What the Casimir-Effect really is telling about Zero-Point Energy

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The attractive force between metallic surfaces, predicted by Casimir in 1948, seems to indicate the physical existence and measurability of the quantized electromagnetic field's zero-point energy. It is shown in this article, that Casimir's derivation depends essentially on a misleading idealization. When that idealization is replaced by a realistic assumption, Casimir's argument turns to the exact opposite: The observed Casimir force does positively prove, that the electromagnetic field's zero-point energy does *not* exert forces onto metallic surfaces.

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1. Is the Zero-Point Energy of Quantized Fields Observable?

When Heisenberg discovered quantum mechanics in 1925 [1], harmonic and anharmonic oscillators were the first systems, to which he applied his novel formalism. His equations led to the quantized energy-spectrum

$$E = \left(n + \frac{1}{2}\right) h\nu \quad \text{with } n = 0, 1, 2, 3, \ldots \tag{1}$$

The zero-point energy $h\nu/2$ per degree of freedom had been established experimentally already before due to the analysis of the vibrational spectra of molecules [2]. Further experimental evidence for the physical existence of zero-point energy arose in the following years for example from the scattering of X-rays by crystals at low temperature [3] and from the observation that ^{4}He stays liquid at normal pressure even near $T = 0$ [4]. Thus the physical existence of zero-point energy in systems with a finite number of degrees of freedom was well established already in the early years of quantum theory both experimentally and theoretically.

In contrast, there were at that time — and still are by today — severe doubts regarding the physical existence of the infinitely large zero-point energy resulting from the quantization of continuous fields. It was not only the infinitely large value of the energy, which caused concern; that could be reduced to finite values by some appropriate regularization method. But there were simply no positive indications of it's existence known from observations. Quite the contrary: Zero-point energy, like any form of energy, should gravitate and thus, due to it's huge value, result in observable curvature of the intergalactic space, provided that General Relativity Theory is correct. Pauli made an estimation of the curvature of space, which was to be expected due to the electromagnetic field's zero-point energy. "The result was, that the radius of the universe (if short wavelengths are cut-off at the classical electron radius) 'would not even reach to the moon'." [5, page 842]

In an 1928 article on the quantization of the electromagnetic field, Jordan and Pauli concluded (my translation): "It seems to us, that several considerations are indicating, that — in contrast to the eigen-oscillations in the crystal grid (where both theoretical and empirical reasons are indicating the existence of a zero-point energy) — no reality can be assigned to that 'zero-point energy' $h\nu/2$ per degree of freedom in case of the eigen-oscillations of the radiation. As one is dealing with regard to the latter with strictly harmonic oscillators, and as that 'zero-point radiation' can neither be absorbed nor scattered nor reflected, it seems to elude, including it's energy or mass, any method of detection. Therefore it may be the simplest and most satisfactory conception, that in case of the electromagnetic field that zero-point radiation does not exist at all." [6, page 154]

By a publication of Casimir [7] in 1948, the opinion of Jordan and Pauli seemed to be refuted. According to Casimir's computation (on which we will dwell below), the quantized electromagnetic field's zero-point energy does cause an attractive force

$$F_{\text{Casimir}} = -\frac{\pi^2 \hbar c}{240}\,\frac{XY}{D^4} = -1.3\cdot 10^{-9}\mathrm{N}\cdot\frac{XY/\mathrm{mm}^2}{D^4/\mu\mathrm{m}^4} \tag{2}$$

between two parallel metallic plates with area XY, which are separated by a gap of width D. Hence the zero-point energy would be physically measurable and testable, after all. Since then, the Casimir-force has many times been observed experimentally, and the — at least approximate — correctness of equation (2) has been confirmed [8].

The debate about the existence and effectiveness of the zero-point energy of quantum fields was however still not definitely settled by the positive results of the experiments measuring the Casimir-force, because some years after Casimir's publication an alternative explanation for that force was found: Lifshitz [9] and Dzyaloshinskii, Lifshitz, und Pitaevskii [10] derived a formula for the long-range van der Waals-forces, which are acting between two infinitely extended half-spaces with relative dielectric constants ϵ_1 and ϵ_3, which are separated by a gap filled with a material with relative dielectric constant ϵ_2. Schwinger, DeRaad, and Milton [11] reproduced and confirmed the results of Lifshitz et al. by means of an other method. They also considered the limit $\epsilon_1 = \epsilon_3 \to \infty$, $\epsilon_2 \to 1$, i.e. the limit of two metal plates with infinite conductivity, separated by a vacuum gap. It turned out that the formula of Lifshitz simplifies in this limit to the Casimir-force (2).

