

KILLING TIME

James F. Woodward

*Departments of History and Physics
California State University
Fullerton, California 92634*

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Theoretical consequences of the gravitational origin of inertial reaction forces, that is, Mach's principle, are explored. It is argued that Mach's principle leads to the conclusion that time, as we normally treat it in our common experience and physical theory, is not a part of fundamental reality; the past and future have a real, objective existence, as is already suggested by both special and general relativity theory. A laboratory scale experiment whereby Mach's principle, and thus radical timelessness, can be established is mentioned.

Key words: times, relativity, Mach's principle.

1. INTRODUCTION

In the past several years the issue of the origin of mass and inertia has assumed an increasingly central role among the outstanding problems of physical theory. As a result, the once hotly debated "Mach's principle", a topic largely dormant in the 1980s has again attracted serious interest. And just as there was little agreement on precisely what Mach's principle is back in the 1950s, '60s and '70s, the same situation seems to prevail today. (In this connection, see the proceedings of the 1993 Tübingen conference on Mach's principle [Barbour and Pfister (eds.), 1995].) A broad consensus exists that Mach's principle suggests that local inertial properties arise as a consequence of the existence, distribution, and currents of mass-energy throughout the cosmos. And most of those interested in this issue would probably agree that any detailed realization of Mach's principle should at the very least be compatible with general relativity theory (GRT). Indeed, the major variants of Mach's principle, save one, extant in the literature today take the form of the stipulation

of boundary and/or initial conditions on the acceptable cosmological models of GRT. The boundary conditions selected differ in details – very important details with profound implications for our understanding of reality – but it is widely assumed that the origin of inertia is to be found in gravity.

In some measure the variation in the positions affected by interested parties can be traced to their views on whether matter or spacetime is more primordial. That is, some argue that matter elsewhere determines the properties of spacetime here, and in so doing establishes local inertial frames of reference. This, they claim is the essential physical content of Mach's principle. Others take the view that inertial frames of reference are figments of our imaginations; inertial reaction forces are the facts of our experience, and it is they that are to be explained in terms of the distant matter in the cosmos if Mach's principle is to be properly realized. These views are opposite sides of the same coin, of course, and are motivated by the inclination to emphasize, respectively, either Newton's first law (the operational definition of inertial frames of reference), or the second and third laws (that deal with forces and their reactions). As Gribbin [1995], among others, has recently pointed out, however, how one chooses to interpret principles, theories and formalisms often has profound consequences for one's physical understanding.

The interpretational difference just noted, for example, can lead to rather different ideas about what should happen in the hypothetical case where one considers the behavior of one or two objects in an otherwise empty universe. This is one of the central problems addressed by Mach's principle – to be found already in the work of Berkeley in Newton's time. If we accept in broad terms the relativity and gravitational induction of inertia, this becomes the problem of the inertial properties and behavior of objects as they are gravitationally decoupled from the rest of the matter in the cosmos. Since we are killing trees as well as time here, I remark that I have already addressed this issue, at least in part, in the context of the practical feasibility of traversible wormholes in spacetime (TWIST) [traversible wormholes in time (TWITs) being the really interesting subgroup of TWISTs] [Woodward, 1995, hereafter MUSH,¹ esp. section 6].

The interpretational difference under consideration also has consequences for the topic that is the chief subject of this paper: the implications of Mach's principle regarding the nature of time. Inasmuch as Mach's principle is the most extreme statement of the principle of relativity, it is to be expected that it should have profound consequences for our conceptualization of time. But even when the principle of relativity is construed narrowly, as in special relativity theory (SRT) so that its GRT generalization and Mach's principle are excluded, one is led from the absence of absolute simultaneity to

the view that, in Weyl's [1949] words, "Reality simply *is*, it does not *happen*." That is, the past, present, and future all objectively exist. It is all fixed. There is no "free will". [For a popular exposition of this point see Davies, 1995, pp. 59-77.] Refuge from this stark and unappealing reality is easily found in quantum mechanics as it is almost universally interpreted: reality is probabalistic, uncertain, and only progressively actualized by "measurements" as time "goes on". Mach's principle denies us this refuge; if inertia is relative and gravitationally induced, quantum mechanics notwithstanding, it is difficult to avoid killing time.

2.0 KILLING TIME

How does Mach's principle kill our conventional conception of time? By making it necessary to take advanced, as well as retarded gravitational effects into account to correctly describe elementary inertial reaction forces. The advanced effects propagate either from the actualized future, the existence of which we – determined to believe in "free will" – prefer to deny, or into the past which putatively should not be influenced by the present or future. This counter-intuitive consequence of Mach's principle may be shown with the greatest physical transparency employing the vector formalism for gravity – the formalism used by Sciama [1953] nearly a half century ago to illustrate the gravitational induction of inertia. While it is well-known that the vector formalism for gravity is not completely correct, that formalism accurately captures elementary inertial reaction effects and makes it especially easy to see what is going on.

For our purpose we need only consider the simplest of all possible circumstances: the translational acceleration of a test particle by an external force in a universe of otherwise constant matter density ρ . We ignore Hubble flow and suchlike, for they are extraneous to the argument. The gravitational field \mathbf{F} that the matter in the universe exerts on our test particle is:

$$\mathbf{F} = -\nabla\phi - (1/c)\partial\mathbf{A}/\partial t, \quad (2.1)$$

where ϕ and \mathbf{A} are the gravitational scalar and three-vector potentials respectively. Following, roughly, Sciama's development,² we remark that

$$\phi = -G \int_V (\rho/r) dV, \quad (2.2)$$

which, neglecting factors of order of unity, integrates to GM/R , M and R being the mass and radius [particle horizon] of the universe respectively. Since this is true for arbitrary points in the universe, $\nabla\phi$ vanishes everywhere and no Newtonian interaction of the test

particle with the rest of the universe occurs. Inertial reaction forces, we see therefore, must arise from the three-vector part of the full four-potential.

