BROKEN SU(3) x SU(3) x SU(3) x SU(3) SYMMETRY

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I. INTRODUCTION

There are already several pieces of decisive evidence supporting the "eight-fold way" version of SU(3) symmetry, as proposed by Gell-Mann and Ne'eman,[1] to be the dominant feature of the strong interactions.

The symmetry is recognized by the fact that particles and resonances can be grouped into multiplets, and that certain regularities hold among various reaction amplitudes, decay widths, etc. We know so far the existence of a baryon octet B_8, a baryon resonance decuplet B^*_{10}, a pseudoscalar meson octet P_8, and a vector meson octet (plus a singlet?) V_8 ($V_8 + V_1$).

The SU(3) symmetry is a natural enlargement of the well-established SU(2) (isotopic spin) symmetry. A striking feature of these internal symmetries is that they are not exact, yet the

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deviation from perfect symmetry also follows a regular pattern. It was primarily on this basis that the various multiplets have been identified.

To understand the origin of these internal symmetries as well as their deviations will be one of the fundamental problems of elementary particle physics. There seem to be three important notions which are relevant to this problem.

1. Symmetries of fundamental fields. Suppose that there is a basic law governing all the elementary particles, in the form of equations of motion involving some fundamental fields. All the information about the actual world is explicitly or implicitly contained in such equations, and especially the fundamental symmetry should be manifest in them. If an equation (or the Hamiltonian) can be divided in two parts, one of them having the symmetry while the other not, and if the latter is "small" compared to the former in some sense, then we expect that symmetry to be manifest in actual phenomena with a definite pattern of deviation due to the existence of the second term. The question whether there can be higher symmetries, e.g. SU(4), is definitely related to how many fundamental fields there exist.

2. Spontaneous breakdown of symmetry. There is a possibility that not all the symmetries, exact or approximate, that are present in the equations of motion, are manifestly present in the world of observed phenomena. This is due to the fact that the same system of equations of motion for quantized fields may be

realizable in a number of different Hilbert spaces which are not invariant under the symmetry in question. In such a case, the ground state of a possible world (the "vacuum" as usually referred to) as well as its low lying excited states (the "elementary particles" or "elementary excitations") will not show the normal multiplet structure corresponding to a representation of the symmetry group. Examples of this kind are abundant in many-body problems.

3. Spontaneous creation of symmetry. The so-called bootstrap mechanism of creating particles and resonances is not easy to characterize precisely because it is not based on well-defined equations of motion (Hamiltonian) but rather a system of truncated S-matrices. One rather seeks a possible set of states and symmetries which are self-consistently closed within themselves. If, however, such a possible solution can also be derived from some form of Hamiltonian or equations of motion, then this procedure will become equivalent to finding the basic equations of motion having the desired properties, and the remarks made in 1. and 2. will apply here too.

Our primary concern is whether the SU(3) symmetry is the ultimate one in the sense that it is the one exhibited by the fundamental fields and that no higher symmetries are meaningful. Such would be the case if, for example, the "quark" or "ace" models were true, except that with six two-component spinors there is a room for the "chiral" or γ_5 SU(3) symmetry in addition. The fact that such a symmetry indeed has a meaning is well known.

On the other hand, if the number of fundamental fields were allowed to increase, certainly we have a possibility for even higher symmetries. For example, the SU(4) symmetry[2] with four fermion fields, and SU(3) x SU(3) symmetry,[3] with combinations of fundamental boson and fermion fields, have been proposed recently, which have the virtue of avoiding non-integral charge units. It is indeed puzzling why the octet model rather than the logically simple Sakata model seems to be adopted by nature.

Since we do not yet know completely what the fundamental fields and symmetries are, we will attack this problem step by step, starting with empirical evidences. Now any symmetry, rigorous or approximate, will normally manifest itself as a multiplet structure of particle levels, so that it would be natural to exploit all possible symmetries that may be realized by known low lying multiplets of particles.

To this end we make, in particular, the following two observations.

1) There exist an octet of baryons B_8 (spin-parity $1/2^+$) corresponding to a tensor representation $B_\mu(\mu, \nu = 1,2,3)$, $B^\mu_\mu = 0$, of SU(3). There is an indication that the 1405 Mev Y_o^* is $1/2^{\pm}$. Other known resonances have higher spins. These nine baryons may be combined to form a nonet B^ν_μ ($Y_o^* = \frac{1}{\sqrt{3}} B^\nu_\mu$). (The case $1/2^-$ may be understood from symmetry violation as will be shown later.) Now it would be easy to understand the occurrence of a tensor representation for the "ground state" baryons if we could

consider B to be the representation $(3,3^*)$ of a larger group $SU(3)_1 \times SU(3)_2$ so that the lower and upper indices of B^{ν}_{μ} actually correspond to $SU(3)_1$ and $SU(3)_2$ respectively. The usual SU(3) means the simultaneous transformation in both group spaces with identical parameters. However, the fundamental Hamiltonian, whatever its actual form, may possess symmetry under $SU(3)_1$ or $SU(3)_2$ alone provided that certain "small" terms are neglected. An analogous situation exists in atomic electrons in which orbital and spin angular momenta are separately conserved under neglect of the spin-orbit coupling.

2) As was already mentioned, there exists hidden "chiral" or γ_5 symmetry, which is revealed by the existence of partially conserved axial vector currents and the existence of pseudoscalar mesons. Such symmetry can be most readily explained as a spontaneous breakdown phenomenon in a γ_5-invariant Hamiltonian. Actually the pseudoscalar mesons should be expected to be massless if the symmetry were perfect. The finite, but relatively small masses of the P mesons (they are the lowest meson multiplet) would imply that the γ_5 symmetry is not rigorous but still the deviation is small. We do not wish to insist the symmetry to be rigorous, with the massless mesons being eliminated by some devious mechanisms. Such a point of view is neither necessary, nor natural, nor easy to realize. A spontaneous breakdown essentially means dynamical instability of a symmetric solution against infinitesimal perturbations acting as catalysts. In reality these may very well be small but finite rather than infinitesimal. With the introduction of chiral symmetry, each SU(3) of 1) now generates two

separate symmetries, to be applied to the left-handed and right-handed components of B. Thus we are led to a group

$$G = SU(3)_{1L} \times SU(3)_{1R} \times SU(3)_{2L} \times SU(3)_{2R}$$

This is the minimum extention of SU(3) symmetry that follows from the above two remarks.[4] The rest of the paper will be devoted to the study of the structure and consequences of this symmetry group, avoiding for the moment as much as possible the question of what the underlying fundamental fields and their equations will be.