The conclusiveness of (2) as a proof of the measurable effectiveness of zero-point energy was considerably delimited by Lifshitz' alternative theory. The measured forces could always *as well* be interpreted as van der Waals forces. But only "as well". Many researchers in the field sticked to the point of view, that the Casimir force is to be considered as a direct macroscopic manifestation of the electromagnetic quantum field's zero-point energy. Should not the fact, that the observed forces could be interpreted *as well* as an action of zero-point energy, at least be acknowledged as a distinct indication of the physically observable existence of zero-point energy? Should it really be nothing than an odd coincidence, that Casimir's computation of the effectiveness of zero-point energy, and Lifshitz's computation of van der Waals-forces, had led to the identical result (2)?

The answer on these questions, which will be justified below, is surprising: Casimir's formula (2) is absolutely conclusive. But it's experimental confirmation does prove — exactly opposite to Casimir's intention — that the electromagnetic field's zero-point energy does *not* exert forces onto metallic plates, i.e. that the observed forces by no means are related to the electromagnetic field's zero-point energy.

2. Casimir's Computation

Casimir considered a resonator as sketched in figure 1. The rectangular cavity's size is $X \times Y \times (Z + P)$. Inside the cavity there is a plate of thickness P, which is aligned parallel to the cavity's XY-face and movable in Z-direction. The plate's area is only minimally smaller than $X \times Y$. The plate's distance from one side wall of the cavity is D, it's distance from the opposite side wall is $Z - D$.

We firstly consider the left cavity. At temperature T, it's walls and the plate are in thermodynamic equilibrium with the electromagnetic blackbody radiation within the cavity. The spectrum of the radiation's wave numbers

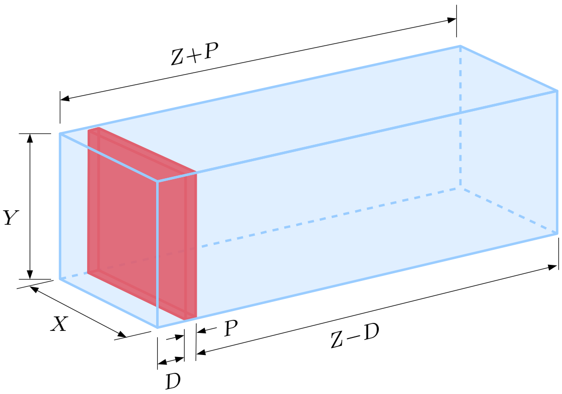


Fig. 1. Cavity resonator with movable plate

is discrete:

$$k_{rst} = \frac{2\pi}{\lambda_{rst}} = \sqrt{\left(\frac{r\pi}{X}\right)^2 + \left(\frac{s\pi}{Y}\right)^2 + \left(\frac{t\pi}{D}\right)^2} \quad \text{with } r,s,t \in \mathbb{N} \tag{3}$$

There are 2 modes each with $r,s,t = 1,2,3,\ldots$ and 1 mode each with one of the indices 0 and the both other indices $1,2,3,\ldots$ [12, chap. D.II.2.b.].

λ_{rst} are the wavelengths of the possible radiation modes. This equation does not hold for arbitrary wave numbers, because any metal becomes transparent for radiation of sufficiently high frequency. The transparency of metals at very high frequency is a basic condition for Casimir's derivation of formula (2). Casimir was well aware of this condition, which he emphasized in his publication. He introduced "a function $f(k_{rst}/k_{\mathrm{M}})$, which is unity for $k_{rst} \ll k_{\mathrm{M}}$ but tends to zero sufficiently rapidly for $k_{rst} \to \infty$". But he did not specify the concrete form of $f(k_{rst}/k_{\mathrm{M}})$ explicitly.

To design $f(k_{rst}/k_{\mathrm{M}})$ as realistic as possible, we assume the cavity and the plate to be made of copper. The reflectivity R (i.e. the ratio of reflected radiation intensity versus incoming intensity) as a function of wave number is quite complicated, see [13, fig. 1b]. For the purpose of our investigation, the rough approximation

$$R = \exp\{-k_{rst}/k_{\mathrm{M}}\} \quad \text{with } k_{\mathrm{M}} = 38 \cdot 10^6 \mathrm{m}^{-1} \tag{4}$$

for the reflectivity of copper is completely sufficient. (k_{M} is about $0.8\times$ the plasma-wavenumber of copper [13] according to the Drude-model.)