The vector potential is obtained by taking the volume integral of the matter-charge current out to the particle horizon, and if our test particle is at rest relative to the remainder of the universe, \mathbf{A} and its time derivative vanish. If the test particle moves with some velocity \mathbf{v} with respect to the uniformly distributed matter (Berkeley and Mach's "fixed stars"), \mathbf{A} no longer vanishes. In the frame of reference of instantaneous rest for the test particle, the universe appears to be moving rigidly with velocity $-\mathbf{v}$, so,

$$\mathbf{A} = -(G/c) \int_V (\rho \mathbf{v}/r) dV. \quad (2.3)$$

Since in the frame of the test particle the matter everywhere in the universe appears to have velocity $-\mathbf{v}$, we may take \mathbf{v} to be a constant and remove it from the integration. Doing this we immediately obtain:

$$\mathbf{A} = (\phi/c) \mathbf{v}. \quad (2.4)$$

We note in passing that \mathbf{A} given by Eq. (2.4) displays $1/r$ dependence rather than the $1/r^2$ dependence of the $\nabla\phi$ term in Eq. (2.1). This means that inertial reaction forces are dominated by the effect of the more distant matter in the cosmos, and that they are radiative interactions. Substituting \mathbf{A} from Eq. (2.4) into Eq. (2.1), we find that

$$\mathbf{F} = -(\phi/c^2) \partial \mathbf{v} / \partial t, \quad (2.5)$$

and, if $\phi/c^2 = 1$, then the inertial reaction force an accelerated body experiences is just the gravitational force exerted on it by the rest of the matter in the Universe. We have obtained the right answer in this representation. But is this also true in GRT?

The gravitational induction of inertia in GRT is especially easy to recover using Nordtvedt's [1988] "PPN" formalism [Woodward, 1990 and 1991]. As Nordtvedt showed, an extended massive object translationally accelerated by an external force suffers a transient change in its mass due to the fact that the object drags inertial space along with it. The dragging of inertial space with acceleration $\delta \mathbf{a}$ at each point \mathbf{x} within the object depends on the gravitational potential ϕ due to the rest of the object and the acceleration of the object \mathbf{a} as

$$\delta \mathbf{a} = [4\phi(\mathbf{x})/c^2] \mathbf{a}. \quad (2.6)$$

We now ask: If the object accelerated by the external force is the Universe itself,³ then what is the condition implied by Eq. (2.6) for all of its parts to be dragged rigidly with it so that such hypothetical

accelerations are undetectable for observers in the Universe? $\delta \mathbf{a}$ must be equal to \mathbf{a} if rigid dragging of local observers and the principle of relativity are to be preserved. So we find that $4\phi(\mathbf{x}) = c^2$ locally in the PPN representation of GRT. Up to the factor of four, this is just the condition recovered in Sciama's vector potential analysis.

Consistency with GRT can also be established by comparing Eqs. 2.1 to 2.3 with the equations (118) obtained by Einstein in his discussion of Mach's principle in *The Meaning of Relativity* [1956, p. 102]. Neglecting a rotational term he includes, they are the same.⁴ Note that this calculation shows that if inertial reaction forces are to be ascribed to gravity, we must accept that the absolute value of ϕ is not arbitrary – it cannot be adjusted by an additive constant – for \mathbf{A} depends on ϕ , not merely the gradient of ϕ . Thus we see that Mach's principle is satisfied in relativistic theories of gravity in general, and GRT in particular, as long as we are willing to admit that absolute gravitational potentials have real, observable effects. But what does this have to do with killing time?

2.1 Time and Causation

Note that by taking ϕ to be determined by Eq. (2.2) we have fixed the gauge in the calculation of the previous subsection because ϕ is implicitly assumed to be propagated instantaneously. No delta function is included in the integrand to make the calculation advanced or retarded. It appears that we have adopted the Coulomb gauge [$\nabla \cdot \mathbf{A} = 0$]. We will explore the consequences of this choice of gauge presently, but now we remark that the notation used above is misleading. In fact, the calculation is based on the assumption of the Lorentz gauge, and the integrations that recover ϕ and \mathbf{A} are done along the past lightcone, notwithstanding the absence of appropriate notation. That is why we literally see the luminous matter in the universe move apparently rigidly with respect to us when we move with respect to, say, some suitably defined mean cosmic rest frame.

As others have remarked, notably Ciufolini and Wheeler in their recent book *Gravitation and Inertia* [1995, esp. Chap. 7], this explanation of inertia raises serious problems of causation. Inertial reaction forces, instantaneous though they may be, evidently are communicated radiatively given their acceleration and characteristic $1/r$ dependence. The problem seems to be: how does the matter along the past lightcone “know” when to radiate the appropriate field to produce the right force at the right time on the body we choose to accelerate? One way to answer this question is to note that accelerating the body will produce both a retarded *and* an advanced disturbance. If we take the advanced signal seriously, then as it propagates down the past lightcone it triggers the emission of retarded radiation by interacting matter, all of which arrives from the

past in the present at the same instant. That, presumably, produces the reaction force.

From Wheeler and Feynman's analysis of the electrodynamic case [1945 and 1949], a problem here can be quickly identified. The retarded radiation, stimulated by the advanced disturbance, propagating along the past lightcone to produce the reaction force in our present will cancel the retarded wave emitted by the accelerated object itself as it is accelerated. As a result, no net future directed retarded disturbance will be produced by the acceleration. And gravitational radiation would be purely advanced, thus carrying negative energy into the past. Should we insist that the radiation be taken to be future directed, the energy proceeding from the system would nonetheless be negative. So the system would acquire energy in time, not dissipate it. This is inconsistent with the behavior of the binary pulsar system where gravitational radiation carries energy out of the system.

