

A Goldstone Boson Primer

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These lectures are an extremely condensed version of the theory of Goldstone bosons, with general features illustrated using a simple model. A more comprehensive version of these lectures, which includes a general discussion of effective theories of Goldstone bosons, including applications to the low-energy behaviour of pions, spin waves (in antiferromagnets and ferromagnets), and to the $SO(5)$ proposal for high- T_c superconductors may be found in [hep-th/9808176](#).

1 Introduction

George Bernard Shaw once observed that England and America were divided by a common language. The same might be said about the fields of theoretical high-energy, nuclear and condensed-matter physics. Since their joint start with the birth of quantum mechanics, these three disciplines have diverged so far from one another that it is very difficult for the practitioners of one of these fields to follow in detail the developments and techniques which are common in the others. This divergence is unfortunate, since the cross-fertilization of ideas between these fields has been a rich source of progress to all three.

And yet, their languages are very much the same. There is, after all, considerable overlap in the theoretical techniques used in all three of these disciplines. On the broadest level (for very good reasons^{1,2}), all three heavily rely on field theory — both classical

and quantum. Other similarities also arise when they are inspected in more detail, two of which play a significant role in these lectures.

1. All of these disciplines rely heavily on the appearance and utility of symmetries to analyze the behaviour of complicated processes.
2. All of these fields exploit low-energy expansions to take advantage of the simplifications which often accompany large hierarchies of scale. They also frequently use renormalization-group techniques to resum singularities and identify scaling behaviour away from characteristic energy scales.

The lectures summarized here describe a powerful theoretical technique which is based on the exploitation of symmetries and the simplicity of the low-energy limit, and so which has wide applications within the above-mentioned disciplines, as well as more widely throughout physics. The technique is the use of effective field theories to describe low-energy behaviour, specifically applied to the low-energy states — Goldstone bosons — which arise whenever a system's ground state does not share all of the symmetries of its Hamiltonian.

For brevity's sake, the main ideas are presented here purely within the context of a very simple model. The reader is referred to ref. ³ for all of the technical details, including the general formulation of the low-energy theory of Goldstone bosons as well as its application to several examples from nuclear and condensed-matter physics. Although the model used is Lorentz invariant for simplicity, the consequences drawn are not limited to this case.

2 A Model

Consider the model defined by the following Lagrangian density for a complex scalar field, ϕ :

$$\mathcal{L} = -\partial_\mu\phi^*\partial^\mu\phi - V(\phi^*\phi),$$

$$\text{with } V = \frac{\lambda}{4} \left(\phi^* \phi - \frac{\mu^2}{\lambda} \right)^2. \quad (1)$$

2.1 Symmetries and Conservation Laws

This theory describes two spinless particles which are related to one another by a continuous $U(1)$ symmetry of the form

$$\phi \rightarrow e^{i\omega} \phi. \quad (2)$$

The variation of \mathcal{L} under an infinitesimal symmetry transformation, $\delta\phi = i\omega \phi$, is:

$$\delta\mathcal{L} = -i\partial^\mu \phi \partial_\mu \omega, \quad (3)$$

which shows that this transformation is a global (or rigid) symmetry because it leaves the Lagrangian density invariant only if its parameter, ω , is a constant.

Continuous symmetries imply conservation laws. In ordinary quantum mechanics the $U(1)$ symmetry considered here would guarantee the time-independence (*i.e.* conservation) of a hermitian charge operator Q whose commutator with any operator gives its transformation under the symmetry. In the present example:

$$i\omega [Q, \phi(x)] = \delta\phi(x) = i\omega \phi(x). \quad (4)$$

In field theories continuous symmetries carry an additional implication. Besides implying the overall conservation of the charge Q , the conservation laws must in addition hold locally. This implies the existence of a local (Noether) current, $j^\mu(x)$, for which conservation is the differential condition $\partial_\mu j^\mu = 0$. This expresses conservation because it implies the time independence of Q , which may be defined in terms of j^μ by $Q = \int j^0(\mathbf{x}, t) d^3\mathbf{x}$. The Noether current for the model under consideration is:

$$j_\mu = -i(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*). \quad (5)$$

2.2 Semiclassical Ground State

The next question is to determine the ground state and energy spectrum in this model. This may be done explicitly if $\lambda \ll 1$ since this condition justifies a semiclassical calculation of these quantities.