At temperature $T = 0$, only the zero-point oscillations of the electromagnetic field are excited. The energy per mode then is $\hbar c k_{rst}/2$, and the zero-point energy enclosed within the cavity is

$$U_0 = 2 \sum_{r,s,t=0}^{\infty}{}' \frac{\hbar c k_{rst}}{2} \exp\{-k_{rst}/k_{\mathrm{M}}\} = \sum_{r,s,t=0}^{\infty}{}' \hbar c \cdot$$
$$\cdot \sqrt{\left(\frac{r\pi}{X}\right)^2 + \left(\frac{s\pi}{Y}\right)^2 + \left(\frac{t\pi}{D}\right)^2} \exp\left\{ -\frac{1}{k_{\mathrm{M}}} \sqrt{\left(\frac{r\pi}{X}\right)^2 + \left(\frac{s\pi}{Y}\right)^2 + \left(\frac{t\pi}{D}\right)^2} \right\} . \quad (5)$$

The prime$'$ at the summation symbol is a reminder, that the multiplicity of polarizations, as indicated in (3), has to be considered. At most one of the numbers r, s, t of an oscillation mode can be zero, and terms with one zero index get a factor $1/2$.

Once U_0 is known as a function of D, the force

$$F = -\frac{\mathrm{d}}{\mathrm{d}D} \Big(U_0(D) + U_0(Z - D) \Big) \quad (6)$$

can be computed, which is exerted by the electromagnetic field's zero-point energy onto the movable plate. Which result is to be expected? When D is increased, then the energy of each mode in the left cavity, and thus the overall energy in the left cavity, will become lower. On the other hand, some high-energy modes, which have been only poorly reflected before, now are better reflected due to their reduced energy, thus increasing the energy content of the left cavity. Just the opposite is happening in the right cavity. Furthermore, the density of modes in the large right cavity is higher than in the small left cavity, which has some further impact onto the result. The net effect of all these different factors is not easy to guess, and a detailed computation of the force acting onto the plate is necessary.

For that computation, we consider — like Casimir — the limit $X \to \infty$ and $Y \to \infty$. Thus the sums over the discrete indices r and s may be replaced by integrals. Only the sum over t is still considered discrete. It has been elaborated elsewhere [14, sect. 4] in very detail, how (5) then can be transformed into

$$U_0 = \frac{\pi^2 \hbar c X Y}{2} \left(\frac{6 D k_{\mathrm{M}}^4}{\pi^4} - \frac{1}{360\, D^3} + \sum_{j=6}^{\infty} \frac{B_j}{j!} \, \frac{(j^2 - 5j + 6)\, \pi^{j-4}}{k_{\mathrm{M}}^{j-4} D^{j-1}} \right) . \quad (7)$$

The coefficients B_j are the Bernoulli-numbers

$$B_0 = 1 \quad , \quad B_j = -\sum_{n=0}^{j-1} \frac{j!}{n!(j-n+1)!} \, B_n \quad \text{for } j > 0 \, . \quad (8)$$

The expansion, which led to the series with the Bernoulli coefficients does converge only for

$$D > \frac{1}{2k_{\mathrm{M}}} \overset{(4)}{=} 13.2\,\mathrm{nm}\ . \tag{9}$$

For smaller distance D between the movable plate and the cavity wall, (7) is not valid.

(7) is the zero-point energy enclosed within the left cavity of figure 1. To find the energy within the right cavity, Casimir replaced D everywhere by $Z - D$. Thus the total zero-point energy within both cavities of the resonator becomes

$$\begin{aligned} U_{0,\mathrm{total}} = \frac{\pi^2 \hbar c XY}{2} \Bigg(& \frac{6Z k_{\mathrm{M}}^4}{\pi^4} - \frac{1}{360\, D^3} - \frac{1}{360\,(Z-D)^3} + \\ & + \sum_{j=6}^{\infty} \frac{B_j}{j!} \frac{(j^2 - 5j + 6)}{k_{\mathrm{M}}^{j-4}\, \pi^{4-j}} \left(\frac{1}{D^{j-1}} + \frac{1}{(Z-D)^{j-1}} \right) \Bigg)\ . \end{aligned} \tag{10}$$