Another way to approach the problem is to observe that the particular value of v in Eqs. (2.4) and (2.5) is irrelevant; only $\partial v / \partial t$ matters. So we can simply assert that all advanced effects are to be rejected as unphysical. The matter along the past lightcone then propagates a ϕ field into its future to our present (the absolute value of which in the present, measured locally, must be $\approx c^2$). Motion with respect to the ϕ field then leads to an A field of unknown strength (assuming that we do not know the "real" value of v). Since only $\partial A / \partial t$ matters, not knowing the value of A *per se* is not important. And inertial reaction forces result whenever we accelerate objects with respect to the matter along the past lightcone that generates the ϕ field. At least in the regime where $v \ll c$ this will work. It isn't really right though. We have arbitrarily suppressed the advanced solutions on the grounds of "causality". And we have made the ϕ field stand in as an improper surrogate for a proper A field. Were this the only way to recover the behavior we actually observe in the Lorentz gauge, this "causality" justified fudging might be defensible. But it isn't.

If we use the Wheeler-Feynman "absorber" approach we can get the same result. All we have to do is let the future directed retarded waves produced by the acceleration of the body stimulate advanced waves from the matter along the future lightcone. They will converge in the present to produce the inertial reaction force, and cancel the advanced wave produced by the acceleration of the body that would otherwise propagate into the past. When A is produced by a retarded interaction along the future lightcone, rather than an advanced interaction along the past lightcone, energy flow in gravitational radiation proceeds as observed in the binary pulsar system. All is well. We have not had to suppress any solutions as unphysical or invoke a "causality" postulate. But the future exists

and inertial reaction forces are naked advanced signals from that “already” actualized future.

Familiar with the Wheeler-Feynman treatment of radiation reaction in electrodynamics, one might object that electromagnetic radiation reaction forces, typically, are miniscule, so in the gravitationally analogous case, gravity being so much weaker than electromagnetism, they cannot possibly account for gross inertial reaction forces. But, even in the vector potential formulation, gravity and electromagnetism are different. In electrodynamics the electric charge density, globally, on average, is zero. So the lowest order contribution to the three-vector potential [Eq. (2.3)] vanishes. Only terms involving the square of the electric charge density (and third and higher order time derivatives of position) contribute to radiation resistance. The mean matter-charge density for gravity, however, does not vanish globally; and the comparatively enormous effect contained in Eq. (2.3) cannot be ignored. I note in passing a point to which we will return: Some physically sensible criterion must be invoked in this explanation of inertial reaction forces to make the volume integral along the future lightcone converge and suppress contributions from the surface integral (possibly at infinity). This corresponds to the bound set by the particle horizon (and the vanishing of the surface integral on that horizon) in the first case examined.

2.2 Another Gauge

Perhaps we can avoid the peculiar conclusions of the previous section by choosing a different gauge. The obvious choice is the Coulomb gauge. The integration for ϕ now proceeds over a three dimensional space-like hypersurface. We can choose it to be the hypersurface of constant cosmic time wherein contributions to ϕ are propagated instantaneously from matter at arbitrarily large distances. (Cosmic time is “York time”. See Ciufolini and Wheeler [1995], pp. 276-277.) In this gauge, however, \mathbf{A} does not share the property of instantaneous propagation. It must satisfy an inhomogenous hyperbolic wave equation, and its effects propagate at light speed.⁵ So Eq. (2.3) used to calculate \mathbf{A} must include a delta function that makes the calculation explicitly either advanced or retarded. Practically speaking, this doesn’t matter, since the integration for \mathbf{A} must yield Eq. (2.4). But notwithstanding the instantaneous propagation of ϕ through the hypersurface, \mathbf{A} is not instantaneously propagated. So, using the Coulomb gauge hasn’t bought us anything. To get inertial reaction forces we are still doing our integration down the past lightcone. And any resemblance between ϕ obtained in this integration and that obtained by integrating over the spacelike hypersurface is either coincidental, or provided by suitable constraints. If we arbi-

trarily demand that \mathbf{A} propagate instantaneously, then $\nabla \cdot \mathbf{A} \neq 0$, Eq. (2.1) no longer obtains, and Eqs. (2.3) through (2.5) are not recovered. But we must recover Eq. (2.5). We seem to be stuck.

Physically, what all of this means is that the matter in the universe does not generate an \mathbf{A} field present everywhere that is ready to spring instantaneously into action to produce inertial reaction forces every time a local object is accelerated. We can make it appear to be that way by invoking "causality" to suppress advanced solutions of wave equations. Then the retarded ϕ field produced in the present by the matter in the past will look like an \mathbf{A} field to accelerated matter. But with out positing "causality", doing our integrations along the future lightcone (suitably bounded), we get the same result. And in the Coulomb gauge \mathbf{A} really propagates at light speed. So the appearance of causality evidently is not real.

Looking at this business in the Coulomb gauge brings to light another problem, first pointed out by Ellis and Sciamia [1972]. As they remarked, an electric charge, even if it lies outside of a particle horizon, contributes to the local electric potential and thus its gradient. As they put it, we can "feel" its presence through the static Coulomb force even though we cannot "see" it should it be accelerated and radiate in our direction. Similarly, in the case of gravity, should a matter density anomaly exist outside our particle horizon, we should "feel" it too. In the retarded potential formulation this would appear as a non-vanishing contribution from the surface integral at the particle horizon. Only in cosmologies where such contributions are uniform over the horizon surface (like Robertson-Walker models), leading to a uniform potential (without gradient) within the horizon can they be ignored.

This sort of problem is equally important when an event horizon is invoked to cut off interactions with more distant matter. Any "nonlocal" interaction that appears to propagate instantaneously will no more be suppressed by an event horizon than by a particle horizon. For example, black holes are separated from us by event horizons. But they participate in elementary gravitational/inertial and electrical interactions with exterior matter without infinite time delays. (We promptly detect their mass and charge, despite not being able to detect gravitational or electromagnetic waves from within the horizon.) So an event horizon cannot be used to bound a retarded/advanced integration along the future lightcone when computing \mathbf{A} for inertial reaction forces in an absorber theory.