II. ALGEBRA AND PARTICLES

We start from the baryon fields. Consider 18 two-component spinor fields $B_{L\mu}$, $B_{R\mu}$; B_L and B_R respectively form a representation $(3_L, 3_L^*)$ and $(3_R, 3_R^*)$ of the groups $SU(3)_{1L} \times SU(3)_{2L}$ and $SU(3)_{1R} \times SU(3)_{2R}$, where the lower and upper indices of B_μ correspond respectively to $SU(3)_1$ and $SU(3)_2$. Thus if we introduce the eight generators $\lambda^i (i = 1, . . 8)$ as defined in Ref. 1, we can write the four infinitesimal transformations of B as

$$SU(3)_{1L}: \qquad \delta B_{L\mu}^{\nu} = i\alpha^i \lambda^i_{\mu\rho} B_{L\rho}^{\nu}$$

$$\delta B_{R\mu}^{\nu} = 0$$

$$SU(3)_{2L}: \qquad \delta B_{L\mu}^{\nu} = i\alpha^i \lambda^{i*}_{\nu\rho} B_{L\mu}^{\rho} = -i\alpha^i B_{L\mu}^{\rho} \lambda_{\rho\nu}$$

$$\delta B_{R\mu}^{\nu} = 0$$

$$SU(3)_{R1}: \quad \delta B_{L\mu}^{\ \nu} = 0$$

$$\delta B_{R\mu}^{\ \nu} = i\alpha^i \lambda_{\mu\rho} B_{R\rho}^{\ \nu}$$

$$SU(3)_{R2}: \quad \delta B_{L\mu}^{\ \nu} = 0$$

$$\delta B_{R\mu}^{\ \nu} = -i\alpha^i \lambda^{i*}_{\nu\rho} B_{R\mu}^{\ \rho} = -i\alpha^i B_{R\mu}^{\ \rho} \lambda_{\rho\nu} \tag{2.1}$$

There are thus four sets of 8 infinitesimal generators $\lambda^i/2$, $i = 1 \ldots 8$ operating in 4 different spaces. The second way of writing shows that it is convenient to regard B_μ^ν as a matrix $B_{\mu\nu}$. This will be adopted from here on. Under a finite transformation, B_μ transforms according to

$$B_L \longrightarrow S_{1L}\ B_L\ S_{2L}$$

$$B_R \longrightarrow S_{2R}\ B_R\ S_{2R} \tag{2.2}$$

where each S is of the form

$$S = \exp[\ \frac{i}{2} \sum_i \alpha^i \lambda^i\ ] \tag{2.3}$$

Let us call these four sets of infinitesimal generators G_L, H_L, G_R, H_R. In each set, the generators satisfy the commutation rule

$$[G_{Li},\ G_{Lj}] = i f_{ijk} G_{Lk} \text{ etc.} \tag{2.4}$$

while different sets commute. It is also convenient to introduce the following linear combinations

$$F^{Vi} = G_L^i + G_R^i - H_L^i - H_R^i$$

$$D^{Vi} = G_L^i + G_R^i + H_L^i + H_R^i$$

$$F^{Ai} = G_L^i - G_R^i - H_L^i + H_R^i$$

$$D^{Ai} = G_L^i - G_R^i + H_L^i - H_R^i \tag{2.5}$$

with the commutation relations

$$\begin{aligned}
[F^{Vi}, F^{Vj}] &= if_{ijk}F^{Vk} & \qquad [F^{Ai}, D^{Aj}] &= if_{ijk}D^{Fk} \\
[F^{Vi}, D^{Vj}] &= if_{ijk}D^{Vk} & \qquad [F^{Ai}, D^{Vj}] &= if_{ijk}D^{Ak} \\
[F^{Vi}, F^{Aj}] &= if_{ijk}F^{Ak} & \qquad [D^{Vi}, D^{Vj}] &= if_{ijk}F^{Vk} \\
[F^{Vi}, D^{Aj}] &= if_{ijk}D^{Ak} & \qquad [D^{Ai}, D^{Aj}] &= if_{ijk}F^{Vk} \\
[F^{Ai}, F^{Aj}] &= if_{ijk}F^{Vk} & \qquad [D^{Vi}, D^{Aj}] &= if_{ijk}F^{Ak}
\end{aligned} \tag{2.6}$$

To each generator corresponds a four-vector current density j_μ^i which will satisfy $\partial j_\mu^i/\partial x_\mu^i = 0$ if the Hamiltonian is invariant under this symmetry. These currents are simply

$$\begin{aligned}
(j_{1L})_\mu^i &= T_r\bar{B}_L \frac{\lambda^i}{2} \gamma_\mu B_L \;, & \qquad (j_{2L})_\mu^i &= T_r\bar{B}_L\gamma_\mu B_L \frac{\lambda^i}{2} \;, \\
(j_{1R})_\mu^i &= T_r\bar{B}_R \frac{\lambda^i}{2} \gamma_\mu B_R \;, & \qquad (j_{2R})_\mu^i &= T_r\bar{B}_R\gamma_\mu B_R \frac{\lambda^i}{2} \;,
\end{aligned} \tag{2.7}$$

where the trace is taken with respect to the SU(3) tensor indices. The currents corresponding to the generators (2.5) are

