

## The Mechanism of Nuclear Fission

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On the basis of the liquid drop model of atomic nuclei, an account is given of the mechanism of nuclear fission. In particular, conclusions are drawn regarding the variation from nucleus to nucleus of the critical energy required for fission, and regarding the dependence of fission cross section for a given nucleus on energy of the exciting agency. A detailed discussion of the observations is presented on the basis of the theoretical considerations. Theory and experiment fit together in a reasonable way to give a satisfactory picture of nuclear fission.

### INTRODUCTION

THE discovery by Fermi and his collaborators that neutrons can be captured by heavy nuclei to form new radioactive isotopes led especially in the case of uranium to the interesting finding of nuclei of higher mass and charge number than hitherto known. The pursuit of these investigations, particularly through the work of Meitner, Hahn, and Strassmann as well as Curie and Savitch, brought to light a number of unsuspected and startling results and finally led Hahn and Strassmann<sup>1</sup> to the discovery that from uranium elements of much smaller atomic weight and charge are also formed.

The new type of nuclear reaction thus discovered was given the name "fission" by Meitner and Frisch,<sup>2</sup> who on the basis of the liquid drop model of nuclei emphasized the analogy of the process concerned with the division of a fluid sphere into two smaller droplets as the result of a deformation caused by an external disturbance. In this connection they also drew attention to the fact that just for the heaviest nuclei the mutual repulsion of the electrical charges will to a large extent annul the effect of the short range nuclear forces, analogous to that of surface tension, in opposing a change of shape of the nucleus. To produce a critical deformation will therefore require only a comparatively small energy, and by the subsequent division of the nucleus a very large amount of energy will be set free.

<sup>1</sup> O. Hahn and F. Strassmann, *Naturwiss.* **27**, 11 (1939); see, also, P. Abelson, *Phys. Rev.* **55**, 418 (1939).

<sup>2</sup> L. Meitner and O. R. Frisch, *Nature* **143**, 239 (1939).

Just the enormous energy release in the fission process has, as is well known, made it possible to observe these processes directly, partly by the great ionizing power of the nuclear fragments, first observed by Frisch<sup>3</sup> and shortly afterwards independently by a number of others, partly by the penetrating power of these fragments which allows in the most efficient way the separation from the uranium of the new nuclei formed by the fission.<sup>4</sup> These products are above all characterized by their specific beta-ray activities which allow their chemical and spectrographic identification. In addition, however, it has been found that the fission process is accompanied by an emission of neutrons, some of which seem to be directly associated with the fission, others associated with the subsequent beta-ray transformations of the nuclear fragments.

In accordance with the general picture of nuclear reactions developed in the course of the last few years, we must assume that any nuclear transformation initiated by collisions or irradiation takes place in two steps, of which the first is the formation of a highly excited compound nucleus with a comparatively long lifetime, while

<sup>3</sup> O. R. Frisch, *Nature* **143**, 276 (1939); G. K. Green and Luis W. Alvarez, *Phys. Rev.* **55**, 417 (1939); R. D. Fowler and R. W. Dodson, *Phys. Rev.* **55**, 418 (1939); R. B. Roberts, R. C. Meyer and L. R. Hafstad, *Phys. Rev.* **55**, 417 (1939); W. Jentschke and F. Prankl, *Naturwiss.* **27**, 134 (1939); H. L. Anderson, E. T. Booth, J. R. Dunning, E. Fermi, G. N. Glasoe and F. G. Slack, *Phys. Rev.* **55**, 511 (1939).

<sup>4</sup> F. Joliot, *Comptes rendus* **208**, 341 (1939); L. Meitner and O. R. Frisch, *Nature* **143**, 471 (1939); H. L. Anderson, E. T. Booth, J. R. Dunning, E. Fermi, G. N. Glasoe and F. G. Slack, *Phys. Rev.* **55**, 511 (1939).

the second consists in the disintegration of this compound nucleus or its transition to a less excited state by the emission of radiation. For a heavy nucleus the disintegrative processes of the compound system which compete with the emission of radiation are the escape of a neutron and, according to the new discovery, the fission of the nucleus. While the first process demands the concentration on one particle at the nuclear surface of a large part of the excitation energy of the compound system which was initially distributed much as is thermal energy in a body of many degrees of freedom, the second process requires the transformation of a part of this energy into potential energy of a deformation of the nucleus sufficient to lead to division.<sup>5</sup>

Such a competition between the fission process and the neutron escape and capture processes seems in fact to be exhibited in a striking manner by the way in which the cross section for fission of thorium and uranium varies with the energy of the impinging neutrons. The remarkable difference observed by Meitner, Hahn, and Strassmann between the effects in these two elements seems also readily explained on such lines by the presence in uranium of several stable isotopes, a considerable part of the fission phenomena being reasonably attributable to the rare isotope  $U^{235}$  which, for a given neutron energy, will lead to a compound nucleus of higher excitation energy and smaller stability than that formed from the abundant uranium isotope.<sup>6</sup>

In the present article there is developed a more detailed treatment of the mechanism of the fission process and accompanying effects, based on the comparison between the nucleus and a liquid drop. The critical deformation energy is brought into connection with the potential energy of the drop in a state of unstable equilibrium, and is estimated in its dependence on nuclear charge and mass. Exactly how the excitation energy originally given to the nucleus is gradually exchanged among the various degrees of freedom and leads eventually to a critical deformation proves to be a question which needs not be discussed in order to determine the fission probability. In fact, simple statistical con-

siderations lead to an approximate expression for the fission reaction rate which depends only on the critical energy of deformation and the properties of nuclear energy level distributions. The general theory presented appears to fit together well with the observations and to give a satisfactory description of the fission phenomenon.

For a first orientation as well as for the later considerations, we estimate quantitatively in Section I by means of the available evidence the energy which can be released by the division of a heavy nucleus in various ways, and in particular examine not only the energy released in the fission process itself, but also the energy required for subsequent neutron escape from the fragments and the energy available for beta-ray emission from these fragments.

In Section II the problem of the nuclear deformation is studied more closely from the point of view of the comparison between the nucleus and a liquid droplet in order to make an estimate of the energy required for different nuclei to realize the critical deformation necessary for fission.

In Section III the statistical mechanics of the fission process is considered in more detail, and an approximate estimate made of the fission probability. This is compared with the probability of radiation and of neutron escape. A discussion is then given on the basis of the theory for the variation with energy of the fission cross section.

In Section IV the preceding considerations are applied to an analysis of the observations of the cross sections for the fission of uranium and thorium by neutrons of various velocities. In particular it is shown how the comparison with the theory developed in Section III leads to values for the critical energies of fission for thorium and the various isotopes of uranium which are in good accord with the considerations of Section II.

In Section V the problem of the statistical distribution in size of the nuclear fragments arising from fission is considered, and also the questions of the excitation of these fragments and the origin of the secondary neutrons.

Finally, we consider in Section VI the fission effects to be expected for other elements than thorium and uranium at sufficiently high neutron velocities as well as the effect to be anticipated in

<sup>5</sup> N. Bohr, *Nature* **143**, 330 (1939).

<sup>6</sup> N. Bohr, *Phys. Rev.* **55**, 418 (1939).

thorium and uranium under deuteron and proton impact and radiative excitation.

### I. ENERGY RELEASED BY NUCLEAR DIVISION

The total energy released by the division of a nucleus into smaller parts is given by

$$\Delta E = (M_0 - \sum M_i)c^2, \quad (1)$$

where  $M_0$  and  $M_i$  are the masses of the original and product nuclei at rest and unexcited. We have available no observations on the masses of nuclei with the abnormal charge to mass ratio formed for example by the division of such a heavy nucleus as uranium into two nearly equal parts. The difference between the mass of such a fragment and the corresponding stable nucleus of the same mass number may, however, if we look apart for the moment from fluctuations in energy due to odd-even alternations and the finer details of nuclear binding, be reasonably assumed, according to an argument of Gamow, to be representable in the form

$$M(Z, A) - M(Z_A, A) = \frac{1}{2}B_A(Z - Z_A)^2, \quad (2)$$

where  $Z$  is the charge number of the fragment and  $Z_A$  is a quantity which in general will not be an integer. For the mass numbers  $A = 100$  to  $140$  this quantity  $Z_A$  is given by the dotted line in Fig. 8, and in a similar way it may be determined for lighter and heavier mass numbers.

$B_A$  is a quantity which cannot as yet be determined directly from experiment but may be estimated in the following manner. Thus we may assume that the energies of nuclei with a given mass  $A$  will vary with the charge  $Z$  approximately according to the formula

TABLE I. Values of the quantities which appear in Eqs. (6) and (7), estimated for various values of the nuclear mass number  $A$ . Both  $B_A$  and  $\delta_A$  are in Mev.

$A$	$Z_A$	$B_A$	$\delta_A$	$A$	$Z_A$	$B_A$	$\delta_A$
50	23.0	3.5	2.8	150	62.5	1.2	1.5
60	27.5	3.3	2.8	160	65.4	1.1	1.3
70	31.2	2.5	2.7	170	69.1	1.1	1.2
80	35.0	2.2	2.7	180	72.9	1.0	1.2
90	39.4	2.0	2.7	190	76.4	1.0	1.1
100	44.0	2.0	2.6	200	80.0	0.9 <sub>5</sub>	1.1
110	47.7	1.7	2.4	210	83.5	0.9 <sub>2</sub>	1.1
120	50.8	1.5	2.1	220	87.0	0.8 <sub>8</sub>	1.1
130	53.9	1.3	1.9	230	90.6	0.8 <sub>6</sub>	1.0
140	58.0	1.2	1.8	240	93.9	0.8 <sub>3</sub>	1.0

$$M(Z, A) = C_A + \frac{1}{2}B_A'(Z - \frac{1}{2}A)^2 + (Z - \frac{1}{2}A)(M_p - M_n) + 3Z^2e^2/5r_0A^{\frac{1}{3}}. \quad (3)$$

Here the second term gives the comparative masses of the various isobars neglecting the influence of the difference  $M_p - M_n$  of the proton and neutron mass included in the third term and of the pure electrostatic energy given by the fourth term. In the latter term the usual assumption is made that the effective radius of the nucleus is equal to  $r_0A^{\frac{1}{3}}$ , with  $r_0$  estimated as  $1.48 \times 10^{-13}$  from the theory of alpha-ray disintegration. Identifying the relative mass values given by expressions (2) and (3), we find

$$B_A' = (M_p - M_n + 6Z_Ae^2/5r_0A^{\frac{1}{3}})/(A - Z_A) \quad (4)$$

and

$$B_A = B_A' + 6e^2/5r_0A^{\frac{1}{3}} = (M_p - M_n + 3A^{\frac{2}{3}}e^2/5r_0)/(A - Z_A). \quad (5)$$

The values of  $B_A$  obtained for various nuclei from this last relation are listed in Table I.