2.3 The Serious Stuff

So far we have been using what has on occasion been called a "toy" model of inertia induction. It is reasonable to ask if this corresponds to "real" models that conform to GRT. In fact, it does

pretty well. In the case of the retarded potential plus “causality” postulate model – which I shall call the *retarded* model for short – it mimics the approach to Mach’s principle pursued most recently chiefly by Raine and several others before him (see his contribution to Barbour and Pfister [1995], pp. 274-292). The approach advocated by Hoyle and Narlikar based on absorber theory – the *advanced* model – corresponds to doing one’s integrations to get A along the future lightcone. They are at pains to show that their theory yields the field equations of GRT (see, for example, Narlikar’s contribution to Barbour and Pfister [1995], pp. 250-261). They, however, go farther to argue for a modified version of steady state cosmology.

The model that more-or-less corresponds to the choice of the Coulomb gauge in the foregoing discussion is that long advocated by Wheeler. Agressively argued by Ciufolini and Wheeler [1995], in this *immediate* model, based on Wheeler’s three-space “geometro-dynamics,” the instantaneity of inertial reaction forces is accounted for by demanding that initial data be specified in a compact three-dimensional space-like hypersurface.⁶ The freely specifiable initial data include both matter and matter currents. The constraint equations that the initial data must satisfy on the hypersurface are elliptic (Coulomb-like), not hyperbolic, so they yield the local value of the gravitomagnetic vector potential in terms of the matter currents throughout the hypersurface. That potential leads to inertial frames of reference and by inference to inertia induction. So they claim that it is reasonable in some sense to assume that inertial reaction forces are propagated instantaneously through the hypersurface via the agency of the constraint equations. In addition to the constraint equations, of course, they also have the field equations of GRT to propagate the initial configuration onto past or future hypersurfaces. Usually one takes physical influences to be transferred from one place to another via fields, not constraints. But subtle, perhaps even desperate, measures are required to account for instantaneous propagation of influences in locally Lorentz-invariant theories, especially if you want to avoid killing time.

2.4 Reprise

Let us sum up what our “toy” model of inertia has revealed about the origin of inertial reaction forces to this point. In the *retarded* interpretation we had to invoke the “causality” condition to suppress advanced solutions of our evolution equations so that energy flow corresponds to observation. This had the effect of creating the *illusion* that accelerations with respect to a scalar potential field produce inertial reaction forces. Since inertial reaction forces are local field-source interactions in this view, they are instantaneous. But evidence for the existence of such a real scalar field, now long

sought, is absent. In the *immediate* interpretation the instantaneity of inertial reaction forces is accounted for by choosing boundary conditions on a space-like hypersurface where initial data are stipulated that allow one to recover inertial frames of reference (not reaction forces *per se*) as the solution of elliptic, instantaneous constraint equations. “Causality” is implicitly posited in this interpretation, for the boundary conditions must be chosen so that the constraint equations give back the result obtained in an integration down the past lightcone for the transverse component of \mathbf{A} .

The *advanced* interpretation requires neither the “causality” condition nor the artifice of using constraint equations to explain the behavior of what is really an integration along the past lightcone. Instantaneous inertial reaction forces are recovered without suppressing solutions of field equations and energy flow is everywhere reasonable. Boundary conditions, however, must be assumed that lead to convergence of the integrals for the sources of the potentials along the future lightcone, and a simple event horizon is not necessarily sufficient to guarantee such convergence. Also, the future “already” objectively exists in this interpretation. As regards time, all of these interpretations, being classical, are completely equivalent despite the differences noted above. They are deterministic and thus acausal. So, if we choose, we can assert the objective reality of the past and future, for they are inexorable. But one would not be well-advised to take this too seriously yet. Not only are we dealing with a “toy” model of inertia, we have ignored quantum mechanics.

2.5 Quantum Mechanics

If we adopt either the *retarded* or *immediate* interpretation of inertial reaction forces in our toy model of inertia induction, then we are free to deny the existence of an already actualized future and assert the validity of the Copenhagen interpretation of quantum mechanics (or one of its variants).⁷ Doing so, however, leads to some fairly serious problems, as discussed at some length by Kuchar [1992] in his review of time and interpretations of quantum gravity. Adopting the “many worlds” interpretation of quantum mechanics leads to curious results, noted several years ago by Page and Wothers [1983], recently argued by Page [1994] and Barbour [1994a-c, 1995]. They suggest that “many worlds” become “many instants” in the *immediate* interpretation. Barbour then argues that this forces one to accept “radical timelessness” since quantum cosmological solutions of the Wheeler-deWitt equation [the quantum gravity analog of the Schrödinger equation] are static. “Clock time” is an internal construction of our consciousness in this view, not a fundamental physical entity.

I suspect that if the serious (superspace) version of the *imme-*

diate interpretation of Mach's principle is accepted, then the weird consequences regarding timelessness will prove unavoidable. If this is so, then the *advanced* interpretation should lead to the same conclusion. (The *retarded* interpretation is flawed by the arbitrary suppression of advanced waves via the causality postulate, which leads to the fictitious action of the scalar field that mimics a causeless retarded *A* field propagating along the past lightcone that fortuitously shows up at just the right time to produce observed inertial reaction forces. So one should not expect the *retarded* interpretation to yield radical timelessness.) The really counter-intuitive aspect of the *advanced* interpretation is the real, objective existence of the future as already actualized. Perhaps we can deal with the problem of the existence of the future by assuming that in some statistical sense it exists to absorb retarded radiation from the past and send back advanced signals, but that it is not yet really either determined or actualized. After all, the future will eventually occur no matter what. Making this assumption allows us to account for the instantaneity of inertial reaction forces, and to preserve our belief in "free will". Our free will, of course, is the bottom-line issue in this business. And the Copenhagen interpretation of quantum mechanics falls right in with this sort of approach to the problem. Reality is only determined and actualized through the "measurement" process, and processes require time to occur, so the future progressively unfolds in time. Our future, in this view, literally does not yet exist, and it has not yet been inexorably determined.⁸

This scenario isn't as good as it may sound. As the retarded waves (of whatever sort) produced in the present go propagating off into the progressively actualized future, the interactions they have in the future that stimulate the advanced waves that return to us now are with the actual future, not some "statistical" future. Taking the waves to propagate at light speed along the future lightcone, from our perspective this may not matter too much, because that future is *somewhere* else. It is not our future (which lies along timelike, not null, worldlines). But for the intelligent aliens in the Andromeda galaxy who share our cosmic time, their advanced signals, in part, originate in our actual future. They cannot tell us what that future is, however, even if they have learned how to decipher the advanced signals manifest in their inertial reaction forces – unless they have figured out how to make TWISTs.