The field configuration which corresponds to the semiclassical ground state, or vacuum, is found by minimizing the system's energy density, which is $\mathcal{H} = \frac{1}{2} \partial_t \phi^* \partial_t \phi + \frac{1}{2} \nabla \phi^* \cdot \nabla \phi + V(\phi^* \phi)$. Being the sum of non-negative terms, this is easy to minimize. The vacuum configuration is found in this way to be a constant, $\dot{\phi} = \nabla \phi = 0$, whose value, $\phi = v$, minimizes the classical potential: $V(v^*v) = 0$. Using the $U(1)$ symmetry to make v real (and assuming μ^2 is positive) gives the solution $v = \mu/\sqrt{\lambda}$.

The low-energy degrees of freedom in the semiclassical approximation consists of small harmonic oscillations of the fields about the minimum of the scalar potential. The low energy spectrum is simply the energy eigenvalues for each of these harmonic normal modes. Using the smallness of λ to drop cubic and higher powers of the fluctuation, $\phi - v$, and writing separately its real and imaginary part ($\mathcal{R} \equiv \sqrt{2} \operatorname{Re}(\phi - v)$ and $\mathcal{I} \equiv \sqrt{2} \operatorname{Im} \phi$) gives the harmonic Lagrangian $\mathcal{L}_h = -\frac{1}{2} (\partial_\mu \mathcal{R} \partial^\mu \mathcal{R} + m_R^2 \mathcal{R}^2) - \frac{1}{2} \partial_\mu \mathcal{I} \partial^\mu \mathcal{I}$, where $m_R^2 = \lambda v^2$.

This gives the usual result: two particle types with a relativistic dispersion relation $E(p) = \sqrt{p^2 + m^2}$, with the particle associated with the field \mathcal{R} having rest mass m_R and the particle associated with \mathcal{I} have zero mass.

2.3 Particle Interactions

For small λ the interactions amongst these particles may be found perturbatively by expanding the scalar potential in powers of \mathcal{R}

and \mathcal{I} :

$$V = \frac{m_R^2}{2} \mathcal{R}^2 + \frac{g_{30}}{3!} \mathcal{R}^3 + \frac{g_{12}}{2} \mathcal{R} \mathcal{I}^2 + \frac{g_{40}}{4!} \mathcal{R}^4 + \frac{g_{22}}{4} \mathcal{R}^2 \mathcal{I}^2 + \frac{g_{04}}{4!} \mathcal{I}^4, \quad (6)$$

where the couplings in this potential are given in terms of the original parameters, λ and v , by:

$$\frac{g_{30}}{3!} = \frac{g_{12}}{2} = \frac{\lambda v}{2\sqrt{2}}, \quad \frac{g_{40}}{4!} = \frac{g_{04}}{4!} = \frac{g_{22}}{8} = \frac{\lambda}{16}. \quad (7)$$

An interesting point can be made if the amplitude for \mathcal{R} - \mathcal{I} scattering is computed to lowest-order in perturbation theory using these interactions. The four Feynman diagrams which contribute to this order are given in Figure 1.

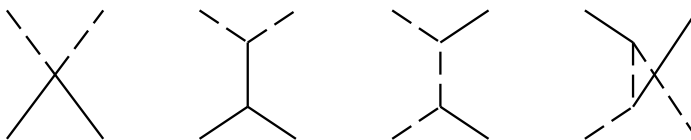


Figure 1: The Feynman graphs which describe \mathcal{R} - \mathcal{I} scattering at tree level in this model.

The S -matrix at tree which results from the evaluation of these graphs is:

$$S[\mathcal{R}(r) + \mathcal{I}(s) \rightarrow \mathcal{R}(r') + \mathcal{I}(s')] = \frac{i\mathcal{A} \delta^4(r + s - r' - s')}{4(2\pi)^2 \sqrt{s^0 r^0 s'^0 r'^0}}, \quad (8)$$

with

$$\mathcal{A} = -g_{22} + \frac{g_{12}g_{30}}{(s + s')^2 + m_R^2 - i\epsilon} + g_{12}^2 \left[\frac{1}{(s + r)^2 - i\epsilon} + \frac{1}{(s - r')^2 - i\epsilon} \right]. \quad (9)$$