On the basis just discussed, we shall be able to estimate the mass of the nucleus  $(Z, A)$  with the help of the packing fraction of the known nuclei. Thus we may write

$$M(Z, A) = A(1 + f_A) + 0 \left\{ \begin{array}{l} A \text{ odd} \\ A \text{ even, } Z \text{ even} \\ A \text{ even, } Z \text{ odd} \end{array} \right\} + \frac{1}{2}B_A(Z - Z_A)^2 - \frac{1}{2}\delta_A \left\{ \begin{array}{l} A \text{ even, } Z \text{ even} \\ A \text{ even, } Z \text{ odd} \end{array} \right\}, \quad (6)$$

where  $f_A$  is to be taken as the average value of the packing fraction over a small region of atomic weights and the last term allows for the typical differences in binding energy among nuclei according to the odd and even character of their neutron and proton numbers. In using Dempster's measurements of packing fractions we must recognize that the average value of the second term in (6) is included in such measurements.<sup>7</sup> This correction, however, is, as may be read from Fig. 8, practically compensated by the influence of the third term, owing to the fact that the great majority of nuclei studied in the mass spectrograph are of even-even character.

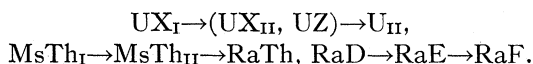
From (6) we find the energy release involved in electron emission or absorption by a nucleus unstable with respect to a beta-ray

<sup>7</sup> A. J. Dempster, Phys. Rev. **53**, 869 (1938).

transformation:

$$E_{\beta} = B_A \left\{ |Z_A - Z| - \frac{1}{2} \right\} - \delta_A \left\{ \begin{array}{l} +0 \quad \{ A \text{ odd} \\ A \text{ even, } Z \text{ even} \\ +\delta_A \quad \{ A \text{ even, } Z \text{ odd} \} \end{array} \right\}. \quad (7)$$

This result gives us the possibility of estimating  $\delta_A$  by an examination of the stability of isobars of even nuclei. In fact, if an even-even nucleus is stable or unstable, then  $\delta_A$  is, respectively, greater or less than  $B_A \{ |Z_A - A| - \frac{1}{2} \}$ . For nuclei of medium atomic weight this condition brackets  $\delta_A$  very closely; for the region of very high mass numbers, on the other hand, we can estimate  $\delta_A$  directly from the difference in energy release of the successive beta-ray transformations



The estimated values of  $\delta_A$  are collected in Table I.

Applying the available measurements on nuclear masses supplemented by the above considerations, we obtain typical estimates as shown in Table II for the energy release on division of a nucleus into two approximately equal parts.<sup>8</sup>

Below mass number  $A \sim 100$  nuclei are energetically stable with respect to division; above this limit energetic instability sets in with respect

TABLE II. Estimates for the energy release on division of typical nuclei into two fragments are given in the third column. In the fourth is the estimated value of the total additional energy release associated with the subsequent beta-ray transformations. Energies are in Mev.

ORIGINAL	TWO PRODUCTS	DIVISION	SUBSEQUENT
${}_{28}\text{Ni}^{61}$	${}_{14}\text{Si}^{30}, {}_{31}$	-11	2
${}_{50}\text{Sn}^{117}$	${}_{25}\text{Mn}^{58}, {}_{59}$	10	12
${}_{68}\text{Er}^{167}$	${}_{34}\text{Se}^{83}, {}_{84}$	94	13
${}_{82}\text{Pb}^{206}$	${}_{41}\text{Nb}^{103}, {}_{103}$	120	32
${}_{92}\text{U}^{239}$	${}_{46}\text{Pd}^{119}, {}_{120}$	200	31

to division into two nearly equal fragments, essentially because the decrease in electrostatic

<sup>8</sup> Even if there is no question of actual fission processes by which nuclei break up into more than two comparable parts, it may be of interest to point out that such divisions in many cases would be accompanied by the release of energy. Thus nuclei of mass number greater than  $A = 110$  are unstable with respect to division into three nearly equal parts. For uranium the corresponding total energy liberation will be  $\sim 210$  Mev, and thus is even somewhat greater than the release on division into two parts. The energy evolution on division of  $\text{U}^{239}$  into four comparable parts will, however, be about 150 Mev, and already division into as many as 15 comparable parts will be endothermic.

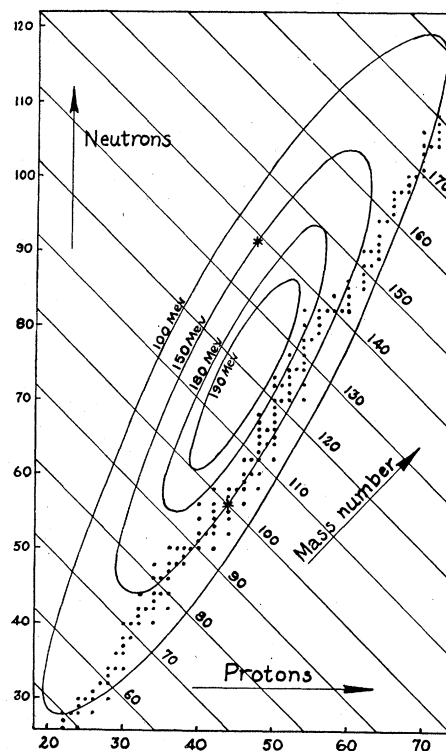


FIG. 1. The difference in energy between the nucleus  ${}_{92}\text{U}^{239}$  in its normal state and the possible fragment nuclei  ${}_{44}\text{Ru}^{100}$  and  ${}_{48}\text{Cd}^{139}$  (indicated by the crosses in the figure) is estimated to be 150 Mev as shown by the corresponding contour line. In a similar way the estimated energy release for division of  $\text{U}^{239}$  into other possible fragments can be read from the figure. The region in the chart associated with the greatest energy release is seen to be at a distance from the region of the stable nuclei (dots in the figure) corresponding to the emission of from three to five beta-rays.

energy associated with the separation overcompensates the desaturation of short range forces consequent on the greater exposed nuclear surface. The energy evolved on division of the nucleus  $\text{U}^{239}$  into two fragments of any given charge and mass numbers is shown in Fig. 1. It is seen that there is a large range of atomic masses for which the energy liberated reaches nearly the maximum attainable value 200 Mev; but that for a given size of one fragment there is only a small range of charge numbers which correspond to an energy release at all near the maximum value. Thus the fragments formed by division of uranium in the energetically most favorable way lie in a narrow band in Fig. 1, separated from the region of the stable nuclei by an amount which corresponds to the change in nuclear charge

associated with the emission of three to six beta-particles.

The amount of energy released in the beta-ray transformations following the creation of the fragment nuclei may be estimated from Eq. (7), using the constants in Table I. Approximate values obtained in this way for the energy liberation in typical chains of beta-disintegrations are shown on the arrows in Fig. 8.

The magnitude of the energy available for beta-ray emission from typical fragment nuclei does not stand in conflict with the stability of these nuclei with respect to spontaneous neutron emission, as one sees at once from the fact that the energy change associated with an increase of the nuclear charge by one unit is given by the *difference* between binding energy of a proton and of a neutron, plus the neutron-proton mass difference. A direct estimate from Eq. (6) of the binding energy of a neutron in typical nuclear fragments lying in the band of greatest energy release (Fig. 1) gives the results summarized in the last column of Table III. The comparison of the figures in this table shows that the neutron binding is in certain cases considerably smaller than the energy which can be released by beta-ray transformation. This fact offers a reasonable explanation as we shall see in Section V for the delayed neutron emission accompanying the fission process.

## II. NUCLEAR STABILITY WITH RESPECT TO DEFORMATIONS

According to the liquid drop model of atomic nuclei, the excitation energy of a nucleus must be

TABLE III. *Estimated values of energy release in beta-ray transformations and energy of neutron binding in final nucleus, in typical cases; also estimates of the neutron binding in the dividing nucleus. Values in Mev.*

BETA-TRANSITION		RELEASE	BINDING
$^{40}\text{Zr}^{99}$	$^{41}\text{Nb}^{99}$	6.3	8.2
$^{41}\text{Nb}^{100}$	$^{42}\text{Mo}^{100}$	7.8	8.6
$^{46}\text{Pd}^{125}$	$^{47}\text{Ag}^{125}$	7.8	6.7
$^{47}\text{Ag}^{125}$	$^{48}\text{Cd}^{125}$	6.5	5.0
$^{49}\text{In}^{130}$	$^{50}\text{Sn}^{130}$	7.6	7.1
$^{52}\text{Te}^{140}$	$^{53}\text{I}^{140}$	5.0	3.5
$^{53}\text{I}^{140}$	$^{54}\text{Xe}^{140}$	7.4	5.9
<i>Compound Nucleus</i>			
	$^{92}\text{U}^{235}$		5.4
	$^{92}\text{U}^{236}$		6.4
	$^{92}\text{U}^{239}$		5.2
	$^{90}\text{Th}^{233}$		5.2
	$^{91}\text{Pa}^{232}$		6.4

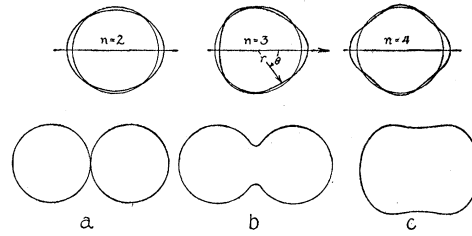


FIG. 2. Small deformations of a liquid drop of the type  $\delta r(\theta) = \alpha_n P_n(\cos \theta)$  (upper portion of the figure) lead to characteristic oscillations of the fluid about the spherical form of stable equilibrium, even when the fluid has a uniform electrical charge. If the charge reaches the critical value  $(10 \times \text{surface tension} \times \text{volume})^{1/3}$ , however, the spherical form becomes unstable with respect to even infinitesimal deformations of the type  $n=2$ . For a slightly smaller charge, on the other hand, a finite deformation (c) will be required to lead to a configuration of *unstable equilibrium*, and with smaller and smaller charge densities the critical form gradually goes over (c, b, a) into that of two uncharged spheres an infinitesimal distance from each other (a).

expected to give rise to modes of motion of the nuclear matter similar to the oscillations of a fluid sphere under the influence of surface tension.<sup>9</sup> For heavy nuclei the high nuclear charge will, however, give rise to an effect which will to a large extent counteract the restoring force due to the short range attractions responsible for the surface tension of nuclear matter. This effect, the importance of which for the fission phenomenon was stressed by Frisch and Meitner, will be more closely considered in this section, where we shall investigate the stability of a nucleus for small deformations of various types<sup>10</sup> as well as for such large deformations that division may actually be expected to occur.

Consider a small arbitrary deformation of the liquid drop with which we compare the nucleus such that the distance from the center to an arbitrary point on the surface with colatitude  $\theta$  is changed (see Fig. 2) from its original value  $R$

<sup>9</sup> N. Bohr, *Nature* **137**, 344 and 351 (1936); N. Bohr and F. Kalckar, *Kgl. Danske Vid. Selskab., Math. Phys. Medd.* **14**, No. 10 (1937).

<sup>10</sup> After the formulae given below were derived, expressions for the potential energy associated with spheroidal deformations of nuclei were published by E. Feenberg (*Phys. Rev.* **55**, 504 (1939)) and F. Weizsäcker (*Naturwiss.* **27**, 133 (1939)). Further, Professor Frenkel in Leningrad has kindly sent us in manuscript a copy of a more comprehensive paper on various aspects of the fission problem, to appear in the U.S.S.R. "*Annales Physicae*," which contains a deduction of Eq. (9) below for nuclear stability against arbitrary small deformations, as well as some remarks, similar to those made below (Eq. (14)) about the shape of a drop corresponding to unstable equilibrium. A short abstract of this paper has since appeared in *Phys. Rev.* **55**, 987 (1939).