Evidently, if we admit the reality of advanced effects, we cannot avoid killing time. This has long been suspected by at least a few of those interested in the "interpretation" of quantum mechanics. For example, O. Costa de Beauregard [1994, and refs. therein], since the 1950s, has advocated taking advanced effects seriously and looking for signs of "retrocausation". More recently, others have taken up this cause, notably Aharonov and various colleagues [e.g.,

Aharonov and Vaidman, 1995 and refs. therein], and this approach has been endorsed by, among others, Penrose [1994, pp. 388-391]. Perhaps the most systematic exposition of this idea, taken as a general interpretation of quantum mechanics, has been given by Cramer [1988, 1986 and refs. therein] in his "transactional" interpretation.

Costa de Beauregard argues that the inclusion of advanced effects should have detectable consequences not expected if the Copenhagen interpretation is right. Cramer, however, claims that his transactional interpretation is in all ways completely equivalent to the Copenhagen interpretation, and so experimentally indistinguishable therefrom. The future, however, is fully actualized in the transactional interpretation [MUSH, section 3]. So the putative equivalence cannot be right. In the transactional interpretation the explicit inclusion of advanced effects makes all of the weirdness and mysteries of quantum mechanics disappear in much the same way as advanced effects allow one to account for instantaneous inertial reaction effects in the *advanced* interpretation above [Gribbin, 1995, epilog]. The price for this clarity of understanding is accepting radical timelessness.

2.6 The Vacuum

What about the quantum mechanical vacuum? In it random fluctuations of zero point fields of all types are present everywhere in the absence of normal matter. We know they exist because we have detected and measured their effects: the Lamb shift, the Casimir effect, and so forth. The fluctuations are justified by the Uncertainty Principle, and the Uncertainty Principle implies that the future is not yet actualized. So the experiments that reveal the quantum mechanical vacuum seem to be compelling evidence that the future does not have an inalterable objective existence.

The obvious comment to make is that the quantum mechanical vacuum must be the aether of our era. If it really did exist, I would not have written this, and you would not be reading it. The universe would be curled up into an idiotically small ball because of the incredibly large energy density of the vacuum. (For a review of the various attempts to deal with this problem chiefly in the context of relativistic quantum field theory and quantum cosmology see Weinberg [1989].) How then do we account for its predicted and observed effects? As advanced effects [and thus seemingly instantaneous and local] from the distant matter in the universe. Oddly enough, quantum vacuum effects may well be evidence for the correctness of Mach's principle, not for a real, seething structure to emptiness. The calculations attendant to an argument for this claim have already been done by several of those interested in the quantum vacuum, cosmology, and the nature of time. I shall just coalesce and

briefly summarize their work here.

If one assumes that the vacuum is truly empty, not filled with virtual, fluctuating fields, then all interactions must be between sources. Given the constraint of local Lorentz-invariance, hyperbolic wave equations must obtain in any consistent theory to describe the delayed actions of sources on each other. If an accurate description of reality is demanded, then advanced effects must be incorporated to account for apparently instantaneous, non-local interactions.⁹ How can this be compatible with vacuum effects? Milonni's incisive work [1988 and refs. therein] on the vacuum of quantum electrodynamics (QED) provides a crucial insight in this business. The crux of Milonni's arguments is that when proper attention is paid to the environment in which "vacuum" effects occur, they can all either be interpreted as due to vacuum fluctuations, or to radiation reaction, or to a linear combination of the two. The sign that radiation reaction and vacuum fluctuations are, to mix a metaphor, opposite sides of the same coin is that radiation reaction depends on the third time derivative of position, and the spectral energy density of vacuum fluctuations goes as ω^3 (the only spectral dependence invariant under Lorentz boosts, making it impossible to define a preferred reference frame on the basis of the vacuum). As Milonni remarks, which particular interpretation is emphasized depends on the ordering of commuting operators in the calculations (which has no effect on the results of the calculation).

The fact that the effects usually attributed to local vacuum fluctuations can be dealt with as radiation reaction effects tells us that, at least in principle, they can be regarded as the direct interaction with a possibly *very* distant "environment". Recall that purely retarded waves and radiation reaction are the residue of the Wheeler-Feynman absorber theory of electrodynamics. In this theory only delayed direct particle interactions take place. Fields are the formalism to describe the transfer of physical effects between sources only. They have no independent degrees of freedom assigned to them and they have no existence independent of the sources that emit and absorb them. So an excited atom cannot radiate until the absorption by another atom of the photon to be emitted is irreversibly established. Radiation reaction, it follows, is *not* a local process involving emission of radiation only, it is determined by the spatio-temporally removed absorber whose effects, nonetheless, are felt instantaneously because they are communicated by advanced waves. Put another way, radiation reaction is another of the weird (but neat) non-local effects most obviously encountered in quantum measurement theory. And since the complete absorber is of cosmic dimensions, we once again encounter Mach's principle, albeit in a situation far removed from any Mach would have anticipated.

Two questions arise here: 1. If the radiation reaction in-

terpretation is equivalent to the vacuum fluctuation interpretation, then both are equally “real” descriptions of reality, so we can still believe that the future has not yet happened, right? 2. The Wheeler-Feynman theory is an idealized classical theory, so it doesn’t really work, does it? The answer to the first question is: If QED were all there were, yes we might still be able to believe that the future had not yet happened. But QED isn’t all there is. Gravity breaks the interpretational equivalence. Local vacuum fluctuations are not consistent with the reality in which we exist (unless they are exactly balanced by negative energy density of unknown origin to a part in many, many orders of magnitude). And (gravitational/inertial) reaction forces are naked advanced effects in the *advanced* interpretation. If they occur in gravity, as they evidently do, it would seem unlikely they would be absent in electrodynamics.