$$
\begin{aligned}
(\mathcal{V}_F)^i_\mu &= \mathrm{Tr}\bar{B}\gamma_\mu[\lambda^i, B]/2 \\
(\mathcal{V}_D)^i_\mu &= \mathrm{Tr}\bar{B}\gamma_\mu\{\lambda^i, B\}/2 \\
(a_F)^i_\mu &= \mathrm{Tr}\bar{B}\gamma_\mu\gamma_5[\lambda^i, B]/2 \\
(a_D)^i_\mu &= \mathrm{Tr}\bar{B}\gamma_\mu\gamma_5\{\lambda^i, B\}/2
\end{aligned} \tag{2.8}
$$

which justifies the notation F^V, etc. F and D corresponds to the familiar antisymmetric and symmetric types of octets contained in the product $\bar{B} \times B \sim 8 \times 8$ under the ordinary SU(3). Now the algebra generated by the 32 generators F^V, F^A, D^V, D^A contains various subalgebras corresponding to subgroups of $[SU(3)]^4$. Each such subgroup is a possible candidate of symmetry realized with a varying degree of perfectness. Larger the group, the more approximate the symmetry will be. Thus we expect a hierarchy of approximate symmetries, all contained in our largest group $[SU(3)]^4$. These subgroups and corresponding subalgebras A may be classified as follows:

$$
\begin{aligned}
A_{10} &= \{F^{Vi}\} \\
A^D_{11} &= \{F^{Vi}, D^{Aj}\} \\
A^F_{11} &= \{F^{Vi}, F^{Aj}\} \\
A_{20} &= \{F^{Vi}, D^{Vj}\} \\
A^{\pm}_{21} &= \{F^{Vi}, D^{Vj}, F^{Ak} \pm D^{Ak}\} \\
A_{22} &= \{F^{Vi}, D^{Vj}, F^{Ak}, D^{Al}\}
\end{aligned} \tag{2.9}
$$

Here A_{mn} means that it contains m vector octets and n axial vector octets of conserved currents, corresponding to a group $\sim [SU(3)]^{m+n}$. We note that $A_{mn} \subset A_{m'n'}$ if $m < m'$, $n < n'$ (but not necessarily if $m \leq m'$, $n \leq n'$). At this point it is useful to consider extra discreet symmetries which can be naturally incorporated in our scheme by slightly enlarging $[SU(3)]^4$. Namely, we consider the permutation group g_4 of the four SU(3) groups among themselves. In particular, let us define the operations

$$\mathcal{R}_L: \quad SU(3)_{1L} \longleftrightarrow SU(3)_{2L}$$

$$\mathcal{R}_R: \quad SU(3)_{1R} \longleftrightarrow SU(3)_{2R}$$

$$\mathcal{P}_1: \quad SU(3)_{1L} \longleftrightarrow SU(3)_{1R}$$

$$\mathcal{P}_2: \quad SU(3)_{2L} \longleftrightarrow SU(3)_{2R}$$

$$\mathcal{R} = \mathcal{R}_1 \mathcal{R}_2 = \mathcal{R}_2 \mathcal{R}_1, \quad \mathcal{P} = \mathcal{P}_1 \mathcal{P}_2 = \mathcal{P}_2 \mathcal{P}_1 .$$

$\mathcal{R}$ corresponds to the interchange of the two indices of B_μ, whereas $\mathcal{P}$ corresponds to the interchange $B_L \longleftrightarrow B_R$. $\mathcal{R}$ can be identified with the "R-parity" (R-symmetry). It is easy to see that the four generators (2.3) (and the corresponding currents (2.8) constitute a representation of g_4, and in particular

$$\mathcal{R}: \quad F^V \longrightarrow -F^V, \quad F^A \longrightarrow -F^A$$
$$\qquad D^V \longrightarrow D^V, \quad D^A \longrightarrow D^A$$

$$\mathcal{P}: \quad F^V \longrightarrow F^V, \quad D^V \longrightarrow D^V$$
$$\qquad F^A \longrightarrow -F^A, \quad D^A \longrightarrow -D^A$$

Let us next see if any of these subalgebras can be discarded as irrelevant in the face of observed evidence. On this point we note the following.

1. The coupling of $\mathcal{P}$ octet to the baryons as well as the axial vector weak current is of the type $\alpha D + (1-\alpha)F$, where $\alpha \approx 2/3$.[6]

In fact these two must have the same structure if the Goldberger-Treiman type relation is to hold as a result of spontaneous breakdown of symmetry. Now the only subalgebras of A_{22} which contain axial currents close to this ratio are A_{11}^{D} and A_{21}^{+}, respectively with $\alpha = 1$ and $\alpha = 1/2$. We then obtain the following two sequences of algebras

$$\begin{aligned} C_{1/2} &= A_{22} \supset A_{21}^{+} \supset A_{20} \supset A_{10} \,, \\ C_{1} &= A_{22} \supset A_{11}^{D} \supset A_{10} \,. \end{aligned} \tag{2.10}$$

Lacking further information, we shall study them in the following classification of particles. We shall denote a representation of the $[SU(3)]^4$ group by the symbol (p,q,r,s) where the indices run in the order $SU(3)_{1L}$, $SU(3)_{1R}$, $SU(3)_{2L}$, $SU(3)_{2R}$. The nine baryons (18 components) are assigned to $(313^*1) + (1313^*)$. The generators G_L, G_R, H_L, H_R belong to (8111), (1811), (1181), (1118) respectively. Under restriction to a subgroup, a representation is contracted by multiplication. The 6 subalgebras listed above reduce a representation (pqrs) as follows:

$$\begin{aligned} A_{21}^{+}: &\quad (pqrs) \longrightarrow (p,q,r\times s) \\ A_{21}^{-}: &\quad (pqrs) \longrightarrow (p\times q,r,s) \\ A_{20}: &\quad (pqrs) \longrightarrow (p\times q,\ r\times s) \\ A_{11}^{D}: &\quad (pqrs) \longrightarrow (p\times s,q\times r) \\ A_{11}^{F}: &\quad (pqrs) \longrightarrow (p\times r,q\times s) \\ A_{10}: &\quad (pqrs) \longrightarrow (p\times q\times r\times s) \end{aligned} \tag{2.11}$$

(The order of the indices after reduction is arbitrarily taken.) The baryon resonances and mesons will be represented in a similar fashion. We will discuss below the properties of possible particles in each of the algebras (2.10).