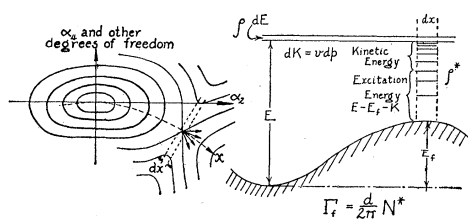


FIG. 3. The potential energy associated with any arbitrary deformation of the nuclear form may be plotted as a function of the parameters which specify the deformation, thus giving a contour surface which is represented schematically in the left-hand portion of the figure. The pass or saddle point corresponds to the critical deformation of unstable equilibrium. To the extent to which we may use classical terms, the course of the fission process may be symbolized by a ball lying in the hollow at the origin of coordinates (spherical form) which receives an impulse (neutron capture) which sets it to executing a complicated Lissajous figure of oscillation about equilibrium. If its energy is sufficient, it will in the course of time happen to move in the proper direction to pass over the saddle point (after which fission will occur), unless it loses its energy (radiation or neutron re-emission). At the right is a cross section taken through the fission barrier, illustrating the calculation in the text of the probability per unit time of fission occurring.

to the value

$$r(\theta) = R[1 + \alpha_0 + \alpha_2 P_2(\cos \theta) + \alpha_3 P_3(\cos \theta) + \dots], \quad (8)$$

where the  $\alpha_n$  are small quantities. Then a straightforward calculation shows that the surface energy plus the electrostatic energy of the comparison drop has increased to the value

$$E_{S+E} = 4\pi(r_0 A^{1/3})^2 O [1 + 2\alpha_2^2/5 + 5\alpha_3^2/7 + \dots + (n-1)(n+2)\alpha_n^2/2(2n+1) + \dots] + 3(Ze)^2/5r_0 A^{1/3} [1 - \alpha_2^2/5 - 10\alpha_3^2/49 - \dots - 5(n-1)\alpha_n^2/(2n+1)^2 - \dots], \quad (9)$$

where we have assumed that the drop is composed of an incompressible fluid of volume  $(4\pi/3)R^3 = (4\pi/3)r_0^3 A$ , uniformly electrified to a charge  $Ze$ , and possessing a surface tension  $O$ . Examination of the coefficient of  $\alpha_2^2$  in the above expression for the distortion energy, namely,

$$4\pi r_0^2 O A^{1/3} (2/5) \{1 - (Z^2/A) \times [e^2/10(4\pi/3)r_0^3 O]\} \quad (10)$$

makes it clear that with increasing value of the ratio  $Z^2/A$  we come finally to a limiting value

$$(Z^2/A)_{\text{limiting}} = 10(4\pi/3)r_0^3 O/e^2, \quad (11)$$

beyond which the nucleus is no longer stable with respect to deformations of the simplest type. The actual value of the numerical factors can be calculated with the help of the semi-empirical formula given by Bethe for the respective contributions to nuclear binding energies due to electrostatic and long range forces, the influence of the latter being divided into volume and surface effects. A revision of the constants in Bethe's formula has been carried through by Feenberg<sup>11</sup> in such a way as to obtain the best agreement with the mass defects of Dempster; he finds

$$r_0 \doteq 1.4 \times 10^{-13} \text{ cm}, \quad 4\pi r_0^2 O \doteq 14 \text{ Mev}. \quad (12)$$

From these values a limit for the ratio  $Z^2/A$  is obtained which is 17 percent greater than the ratio  $(92)^2/238$  characterizing  $U^{238}$ . Thus we can conclude that nuclei such as those of uranium and thorium are indeed near the limit of stability set by the exact compensation of the effects of electrostatic and short range forces. On the other hand, we cannot rely on the precise value of the limit given by these semi-empirical and indirect determinations of the ratio of surface energy to electrostatic energy, and we shall investigate below a method of obtaining the ratio in question from a study of the fission phenomenon itself.

Although nuclei for which the quantity  $Z^2/A$  is slightly less than the limiting value (11) are stable with respect to small arbitrary deformations, a larger deformation will give the long range repulsions more advantage over the short range attractions responsible for the surface tension, and it will therefore be possible for the nucleus, when suitably deformed, to divide spontaneously. Particularly important will be that critical deformation for which the nucleus is just on the verge of division. The drop will then possess a shape corresponding to unstable equilibrium: the work required to produce any infinitesimal displacement from this equilibrium configuration vanishes in the first order. To examine this point in more detail, let us consider the surface obtained by plotting the potential energy of an arbitrary distortion as a function of the parameters which specify its form and magnitude. Then we have to recognize the fact that the

<sup>11</sup> E. Feenberg, Phys. Rev. 55, 504 (1939).

potential barrier hindering division is to be compared with a pass or saddle point leading between two potential valleys on this surface. The energy relations are shown schematically in Fig. 3, where of course we are able to represent only two of the great number of parameters which are required to describe the shape of the system. The deformation parameters corresponding to the saddle point give us the critical form of the drop, and the potential energy required for this distortion we will term the critical energy for fission,  $E_f$ . If we consider a continuous change in the shape of the drop, leading from the original sphere to two spheres of half the size at infinite separation, then the critical energy in which we are interested is the lowest value which we can at all obtain, by suitable choice of this sequence of shapes, for the energy required to lead from the one configuration to the other.

Simple dimensional arguments show that the critical deformation energy for the droplet corresponding to a nucleus of given charge and mass number can be written as the product of the surface energy by a dimensionless function of the charge mass ratio:

$$E_f = 4\pi r_0^2 O A^{\frac{2}{3}} f\{(Z^2/A)/(Z^2/A)_{\text{limiting}}\}. \quad (13)$$

We can determine  $E_f$  if we know the shape of the nucleus in the critical state; this will be given by solution of the well-known equation for the form of a surface in equilibrium under the action of a surface tension  $O$  and volume forces described by a potential  $\varphi$ :

$$\kappa O + \varphi = \text{constant}, \quad (14)$$

where  $\kappa$  is the total normal curvature of the surface. Because of the great mathematical difficulties of treating large deformations, we are however able to calculate the critical surface and the dimensionless function  $f$  in (13) only for certain special values of the argument, as follows: (1) if the volume potential in (14) vanishes altogether, we see from (14) that the surface of unstable equilibrium has constant curvature; we have in fact to deal with a division of the fluid into spheres. Thus, when there are no electrostatic forces at all to aid the fission, the critical energy for division into two equal fragments will just

equal the total work done against surface tension in the separation process, i.e.,

$$E_f = 2 \cdot 4\pi r_0^2 O (A/2)^{\frac{2}{3}} - 4\pi r_0^2 O A^{\frac{2}{3}}. \quad (15)$$

From this it follows that

$$f(0) = 2^{\frac{2}{3}} - 1 = 0.260. \quad (16)$$

(2) If the charge on the droplet is not zero, but is still very small, the critical shape will differ little from that of two spheres in contact. There will in fact exist only a narrow neck of fluid connecting the two portions of the figure, the radius of which,  $r_n$ , will be such as to bring about equilibrium; to a first approximation

$$2\pi r_n O = (Ze/2)^2 / (2r_0(A/2)^{\frac{1}{3}})^2 \quad (17)$$

or

$$r_n/r_0 A^{\frac{1}{3}} = 0.66 \left(\frac{Z^2}{A}\right) / \left(\frac{Z^2}{A}\right)_{\text{limiting}} \quad (18)$$

To calculate the critical energy to the first order in  $Z^2/A$ , we can omit the influence of the neck as producing only a second-order change in the energy. Thus we need only compare the sum of surface and electrostatic energy for the original nucleus with the corresponding energy for two spherical nuclei of half the size in contact with each other. We find

$$\begin{aligned} E_f = & 2 \cdot 4\pi r_0^2 O (A/2)^{\frac{2}{3}} - 4\pi r_0^2 O A^{\frac{2}{3}} \\ & + 2 \cdot 3(Ze/2)^2 / 5r_0(A/2)^{\frac{1}{3}} \\ & + (Ze/2)^2 / 2r_0(A/2)^{\frac{1}{3}} - 3(Ze)^2 / 5r_0 A^{\frac{1}{3}}, \end{aligned} \quad (19)$$

from which

$$E_f / 4\pi r_0^2 O A^{\frac{2}{3}} \equiv f(x) = 0.260 - 0.215x, \quad (20)$$

provided

$$x = \left(\frac{Z^2}{A}\right) / \left(\frac{Z^2}{A}\right)_{\text{limiting}} = (\text{charge})^2 / \text{surface tension} \times \text{volume} \times 10 \quad (21)$$

is a small quantity. (3) In the case of greatest actual interest, when  $Z^2/A$  is very close to the critical value, only a small deformation from a spherical form will be required to reach the critical state. According to Eq. (9), the potential energy required for an infinitesimal distortion will increase as the square of the amplitude, and

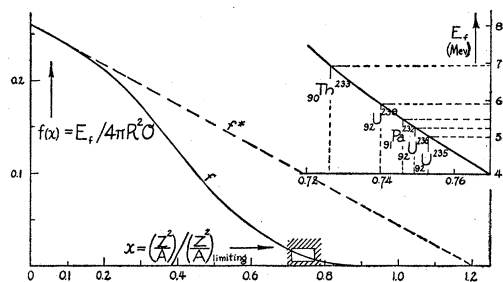


FIG. 4. The energy  $E_f$  required to produce a critical deformation leading to fission is divided by the surface energy  $4\pi R^2\sigma$  to obtain a dimensionless function of the quantity  $x = (\text{charge})^2 / (10 \times \text{volume} \times \text{surface tension})$ . The behavior of the function  $f(x)$  is calculated in the text for  $x=0$  and  $x=1$ , and a smooth curve is drawn here to connect these values. The curve  $f^*(x)$  determines for comparison the energy required to deform the nucleus into two spheres in contact with each other. Over the cross-hatched region of the curve of interest for the heaviest nuclei the surface energy changes but little. Taking for it a value of 530 Mev, we obtain the energy scale in the upper part of the figure. In Section IV we estimate from the observations a value  $E_f \sim 6$  Mev for  $\text{U}^{239}$ . Using the figure we thus find  $(Z^2/A)_{\text{limiting}} = 47.8$  and can estimate the fission barriers for other nuclei, as shown.

will moreover have the smallest possible value for a displacement of the form  $P_2(\cos \theta)$ . To find the deformation for which the potential energy has reached a maximum and is about to decrease, we have to carry out a more accurate calculation. We obtain for the distortion energy, accurate to the fourth order in  $\alpha_2$ , the expression

$$\begin{aligned} \Delta E_{S+E} = 4\pi r_0^2 O A^{\frac{2}{3}} [ & 2\alpha_2^2/5 + 116\alpha_2^3/105 \\ & + 101\alpha_2^4/35 + 2\alpha_2^2\alpha_4/35 + \alpha_4^2 ] \\ & - 3(Ze)^2/5r_0 A^{\frac{2}{3}} [ \alpha_2^2/5 + 64\alpha_2^3/105 \\ & + 58\alpha_2^4/35 + 8\alpha_2^2\alpha_4/35 + 5\alpha_4^2/27 ], \quad (22) \end{aligned}$$

in which it will be noted that we have had to include the terms in  $\alpha_4^2$  because of the coupling which sets in between the second and fourth modes of motion for appreciable amplitudes. Thus, on minimizing the potential energy with respect to  $\alpha_4$ , we find

$$\alpha_4 = -(243/595)\alpha_2^2 \quad (23)$$

in accordance with the fact that as the critical form becomes more elongated with decreasing  $Z^2/A$ , it must also develop a concavity about its equatorial belt such as to lead continuously with variation of the nuclear charge to the dumb-bell shaped figure discussed in the preceding paragraph.