The answer to the second question was provided many years ago by Hoyle and Narlikar [1995 and refs. therein]. Building on Hogarth’s work on absorber theory and cosmology, employing the path integral formalism of Feynman, they were able to show that the various results of QED can be recovered in an action-at-a-distance (no second quantization) quantum mechanical formalism. (Davies [1972 and refs. therein] did much the same demonstration using the S -matrix, instead of path integral, formulation of QED.) If this formalism allows one to get rid of a physical vacuum that is plainly incompatible with GRT, one may ask: Why hasn’t Hoyle and Narlikar’s work received more serious attention? Cosmology. They are steady-staters, and most everybody else are big-bangers. This despite the problem of the “missing” mass – now fashionably relabeled cosmic “dark” matter. (I mention in passing that Barry [1995] has given a compelling explanation of the missing mass.) Since Hoyle and Narlikar tie their version of “absorber” quantum mechanics to their cosmology, it is hardly surprising that big-bangers, confident of the basic correctness of their theory, would not show much interest.

Inspection of Hoyle and Narlikar’s work reveals that their arguments for steady state cosmology based on absorber theory are, with one exception, classical, not quantum mechanical. The one argument that is quantum mechanical is their claim that the cut-offs needed in relativistic quantum field theory are accounted for by the existence of the future event horizon that occurs in steady state cosmology [Hoyle and Narlikar, 1995, pp. 150-151]. If one, with Ashtekar [1989] for example, believes that in a really good theory everything will remain finite and well-defined without *ad hoc* cutoffs, then this argument loses much of its force. Since the remainder of Hoyle and Narlikar’s analysis is independent of specifically cosmological model-dependent assumptions, it follows that “absorber” QED does not require that one believe steady state cosmology. And as they point out [1995, p. 153-154]:

The discussions of [earlier parts of their paper] tell us that it is not proper to talk of a probability amplitude for a local microscopic system. The correct description of the physical behavior of the system follows from the probability calculation that includes the [future] response of the universe. . . . This may explain the mystery that surrounds such epistemological issues like the *collapse of the wave function*. What is missing from the usual discussion of the problem is the response of the universe. . . . We suggest this idea as a way of understanding many other conceptual issues of quantum mechanics. It may well be that the real nonlocal “hidden variables” are contained in the response of the universe.

Since most of the response of the universe occurs in the far future, radical timelessness obtains. All of this may strike you as pretty “far out”. I suppose that is to be expected if the truth is “out there”. The only really interesting question, I think, is: Is there any tangible evidence that suggests anyone should take any of this stuff seriously?

3. SOURCERY AND MACH'S PRINCIPLE

If inertial reaction forces are induced by the distant matter in the cosmos (past, present, or future), then every such force experienced is evidence for the truth of Mach's principle. But if we want to establish the truth of Mach's principle independently of inertial reaction forces to show that they really are generated in this way, we face a much more difficult situation: finding some effect other than inertial reaction forces per se that only exists if Mach's principle is true. Such effects do exist. Those that are widely known are the precessional frame dragging effects involving the gravitomagnetic vector potential, reviewed at some length in Ciufolini and Wheeler [1995, esp. Chap. 3] and Barbour and Pfister [1995, esp. 386-402]. They are exceedingly small and very hard (expensive) to detect since they are produced by local matter currents under normal circumstances. Another effect – testable in the laboratory at reasonable expense – follows from the assumptions of local Lorentz-invariance and that inertial reaction forces arise from the interaction of accelerated objects with an external “field”. I have advocated its exploration for some time now [e.g., Woodward, 1992, 1994, and MUSH]. Before briefly recapitulating that effect, we first consider some related matters involving the sources of gravity and mass.

As mentioned at the end of Sec. 2.0 above, Einstein, in his

discussion of Mach's principle in *The Meaning of Relativity* [1956], showed that inertia induction, like that in Sciama's "toy" model used here, follows from the weak field limit of the field equations of GRT. In the weak field limit the metric tensor $g_{\mu\nu}$ is approximated:

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}, \quad (3.1)$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ is a small perturbation of the metric caused by local accumulations of matter. In addition to the inertia induction effects discussed above, Einstein found that the inertial masses m of bodies appeared to be affected by the presence of surrounding matter. In particular, he found that:

$$m = m_o(1 + \phi/c^2), \quad (3.2)$$

where m_o is the restmass of the body in question in the absence of local concentrations of matter and ϕ is now the gravitational potential at the body due to the local surrounding matter. For those with Machian inclinations, this effect of nearby masses on the mass of a body is a satisfying result. It is a step in the direction of the gravitational induction of mass.

For the dyed-in-the-wool Machian the likely reaction to this result is: Where the devil did the 1 in Eq. (3.2) come from? From the Minkowski metric of course. Empty, infinite, flat spacetime endowed with inertial properties that swamp the effects of reasonable concentrations of local matter. For this reason Minkowski spacetime is routinely held up as the epitome of an "anti-Machian" spacetime. The unreconstructed Machian might answer that, mathematics notwithstanding, the inertial properties of Minkowski spacetime *physically* must arise from a spherical shell of matter (particles, gravitational waves, other radiation, black holes, whatever) at spatial infinity. That is, the volume integral over sources for ϕ may vanish because the spacetime is empty out to infinity, but ϕ does not vanish because the surface integral, even though the surface is an infinite distance away, does not vanish putatively since the total mass in the surface is infinite. The 1 in Eq. (3.2) in this view then should be replaced by ϕ_s/c^2 , where ϕ_s is the potential due to the surface integral, which is everywhere equal to c^2 . Einstein followed a similar boundary condition path in 1916 [Hofer, in: Barbour and Pfister, 1995, pp. 67-90, esp. pp. 74-79 and 88]. In it the "natural" metric of empty spacetime far from any matter is degenerate; and elsewhere, even if no normal matter is present, "dark matter" causes this metric to become Minkowskian. He abandoned it in 1917 for the closed universe approach.