1) A_{20}. This algebra does not contain the chiral (γ_5) SU(3) symmetry. The baryons belong to a representation $(3,3^*)$ according to Eq.(2.11). Since we have an $[SU(3)]^2$ group, there exist four commuting quantum numbers, F^{V3}, F^{V8}, D^{V3}, D^{V8}, which we call I_3, Y, I_3' and Y'. I_3 and Y are the ordinary isospin and hypercharge. The assignment of those numbers to various baryons is given in Table 1. We recall that F^V and D^V behave like the F- and D-type couplings of baryons and octet mesons. It is clear then that Σ^0, Λ^0 and Y^0 cannot be diagonal with respect to I_3' and Y'. With respect to these three states, I_3' and Y' form the matrices

$$I_3' = \begin{pmatrix} 0 & \frac{1}{\sqrt{3}} & \frac{2}{3} \\ \frac{1}{\sqrt{3}} & 0 & 0 \\ \frac{2}{3} & 0 & 0 \end{pmatrix} \qquad Y' = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -\frac{2}{3} & \frac{\sqrt{8}}{3} \\ 0 & \frac{\sqrt{8}}{3} & 0 \end{pmatrix}$$

Their eigenstates are

$$I_3' = \pm 1, \quad Y' = \frac{2}{3} \quad : \quad \frac{1}{\sqrt{6}} (\pm \sqrt{3}\,\Sigma^0 + \Lambda + \sqrt{2}\, Y^0)$$

$$I_3' = 0, \quad Y' = -\frac{4}{3} : \quad \frac{1}{\sqrt{3}} (\sqrt{2}\,\Lambda - Y^0)$$

The D type symmetry is of course badly broken by the mass differences. Otherwise the quantum numbers I_3' and Y' would imply additional selection rules[4] which are not actually observed. However, they might become manifest at extremely high energies.

If vector gauge fields are assumed for this symmetry, they must behave like the 16 vector currents of A_{20}. In other words, there will be 16 vector fields V_1^i and V_2^i. They are associated respectively with $SU(3)_1$ and $SU(3)_2$ and correspond to representations (8,1) and (1,8). From these we can construct also $V_F^i = V_1^i - V_2^i$ and $V_D^i = V_1^i + V_2^i$. These behave like vector currents v_F^i and v_D^i of Eq. (2.8). From the standpoint of group representation alone, however, we can also consider other representations, such as $(3,3^*)$ and $(3^*,3)$ behaving like B and $\bar{B}$. A self-conjugate combination $(3,3^*) + (3^*,3)$ reduces to $8 + 8 + 1 + 1$ under ordinary SU(3). Since $\bar{B} \times B$ does not contain $(3,3^*)$ or $(3^*,3)$, these mesons cannot couple to the baryons trilinearly under A_{20}.

Pseudoscalar mesons (and possibly other mesons) may be discussed in a similar way, except that they are not related to γ_5 symmetry within this algebra. The baryon resonances can be obtained from baryon and meson representations if the resonances remain resonances (i.e. the decay coupling exists) under A_{20}. With the assignment (8,1) and (1,8) for the meson M(= P or V), we get

$$(8,1) \times (3,3^*) = (3,3^*) + (6^*,3^*) + (15,3^*)$$
$$(1,8) \times (3,3^*) = (3,3^*) + (3,6) + (3,15^*)$$

The decuplet is contained in (3,6) and $(15,3^*)$ which reduce respectively to $10 + 8$ and $8 + 10 + 27$ under SU(3).

The assignment $(3,3^*) + (3^*,3)$ to the mesons yields

$$M \times B = (3^*,3) + (3^*,6^*) + (6,3) + (6,6^*)$$
$$+ (8,8) + (8,1) + (1,8) + (1,1)$$

The 10 is contained in $(6,3)$ and $(8,8)$.

2) A_{11}^D. We consider this as the minimum symmetry comprising the γ_5 type one.[7] Although the group structure is again $[SU(3)]^2$, the physical manifestations are different from the A_{20} case. If the γ_5 symmetry is spontaneously broken by the baryon mass, there will be no extra quantum numbers like I_3' and Y' for any particle. Parity conservation also demands a pair of conjugate representations to appear simultaneously. Thus the baryons $B_L + B_R$ belong to $(3,3^*) + (3^*,3)$. Under the gauge field principle, there will be 8 vector (V_F) and 8 axial vector (A_D) mesons coupled to conserved currents v_F and a_D respectively.[4,8] These correspond to $(8,1) \pm (1,8)$. As the common baryon mass is created, we also induce a massless pseudoscalar octet P coupled to the divergence of a_D. These "zerons"[9] must exist, as an "incomplete" representation of A_{11}^D, in addition to any other bona fide pseudoscalar multiplets, if the spontaneous breakdown is invoked. In general, it does not make sense to talk about a representation (p,q) of A_{11}^D any more, so (p,q) collapses into $(p \times q)$ of A_{10}. The algebra A_{11}^D has been emphasized by Gell-Mann[7] because of its minimal character. Our point of view becomes somewhat different when we treat symmetry violations.

A^+_{21}. This algebra combines two conserved vector current octets v_F and v_D, and one axial vector current $a_F + a_D$. Alternatively speaking, these have both ordinary and γ_5 symmetry with respect to $SU(3)_1$, and only ordinary symmetry with respect to $SU(3)_2$. It will be naturally realized in a model like Schwinger's[3] where both spinor and vector fields are introduced as fundamental fields. The baryons, according to (2,11), belong to the representation $(P_{1L}, q_{1R}, r_2) = (3,1,3^*) + (1,3,3^*)$. The gauge fields will be of the type (8,1,1), (1,8,1) and (1,1,8). The axial vector mesons are coupled to the baryons with the ratio F/D = 1, which means, e.g., $g_{\overline{\Xi}\Xi\pi} = g_{\overline{N}\Sigma K} = 0$. After spontaneous breakdown, the representation (p,q,r) of A^+_{21} collapses into (p x q, r) of A_{20}. Thus the induced P as well as A become (8,1), with the F/D = 1 coupling ratio. So this algebra is characterized by the lack of R-symmetry as $F + D \rightarrow -F + D$ under R.

