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PHYSICS AT THE PLANCK LENGTH

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Results of the conference, "String Quantum Gravity and Physics at the Planck Energy Scale," organized by Dr. Norma Sanchez, in Erice, Sicily, June 21–28, 1992 will appear in *Proceedings of the International Workshop of Theoretical Physics, 6th Session*, World Scientific Publishing Co. Pte. Ltd., Singapore, 1993. The conference has given new occasion to ask what linkage, if any, ties together particle physics and the quantum theory of gravitation, where this fantastically short distance first displayed its relevance.

"I had always regarded the search for the absolute as the loftiest goal of all scientific activity... by nature I am peaceful and against dubious adventures... But I had been fighting for six years from 1894 on with the problem of the equilibrium between radiation and matter without having any success. I knew that this problem is of fundamental significance for physics... A theoretical explanation, therefore, had to be found at all costs whatever the price." It cost him two weeks of the most intense work in his life.[1] This was Planck's road to his famous May 1899, 19 October 1900 and 14 December 1900 formula,[2]

$$E = h\nu\,,$$

as he described it in a letter[3] to Robert W. Wood, my Johns Hopkins professor of physical optics. Didn't this formula exist before the Planck length was conceived? So it is easy but incorrect to presuppose. It demolishes this misconception to recall that today's formula for the Planck length,

$$L^* = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}\,\mathrm{cm}\,, \tag{1}$$

contains, in addition to Newton's $G = 6.673 \times 10^{-8}\,\mathrm{cm}^3/\mathrm{g\ sec}^2$ and Michelson's $c = 2.997 \times 10^{10}$ cm/sec, not Planck's 1900 h, but Dirac's 1922 $\hbar$. It gives perspective to recall that Planck's quantum was not the fruit of a momentary inspiration, but the heart-sought yield of a lifetime search for the deep and fundamental features of nature and the conviction that they could be found in the field of heat because heat radiation is independent of the nature of the container wall and free of all the complications of the structure of atoms and of matter. No wonder Planck held up[4] as beacon light the Wien displacement law,[5]

$$\text{(wave length of peak energy)(temperature)} = 0.4818 \times 10^{-10}\,\text{sec deg C}\,, \qquad (2)$$

connecting the wave length of peak black-body radiation with the temperature.

Already in May 1899, the analysis that led him to the quantum of action yielded a value for this not-yet-identified quantity:

$$b = 6.885 \cdot 10^{-27}\,\text{g cm}^2/\text{s}\,. \qquad (3)$$

With it, he went on to say,[6] "The possibility exists to set up units for length, mass, [and] time... which, independent of special bodies or substances [such as the platinum standard meter at the International Bureau of Weights and Measures in Sèvres, outside of Paris], remain significant for all times and for all cultures, whether extraterrestrial or nonanthropic, and which therefore can be designated as 'natural units of measure'" [in contrast to the meter, originally defined in terms of the distance from the North Pole to Earth's equator, the second, originally derived from the rate of Earth's spin of our planet about its axis, and the gram, derived from the density of water, the liquid rained down by our clouds]. "The means to determine the three [Planck makes it four] standards of length, mass, [and] time [Planck includes temperature] are given by the two constants already mentioned, a [in the Wien displacement law] and b [Planck's symbol for what he ultimately called h], plus the magnitude of the speed of light, c, *in vacuo*, and the constant of gravitation, f. Referred to the centimeter, gram, second and to the degree Celsius, the numerical values of these quantities are

$$\begin{aligned} a &= 0.4818 \times 10^{-10}\,\text{s deg C}\,, \\ b &= 6.885 \times 10^{-27}\,\text{g cm}^2/\text{s}\,, \\ c &= 3.00 \times 10^{10}\,\text{cm/s}\,, \\ f &= 6.685 \times 10^{-8}\,\text{cm}^3/\text{gs}^2\,.\text{''} \end{aligned} \qquad (4)$$

"If one now chooses natural units such that in the new system of measurement every one of the foregoing constants takes the value unity, then one gets as unit of length the quantity

$$\sqrt{\frac{bf}{c^3}} = 4.13 \times 10^{-33}\,\text{cm} : \qquad (5)$$

as unit of mass,

$$\sqrt{\frac{bc}{f}} = 5.56 \times 10^{-5}\,\text{g}\,; \qquad (6)$$

as unit of time,

$$\sqrt{\frac{bf}{c^5}} = 1.38 \times 10^{-43}\,\text{sec}\,; \tag{7}$$

as unit of temperature,

$$\sqrt{\frac{c^5}{bf}} = 3.50 \times 10^{32}\,{}^{\circ}\text{C}\,.\text{''} \tag{8}$$

"These quantities preserve their natural significance as long as the laws of gravitation, of propagation of light in a vacuum and the two principal laws of heat theory remain valid; they must, therefore, be measured [as the same] by the most different intelligences by the most different methods."