With the help of (23) we obtain the deformation energy as a function of  $\alpha_2$  alone. By a straightforward calculation we then find its maximum value as a function of  $\alpha_2$ , thus determining the energy required to produce a distortion on the verge of leading to fission:

$$\begin{aligned} E_f/4\pi r_0^2 O A^{\frac{2}{3}} = f(x) = & 98(1-x)^3/135 \\ & - 11368(1-x)^4/34425 + \dots \quad (24) \end{aligned}$$

for values of  $Z^2/A$  near the instability limit.

Interpolating in a reasonable way between the two limiting values which we have obtained for the critical energy for fission, we obtain the curve of Fig. 4 for  $f$  as a function of the ratio of the square of the charge number of the nucleus to its mass number. The upper part of the figure shows the interesting portion of the curve in enlargement and with a scale of energy values at the right based on the surface tension estimate of Eq. (12) and a nuclear mass of  $A=235$ . The slight variation of the factor  $4\pi r_0^2 O A^{\frac{2}{3}}$  among the various thorium and uranium isotopes may be neglected in comparison with the changes of the factor  $f(x)$ .

In Section IV we estimate from the observations that the critical fission energy for  $\text{U}^{239}$  is not far from 6 Mev. According to Fig. 4, this corresponds to a value of  $x=0.74$ , from which we conclude that  $(Z^2/A)_{\text{limiting}} = (92)^2/239 \times 0.74 = 47.8$ . This result enables us to estimate the critical energies for other isotopes, as indicated in the figure. It is seen that protactinium would be particularly interesting as a subject for fission experiments.

As a by product, we are also able from Eq. (12) to compute the nuclear radius in terms of the surface energy of the nucleus; assuming Feenberg's value of 14 Mev for  $4\pi r_0^2\sigma$ , we obtain  $r_0 = 1.47 \times 10^{-13}$  cm, which gives a satisfactory and quite independent check on Feenberg's determination of the nuclear radius from the packing fraction curve.

So far the considerations are purely classical, and any actual state of motion must of course be described in terms of quantum-mechanical concepts. The possibility of applying classical pictures to a certain extent will depend on the smallness of the ratio between the zero point amplitudes for oscillations of the type discussed above and the nuclear radius. A simple calcu-



lation gives for the square of the ratio in question the result

$$\begin{aligned} \langle \alpha_n^2 \rangle_{Av; \text{zero point}} &= A^{-7/6} \\ &\times \{ (\hbar^2/12M_p r_0^2)/4\pi r_0^2 O \}^{1/2} n^{1/2} (2n+1)^{1/2} \\ &\times \{ (n-1)(n+2)(2n+1) - 20(n-1)x \}^{-1/2}. \end{aligned} \quad (25)$$

Since  $\{ (\hbar^2/12M_p r_0^2)/4\pi r_0^2 O \}^{1/2} \doteq \frac{1}{3}$ , this ratio is indeed a small quantity, and it follows that deformations of magnitudes comparable with nuclear dimensions can be described approximately classically by suitable wave packets built up from quantum states. In particular we may describe the critical deformations which lead to fission in an approximately classical way. This follows from a comparison of the critical energy  $E_f \sim 6$  Mev required, as we shall see in Section IV, to account for the observations on uranium, with the zero point energy

$$\begin{aligned} \frac{1}{2} \hbar \omega_2 &= A^{-1/2} \{ 4\pi r_0^2 O \cdot 2(1-x) \hbar^2 / 3M_p r_0^2 \}^{1/2} \\ &\sim 0.4 \text{ Mev} \end{aligned} \quad (26)$$

of the simplest mode of capillary oscillation, from which it is apparent that the amplitude in question is considerably larger than the zero point disturbance:

$$\langle \alpha_2^2 \rangle_{Av} / \langle \alpha_2^2 \rangle_{Av; \text{zero point}} \approx E_f / \frac{1}{2} \hbar \omega_2 \sim 15. \quad (27)$$

The drop with which we compare the nucleus will also in the critical state be capable of executing small oscillations about the shape of unstable equilibrium. If we study the distribution in frequency of these characteristic oscillations, we must expect for high frequencies to find a spectrum qualitatively not very different from that of the normal modes of oscillation about the form of stable equilibrium. The oscillations in question will be represented symbolically in Fig. 3 by motion of the representative point of the system in configuration space normal to the direction leading to fission. The distribution of the available energy of the system between such modes of motion and the mode of motion leading to fission will be determining for the probability of fission if the system is near the critical state. The statistical mechanics of this problem is considered in Section III. Here we would only like to point out that the fission process is from a practical point of view a nearly irreversible process. In fact if we imagine the fragment nuclei resulting from a fission to be

reflected without loss of energy and to run directly towards each other, the electrostatic repulsion between the two nuclei will ordinarily prevent them from coming into contact. Thus, relative to the original nucleus, the energy of two spherical nuclei of half the size is given by Eq. (19) and corresponds to the values  $f^*(x)$  shown by the dashed line in Fig. 4. To compare this with the energy required for the original fission process (smooth curve for  $f(x)$  in the figure), we note that the surface energy  $4\pi r_0^2 O A^{1/3}$  is for the heaviest nuclei of the order of 500 Mev. We thus have to deal with a difference of  $\sim 0.05 \times 500$  Mev = 25 Mev between the energy available when a heavy nucleus is just able to undergo fission, and the energy required to bring into contact two spherical fragments. There will of course be appreciable tidal forces exerted when the two fragments are brought together, and a simple estimate shows that this will lower the energy discrepancy just mentioned by something of the order of 10 Mev, which is not enough to alter our conclusions. That there is no paradox involved, however, follows from the fact that the fission process actually takes place for a configuration in which the sum of surface and electrostatic energy has a considerably smaller value than that corresponding to two rigid spheres in contact, or even two tidally distorted globes; namely, by arranging that in the division process the surface surrounding the original nucleus shall not tear until the mutual electrostatic energy of the two nascent nuclei has been brought down to a value essentially smaller than that corresponding to separated spheres, then there will be available enough electrostatic energy to provide the work required to tear the surface, which will of course have increased in total value to something more than that appropriate to two spheres. Thus it is clear that the two fragments formed by the division process will possess internal energy of excitation. Consequently, if we wish to reverse the fission process, we must take care that the fragments come together again sufficiently distorted, and indeed with the distortions so oriented, that contact can be made between projections on the two surfaces and the surface tension start drawing them together while the electrostatic repulsion between the effective electrical centers of gravity of the two parts is

still not excessive. The probability that two atomic nuclei in any actual encounter will be suitably excited and possess the proper phase relations so that union will be possible to form a compound system will be extremely small. Such union processes, converse to fission, can be expected to occur for unexcited nuclei only when we have available much more kinetic energy than is released in the fission processes with which we are concerned.

The above considerations on the fission process, based on a comparison between the properties of a nucleus and those of a liquid drop, should be supplemented by remarking that the distortion which leads to fission, although associated with a greater effective mass and lower quantum frequency, and hence more nearly approaching the possibilities of a classical description than any of the higher order oscillation frequencies of the nucleus, will still be characterized by certain specific quantum-mechanical properties. Thus there will be an essential ambiguity in the definition of the critical fission energy of the order of magnitude of the zero point energy,  $\hbar\omega_2/2$ , which however as we have seen above is only a relatively small quantity. More important from the point of view of nuclear stability will be the possibility of quantum-mechanical tunnel effects, which will make it possible for a nucleus to divide even in its ground state by passage through a portion of configuration space where classically the kinetic energy is negative.

An accurate estimate for the stability of a heavy nucleus against fission in its ground state will, of course, involve a very complicated mathematical problem. In natural extension of the well-known theory of  $\alpha$ -decay, we should in principle determine the probability per unit time of a fission process,  $\lambda_f$ , by the formula

$$\lambda_f (= \Gamma_f/\hbar) = 5(\omega_f/2\pi) \times \exp -2 \int_{P_1}^{P_2} \{2(V-E) \sum_i m_i (dx_i/d\alpha)^2\}^{1/2} d\alpha/\hbar. \quad (28)$$

The factor 5 represents the degree of degeneracy of the oscillation leading to instability. The quantum of energy characterizing this vibration is, according to (26),  $\hbar\omega \sim 0.8$  Mev. The integral in

the exponent leads in the case of a single particle to the Gamow penetration factor. Similarly, in the present problem, the integral is extended in configuration space from the point  $P_1$  of stable equilibrium over the fission saddle point  $S$  (as indicated by the dotted line in Fig. 3) and down on a path of steepest descent to the point  $P_2$  where the classical value of the kinetic energy,  $E-V$ , is again zero. Along this path we may write the coordinate  $x_i$  of each elementary particle  $m_i$  in terms of a certain parameter  $\alpha$ . Since the integral is invariant with respect to how the parameter is chosen, we may for convenience take  $\alpha$  to represent the distance between the centers of gravity of the nascent nuclei. To make an accurate calculation on the basis of the liquid-drop model for the integral in (28) would be quite complicated, and we shall therefore estimate the result by assuming each elementary particle to move a distance  $\frac{1}{2}\alpha$  in a straight line either to the right or the left according as it is associated with the one or the other nascent nucleus. Moreover, we shall take  $V-E$  to be of the order of the fission energy  $E_f$ . Thus we obtain for the exponent in (28) approximately

$$(2ME_f)^{1/2} \alpha/\hbar. \quad (29)$$

With  $M = 239 \times 1.66 \times 10^{-24}$ ,  $E_f \sim 6$  Mev  $= 10^{-5}$  erg, and the distance of separation intermediate between the diameter of the nucleus and its radius, say of the order  $\sim 1.3 \times 10^{-13}$  cm, we thus find a mean lifetime against fission in the ground state equal to

$$1/\lambda_f \sim 10^{-21} \exp [(2 \times 4 \times 10^{-22} \times 10^{-5})^{1/2} 1.3 \times 10^{-12}/10^{-27}] \sim 10^{30} \text{ sec.} \sim 10^{22} \text{ years.} \quad (30)$$

It will be seen that the lifetime thus estimated is not only enormously large compared with the time interval of the order  $10^{-15}$  sec. involved in the actual fission processes initiated by neutron impacts, but that this is even large compared with the lifetime of uranium and thorium for  $\alpha$ -ray decay. This remarkable stability of heavy nuclei against fission is as seen due to the large masses involved, a point which was already indicated in the cited article of Meitner and Frisch, where just the essential characteristics of the fission effect were stressed.