Now the baryon resonances may be constructed out of P x B which decomposes as

$$\begin{aligned}(8,1) \times (3,3^*) &= (24,3^*)\\ &= (15,3^*) + (6^*,3) + (3,3^*)\end{aligned}$$

Because of the non R-invariance, we have obtained $(15,3^*)$ which contains the SU(3) decuplet 10, but no 10^* is found here.

III. SYMMETRY BREAKING AND MASS FORMULAS

We can contemplate four kinds of symmetry breaking within our context.

1. Breaking which destroys non-γ_5 $SU(3)_1 \times SU(3)_2$, reducing it to simple SU(3).

2. The Gell-Mann-Okubo breaking of simple SU(3).

3. Spontaneous breaking of γ_5 SU(3) groups and possibly non-γ_5 groups.

4. Non-spontaneous breaking which violates γ_5 symmetry and gives zerons finite masses.

The last one is a rather obscure problem at the moment, and we will not go into it in this paper. We start with the first kind.

1. Lifting of degeneracy. The pattern of breakdown of $SU(3)_1 \times SU(3)_2$ is similar to that due to spin-orbit coupling for angular momentum. A multiplet (ℓ_1, ℓ_2) of product rotation group $O(3)_1 \times O(3)_2$ will split, as the coupling between $O(3)_1$ and $O(3)_2$ is turned on, into irreducible representations of an O(3) characterized by the total angular momentum $(\underset{\sim}{\ell}_1 + \underset{\sim}{\ell}_2)^2$. In the present case, the generators $F_1^i = G_L^i + G_R^i$, $F_2^i = H_L^i + H_R^i$ correspond to $\underset{\sim}{\ell}_1$ and $\underset{\sim}{\ell}_2$. In contrast to O(3), however, two scalars (Casimir operators) may be constructed out of them. These are

$$F^2 = \sum_{i=1}^{8} F^i F^i ,$$

$$G^3 = \sum_{i,j,k} d_{ijk} F^i F^j F^k , \qquad F^i = F_1^i + F_2^i \tag{3.1}$$

Subtracting from them corresponding invariants for $SU(3)_1$ and $SU(3)_2$, we obtain

$$\frac{1}{2}(F^2 - F_1^2 - F_2^2) = \sum_i F_1^i \cdot F_2^i = \mathcal{F}$$

$$\frac{1}{3}(G^3 - G_1^3 - G_2^3) = \sum_{ijk} d_{ijk}(F_1^i F_1^j F_2^k + F_1^i F_2^j F_2^k) = \mathcal{G} \qquad (3.2)$$

These are the analogs of "spin-orbit" coupling. In terms of F^{Vi} and D^{Vi} of Eq. (2.3), they may also be written

$$\mathcal{F} = -\frac{1}{4}\sum_i (F^{Vi}F^{Vi} - D^{Vi}D^{Vi})$$

$$\mathcal{G} = \frac{1}{4}\sum_{ijk} d_{ijk}(D^{Vi}D^{Vi}D^{Vk} - D^{Vi}F^{Vj}F^{Vk}) \qquad (3.3)$$

The values of F^2 and G^3 for each SU(3) multiplet are tabulated in Table II. The mass splitting in an $SU(3)_1 \times SU(3)_2$ supermultiplet will then be characterized by a function $f(\mathcal{F},\mathcal{G})$, which is determined by the dynamics of the system. A simplest assumption will be to take a linear form in $\mathcal{F}$ and $\mathcal{G}$, in analogy to spin-orbit splitting. The mass formula is then

$$\Delta m = a\mathcal{F} + b\mathcal{G} \qquad (3.4)$$

(As usual we should rather take Δm^2 for bosons.) It is also conceivable that the $\mathcal{G}$ term is small since it is trilinear in the generators. In this case

$$\Delta m = a\mathcal{F} \qquad (3.5)$$

Eq. (3.4) (or(3.5)) gives a useful sum rule only if there are more than three (or two) SU(3) multiplets in a supermultiplet.

Applied to the baryon nonet (3.3^*), it splits the octet B_8 and the singlet $B_1 = Y^o$. An interesting possibility is that Δm is large compared to the common mass so that one of them may become negative. In this case we interpret the negative mass particle to have a relative odd parity since the transformation $B \rightarrow i\gamma_5 B$ restores the positive sign of mass. Experimentally the parity of $Y_o^*(1405)$ is not well established, but either parity can be accommodated in our scheme. In other words,

$$\begin{aligned} \Delta m = m_8 - m_1 &\approx 1150-1400 = -250 \text{ Mev if } B_1 \text{ is } 1/2^+, \\ &\approx 1150+1400 = +2550 \text{ Mev if } B_1 \text{ is } 1/2^-, \end{aligned} \tag{3.6}$$

where m_8 is the octet mass before Okubo-Gell-Mann splitting. Obviously a negative m^2 is not allowed for the bosons.

Turning to the baryon resonances, Eq. (3.5) yields a sum rule if the decuplet belongs to $(15,3^*) = 10 + 8 + 27$. Namely

$$3\, m_{27} - 5\, m_{10} + 2\, m_8 = 0 \tag{3.7}$$

Suppose we identify 8 with the second πN resonance[10] with the assignment $j^P = \frac{3}{2}^+$. Then we would also expect a pretty low energy ($<$ 2 Bev taking $m_{10} \sim m_8 \sim 1500$ Mev) 27 multiplet with $j^P = \frac{3}{2}^+$, which actually is unlikely to exist. If the second resonance is $\frac{3}{2}^-$, 27 is still $\frac{3}{2}^+$ but the mass is a little higher: $m_{27} \sim 3$ Bev. With the two-parameter formula (3.4) of course no prediction can be made.