The discovery of the quantum of action, while the greatest force setting the direction of the ensuing development of physics, was not the only influence. It was not foreordained that the physics of atoms should come before the quantum physics of fields, but that is the way the accidents of history worked out. Out of the analysis of a system so simple as the harmonic oscillator came the concept of the root mean square "zero point oscillation amplitude,"

$$\Delta x \sim (\hbar/m\omega)^{1/2}\,. \tag{9}$$

From these fluctuations in the position of one oscillator, it was a natural step to figure zero point oscillations in such a system of oscillators as the electromagnetic field presents, and thus the fluctuations in the electric field, and in the magnetic field, in a region of extension L,

$$\Delta E \sim (\hbar c)^{1/2}/L^2\,, \tag{10}$$

$$\Delta B \sim (\hbar c)^{1/2}/L^2\,. \tag{11}$$

"In other words, the field 'resonates' between one configuration and another with the range of configurations of significance given by (11). Moreover, the smaller is the region of space under consideration, the larger are the field magnitudes that occur with appreciable probability.[7] "

"Of all the remarkable developments of physics since World War II, none was more impressive than the prediction and verification of the effects of the vacuum fluctuations in the electromagnetic field on the motion of the electron in the hydrogen atom.[1] The fluctuation field brings about in the most elementary approximation the displacement $\Delta x = \int (e/m\omega^2) E_x(\omega) e^{-i\omega t}\, d\omega$. The average vanishes but the root mean square $\langle \Delta x^2 \rangle$ does not. In consequence the electron feels an effective atomic potential altered from the expected value $V(x,y,z)$ by the amount

$$\Delta V(x,y,z) = (l/2)\langle (\Delta x)^2 \rangle\, \nabla^2 V(x,y,z)\,. \tag{12}$$

The average of this perturbation over the unperturbed motion accounts for the major part of the observed Lamb–Retherford shift $\Delta E = \langle \Delta V(x,y,z) \rangle$ in the energy level. Conversely, the observation of the expected shift makes the reality of the vacuum fluctuations inescapably evident.[8] "

"Similar considerations apply [to the geometry of space–time. Quantum fluctuations in the geometry are superposed on and coexist with the large-scale, slowly varying curvature predicted by classical deterministic general relativity. Thus, in a region of dimension L, where in a local Lorentz frame the normal values of the metric coefficients will be -1, 1, 1, 1, there will occur fluctuations in these coefficients of the order

$$\Delta x \sim L^*/L\,, \tag{13}$$

fluctuations in the first derivatives of the g_{ik}'s of the order

$$\Delta\Gamma \sim \Delta g/L \sim L^*/L^2\,, \tag{14}$$

and fluctuations in the curvature of space of the order

$$\Delta R \sim \Delta g/L^2 \sim L^*/L^3\,.\text{"} \tag{15}$$

Particles and Planck's Length

Past descriptions of electricity postulate[9] "a breakdown in Maxwell's field equations for the vacuum at a site where charge is located, or postulate the existence of some foreign and "physical" electric jelly embedded in space, or both. No one has ever found a way to describe electricity free of these unhappy features except to say that the quantum fluctuations in the geometry of space are so great at small distances that even the topology fluctuates, makes "wormholes," and traps lines of force. These fluctuations have to be viewed, not as tied to particles, and endowed with the scale of distances associated with particle physics ($\sim 10^{13}$ cm) but as pervading all space ("foam-like structure of geometry") and characterized by the Planck distance ($\sim 10^{-33}$ cm)." [10] Ordinary matter is as insignificant in its effective density of energy relative to these fluctuations in space–time geometry as a cloud is insignificant in density relative to the sky. The ordinary matter, nevertheless, like the cloud, is what we see.