### III. BREAK-UP OF THE COMPOUND SYSTEM AS A MONOMOLECULAR REACTION

To determine the fission probability, we consider a microcanonical ensemble of nuclei, all having excitation energies between  $E$  and  $E+dE$ . The number of nuclei will be chosen to be exactly equal to the number  $\rho(E)dE$  of levels in this energy interval, so that there is one nucleus in each state. The number of nuclei which divide per unit time will then be  $\rho(E)dE\Gamma_f/\hbar$ , according to our definition of  $\Gamma_f$ . This number will be equal to the number of nuclei in the transition state which pass outward over the fission barrier per unit time.<sup>11a</sup> In a unit distance measured in the direction of fission there will be  $(dp/h)\rho^*(E-E_f-K)dE$  quantum states of the microcanonical ensemble for which the momentum and kinetic energy associated with the fission distortion have values in the intervals  $dp$  and  $dK=vd p$ , respectively. Here  $\rho^*$  is the density of those levels of the compound nucleus in the transition state which arise from excitation of all degrees of freedom other than the fission itself. At the initial time we have one nucleus in each of the quantum states in question, and consequently the number of fissions per unit time will be

$$dE \int v(dp/h)\rho^*(E-E_f-K) = dEN^*/h, \quad (31)$$

where  $N^*$  is the number of levels in the transition state available with the given excitation. Comparing with our original expression for this number, we have

$$\Gamma_f = N^*/2\pi\rho(E) = (d/2\pi)N^* \quad (32)$$

for the fission width expressed in terms of the level density or the level spacing  $d$  of the compound nucleus.

The derivation just given for the level width will only be valid if  $N^*$  is sufficiently large compared to unity; that is, if the fission width is comparable with or greater than the level spacing. This corresponds to the conditions under which a correspondence principle treatment of the fission distortion becomes possible. On the other hand, when the excitation exceeds by only a

<sup>11a</sup> For a general discussion of the ideas involved in the concept of a transition state, reference is made to an article by E. Wigner, *Trans. Faraday Soc.* **34**, part 1, 29 (1938).

little the critical energy, or falls below  $E_f$ , specific quantum-mechanical tunnel effects will begin to become of importance. The fission probability will of course fall off very rapidly with decreasing excitation energy at this point, the mathematical expression for the reaction rate eventually going over into the penetration formula of Eq. (28); this, as we have seen above, gives a negligible fission probability for uranium.

The probability of neutron re-emission, so important in limiting the fission yield for high excitation energies, has been estimated from statistical arguments by various authors, especially Weisskopf.<sup>12</sup> The result can be derived in a very simple form by considering the microcanonical ensemble introduced above. Only a few changes are necessary with respect to the reasoning used for the fission process. The transition state will be a spherical shell of unit thickness just outside the nuclear surface  $4\pi R^2$ ; the critical energy is the neutron binding energy,  $E_n$ ; and the density  $\rho^{**}$  of excitation levels in the transition state is given by the spectrum of the residual nucleus. The number of quantum states in the microcanonical ensemble which lie in the transition region and for which the neutron momentum lies in the range  $p$  to  $p+dp$  and in the solid angle  $d\Omega$  will be

$$(4\pi R^2 \cdot p^2 dp d\Omega / h^3) \rho^{**}(E-E_n-K) dE. \quad (33)$$

We multiply this by the normal velocity  $v \cos \theta = (dK/dp) \cos \theta$  and integrate, obtaining

$$dE(4\pi R^2 \cdot 2\pi m / h^3) \int \rho^{**}(E-E_n-K) K dK \quad (34)$$

for the number of neutron emission processes occurring per unit time. This is to be identified with  $\rho(E)dE(\Gamma_n/\hbar)$ . Therefore we have for the probability of neutron emission, expressed in energy units, the result

$$\begin{aligned} \Gamma_n &= (1/2\pi\rho)(2mR^2/\hbar^2) \int \rho^{**}(E-E_n-K) K dK \\ &= (d/2\pi)(A^{3/2}/K') \sum_i K_i \end{aligned} \quad (35)$$

in complete analogy to the expression

$$\Gamma_f = (d/2\pi) \sum_i 1 \quad (36)$$

<sup>12</sup> V. Weisskopf, *Phys. Rev.* **52**, 295 (1937).

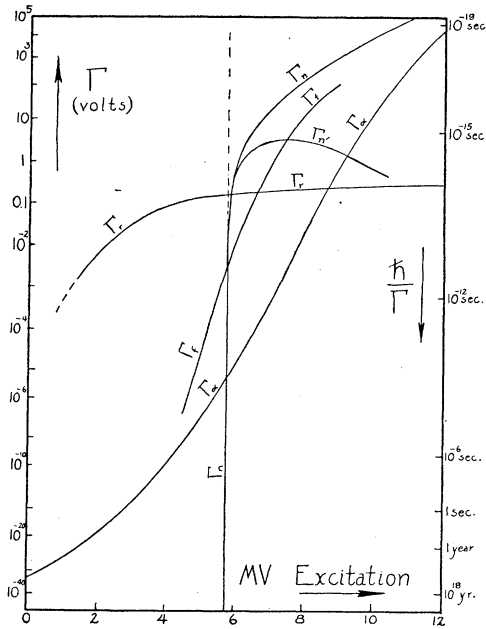


FIG. 5. Schematic diagram of the partial transition probabilities (multiplied by  $\hbar$  and expressed in energy units) and their reciprocals (dimensions of a mean lifetime) for various excitation energies of a typical heavy nucleus.  $\Gamma_r$ ,  $\Gamma_f$ , and  $\Gamma_\alpha$  refer to radiation, fission, and alpha-particle emission, while  $\Gamma_{n'}$  and  $\Gamma_n$  determine, respectively, the probability of a neutron emission leaving the residual nucleus in its ground state or in any state. The latter quantities are of course zero if the excitation is less than the neutron binding, which is taken here to be about 6 Mev.

for the fission width. Just as the summation in the latter equation goes over all those levels of the nucleus in the transition state which are available with the given excitation, so the sum in the former is taken over all available states of the residual nucleus,  $K_i$  denoting the corresponding kinetic energy  $E - E_n - E_i$  which will be left for the neutron.  $K'$  represents, except for a factor, the zero point kinetic energy of an elementary particle in the nucleus; it is given by  $A^{3/2}\hbar^2/2mR^2$  and will be 9.3 Mev if the nuclear radius is  $A^{1/3}1.48 \times 10^{-13}$  cm.

No specification was made as to the angular momentum of the nucleus in the derivation of (35) and (36). Thus the expressions in question give us averages of the level widths over states of the compound system corresponding to many different values of the rotational quantum number  $J$ , while actually capture of a neutron of one- or two-Mev energy by a normal nucleus will give rise only to a restricted range of values

of  $J$ . This point is of little importance in general, as the widths will not depend much on  $J$ , and therefore in the following considerations we shall apply the above estimates of  $\Gamma_f$  and  $\Gamma_n$  as they stand. In particular,  $d$  will represent the average spacing of levels of a given angular momentum. If, however, we wish to determine the partial width  $\Gamma_{n'}$  giving the probability that the compound nucleus will break up leaving the residual nucleus in its ground state and giving the neutron its full kinetic energy, we shall not be justified in simply selecting out the corresponding term in the sum in (35) and identifying it with  $\Gamma_{n'}$ . In fact, a more detailed calculation along the above lines, specifying the angular momentum of the microcanonical ensemble as well as its energy, leads to the expression

$$\Sigma(2J+1)\Gamma_{n'}^J = (2s+1)(2i+1)(d/2\pi)(R^2/\lambda^2) \quad (37)$$

for the partial neutron width, where the sum goes over those values of  $J$  which are realized when a nucleus of spin  $i$  is bombarded by a neutron of the given energy possessing spin  $s = \frac{1}{2}$ .

The smallness of the neutron mass in comparison with the reduced mass of two separating nascent nuclei will mean that we shall have in the former case to go to excitation energies much higher relative to the barrier than in the latter case before the condition is fulfilled for the application of the transition state method. In fact, only when the kinetic energy of the emerging particle is considerably greater than 1 Mev does the reduced wave-length  $\lambda = \hbar/2\pi p$  of the neutron become essentially smaller than the nuclear radius, allowing the use of the concepts of velocity and direction of the neutron emerging from the nuclear surface.

The absolute yield of the various processes initiated by neutron bombardment will depend upon the probability of absorption of the neutron to form a compound nucleus; this will be proportional to the converse probability  $\Gamma_{n'}/\hbar$  of a neutron emission process which leaves the residual neutron emission process which leaves the residual nucleus in its ground state.  $\Gamma_{n'}$  will vary as the neutron velocity itself for low neutron energies; according to the available information about nuclei of medium atomic weight, the width in volts is approximately  $10^{-3}$  times the

square root of the neutron energy in volts.<sup>13</sup> As the neutron energy increases from thermal values to 100 kev, we have to expect then an increase of  $\Gamma_{n'}$  from something of the order of  $10^{-4}$  ev to 0.1 or 1 ev. For high neutron energies we can use Eq. (37), according to which  $\Gamma_{n'}$  will increase as the neutron energy itself, except as compensated by the decrease in level spacing as higher excitations are attained. As an order of magnitude estimate, we can take the level spacing in  $U$  to decrease from 100 kev for the lowest levels to 20 ev at 6 Mev (capture of thermal neutrons) to  $\frac{1}{5}$  ev for  $2\frac{1}{2}$ -Mev neutrons. With  $d = \frac{1}{5}$  ev we obtain  $\Gamma_{n'} = (1/2\pi \times 5)(239^{3/2}/10)2\frac{1}{2} \div \frac{1}{5}$  ev for neutrons from the D+D reaction. The partial neutron width will not exceed for any energy a value of this order of magnitude, since the decrease in level spacing will be the dominating factor at higher energies.

The compound nucleus once formed, the outcome of the competition between the possibilities of fission, neutron emission, and radiation, will be determined by the relative magnitudes of  $\Gamma_f$ ,  $\Gamma_n$ , and the corresponding radiation width  $\Gamma_r$ . From our knowledge of nuclei comparable with thorium and uranium we can conclude that the radiation width  $\Gamma_r$  will not exceed something of the order of 1 ev, and moreover that it will be nearly constant for the range of excitation energies which results from neutron absorption (see Fig. 5). The fission width will be extremely small for excitation energies below the critical energy  $E_f$ , but above this point  $\Gamma_f$  will become appreciable, soon exceeding the radiation width and rising almost exponentially for higher energies. Therefore, if the critical energy  $E_f$  required for fission is comparable with or greater than the excitation consequent on neutron capture, we have to expect that radiation will be more likely than fission; but if the barrier height is somewhat lower than the value of the neutron binding, and in any case if we irradiate with sufficiently energetic neutrons, radiative capture will always be less probable than division. As the speed of the bombarding neutrons is increased, we shall not expect an indefinite rise in the fission yield, however, for the output will be governed by the competition in the compound system between the

possibilities of fission and of neutron emission. The width  $\Gamma_n$  which gives the probability of the latter process will for energies less than something of the order of 100 kev be equal to  $\Gamma_{n'}$ , the partial width for emissions leaving the residual nucleus in the ground state, since excitation of the product nucleus will be energetically impossible. For higher neutron energies, however, the number of available levels in the residual nucleus will rise rapidly, and  $\Gamma_n$  will be much larger than  $\Gamma_{n'}$ , increasing almost exponentially with energy.