With the approximation

$$\left(\frac{D}{Z-D} \right)^4 \ll 1\ , \tag{11}$$

which is well justified for all experimental evaluations of the Casimir force, the force acting onto the movable plate between the both cavities assumes the simple form

$$\begin{aligned} F_{\mathrm{Casimir}} = -\frac{\mathrm{d}U_{0,\mathrm{total}}}{\mathrm{d}D} = -\frac{\pi^2 \hbar c XY}{2} \Bigg(& \frac{1}{120\, D^4} + \\ & + \sum_{j=6}^{\infty} \frac{B_j}{j!} \frac{(-j^3 + 6j^2 - 11j + 6)}{k_{\mathrm{M}}^{j-4}\, \pi^{4-j} D^j} \Bigg)\ . \end{aligned} \tag{12}$$

Casimir considered the limit $k_{\mathrm{M}} \to \infty$, which immediately led to his formula (2). This last simplification is superfluous, because the series with the coefficients B_j is converging rapidly: Inserting $k_{\mathrm{M}} \overset{(4)}{=} 38 \cdot 10^6 \mathrm{m}^{-1}$, it adds for $D > 0.2\,\mu\mathrm{m}$ less than $5\,\%$ and for $D > 0.5\,\mu\mathrm{m}$ even less than $1\,\%$ to the overall force.

3. Inserting Realistic Parameters

The assumption, due to which Casimir's computation lost contact to reality, was not the superfluous simplification of (12). It happened already in (10). The Casimir-force is the tiny difference of two huge forces. We should more clearly discern $k_{\mathrm{M,left}}$ and $k_{\mathrm{M,right}}$, and write (10) in the form

$$U_{0,\mathrm{total}} = \frac{\pi^2\hbar c XY}{2}\Bigg(\frac{6Dk^4_{\mathrm{M,left}}}{\pi^4} + \frac{6(Z-D)k^4_{\mathrm{M,right}}}{\pi^4} - \frac{1}{360\,D^3} - \\ - \frac{1}{360\,(Z-D)^3} + \sum_{j=6}^{\infty}\frac{B_j}{j!}\,\frac{(j^2-5j+6)}{k_{\mathrm{M}}^{j-4}\,\pi^{4-j}}\Big(\frac{1}{D^{j-1}} + \frac{1}{(Z-D)^{j-1}}\Big)\Bigg)\,. \tag{13}$$

With the approximation (11), the force exerted by the zero-point energy onto the movable plate then reads

$$F_{\mathrm{ZPE}} = -\frac{\mathrm{d}U_{0,\mathrm{total}}}{\mathrm{d}D} = -\frac{\pi^2\hbar c XY}{2}\Bigg(\frac{6(k^4_{\mathrm{M,left}} - k^4_{\mathrm{M,right}})}{\pi^4} + \frac{1}{120\,D^4} + \\ + \sum_{j=6}^{\infty}\frac{B_j}{j!}\,\frac{(-j^3+6j^2-11j+6)}{k_{\mathrm{M}}^{j-4}\,\pi^{4-j}\,D^j}\Bigg)\,, \tag{14}$$

where the index $_{\mathrm{ZPE}}$ stands for zero-point energy.

The term $6(k^4_{\mathrm{M,left}} - k^4_{\mathrm{M,right}})/\pi^4$, which Casimir simply dropped due to the idealizing assumption $k_{\mathrm{M,left}} = k_{\mathrm{M,right}}$, is describing the fragile balance of two forces, which are tremendous in comparison to the Casimir-force. Under the realistic assumption $k_{\mathrm{M}} \overset{(4)}{=} 38\cdot 10^6\mathrm{m}^{-1}$, the ratio of these forces is

$$\frac{6k^4_{\mathrm{M}}/\pi^4}{1/(120\,D^4)} \approx \begin{cases} 2.5\cdot 10^4 & \text{for } D = 0.2\,\mu\mathrm{m} \\ 1.5\cdot 10^7 & \text{for } D = 1.0\,\mu\mathrm{m} \\ 9.6\cdot 10^9 & \text{for } D = 5.0\,\mu\mathrm{m} \end{cases} \tag{15}$$

