structure [...]. 2) [...] In a classical curved background one needs positive and negative solutions, but in non-stationary spacetimes it is not clear whether one can define such solutions. *The strategy of particle physicists has been to ignore these problems for the time being, in the hope that they will ultimately be resolved in the final theory. Consequently we will not discuss them any further.*"

On the canonical side[3], Peter Bergmann [120]

> "The world point by itself possess no physical reality. It acquires reality only to the extent that it becomes the bearer of specific properties of the physical fields imposed on the spacetime manifold."

Partially, the divide reflects the different understanding of the world that the particle physics community on the one hand and the relativity community on

[3]For a detailed defense of the relativist point of view in the debate, see [119].

the other hand, have. The two communities have made repeated and sincere efforts to talk to each other and understanding each other. But the divide remains, and, with the divide, the feeling, on both sides, that the other side is incapable of appreciating something basic and essential: On the one side, the structure of quantum field theory as it has been understood in half a century of investigation; on the other side, the novel physical understanding of space and time that has appeared with general relativity. Both sides expect that the point of the other will turn out, at the end of the day, to be not very relevant. One side because all the experience with quantum field theory is on a fixed metric spacetime, and thus is irrelevant in a genuinely background independent context. The other because GR is only a low energy limit of a much more complex theory, and thus cannot be taken too seriously as an indication on the deep structure of Nature. Hopefully, the recent successes of both lines will force the two sides, finally, to face the problems that the other side considers prioritary: background independence on the one hand, control of a perturbation expansion on the other.

So, where are we, after 70 years of research? There are well-developed tentative theories, in particular strings and loops, and several other intriguing ideas. There is no consensus, no established theory, and no theory that has yet received any direct or indirect experimental support. In the course of 70 years, many ideas have been explored, fashions have come and gone, the discovery of the Holly Graal has been several times announced, with much later scorn.

However, in spite of its age, the research in quantum gravity does not seem to have been meandering meaninglessly, when seen in its entirety. On the contrary, one sees a logic that has guided the development of the research, from the early formulation of the problem and the research directions in the fifties to nowadays. The implementation of the programs has been extremely laborious, but has been achieved. Difficulties have appeared, and solutions have been proposed, which, after much difficulty, have lead to the realization, at least partial, of the initial hopes. It was suggested in the early seventies that GR could perhaps be seen as the low energy theory of a theory without uncontrollable divergences; today, 30 years later, such a theory –string theory– is known. In 1957 Charles Misner indicated that in the canonical framework one should be able to compute eigenvalues; and in 1995, 37 years later, eigenvalues were computes –within loop quantum gravity. The road is not yet at the end, much remains to be understood, and some of the current developments might lead nowhere. But it is difficult to deny, looking at the entire development of the subject, that there has been a linear progress. And the road, no doubts, is fascinating.

I am very much indebted to many friends that have contributed to this reconstruction. I am particularly grateful to Augusto Sagnotti, Gary Horowitz,

Ludwig Faddeev, Alejandro Corichi, Jorge Pullin, Lee Smolin, Joy Christian, Bryce DeWitt, Giovanni Amelino-Camelia, Daniel Grumiller, Nikolaos Mavromatos and Ted Newman.

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