Relative to the effective energy density of the typical Planck-scale fluctuation in space–time geometry,

$$M^*/L^{*3} \sim 2.18 \times 10^{-5}\,\text{g}/4.22 \times 10^{-99}\,\text{cm}^3 \sim 0.5 \times 10^{94}\,\text{g/cm}^3\,, \tag{16}$$

the typical density of nuclear matter, $\sim 10^{14}\,\text{g/cm}^3$, is negligible. Even the crash of SSC 20 TeV (2×10^{13} eV) nucleon against 20 TeV nucleon of reputed dimension $\sim 10^{-13}$ cm is reasonably counted on to produce a momentary density "only" of the order of $7 \times 10^{20}\,\text{g/cm}^3$. What about cosmic ray particles? They are known to have energies up to $\sim 10^{20}$ eV. The head-on collision in space of two such particles would give a calculated momentary energy density of the order of magnitude of $10^{86}\,\text{g/cm}^3$, still far short of matching the effective energy density of quantum fluctuations.

Totally inaccessible energy-wise to high-energy-particle experiments would seem to be today's only reasonable assessment of quantum fluctuations in topology and geometry. Ferret out what role such quantum structure may have as the wherewithal from which particles are constructed? By observation of collisions? No. By calculations based on first principles? Yes; or more reasonably, yes, as judged by the history of black holes, neutron stars and the Hubble expansion: first theory; then observation to confirm, motivate and elucidate.

Take what historical example as model? Why not quantum chemistry? "You chemists ought to be paying attention to the Bohr model of the atom if you ever expect to understand chemistry," so went the flavor of an encounter between graduate students in chemistry and those in physics in the 1920's. "You physicists must stop telling us that the all-electric Bohr atom will ever account for features so different as acidic and alkaline compounds, ionic and anionic couplings, homopolar and Van der Waal's forces. No, chemical forces are chemical forces, and physics is physics."

Nothing did so much as quantum mechanics to let electronic orbits account for chemical binding. Nothing does so much as the quantum to challenge the view that particle physics is particle physics and gravitation physics is gravitation physics — two disconnected parts of knowledge. The quantum theory in its most direct translation out of the Hamilton–Jacobi version of Einstein's general relativity theory of gravity puts at the center of attention the probability amplitude, $\Psi({}^{(3)}\mathbf{G})$, for this, that or the other three-geometry, ${}^{3}\mathbf{G}$. However, no one has ever exhibited such a wave function. The infinite-dimensional manifold of three-geometries is too intimidating, or too far from being the item of most direct physical insight, or both.

Quantum mechanics did not have to change its laws and equations to account for the existence of atoms and the interactions between them. This happy circumstance of history invites the hope that quantum geometrodynamics will not have to change its laws and equations to account for the existence and interactions between particles. Moreover, the Ashtekar formulation of quantum gravity[11] by way of the loops and knots that are so central to its formalism, and the weave-like linkages of these knots provide a Planck-scale building structure that promises new and fascinating insights when one has learned to explore and exploit them. Therefore, physics at the Planck length is a topic today endowed with a special aura of promise and enticement to the adventurous.

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11. For an overview, see for example: C. Rovelli, "Ashtekar formulation of general relativity and loop-space non-perturbative quantum gravity: a report," *Class. Quantum Grav.* 8, (1991) 1613–1675.

Note by N. Sánchez

This lecture was given by Prof Wheeler at the International Workshop of Theoretical Physics, 6th Session, "String Quantum Gravity and Physics at the Planck Energy Scale," Erice, 21–28 June 1992. The proceedings of this workshop, where a summary of this lecture appeared, were published by World Scientific Publishing Co., N. Sánchez, Editor (1993), both as a book and as a special issue of *IJMPA (Proc. Suppl.) 4A (1993)*. Unfortunately, it was not possible, due to the delay, to include the present manuscript in the proceedings. However, and in spite of his many commitments, Prof Wheeler was kind enough and strong enough to produce this written version of his lecture, and to send it to me for publication. We are greatly indebted to Prof Wheeler for having contributed so much throughout the whole workshop in Erice, as well as for having written — with so careful thought and structure — the present article for the proceedings.