An interesting feature of this amplitude is that it vanishes in the limit of vanishing momentum for the massless particle, \mathcal{I} . That is, (using the massive-particle dispersion relation, $r^2 = r'^2 = -m_R^2$):

$$\begin{aligned} \mathcal{A} &\rightarrow -g_{22} + \frac{g_{12}g_{30}}{m_R^2} - \frac{2g_{12}^2}{m_R^2}, \\ &= \lambda \left(-\frac{1}{2} + \frac{3}{2} - 1 \right) = 0. \end{aligned} \tag{10}$$

Even more interesting: as may be verified by more complicated calculations, the vanishing of \mathcal{A} in the zero-momentum limit holds also for higher orders in perturbation theory. The same is also true for all other amplitudes involving \mathcal{I} particles. The massless particle of this theory (which remains massless even once interactions are included) completely decouples in the limit of vanishing momentum.

3 The Low-Energy Perspective

The purpose of this section is to show how the remarkable properties of the \mathcal{I} particles just described can be exhibited more transparently, without resorting to detailed calculations.

The key idea is that both the masslessness of \mathcal{I} and its decoupling are properties of the low-energy part of the model. They should be possible to understand purely within the low-energy effective theory which is obtained by ‘integrating out’ all degrees of freedom having energies higher than $O(m_R)$ ^{5,6}.

3.1 The Low-Energy Effective Lagrangian

The interactions amongst \mathcal{I} particles for centre-of-mass energies much smaller than m_R can be described by an ‘effective’ Lagrangian, $\mathcal{L}_{\text{eff}}(\mathcal{I})$ which involves only the field \mathcal{I} . The field \mathcal{R} does not appear in \mathcal{L}_{eff} because if no \mathcal{R} -particles are initially present in a process,

then they are also never produced (so long as $E_{CM} < m_R$) because of energy conservation.

That is not to say that \mathcal{R} is irrelevant to low-energy \mathcal{I} -particle scattering, however, because we know from the full theory that virtual \mathcal{R} -exchange can and does take place. At low energies its influence is suppressed by powers of $1/m_R$, because of the large energy denominators (or propagators) which virtual \mathcal{R} exchange requires. At low energies it is therefore useful to organize the terms in \mathcal{L}_{eff} in powers of derivatives of \mathcal{I} divided by m_R , because this completely captures the influence at low-energies of the higher-energy components of the system.

The result of such a derivative expansion would be:

$$\mathcal{L}_{\text{eff}} = -V_{\text{eff}}(\mathcal{I}) - \frac{1}{2} G(\mathcal{I}) \partial_\mu \mathcal{I} \partial^\mu \mathcal{I} - \frac{H(\mathcal{I})}{m^4} (\partial_\mu \mathcal{I} \partial^\mu \mathcal{I})^2 + \dots, \quad (11)$$

where the ellipses describe further terms in the derivative expansion. (Notice that the resulting lagrangian involves couplings with dimensions of inverse powers of mass – in units for which $\hbar = c = 1$ – and so is not perturbatively renormalizable in the ordinary sense. It nonetheless gives sensible predictions provided one works to a fixed order in powers of $1/m_R$.)

The unknown functions V_{eff} , G and H are determined by comparing \mathcal{I} -particle scattering computed with \mathcal{L}_{eff} to the result computed within the full model, with the Lagrangian of eq. (1).

3.2 A Different Choice of Fields

In the effective-Lagrangian language the masslessness of \mathcal{I} and the vanishing of all S -matrix elements in the zero-energy limit is equivalent to the vanishing of the effective potential:

$$V_{\text{eff}} \equiv 0. \quad (12)$$

The puzzle is to see in a simple way why this should be so.

To this end imagine instead using polar coordinates in field space, rather than the fields \mathcal{I} and \mathcal{R} :

$$\phi(x) = \chi(x) e^{i\theta(x)}. \quad (13)$$

In terms of θ and χ the model's Lagrangian is:

$$\mathcal{L} = -\partial_\mu \chi \partial^\mu \chi - \chi^2 \partial_\mu \theta \partial^\mu \theta - V(\chi^2). \quad (14)$$

Analyzing the spectrum of this theory in the semiclassical approximation about the vacuum $\chi = v$ shows that χ describes the particle with mass m_R and θ represents the massless particle.