In the energy region where the levels of the compound nucleus are well separated, the cross sections governing the yield of the various processes considered above can be obtained by direct application of the dispersion theory of Breit and Wigner.<sup>14</sup> In the case of resonance, where the energy  $E$  of the incident neutron is close to a special value  $E_0$  characterizing an isolated level of the compound system, we shall have

$$\sigma_f = \pi\lambda^2 \frac{2J+1}{(2s+1)(2i+1)} \frac{\Gamma_{n'}\Gamma_f}{(E-E_0)^2 + (\Gamma/2)^2} \quad (38)$$

and

$$\sigma_r = \pi\lambda^2 \frac{2J+1}{(2s+1)(2i+1)} \frac{\Gamma_{n'}\Gamma_r}{(E-E_0)^2 + (\Gamma/2)^2} \quad (39)$$

for the fission and radiation cross sections. Here  $\lambda = \hbar/p = \hbar/(2mE)^{1/2}$  is the neutron wave-length divided by  $2\pi$ ,  $i$  and  $J$  are the rotational quantum numbers of the original and the compound nucleus,  $s = \frac{1}{2}$ , and  $\Gamma = \Gamma_n + \Gamma_r + \Gamma_f$  is the total width of the resonance level at half-maximum.

In the energy region where the compound nucleus has many levels whose spacing,  $d$ , is comparable with or smaller than the total width, the dispersion theory cannot be directly applied due to the phase relations between the contributions of the different levels. A closer discussion<sup>15</sup> shows, however, that in cases like fission and radiative capture, the cross section will be obtained by summing many terms of the form (38) or (39). If the neutron wave-length is large compared with nuclear dimensions, only those states of the compound nucleus will contribute to the

<sup>13</sup> H. A. Bethe, Rev. Mod. Phys. 9, 150 (1937).

<sup>14</sup> G. Breit and E. Wigner, Phys. Rev. 49, 519 (1936). Cf. also H. Bethe and G. Placzek, Phys. Rev. 51, 450 (1937).

<sup>15</sup> N. Bohr, R. Peierls and G. Placzek, Nature (in press).

sum which can be realized by capture of a neutron of zero angular momentum, and we shall obtain

$$\sigma_f = \pi\lambda^2\Gamma_{n'}(\Gamma_f/\Gamma)(2\pi/d) \times \begin{cases} 1 & \text{if } i=0 \\ \frac{1}{2} & \text{if } i>0 \end{cases}, \quad (40)$$

$$\sigma_r = \pi\lambda^2\Gamma_{n'}(\Gamma_r/\Gamma)(2\pi/d) \times \begin{cases} 1 & \text{if } i=0 \\ \frac{1}{2} & \text{if } i>0 \end{cases}. \quad (41)$$

On the other hand, if  $\lambda$  becomes essentially smaller than  $R$ , the nuclear radius (case of neutron energy over a million volts), the summation will give

$$\sigma_f = \frac{\pi\lambda^2 \sum (2J+1)\Gamma_{n'}}{(2s+1)(2i+1)} (\Gamma_f/\Gamma)(2\pi/d) \\ = \pi R^2 \Gamma_f/\Gamma, \quad (42)$$

$$\sigma_r = \pi R^2 \Gamma_r/\Gamma. \quad (43)$$

The simple form of the result, which follows by use of the equation (37) derived above for  $\Gamma_{n'}$ , is of course an immediate consequence of the fact that the cross section for any given process for fast neutrons is given by the projected area of the nucleus times the ratio of the probability per unit time that the compound system react in the given way to the total probability of all reactions. Of course for extremely high bombarding energies it will no longer be possible to draw any simple distinction between neutron emission and fission; evaporation will go on simultaneously with the division process itself; and in general we shall have to expect then the production of numerous fragments of widely assorted sizes as the final result of the reaction.

#### IV. DISCUSSION OF THE OBSERVATIONS

##### A. The resonance capture process

Meitner, Hahn, and Strassmann<sup>16</sup> observed that neutrons of some volts energy produced in uranium a beta-ray activity of 23 min. half-life whose chemistry is that of uranium itself. Moreover, neutrons of such energy gave no noticeable yield of the complex of periods which is produced in uranium by irradiation with either thermal or

fast neutrons, and which is now known to arise from the beta-instability of the fragments arising from fission processes. The origin of the activity in question therefore had to be attributed to the ordinary type of radiative capture observed in other nuclei; like such processes it has a resonance character. The effective energy  $E_0$  of the resonance level or levels was determined by comparing the absorption in boron of the neutrons producing the activity and of neutrons of thermal energy:

$$E_0 = (\pi kT/4) [\mu_{\text{thermal}}(B)/\mu_{\text{res}}(B)]^2 \\ = 25 \pm 10 \text{ ev.} \quad (44)$$

The absorption coefficient in uranium itself for the activating neutrons was found to be 3 cm<sup>2</sup>/g, corresponding to an effective cross section of 3 cm<sup>2</sup>/g  $\times$  238  $\times$  1.66  $\times$  10<sup>-24</sup> g = 1.2  $\times$  10<sup>-21</sup> cm<sup>2</sup>. If we attribute the absorption to a single resonance level with no appreciable Doppler broadening, the cross section at exact resonance will be twice this amount, or 2.4  $\times$  10<sup>-21</sup> cm<sup>2</sup>; if on the other hand the true width  $\Gamma$  should be small compared with the Doppler broadening

$$\Delta = 2(E_0 kT/238)^{\frac{1}{2}} = 0.12 \text{ ev,}$$

we should have for the true cross section at exact resonance 2.7  $\times$  10<sup>-21</sup>  $\Delta/\Gamma$ , which would be even greater.<sup>17</sup> If the activity is actually due to several comparable resonance levels, we will clearly obtain the same result for the cross section of each at exact resonance.

According to Nier<sup>18</sup> the abundances of U<sup>235</sup> and U<sup>234</sup> relative to U<sup>238</sup> are 1/139 and 1/17,000; therefore, if the resonance absorption is due to either of the latter, the cross section at resonance will have to be at least 139  $\times$  2.4  $\times$  10<sup>-21</sup> cm<sup>2</sup> or 3.3  $\times$  10<sup>-19</sup> cm<sup>2</sup>. However, as Meitner, Hahn and Strassmann pointed out, this is excluded (cf. Eq. (39)) because it would be greater in order of magnitude than the square of the neutron wavelength. In fact,  $\pi\lambda^2$  is only 25  $\times$  10<sup>-21</sup> cm<sup>2</sup> for 25-volt neutrons. Therefore we have to attribute the capture to U<sup>238</sup>  $\rightarrow$  U<sup>239</sup>, a process in which the spin changes from  $i=0$  to  $J=\frac{1}{2}$ . We apply the

<sup>16</sup> L. Meitner, O. Hahn and F. Strassmann, *Zeits. f. Physik* **106**, 249 (1937).

<sup>17</sup> We are using the treatment of Doppler broadening given by H. Bethe and G. Placzek, *Phys. Rev.* **51**, 450 (1937).

<sup>18</sup> A. O. Nier, *Phys. Rev.* **55**, 150 (1939).

resonance formula (39) and obtain

$$25 \times 10^{-21} \times 4\Gamma_{n'}\Gamma_r/\Gamma^2 \\ = 2.7 \times 10^{-21}(\Delta/\Gamma) \text{ or } 2.4 \times 10^{-21} \quad (45)$$

according as the level width  $\Gamma = \Gamma_{n'} + \Gamma_r$  is or is not small compared with the Doppler broadening. In any case, we know<sup>13</sup> from experience with other nuclei for comparable neutron energies that  $\Gamma_{n'} \ll \Gamma_r$ ; this condition makes the solution of (45) unique. We obtain  $\Gamma_{n'} = \Gamma_r/40$  if the total width is greater than  $\Delta = 0.12$  ev; and if the total width is smaller than  $\Delta$  we find  $\Gamma_{n'} = 0.003$  ev. Thus in neither case is the neutron width less than 0.003 ev. Comparison with observations on elements of medium atomic weight would lead us to expect a neutron width of  $0.001 \times (25)^{\frac{1}{2}} = 0.005$  ev; and undoubtedly  $\Gamma_{n'}$  can be no greater than this for uranium, in view of the small level spacing, or equivalently, in view of the small probability that enough energy be concentrated on a single particle in such a big nucleus to enable it to escape. We therefore conclude that  $\Gamma_{n'}$  for 25-volt neutrons is approximately 0.003 ev.

Our result implies that the radiation width for the  $U^{239}$  resonance level cannot exceed  $\sim 0.12$  ev; it may be less, but not much less, first, because values as great as a volt or more have been observed for  $\Gamma_r$  in nuclei of medium atomic weight, and second, because values of a millivolt or more are observed in the transitions between individual levels of the radioactive elements, and for the excitation with which we are concerned the number of available lower levels is great and the corresponding radiation frequencies are higher.<sup>13</sup> A reasonable estimate of  $\Gamma_r$  would be 0.1 ev; of course direct measurement of the activation yield due to neutrons continuously distributed in energy near the resonance level would give a definite value for the radiation width.

The above considerations on the capture of neutrons to form  $U^{239}$  are expressed for simplicity as if there were a single resonance level, but the results are altered only slightly if several levels give absorption. However, the contribution of the resonance effect to the radiative capture cross section for *thermal* neutrons does depend essentially on the number of levels as well as their strength. On this basis Anderson and

Fermi have been able to show that the radiative capture of slow neutrons cannot be due to the tail at low energies of only a single level.<sup>19</sup> In fact, if it were, we should have for the cross section from (39)

$$\sigma_r(\text{thermal}) = \pi \lambda_{th}^2 \Gamma_{n'}(\text{thermal}) \Gamma_r / E_0^2; \quad (46)$$

since  $\Gamma_{n'}$  is proportional to neutron velocity, we should obtain at the effective thermal energy  $\pi kT/4 = 0.028$  ev.

$$\sigma_r(\text{thermal}) \sim 23 \times 10^{-18} \\ \times 0.003(0.028/25)^{\frac{1}{2}} 0.1 / (25)^2 \quad (47) \\ \sim 0.4 \times 10^{-24} \text{ cm}^2.$$

Anderson and Fermi however obtain for this cross section by direct measurement  $1.2 \times 10^{-24} \text{ cm}^2$ .

The conclusion that the resonance absorption at the effective energy of 25 ev is actually due to more than one level gives the possibility of an order of magnitude estimate of the spacing between energy levels in  $U^{239}$  if for simplicity we assume random phase relations between their individual contributions. Taking into consideration the factor between the observations and the result (47) of the one level formula, and recalling that levels below thermal energies as well as above contribute to the absorption, we arrive at a level spacing of the order of  $d = 20$  ev as a reasonable figure at the excitation in question.

## B. Fission produced by thermal neutrons

According to Meitner, Hahn and Strassmann<sup>20</sup> and other observers, irradiation of uranium by thermal neutrons actually gives a large number of radioactive periods which arise from fission fragments. By direct measurement the fission cross section for thermal neutrons is found to be between 2 and  $3 \times 10^{-24} \text{ cm}^2$  (averaged over the actual mixture of isotopes), that is, about twice the cross section for radiative capture. No appreciable part of this effect can come from the isotope  $U^{239}$ , however, because the observations on the  $\sim 25$ -volt resonance capture of neutrons by this nucleus gave only the 23-minute activity; the inability of Meitner, Hahn, and Strassmann to find for neutrons of this energy any appreciable yield of the complex of periods

<sup>19</sup> H. L. Anderson and E. Fermi, Phys. Rev. **55**, 1106 (1939).