2. Gell-Mann-Okubo splitting. We next discuss the switching on of the Gell-Mann-Okubo splitting within SU(3) multiplets.

Within the context of $SU(3)_1 \times SU(3)_2$ symmetry, this amounts to the assumption that the G-M-O perturbation behaves like $F^{V8} \sim Y$ and $D^{V8} \sim Y'$. Since D^V couples in general different multiplets in an $SU(3)_1 \times SU(3)_2$ supermultiplet, it causes not only the splitting within each multiplet but also mixing among different multiplets. Take, for example, the baryon nonet. The mass formula takes the form

$$m = aY + bY + c\mathcal{J} + m_o \tag{3.8}$$

where the third term splits m_1 and m_8.

The D term mixes Y^o and Λ so that in the space of bare (Y^o_o, Λ_o), Δm is a matrix

$$\Delta m = \begin{pmatrix} 0 & \sqrt{2/3}\, b \\ \sqrt{2/3}\, b & 3c - \frac{b}{\sqrt{3}} \end{pmatrix} \tag{3.9}$$

with eigenvalues

$$\Delta m = \frac{1}{2}\left[3c - \frac{b}{\sqrt{3}} \pm \left(3b^2 - 2\sqrt{3}\, bc + 9c^2\right)^{1/2}\right] \tag{3.10}$$

The relations following from Eqs. (3.8) and (3.9)

$$\begin{aligned}
(N + \Xi) &= 6c - b/\sqrt{3} + 2m_o \\
\Sigma &= 3c + b/\sqrt{3} + m_o . \\
\Lambda + Y^o &= 3c - b/\sqrt{3} + 2m_o \\
(\Lambda - m_o)\,(Y^o - m_o) &= -\tfrac{2}{3}\, b^2 \\
m_o &= (3\Lambda + 3Y + \Sigma - 2N - 2\Xi)/3
\end{aligned} \tag{3.11}$$

lead to the mass sum rule

$$(3Y^{o} + \Sigma - 2N - 2\Xi)\,(3\Lambda + \Sigma - 2N - 2\Xi) = -2(2\Sigma - N - \Xi)^{2} \qquad (3.12)$$

This gives a Y^{o} mass of $\sim$ + 700 Mev. However, because of the very small second factor ($\sim$25 Mev) on the left-hand side (the Gell-Mann-Okubo relation!), the formula is sensitive to small shifts of the baryon masses, and presumably should not be used in this way to predict the Y^{o} mass.

3. Spontaneous breaking. The motivation for invoking spontaneous breakdown of γ_5 symmetry has been the success of the Goldberger-Treiman relation. However, the mechanism is of such a general character that perhaps it is worthwhile to consider also the spontaneous breakdown of non-γ_5 SU(3) groups. This mechanism, superimposed on the primary breaking of symmetry that exists in the equations of motion themselves, will result in an enhancement of the breaking effect. The characteristic feature of spontaneous breaking is the existence of "zerons" which form an "incomplete multiplet." If the breaking is not purely spontaneous, the zerons will not be massless.

As a practical criterion for the onset of spontaneous breakdown we may take the following two conditions:

1. Existence of an incomplete multiplet.
2. Approximate Goldberger-Treiman type relations.

For example, consider the algebra A_{20}(or A_{21}). The D symmetry is broken by the splitting of singlet and octet baryons, the mass difference behaving like a "spurion." Because of this mass difference, the cross currents $\langle 8|v_\mu^D|1\rangle$ must be supplied with induced scalar terms in order to satisfy the continuity equation. In other words we expect an octet $\{S_D^i\}$ (incomplete "16-plet") of D-type scalar zerons, coupled between baryon singlet and octet, but not between octet members, with the Goldberger-Treiman relation

$$G(m_8-m_1) = g_S\gamma_S \tag{3.13}$$

Here $G = G(0)$ is the normalization of v_D-current between B_1 and B_8; g_S is the $\bar{B}_8\, B_1\, S$ coupling constant; γ_S is the coupling appearing in $\langle 0|v_\mu^i|S^i\rangle = \gamma_S q_\mu$. Let us now introduce the Gell-Mann-Okubo splitting. If this is spontaneous, it will make induced scalar terms appear also for the $Y = \pm 1$ D-type currents $\langle N|v_D|\Sigma\rangle$, $\langle N|v_D|\Lambda\rangle$, $\langle \Xi|v_D|\Sigma\rangle$, $\langle \Xi|v_D|\Lambda\rangle$ (and h.c.), and the $Y = 0$, $T = 1$ current $\langle\Sigma|v_D|\Lambda\rangle$ (and h.c.). In addition, there will be induced scalar terms for the $Y = \pm 1$ F-type currents v_F implying an incomplete SU(3) "octet" of four scalar mesons $\{S_F^i\}$ with $Y = \pm 1$, $T = 1/2$. This dramatic breakdown of SU(3) multiplet structure should be contrasted with the ordinary weak violation in which we would have a complete octet of 8 scalar mesons (with mass splitting, of course). The latter mesons can also exist, but they do not necessarily figure in the Goldberger-Treiman-type relations, and their masses would not tend to zero even when the hidden symmetry becomes perfect.

In this way, we expect an octet of scalar zerons under the algebras A_{20} and A_{21} if spontaneous breakdown applies, and a quartet of $Y = \pm 1$, $T = 1/2$ scalar mesons if the reduction of ordinary SU(3) to SU(2) is also spontaneous. The $\varkappa$(725 Mev) meson could be interpreted in this latter way if its companions were not found to make up an octet.[11]

We make two final remarks. The Goldberger-Treiman type relations predict the coupling of zerons which is not universal but follows a definite pattern depending on the mass splitting. An application of this principle to the $B_{10}{}^* \, B_8 \, P_8$ coupling (using the axial vector current, of course) satisfactorily reproduces the relative decay widths of B^*_{10}.[12] In the same way, we can compute the $Y^o \rightarrow \Sigma \pi$ decay rate, which depends on the parity of Y^o and the D/F ratio of the $\bar{B}BP$ coupling. The result is given in Table III. It favors $J^P = 1/2^+$ assignment.