Now comes the main point. The field θ only appears in \mathcal{L} through its derivative, $\partial_\mu \theta$. Suppose we were now to integrate out the degrees of freedom having energies of order m_R . In the resulting effective lagrangian θ must also appear only differentiated. Using these variables we therefore easily see that $V_{\text{eff}}(\theta) \equiv 0$, and so why the massless particle decouples at low energy.

3.3 A Tradeoff

Weinberg's First Law of Theoretical Physics states⁴: You can use any variables at all to analyze a problem, but if you use the wrong variables you'll be sorry.

In this problem the massless particles decouple at low energy regardless of whether \mathcal{L} is written using the fields \mathcal{I} and \mathcal{R} or the fields χ and θ . The latter pair have the advantage that they display the low-energy decoupling of θ -particles in a transparent way, and so they more clearly exhibit the limits of validity of this decoupling.

There is a price for this clarity, however. This price is most easily seen once the fields are canonically normalized, which is achieved by writing $\chi = v + \frac{1}{\sqrt{2}} \chi'$ and $\theta = \frac{1}{v\sqrt{2}} \varphi$. With these variables the Lagrangian is seen to have acquired complicated, nominally non-renormalizable interactions:

$$\mathcal{L}_{\text{nr}} = - \left[\frac{\chi'^2}{\sqrt{2} v^2} + \frac{\chi'^2}{4v^2} \right] \partial_\mu \varphi \partial^\mu \varphi. \quad (15)$$

Notice this lagrangian only makes sense with this choice of variables if $v \neq 0$.

Of course, the S -matrix for the theory in these variables is identical to that derived from the manifestly renormalizable Lagrangian expressed in terms of the variables \mathcal{R} and \mathcal{I} . So the S -matrix remains renormalizable even when computed using the variables χ' and φ . The same is not true for *off-shell* quantities like Green's functions or the 1PI action, however, since the renormalizability of these quantities need not survive a nonlinear field redefinition.

In this model there is a choice to be made between making the Lagrangian manifestly display either the renormalizability of the theory, or the Goldstone boson nature of the massless particle. Which is best to keep explicit will depend on which is more convenient for the calculation that is of interest. Since, renormalizability is in any case given up when dealing with effective low-energy field theories, it is clear that the variables which keep the Goldstone boson properties explicit are the ones of choice in this case.

4 Naturalness and Goldstone Bosons

Although use of the variables χ and θ clearly display the special properties of the massless particle, it is not yet clear *why* these special properties arise in the first place. This section addresses this issue, first by identifying the $U(1)$ symmetry within the low-energy effective theory. The resulting symmetry argument is then shown to be a the low-energy expression of an exact result: Goldstone's theorem.

4.1 Symmetry Considerations

The key to understanding the properties of the massless particle lie with the model's $U(1)$ symmetry, eq. (2). This is realized on the fields \mathcal{I} and \mathcal{R} as a two-by-two orthogonal rotation, but it is realized on the fields χ and θ *inhomogeneously*. In terms of the

canonically normalized field, φ , this transformation law becomes:

$$\varphi \rightarrow \varphi + \sqrt{2} v \omega. \quad (16)$$

Clearly it is this symmetry which requires θ to appear only differentiated in both \mathcal{L} and \mathcal{L}_{eff} , because only $\partial_\mu \phi$ is invariant with respect to constant shifts of ϕ . This symmetry therefore is also at the root of the masslessness and low-energy decoupling of the particle described by θ .

4.2 *Spontaneously-Broken Symmetries*

An inhomogeneous symmetry of the form $\theta \rightarrow \theta + \omega$ is indicative of a symmetry which does not preserve the system's ground state. Such a symmetry is called *spontaneously broken*.

Recall that if χ is frozen to equal its value in the vacuum, $\chi = v \neq 0$, then a nonzero field configuration for θ corresponds to $\phi(x) = v \exp[i\theta(x)]$. In this sense θ can be regarded as being the result of performing a spacetime-dependent $U(1)$ transformation of the ground state. But this $U(1)$ is a symmetry only when its parameter is a constant, and \mathcal{L} vanishes when evaluated at the vacuum configuration, $\phi = v$. We see that \mathcal{L} is independent of θ as θ tends to a constant, precisely because θ is simply a symmetry transformation of the vacuum in this limit.

If θ varies in space and time it no longer describes a symmetry transformation, and so \mathcal{L} can depend on derivatives of θ . So it is continuity with the constant-field limit, where θ parameterizes a symmetry transformation, which ensures the low-energy decoupling of θ .