<sup>20</sup> L. Meitner, O. Hahn and F. Strassmann, Zeits. f. Physik **106**, 249 (1937).

now known to follow fission indicates that for slow neutrons in general the fission probability for this nucleus is certainly no greater than 1/10 of the radiation probability. Consequently, from comparison of (38) and (39), the fission cross section for this isotope cannot exceed something of the order  $\sigma_f(\text{thermal}) = (1/10)\sigma_r(\text{thermal}) = 0.1 \times 10^{-24} \text{ cm}^2$ . From reasoning of this nature, as was pointed out in an earlier paper by Bohr, we have to attribute practically all of the fission observed with thermal neutrons to one of the rarer isotopes of uranium.<sup>21</sup> If we assign it to the compound nucleus  $\text{U}^{235}$ , we shall have  $17,000 \times 2.5 \times 10^{-24}$  or  $4 \times 10^{-20} \text{ cm}^2$  for  $\sigma_f(\text{thermal})$ ; if we attribute the division to  $\text{U}^{236}$ ,  $\sigma_f$  will be between 3 and  $4 \times 10^{-22} \text{ cm}^2$ .

We have to expect that the radiation width and the neutron width for slow neutrons will differ in no essential way between the various uranium isotopes. Therefore we will assume  $\Gamma_n(\text{thermal}) = 0.003(0.028/25)^{1/2} = 10^{-4} \text{ ev}$ . The fission width, however, depends strongly on the barrier height; this is in turn a sensitive function of nuclear charge and mass numbers, as indicated in Fig. 4, and decreases strongly with decreasing isotopic weight. Thus it is reasonable that one of the lighter isotopes should be responsible for the fission.

Let us investigate first the possibility that the division produced by thermal neutrons is due to the compound nucleus  $\text{U}^{235}$ . If the level spacing  $d$  for this nucleus is essentially greater than the level width, the cross section will be due principally to one level ( $J = \frac{1}{2}$  arising from  $i = 0$ ), and we shall have from

$$\sigma_f = \pi \lambda^2 \frac{2J+1}{(2s+1)(2i+1)} \frac{\Gamma_n \Gamma_f}{(E-E_0)^2 + (\Gamma/2)^2} \quad (38)$$

the equation

$$\Gamma_f / [E_0^2 + \Gamma^2/4] = 4 \times 10^{-20} / 23 \times 10^{-18} \times 10^{-4} = 17(\text{ev})^{-1}. \quad (48)$$

Since  $\Gamma > \Gamma_f$ , this condition can be put as an inequality,

$$E_0^2 < (\Gamma/4)(4/17) - \Gamma \quad (49)$$

from which it follows first, that  $\Gamma \leq 4/17 \text{ ev}$ , and second, that  $|E_0| < 2/17 \text{ ev}$ . Thus the level

would have to be very narrow and very close to thermal energies. But in this case the fission cross section would have to fall off very rapidly with increasing neutron energy; since  $\lambda \propto 1/v$ ,  $E \propto v^2$ ,  $\Gamma_n \propto v$ , we should have according to (38)  $\sigma_f \propto 1/v^5$  for neutron energies greater than about half a volt. This behavior is quite inconsistent with the finding of the Columbia group that the fission cross section for cadmium resonance neutrons ( $\sim 0.15 \text{ ev}$ ) and for the neutrons absorbed in boron (mean energy of several volts) stand to each other inversely in the ratio of the corresponding neutron velocities ( $1/v$ ).<sup>22</sup> Therefore, if the fission is to be attributed to  $\text{U}^{235}$ , we must assume that the level width is greater than the level spacing (many levels effective); but as the level spacing itself will certainly exceed the radiative width, we will then have a situation in which the total width will be essentially equal to  $\Gamma_f$ . Consequently we can write the cross section (40) for overlapping levels in the form

$$\sigma_f = \pi \lambda^2 \Gamma_n 2\pi/d. \quad (50)$$

From this we find a level spacing

$$d = 23 \times 10^{-18} \times 10^{-4} \times 2\pi/4 \times 10^{-20} = 0.4 \text{ ev}$$

which is unreasonably small: According to the estimates of Table III, the nuclear excitations consequent on the capture of slow neutrons to form  $\text{U}^{235}$  and  $\text{U}^{239}$  are approximately 5.4 Mev and 5.2 Mev, respectively; moreover, the two nuclei have the same odd-even properties and should therefore possess similar level distributions. From the difference  $\Delta E$  between the excitation energies in the two cases we can therefore obtain the ratio of the corresponding level spacings from the expression  $\exp(\Delta E/T)$ . Here  $T$  is the nuclear temperature, a low estimate for which is 0.5 Mev, giving a factor of  $\exp 0.6 = 2$ . From our conclusion in IV-A that the order of magnitude of the level spacing in  $\text{U}^{239}$  is 20 ev, we would expect then in  $\text{U}^{235}$  a spacing of the order of 10 ev. Therefore the result of Eq. (51) makes it seem quite unlikely that the fission observed for the thermal neutrons can be due to the rarest uranium isotope; we consequently attribute it almost entirely to the reaction  $\text{U}^{235} + n_{th} \rightarrow \text{U}^{236} \rightarrow \text{fission}$ .

<sup>21</sup> N. Bohr, Phys. Rev. 55, 418 (1939).

<sup>22</sup> Anderson, Booth, Dunning, Fermi, Glasoe and Slack, reference 4.



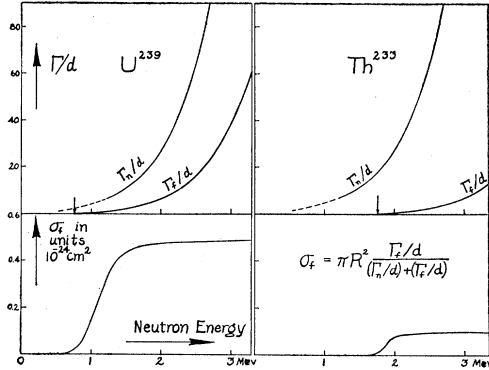


FIG. 6.  $\Gamma_n/d$  and  $\Gamma_f/d$  are the ratios of the neutron emission and fission probabilities (taken per unit of time and multiplied by  $\hbar$ ) to the average level spacing in the compound nucleus at the given excitation. These ratios will vary with energy in nearly the same way for all heavy nuclei, except that the entire fission curve must be shifted to the left or right according as the critical fission energy  $E_f$  is less than or greater than the neutron binding  $E_n$ . The cross section for fission produced by fast neutrons depends on the ratio of the values in the two curves, and is given on the left for  $E_f - E_n = (\frac{3}{2})$  Mev and on the right for  $E_f - E_n = 1\frac{3}{4}$  Mev, corresponding closely to the cases of  $U^{239}$  and  $Th^{235}$ , respectively.

We have two possibilities to account for the cross section  $\sigma_f(\text{thermal}) \sim 3.5 \times 10^{-22}$  presented by the isotope  $U^{235}$  for formation of the compound nucleus  $U^{236}$ , according as the level width is smaller than or comparable with the level spacing. In the first case we shall have to attribute most of the fission to an isolated level, and by the reasoning which was employed previously, we conclude that for this level

$$\frac{\Gamma_f}{\Gamma_n} \left[ \frac{E_0^2 + \Gamma^2}{4} \right] = \left[ \frac{(2s+1)(2i+1)}{(2J+1)} \right] 0.15 (\text{ev})^{-1} = R. \quad (52)$$

If the spin of  $U^{235}$  is  $\frac{3}{2}$  or greater, the right-hand side of (52) will be approximately  $0.30 (\text{ev})^{-1}$ ; but if  $i$  is as low as  $\frac{1}{2}$ , the right side will be either 0.6 or  $0.2 (\text{ev})^{-1}$ . The resulting upper limits on the resonance energy and level width may be summarized as follows:

$$\begin{array}{ccc} i \geq \frac{3}{2} & i = \frac{1}{2}, J=0 & i = \frac{1}{2}, J=1 \\ \Gamma < 4/R = 13 & 7 & 20 \text{ ev} \\ |E_0| < 1/R = 3 & 1.7 & 5 \text{ ev.} \end{array} \quad (53)$$

On the other hand, the indications<sup>22</sup> for low neutron energies of a  $1/v$  variation of fission cross section with velocity lead us as in the discussion of the rarer uranium isotope to the conclusion that either  $E_0$  or  $\Gamma/2$  or both are greater than several electron volts. This allows

us to obtain from (52) a lower limit also to  $\Gamma_f$ :

$$\Gamma_f = R[E_0^2 + \Gamma^2/4] > 10 \text{ to } 400 \text{ ev.} \quad (54)$$

In the present case, the various conditions are not inconsistent with each other, and it is therefore possible to attribute the fission to the effect of a single resonance level.

We can go further, however, by estimating the level spacing for the compound nucleus  $U^{236}$ . According to the values of Table III, the excitation following the neutron capture is considerably greater than in the case  $U^{239}$ , and we should therefore expect a rather smaller level spacing than the value  $\sim 20$  ev estimated in the latter case. On the other hand, it is known that for similar energies the level density is lower in even even than odd even nuclei. Thus the level spacing in  $U^{236}$  may still be as great as 20 ev, but it is undoubtedly no greater. From (54) we conclude then that we have probably to do with a case of overlapping resonance levels rather than a single absorption line, although the latter possibility is not entirely excluded by the observations available.

In the case of overlapping levels we shall have from Eq. (40)

$$\sigma_f = (\pi\lambda^2/2)\Gamma_n(2\pi/d) \quad (55)$$

or consequently a level spacing

$$d = (23 \times 10^{-18}/2) \times 10^{-4} \times 2\pi/3.5 \times 10^{-22} = 20 \text{ ev}; \quad (56)$$

and as we are attributing to the levels an unresolved structure, the fission width must be at least 10 ev. These values for level spacing and fission width give a reasonable account of the fission produced by slow neutrons.

### C. Fission by fast neutrons

The discussion on the basis of theory of the fission produced by fast neutrons is simplified first by the fact that the probability of radiation can be neglected in comparison with the probabilities of fission and neutron escape and second by the circumstance that the neutron wavelength  $/2\pi$  is small in comparison with the nuclear radius ( $R \sim 9 \times 10^{-13}$  cm) and we are in the region of continuous level distribution. Thus the fission cross section will be given by

$$\sigma_f = \pi R^2 \Gamma_f / \Gamma \sim 2.4 \times 10^{-24} \Gamma_f / (\Gamma_f + \Gamma_n), \quad (57)$$

or, in terms of the ratio of widths to level spacing,

$$\sigma_f \sim 2.4 \times 10^{-24} (\Gamma_f/d) / [(\Gamma_f/d) + (\Gamma_n/d)]. \quad (58)$$

According to the results of Section III,

$$\Gamma_n/d = (1/2\pi)(A^{2/3}/10 \text{ Mev}) \sum_i K_i \quad (59)$$

and

$$\Gamma_f/d = (1/2\pi)N^*. \quad (60)$$

In using Eq. (58) it is therefore seen that we do not have to know the level spacing  $d$  of the compound nucleus, but only that of the residual nucleus (Eq. (59)) and the number  $N^*$  of available levels of the dividing nucleus in the transition state (Eq. 60).