For our purpose here it is unimportant whether boundary conditions that relativise inertia in Minkowski spacetimes can be

found. The salient point to be taken is that noted by Brans in [1962]:

... according to general relativity, if small masses, charged or uncharged, are introduced into [a] laboratory, the description of their motions and interactions in [a locally Minkowskian] coordinate system is independent of the rest of the universe. This is due to the fact that once the laboratory is shielded, the only way the rest of the universe could influence it, according to general relativity, is through the metric. If this is sensibly flat within, its influence can be transformed away by a coordinate transformation, thus eliminating any effects from the rest of the universe. [p. 389]

To see the import of these inferences, we write out the fully Machianized version of Eq. (3.2):

$$m = m_o(\phi_s + \phi)/c^2. \quad (3.3)$$

We rearrange this to:

$$m/(\phi_s + \phi) = m_o/c^2. \quad (3.4)$$

Now m_o is invariant and c is a local invariant, so the RHS of Eq. (3.4) is a local invariant. It follows that the LHS of Eq. (3.4) must also be a local invariant. If we assert that $\phi_s = c^2$ and then assign some value to ϕ that allegedly arises from local matter, m must assume a value that preserves the invariance of the LHS of Eq. (3.4). As long as we posit that the passive and active gravitational mass change in proportion to the change in m , the validity of the equivalence principle is preserved and we have a metric theory of gravity.

If asserting the invariance of $m/(\phi_s + \phi)$ and letting ϕ and m vary accordingly were all that needed to be done, we could get some interesting, observable local effects [see, for example, Rindler, in: Barbour and Pfister, pp. 437-441 in this connection], but we would not have GRT.¹⁰ And there is something not quite right about this, for in our enclosed laboratory we could not tell whether the mass increase in objects in the laboratory (which they all experience because coupling to gravity is universal) were due to the presence of a gravitational potential, or it were a kinematic effect, at a given instant, arising from some suitable instantaneous velocity with respect to some Minkowski background. But, after that instant, the difference would be obvious. In the latter case everything would be in motion, in the former it would not. Since absolute gravitational potentials must be observable and not equal to zero to account for

inertial reaction forces, and at the same time it must be impossible to distinguish by any *local* observation between the effect of gravity and some suitable kinematic effect (if kinematic and gravitational effects are to be equivalent as demanded by the equivalence principle), we are forced to the conclusion that: the total gravitational potential *as measured locally* must be an invariant like the speed of light if the principle of relativity is to be preserved.

Brans makes this point, that GRT takes account of this local invariance of the total gravitational potential, by appealing to electromagnetic effects. As Brans remarked, in GRT we find that the charge to mass ratios of charged objects are insensitive to the presence of a gravitational potential due to surrounding local masses. Since electric charge is a local invariant, m must be a local invariant, and therefore $(\phi_s + \phi)$ must be a local invariant too. This peculiar result has profound consequences. It renders untenable all scalar-tensor theories and assures the constancy of physical constants, to mention but two.

The local invariance of the total gravitational potential does not make GRT anti-Machian. As Nordtvedt has commented [in: Barbour and Pfister, 1995, pp. 422-435, esp. p. 428], "*What has been called the curvature of four-dimensional space and time (empirically being the location dependence of laboratory clock rates and ruler sizes), a central feature of metric theories of gravity, is a globally measured, though locally unseen, Machian feature of physical law.*" What it does do is make it impossible to do a local, laboratory experiment to measure the variation of the total gravitational potential from one place to another. One might conclude that gravitomagnetic frame dragging experiments are the only way to test for laboratory Machian effects. But that would be wrong.

4. A CHEAP, LABORATORY SCALE EXPERIMENT

Several years ago I pointed out that a curious inertial reaction effect makes it possible to induce transient fluctuations in the mass of translationally accelerated objects. These fluctuations are large enough to be seen in the laboratory with table-top scale apparatus. And when the effect is combined with a pulsed thrust, a stationary apparent change in the weight of a suitable device, in principle, can be produced [Woodward, 1992]. The effects are large enough to have practical consequences [Woodward, 1994]. Indeed, theoretically, the transient mass fluctuation can be made very large and negative with extant technology, perhaps making it possible to probe the fundamental properties of spacetime and gravity, and make TWISTS [Woodward, 1995]. But certainly we can find out if Mach's principle is right.

I have already given fairly detailed derivations of this effect [Woodward, 1992 and 1995], so I shall only sketch the chief points of those arguments. We assume that special relativity theory (SRT) is valid, that four-forces are just the derivative with respect to proper time of four-momenta, and that inertial reaction forces experienced by accelerating objects are produced by their interaction with an external field. We then examine the case of a translationally accelerated test particle with some small extension and proper density ρ_o in a universe of constant matter density. The inertial reaction force the test particle experiences, by hypothesis, is due to the action of an external field. So, to get the equation for the field we specialize to the frame of instantaneous rest for the test particle, normalize the reaction force by dividing by the mass of the test particle, and take the four-divergence of the field strength (force per unit mass). Since the three-vector part of the field is irrotational in this case, it may be written as the gradient of a scalar field ϕ , and we obtain:

$$\nabla^2 \phi - (1/\rho_o c^2)(\partial^2 E_o/\partial t^2) - (1/\rho_o c^2)^2(\partial E_o/\partial t)^2 = 4\pi Q_o, \quad (4.1)$$

where c is the speed of light, E_o is the proper energy density, and Q_o is the proper source charge. Since all matter experiences inertial reaction forces, Q_o is identified as $G\rho_o$ (G being the Newtonian constant of gravitation). To get a relativistic wave equation we must choose $E_o = \rho_o \phi$, that is, the local proper energy density is the proper gravitational potential energy density. And the inertial reaction force is the gravitational force exerted by the rest of the matter in the universe. That is, we have recovered the gravitational induction of inertia - Mach's principle - from the simple assumption that inertial reaction forces arise from the action of a field.