Only with excellent, and actually not realistic match of reflectivities of the both sides of the plate, the Casimir force can become visible. For $D = 1\,\mu$m, the ratio of the forces is

$$\begin{array}{ccc} \dfrac{k_{\mathrm{M,right}}}{k_{\mathrm{M,left}}} & & \dfrac{F_{\mathrm{Casimir}} = (12)}{F_{\mathrm{ZPE}} = (14)} \\ 1-10^{-4} & \Longrightarrow & 1.6\cdot 10^{-4} \\ 1-10^{-5} & \Longrightarrow & 1.6\cdot 10^{-3} \\ 1-10^{-6} & \Longrightarrow & 1.6\cdot 10^{-2} \\ 1-10^{-7} & \Longrightarrow & 1.4\cdot 10^{-1} \\ 1-10^{-8} & \Longrightarrow & 6.2\cdot 10^{-1} \end{array} \tag{16}$$

Actually, the experimentalists don't even try to achieve good matching surface conditions on the both sides of the movable plate. For example in the Purdue experiment [15] [16], the wall of the cavity is replaced by an Au-coated sphere, and the movable plate is replaced by a 3 μm thick silicon plate, which is Cu- or Au-coated on it's side facing the sphere, but un-coated bare Si on it's rear side. In a Yale experiment [17], the cavity wall again is replaced by an Au-coated sphere, while the movable plate is replaced by a SiN membrane, which is Au-coated on it's side facing the sphere, but un-coated bare SiN on it's rear side.

Still these experiments — like all other experiments, which as well don't pay attention to the surface conditions of their movable plate's rear sides — are seeing the Casimir-force (2), but not the force $F_{\text{ZPE}} = (14)$ caused by zero-point energy.

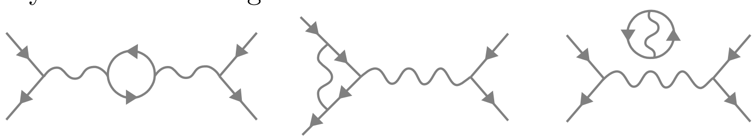
If the zero-point energy would exert forces onto metallic surfaces, then the term $k^4_{\text{M,left}} - k^4_{\text{M,right}}$ would dominate the observed forces in all of these experiments, making the tiny $\sim D^{-4}$ Casimir-force invisible. F_{ZPE} does not depend on D, and it can — depending on the relative reflectivities of the two sides of the movable plate — be attractive or repulsive. No force with that signature has ever been reported from any Casimir-force experiment.

Thus the experimental observations of the Casimir-force (2) (which obviously is the van der Waals-force, and nothing else) do stringently prove, that the force F_{ZPE} (caused by zero-point energy) does not exist.

4. How shall we dispose of that Zero-Point Energy?

With the force exerted onto metallic surfaces, allegedly being the direct macroscopic manifestation of the electromagnetic quantum field's zero-point energy, the only, and anyway faint indication for the measurable existence of a quantum field's zero-point energy is lost. On the other hand, the powerful counter-argument against the existence of that energy, i.e. the absence of it's gravitational effect, is persisting. Therefore one might be inclined to abandon the zero-point energy of quantum fields without further discussion. Still the advise from Jordan and Pauli — "it may be the simplest and most satisfactory conception, that in case of the electromagnetic field that zero-point radiation does not exist at all" — may be acceptable for the moment being, but it does not at all seem "satisfactory" to me.

To make this point clear, consider these three Feynman-diagrams, which are encountered — besides many others — in second order QED perturbation theory of the scattering cross section of two fermions:



The first two diagrams are called "connected", because all of their structures are directly or indirectly connected to the incoming and outgoing lines. All of the impressive achievements of QED like Lamb-shift, electron g-factor, hydrogen hyperfine-splitting, and so on, are described and computed by connected diagrams. The third diagram is "unconnected", because the vacuum bubble is not connected to the in- and outgoing lines. Such diagrams are rarely drawn and almost never computed, because we know upfront that all vacuum bubbles will eventually cancel in all orders of QED perturbation theory.

The essential point is, that we do not need to wait for somebody coming around and tell us that we should consider the vacuum bubbles as not existing. We even don't need feed-back from experiments to find out that the bubbles will cancel. It's a built-in feature of QED, that vacuum bubbles automatically cancel. Thus we would just spent unnecessary work if we started to compute them, but in the end we would arrive at the correct result anyway.

However it's not a built-in feature of any quantum field theory, that vacuum bubbles don't gravitate. This information must come from outside, and must somehow be fit in "by hand". This situation certainly can not be considered "satisfactory".

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