4.3 *Goldstone Bosons*

The argument just given is very general. It states that for *any* continuous symmetry which does not preserve the ground state, there

is a massless degree of freedom which decouples at low energies. This mode is called the Goldstone (or Nambu-Goldstone) particle for the symmetry.

Furthermore, this degree of freedom may be explicitly displayed by performing the symmetry transformation in question on the ground-state field configurations. If the parameters of this transformation are treated as fields they automatically drop out of the Lagrangian (or effective Lagrangian) in the limit of constant fields.

This observation implies a number of other consequences for the Goldstone particles, in addition to their masslessness and low-energy decoupling^{7,8,9}:

1. *Spin and Statistics*: For internal symmetries the Goldstone particles are spinless bosons, since they can be represented by fields which are rotational scalars (*i.e.* the transformation parameters of the symmetry group). An identical argument for spontaneously-broken supersymmetry implies the Goldstone particles are spin-half fermions, and they are spin-one bosons (phonons) for spontaneously-broken translation invariance.
2. *Counting*: There is precisely one Goldstone particle for each symmetry generator which is broken. For example, if $U(N)$ (which has N^2 generators) is spontaneously broken to $U(N')$ then there must be $N^2 - N'^2$ Goldstone bosons.
3. Finally, applying the symmetry transformation of eq. (16) to the Goldstone boson kinetic terms, $-\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi$, implies the corresponding conserved current depends on the Goldstone boson in the following way:

$$j^\mu = \sqrt{2} v \partial^\mu \varphi + \dots \quad (17)$$

The ellipses in this expression represent contributions to j^μ which come from other terms in the Lagrangian besides the φ

kinetic term, and so involve other fields or additional derivatives of powers of φ .

Eq. (17) implies another general property of Goldstone bosons: the matrix element of the current between the ground state, $|\Omega\rangle$ and the Goldstone state, $|G\rangle$, must be nonzero: $\langle G|j^\mu|\Omega\rangle \neq 0$.

5 Discussion

The model examined here illustrates a general property of field theories. When a continuous, global symmetry is spontaneously broken, the spectrum must contain a massless (Goldstone) particle which completely decouples at zero energy, and so is weakly-interacting at low energies.

The special low-energy properties of Goldstone bosons are all consequences of their particular form of inhomogeneous transformation law under the corresponding broken symmetry. (For abelian symmetries this transformation rule is as in eq. (16), but for non-abelian symmetries a more complicated form is required¹⁰.) Because Goldstone-boson properties all can be derived purely on the grounds of their symmetry transformation properties, they do not depend at all (at low energies) on the details of the underlying model which breaks these symmetries.

More generally, dependence on the underlying model appears once predictions are required beyond the leading order in the low-energy derivative expansion. But even once subleading corrections are included, underlying physics affects Goldstone boson interactions only through a comparatively small number of parameters.

To show that this is true, imagine writing an arbitrary effective theory for a real scalar field, φ , subject only to the symmetry of eq. (16) (and, for simplicity, to Poincaré invariance). The most general Lagrangian which is invariant under this transformation is an arbitrary function of the derivatives, $\partial_\mu\varphi$, of the field. An

expansion in interactions of successively higher dimension would be:

$$\mathcal{L}_{\text{eff}}(\varphi) = -\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{a}{v^4} \partial_\mu \varphi \partial^\mu \varphi \partial_\nu \varphi \partial^\nu \varphi + \dots, \quad (18)$$

where a power of v is inserted on as appropriate to ensure that the parameter a is dimensionless. This accords with the expectation that it is the symmetry-breaking scale, v , which sets the natural scale relative to which the low energy limit is to be taken. In the example considered earlier, integrating out the heavy field, χ' produces these powers of v through the appearance of inverse powers of m_R .

It follows that, up to subleading order in $1/m_R$, the mutual scattering of Goldstone bosons in *any* model which spontaneously breaks $U(1)$ depends on the details of the model only through its predictions for the parameter a . This is because the integrating out of all other, heavier, degrees of freedom necessarily must give an effective Lagrangian of the form of eq. (18), but with a specific, calculable coefficient for the parameter a .

Such an understanding of the Goldstone nature of a field, like φ , as an automatic consequence of a symmetry is clearly invaluable when constructing effective Lagrangians for systems subject to spontaneous symmetry breaking.

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