Further interesting results are obtained if we make the assumption that each current is dominated by a vector meson so that the form factor $G(q^2) = m^2/(q^2 + m^2)$. In this case it is possible to derive a relation between the vector (axial vector) meson and scalar (pseudoscalar) zeron coupling constants.[13] This formula takes the following form

$$\left[\frac{(g_V)^i_{jk}}{(m^i_V)^2}\right]^2 = \left[\frac{(g_S)^i_{jk}}{M_j - M_k}\right]^2$$
$$\left[\frac{(g_A)^i_{jk}}{(m^i_A)^2}\right]^2 = \left[\frac{(g_P)^i_{jk}}{M_j + M_k}\right]^2 \qquad (3.14)$$

where $(g_V)^i_{jk}$ is the $\bar{B}_j B_k V_i$ coupling, etc. This enables one to predict g_S and g_A from the known g_V and g_P.

IV. WEAK AND ELECTROMAGNETIC INTERACTIONS

Whether or not weak interactions are mediated by bosons, their structure is essentially defined once the vector and axial vector currents are known. In higher symmetry schemes it is natural to identify the weak currents with the conserved currents derived from symmetry. As we have pointed out earlier A_{22} by itself does not completely determine the weak currents. In order to find them one has to consider a chain of algebras of the type (2.10). The smallest algebras of the chain will then dictate the D/F structure of the weak currents. Because of the presence of A_{10} in both $C_{1/2}$ and C_1 the vector current will be pure F-type. The structure of the axial vector current will be determined by A^{+}_{21} and A^{D}_{11} to be $\alpha = 1/2$ and $\alpha = 1$ for $C_{1/2}$ and C_1, respectively. Of course these values do not take into account symmetry breaking and therefore a raw comparison to the experimental value $\alpha \approx 2/3$ might not be very meaningful. As can be seen from Table IV the difference between $C_{1/2}$ and C_1 for leptonic decays manifests itself in the branding ratios for $\Lambda \rightarrow p$ and $\Sigma^- \rightarrow n$ decays, and the assymmetry parameters for $\Lambda \rightarrow p$, $\Sigma^- \rightarrow n$ and $\Xi^- \rightarrow \Lambda$ decays. The largest of these differences is for $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$. Here C_1 predicts a 5 times larger branching rate than $C_{1/2}$ and a V + 1.15A structure as compared with pure V for $C_{1/2}$. We therefore suggest this decay to be a particularly favorable one for finding which of the chains $C_{1/2}$ and C_1 is more relevant to nature.[14]

The electromagnetic currents are F-type. Invariance under products of SU(3) groups will, however, give us some clues on the electromagnetic and weak form factors. Let us consider the matrix elements of the conserved vector current between baryon states

$$\langle B' | v_\mu^x | B \rangle = \frac{m}{\sqrt{EE'}} \bar{u}_{B'} [\gamma_\mu F^x_{B'B} f_1 + \sigma_{\mu\nu} q_\nu (F^x_{B'B} f_2 + D^x_{B'B} d_2)] u_B \tag{4.1}$$

Since in A_{22}[15] we have two octets of vector mesons we expect both of them to contribute to f_1, f_2 and d_2. If on the basis of experimental indications we accept that

$$t\, f_2(t) \underset{t\to\infty}{\longrightarrow} 0 \quad \text{and} \quad t\, d_2(t) \underset{t\to\infty}{\longrightarrow} 0 \tag{4.2}$$

even in the presence of symmetry breaking effects, then Eq. (4.2) should place severe restrictions on the symmetry breaking. In order to discuss these restrictions let us make the following assumptions:

i) f_2 and d_2 are dominated by single vector meson intermediate states.

ii) The vector meson-baryon Pauli couplings do not appreciably depart from the symmetric couplings even in the presence of symmetry breaking (for both octets of vector mesons).

iii) A singlet vector meson (ω_0) mixes with the lower lying octet $V = (\rho, \varphi_0, K^*)$ but no singlet mixes with the second octet $V' = (\rho', \varphi', K^{*'})$, and there is no appreciable

V-V' mixing.

iv) The V-W(W = photon or intermediate weak boson) and V'-W couplings depart from their symmetric values in direct proportion to $m_V^2/m_{V_o}^2$ and $m_{V'}^2/m_{V'_o}^2$ (m_{V_o} and $m_{V'_o}$ being the common mass of the mesons in the limit of exact symmetry).[17]

With these assumptions Eq. (4.2) leads to

$$\frac{m_\rho^2}{m_{\rho'}^2} = \frac{m_{K^*}^2}{m_{K^{*\prime}}^2} = \frac{4m_{K^*}^2 - m_\rho^2}{3m_{\varphi'}^2} = -\frac{D'}{D} = -\frac{F'}{F} \qquad (4.3)$$

where D and F(D' and F') are the D- and F-type BBV(BBV') Pauli coupling constants. The relations (4.3) are very powerful in that they determine the masses of all V' mesons once the mass of one V' meson is known. The axial vector and scalar mesons and their contributions to weak-interaction form factors have been discussed in Chapter III.

V. CONCLUSIONS

We have presented in this paper a minimal extension of SU(3) symmetry that provides conserved axial vector-currents along with the conserved vector currents and at the same time places the baryons in a more fundamental representation than SU(3). This scheme led us naturally to two hierarchies of symmetries that were capable of fixing such parameters as the D/F ratio of the axial vector current in weak interactions. The breaking of the constituents of these hierarchies is expected to increase with their symmetry.