Keeping only terms up to first order in $1/c^2$, Eq. (4.1) becomes:

$$\nabla^2 \phi - (1/c^2)(\partial^2 \phi/\partial t^2) \approx 4\pi G\rho_o + (\phi/\rho_o c^2)(\partial^2 \rho_o/\partial t^2). \quad (4.2)$$

The transient term on the RHS of Eq. (4.2) is the predicted effect. If the gravitational potential ϕ were taken to be just due to local matter, the effect would be completely negligible. *Or if ϕ were arbitrary up to an additive constant, then the term would have to be set equal to zero since any non-zero value would be in principle observable, but at the same time arbitrary.* Gravity, however, is non-linear, so ϕ cannot be chosen arbitrarily [Peters, 1981]. When all of the matter in the universe is taken into account, one finds that $\phi \approx c^2$ and the transient source term in Eq. (4.2) is not negligible if the local proper matter density is fluctuating quickly. This transient mass fluctuation can be made to appear as a stationary shift in the weight of a suitable device [Woodward, 1992]. I shall not pursue this further

here since the execution and results of such an experiment will be the subject of the sequel to this paper, forthcoming shortly.

To finish up this section I address the question: If local detection of Machian effects that depend upon the absolute value of the gravitational potential are precluded in GRT (as discussed in section 3 above), how is it possible that an effect like that considered in this section could exist? The answer to this question lies in noting that the effect goes as the second order time derivative of the proper energy density. Dimensionally, energy is mass times velocity squared, so its second order time derivative will involve the *third* time derivative of position. This is the signature of a radiation reaction effect. They are normally excluded in dynamical theories – including GRT – by the demand that such terms vanish. So, while this effect is compatible with GRT in some sense because it is a consequence of the local validity of special relativity theory, it is not a formal prediction of GRT. Perhaps, if gravity is dynamically well-behaved, this effect does not exist. Our views on well-behaved dynamical theories notwithstanding, radiation reaction effects, in electrodynamics at least, exist as a *matter of fact*. Given the parallels between gravity and electromagnetism, it would be quite remarkable if this effect did not exist. (But if it doesn't exist, it's probably impossible to make TWITs.)

5. CONCLUSION

To bring things to a close, I shall briefly address a few issues that go beyond the physics discussed above. The almost universal reaction I have found to the irrelevance of “free will” entailed by the physics presented here is: “If the future already objectively exists, why should I bother to do anything about it [exercise judgment and discretion in my actions]? It will happen however it already is anyway.” The exception to this response is that of lawyers, who see a novel defense ploy in claiming non-responsibility due to the inevitability of reality. [“I couldn't help it; it was my fate. The laws of physics are responsible for my actions.”] Does the inevitability of the future absolve us of responsibility for our actions? Of course not. Should we all stop striving to secure whatever future we want, because the future that will occur objectively already exists? No. What we are – past, present and future – is our choices. Choices, however, that in some real sense we have already made. Sounds positively Calvinistic, doesn't it?¹¹ Speaking of Calvin may bring to mind a few vocables.¹²

Why should we care about striving if everything is determined and there is nothing more than reality? Well, although the fixity of the future seems to be the most arresting feature of the physics, we

should not ignore the objective existence of the past. You may think it is all gone, or at least inaccessible. Maybe. Then again, maybe not. It may be that TWITs are looming on the horizon. That is no laughing matter.

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NOTES:

1. That is, "Making the Universe Safe for Historians".
2. Sciama emphasized the relationship between G and the mean matter density of the universe, and so did not include it in his potentials. I do not replicate this aspect of his argument here and thus include G in the expressions for ϕ and A .
3. Yes, this is *the* quintessential *thought* experiment.
4. One may object that the "weak field" approximation was employed by Einstein in this calculation. That is true. But, at the level of approximation invoked here, a "strong field" calculation is not required. It is the presence of very large curvature – which does *not* obtain for a universe as far evolved as ours presently is – that makes "strong field" assumptions necessary.
5. Strictly speaking, in this gauge it is the part of A perpendicular to v (i.e., the transverse part) that must satisfy the wave equation. In the simple case treated here A and its transverse part are the same because the longitudinal part $[\nabla(\partial\phi/\partial t)]$ vanishes since ϕ is a constant, so this distinction is not emphasized.
6. Isenberg [in: Barbour and Pfister, 1995, pp. 188-207] calls these Wheeler-Einstein-Mach spaces. The anachronistic order here covers an acronymistic infelicity. These spaces may be compact, but they are not cuddly.
7. Recent critiques of non-locality in quantum mechanics suggest that not- yet-actualized-future interpretations face some pretty tough sledding (MUSH, section 3).
8. After all, everybody knows that the act of observation screws things up [Morrow, 1995].
9. As Cramer has pointed out, advanced effects are also implicit in normal relativistic quantum field theory, their signatures being negative energies and complex conjugate wavefunctions.
10. The non-induction of locally measured mass in objects by the

gravitational potentials of local surrounding matter in GRT, apart from being unfortunate from a simple Machian point of view, is not entirely obvious. See, for example, Rosen [1981].

11. In lieu of a quotation from book 11 of St. Augustine's *Confessions* here, I offer: "You must remember this: a kiss is but a kiss, ..." (Herman Hupfeld).
12. I borrow "vocale" from the postmodernist lexicon advisedly. In the beginning the *word* was robust, indeed powerful. Of late, it has lost much of its luster. ["What's the word?"] It's not even up to "frequency"; it's just another four letter word. "Vocale" restores the robustness (if not the power) of the original meaning, for more than words can be given voice. My favorite postmodern vocables are: "Yakka foob mog. Grug pubbawup zink wattooom gazork. Chumble spuzz." A Calvinistic explanation of Newton's first law.

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