These higher symmetries lead to new additive quantum numbers for strong interactions. They furthermore produce particle multiplets of high dimensionalities (e.g., the $J^P = 3/2^+$ baryon decuplet now is further included in a 45-dimensional multiplet in the company of a 27-plet and of an octet). To find these extra particles is one immediate possibility to check this scheme. Of course, because of symmetry-breaking effects large mass-splittings are expected within the multiplets. We have presented a specific symmetry-breaking mechanism based on spontaneous breakdown due to vacuum degeneracy. This mechanism then automatically led us to the existence of scalar and pseudoscalar mesons appearing in "incomplete multiplets." The coupling of these mesons to baryons is uniquely determined by the requirement of current conservation. We have also given mass-formulae (3.7 and (4.3) that locate the mass values at which the new particles have to be sought.

It is now an important experimental problem to find the axial vector and scalar mesons that lie in the center of our scheme.

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REFERENCES

1. M. Gell-Mann, Calif. Inst. Technology Report CTSL-20 (1961);
   Phys. Rev. 125, 1067 (1962).
   Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

2. P. Tarjanne and V. L. Teplitz, Phys. Rev. Lett. 11, 447 (1963);
   D. Amati, H. Bacry, J. Nuyts, J. Prentki, Phys. Lett. 11, 190 (1964);
   Z. Maki, Prog. Theor. Phys. 31, 331 and 333 (1964);
   I. Sogami, Prog. Theor. Phys. 31, 725 (1964);
   Y. Hara, Phys. Rev. 134, B701 (1964).

3. J. Schwinger, Phys. Rev. Lett. 12, 237 (1964).
   See also F. Gürsey, T. D. Lee and M. Nauenberg, Phys. Rev. 135, 467 (1964).

4. P.G.O. Freund and Y. Nambu, Phys. Rev. Lett. 13, 221 (1964).
   A similar scheme also has been proposed by A. Salam and J. C. Ward, to be published.

5. R. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963);
   A. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963), Nuovo Cimento 31, 1324 (1964);
   R. H. Capps, Nuovo Cimento 27, 1208 (1963);
   J. J. de Swart and C. K. Iddings, Phys. Rev. 130, 319 (1963).

6. N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963);
   N. Brene, B. Hellesen and M. Roos, Phys. Lett. 11, 344 (1964);
   W. Willis et.al., Phys. Rev. Lett. 13, 291 (1964).

7. M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964).

8. The axial vector gauge field has been considered before by A. Salam and J. C. Ward, Nuovo Cimento 19, 167 (1961). See also P.G.O. Freund and Y. Nambu, Phys. Lett. to be published; Riazuddin and R. E. Marshak, Phys. Lett. 11, 182 (1964).

9. We have proposed these terms in P.G.O. Freund and Y. Nambu, Phys. Rev. Lett. 13, 221 (1964).

10. The parity assignment as well as the identification of the second octet is not definitely established yet. One possibility is $N^*(1518)$, $Y_1^*(1660)$, $Y_0^*(1660)$, $\Xi^*(1810)$ with $J^P = 3/2^-$.

11. Y. Nambu and J. J. Sakurai, Phys. Rev. Lett. 11, 42 (1963).

12. P.G.O. Freund and Y. Nambu, Ref. 9.

13. Riazuddin and R. E. Marshak, Phys. Lett. 11, 182 (1964); P.G.O. Freund and Y. Nambu, Phys. Lett., to be published.

14. The present experimental data[6] seem to be slightly in favor of $C_{1/2}$.

15. Actually we even break A_{10} so that A_{22} is used only to provide the second octet of vector mesons. In Eq. (4.1) B and B' stand for bona fide baryons but the argument could be extended to include Y_0^*.

16. C. P. Bhalla, as quoted by Willis et.al., Ref. 6.

17. See e.g. R. F. Dashen and D. H. Sharp, Phys. Rev. 133, B1581 (1964).

TABLE I.

QUANTUM NUMBERS FOR THE BARYONS

	p	n	Ξ^-	Ξ^0	Σ^+	Σ^-	Σ^0	Λ^0	Y^0
Y	+1	1	-1	-1	0	0	0	0	0
I_3	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	-1	0	0	0
Y'	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	-	-
I'_3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	-	-	-

TABLE II.

EIGENVALUES OF CASIMIR OPERATORS

For a representation $\mathcal{D}(p,q)$,

$$F^2 = \frac{1}{3}(p^2 + q^2 + pq) + p + q$$

$$G^3 = \frac{1}{18}(p-q)(2p + q + 3)(2q + p + 3)$$

Representation	Dimensionality	F^2	G^3
$\mathcal{D}(0,0)$	1	0	0
$\mathcal{D}(1,0)$	3	4/3	10/9
$\mathcal{D}(1,1)$	8	3	0
$\mathcal{D}(2,0)$	6	10/3	35/9
$\mathcal{D}(2,1)$	15	16/3	28/9
$\mathcal{D}(2,2)$	27	8	0
$\mathcal{D}(3,0)$	10	6	9

TABLE III.

TOTAL DECAY WIDTH FOR $Y^{o} \rightarrow \Sigma \pi$

Y^{o}	$\alpha = 1$ (A^{D}_{11})	$\alpha = \frac{1}{2}(A^{+}_{21})$	$\alpha = \frac{2}{3}$	exp
$J^{P} = \frac{1}{2}^{+}$	150 Mev	30	67	50
$J^{P} = \frac{1}{2}^{-}$	450	110	200	

TABLE IV.

THE β DECAY RATES OF BARYONS*

Decay process	$C_{1/2}$ branching ratio X 10^3	$C_{1/2}$ V- A	C_1 branching ratio X 10^3	C_1 V- A
$\Lambda \rightarrow p$	0.79	V-0.77A	0.42	V-0.38A
$\Sigma^- \rightarrow \Lambda$	0.08	A	0.08	A
$\Sigma^- \rightarrow n$	0.84	V	4.17	V+1.15A
$\Xi^- \rightarrow \Lambda$	0.43	V-0.38A	0.43	V+0.38A
$\Xi^- \rightarrow \Sigma^0$	0.06	V-1.15A	0.06	V-1.15A
$\Xi^0 \rightarrow \Sigma^+$	0.23	V-1.15A	0.23	V-1.15A

*Based on V-1.15A for $n \rightarrow p$ [16] and the Cabibbo angle $\theta = 0.26$.