Considered as a function of energy, the ratio of fission width to level spacing will be extremely small for excitations less than the critical fission energy; with increase of the excitation above this value Eq. (60) will quickly become valid, and we shall have to anticipate a rapid rise in the ratio in question. If the spacing of levels in the transition state can be compared with that of the lower states of an ordinary heavy nucleus ( $\sim 50$  to  $100$  kev) we shall expect a value of  $N^* = 10$  to  $20$  for an energy  $1$  Mev above the fission barrier; but in any case the value of  $\Gamma_f/d$  will rise almost linearly with the available energy over a range of the order of a million volts, when the rise will become noticeably more rapid owing to the decrease to be expected at such excitations in the level spacing of the nucleus in the transition state. The associated behavior of  $\Gamma_f/d$  is illustrated in curves in Fig. 6. It should be remarked that the specific quantum-mechanical effects which set in at and below the critical fission energy may even show their influence to a certain extent above this energy and produce slight oscillations in the beginning of the  $\Gamma_f/d$  curve, allowing possibly a direct determination of  $N^*$ . How the ratio  $\Gamma_n/d$  will vary with energy is more accurately predictable than the ratio just considered. Denoting by  $K$  the neutron energy, we have for the number of levels which can be excited in the residual (=original) nucleus a figure of from  $K/0.05$  Mev to  $K/0.1$  Mev, and for the average kinetic energy of the inelastically scattered neutron  $\sim K/2$ , so that the sum  $K_i$  in (59) is

easily evaluated, giving us, if we express  $K$  in Mev,

$$\Gamma_n/d \sim 3 \text{ to } 6 \text{ times } K^2. \quad (61)$$

This formula provides as a matter of fact however only a rough first orientation, since for energies below  $K=1$  Mev it is not justified to apply the evaporation formula (a transition occurring until for slow neutrons  $\Gamma_n/d$  is proportional to velocity) and for energies above  $1$  Mev we have to take into account the gradual decrease which occurs in level spacing in the residual nucleus, and which has the effect of increasing the right-hand side of (61). An attempt has been made to estimate this increase in drawing Fig. 6.

The two ratios involved in the fast neutron fission cross section (58) will vary with energy in the same way for all the heaviest nuclei; the only difference from nucleus to nucleus will occur in the critical fission energy, which will have the effect of shifting one curve with respect to another as shown in the two portions of Fig. 6. Thus we can deduce the characteristic differences between nuclei to be expected in the variation with energy of the fast neutron cross section.

Meitner, Hahn, and Strassmann observed that fast neutrons as well as thermal ones produce in uranium the complex of activities which arise as a result of nuclear fission, and Ladenburg, Kanner, Barschall, and van Voorhis have made a direct measurement of the fission cross section for  $2.5$  Mev neutrons, obtaining  $0.5 \times 10^{-24}$  cm<sup>2</sup> ( $\pm 25$  percent).<sup>23</sup> Since the contribution to this cross section due to the  $U^{235}$  isotope cannot exceed  $\pi R^2/139 \sim 0.02 \times 10^{-24}$  cm<sup>2</sup>, the effect must be attributed to the compound nucleus  $U^{239}$ . For this nucleus however as we have seen from the slow neutron observations the fission probability is negligible at low energies. Therefore we have to conclude that the variation with energy of the corresponding cross section resembles in its general features Fig. 6a. In this connection we have the further observation of Ladenburg *et al.* that the cross section changes little between  $2$  Mev and  $3$  Mev.<sup>23</sup> This points to a value of the critical fission energy for  $U^{239}$  definitely less

<sup>23</sup> R. Ladenburg, M. H. Kanner, H. H. Barschall and C. C. van Voorhis, Phys. Rev. **56**, 168 (1939).

than 2 Mev in excess of the neutron binding. Unpublished results of the Washington group<sup>24</sup> give  $\sigma_f = 0.003 \times 10^{-24}$  at 0.6 Mev and  $0.012 \times 10^{-24}$  cm<sup>2</sup> at 1 Mev. With the Princeton observations<sup>23</sup> we have enough information to say that the critical energy for U<sup>239</sup> is not far from  $\frac{3}{4}$  Mev in excess of the neutron binding ( $\sim 5.2$  Mev from Table III):

$$E_f(\text{U}^{239}) \sim 6 \text{ Mev.} \quad (62)$$

A second conclusion we can draw from the absolute cross section of Ladenburg *et al.* is that the ratio of  $(\Gamma_f/d)$  to  $(\Gamma_n/d)$  as indicated in the figure is substantially correct; this confirms our presumption that the energy level spacing in the transition state of the dividing nucleus is not different in order of magnitude from that of the low levels in the normal nucleus.

The fission cross section of Th<sup>232</sup> for neutrons of 2 to 3 Mev energy has also been measured by the Princeton group; they find  $\sigma_f = 0.1 \times 10^{-24}$  cm<sup>2</sup> in this energy range. On the basis of the considerations illustrated in Fig. 6 we are led in this case to a fission barrier  $1\frac{3}{4}$  Mev greater than the neutron binding; hence, using Table III,

$$E_f(\text{Th}^{232}) \sim 7 \text{ Mev.} \quad (63)$$

A check on the consistency of the values obtained for the fission barriers is furnished by the possibility pointed out in Section II and Fig. 4 of obtaining the critical energy for all nuclei once we know it for one nucleus. Taking  $E_f(\text{U}^{239}) = 6$  Mev as standard, we obtain  $E_f(\text{Th}^{232}) = 7$  Mev, in good accord with (63).

As in the preceding paragraph we deduce from Fig. 4  $E_f(\text{U}^{236}) = 5\frac{1}{4}$  Mev,  $E_f(\text{U}^{235}) = 5$  Mev. Both values are *less* than the corresponding neutron binding energies estimated in Table III,  $E_n(\text{U}^{236}) = 6.4$  Mev,  $E_n(\text{U}^{235}) = 5.4$  Mev. From the values of  $E_n - E_d$  we conclude along the lines of Fig. 6 that for thermal neutrons  $\Gamma_f/d$  is, respectively,  $\sim 5$  and  $\sim 1$  for the two isotopes. Thus it appears that in both cases the level distribution will be continuous. We can estimate the as yet entirely unmeasured fission cross section of the lightest uranium isotope for the thermal neutrons from

$$\sigma_f = \pi \lambda^2 \Gamma_n \cdot 2\pi/d. \quad (64)$$

<sup>24</sup> Reported by M. Tuve at the Princeton meeting of the American Physical Society, June 23, 1939.

$d$  will not be much different from what it is for the similar compound nucleus U<sup>239</sup>, say of the order of 20 ev. Thus

$$\begin{aligned} \sigma_f(\text{thermal, U}^{235}) &\sim 23 \times 10^{-18} \times 10^{-4} \times 2\pi/20 \\ &\sim 500 \text{ to } 1000 \times 10^{-24} \text{ cm}^2, \end{aligned} \quad (65)$$

which is of course practically the same figure which holds for the next heaviest compound nucleus.

The various values estimated for fission barriers and fission and neutron widths are summarized in Fig. 7. The level spacing  $f$  for fast neutrons has been estimated from its value for slow neutrons and the fact that nuclear level densities appear to increase, according to Weisskopf, approximately exponentially as  $2(E/a)^{\frac{1}{2}}$ , where  $a$  is a quantity related to the spacing of the lowest nuclear levels and roughly 0.1 Mev in magnitude.<sup>25</sup> The relative values of  $\Gamma_n$ ,  $\Gamma_f$  and  $d$  for fast neutrons in Fig. 7, being obtained less indirectly, will be more reliable than their absolute values.

## V. NEUTRONS, DELAYED AND OTHERWISE

Roberts, Meyer and Wang<sup>26</sup> have reported the emission of neutrons following a few seconds after the end of neutron bombardment of a

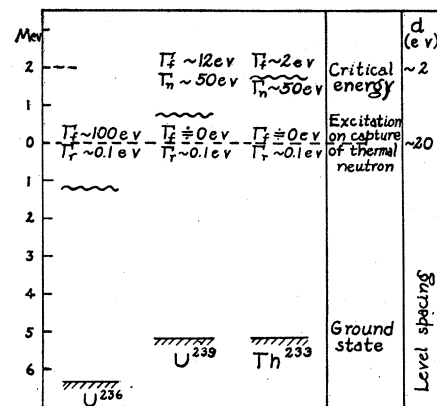


FIG. 7. Summary for comparative purposes of the estimated fission energies, neutron binding energies, level spacings, and neutron and fission widths for the three nuclei to which the observations refer. For fast neutrons the values of  $\Gamma_f$ ,  $\Gamma_n$ , and  $d$  are less reliable than their ratios. The values in the top line refer to a neutron energy of 2 Mev in each case.

<sup>25</sup> V. Weisskopf, Phys. Rev. 52, 295 (1937).

<sup>26</sup> R. B. Roberts, R. C. Meyer and P. Wang, Phys. Rev. 55, 510 (1939).

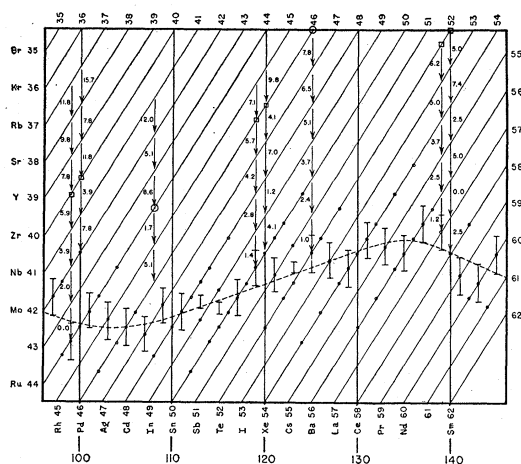


FIG. 8. Beta-decay of fission fragments leading to stable nuclei. Stable nuclei are represented by the small circles; thus the nucleus  ${}_{50}\text{Sn}^{120}$  lies just under the arrow marked 4.1; the number indicates the estimated energy release in Mev (see Section I) in the beta-transformation of the preceding nucleus  ${}_{49}\text{In}^{120}$ . Characteristic differences are noted between nuclei of odd and even mass numbers in the energy of successive transformations, an aid in assigning activities to mass numbers. The dotted line has been drawn, as has been proposed by Gamow, in such a way as to lie within the indicated limits of nuclei of odd mass number; its use is described in Section I.

thorium or uranium target. Other observers have discovered the presence of additional neutrons following within an extremely short interval after the fission process.<sup>27</sup> We shall return later to the question as to the possible connection between the latter neutrons and the mechanism of the fission process. The delayed neutrons themselves are to be attributed however to a high nuclear excitation following beta-ray emission from a fission fragment, for the following reasons:

(1) The delayed neutrons are found only in association with nuclear fission, as is seen from the fact that the yields for both processes depend in the same way on the energy of the bombarding neutrons.

(2) They cannot, however, arise during the fission process itself, since the time required for the division is certainly less than  $10^{-12}$  sec., according to the observations of Feather.<sup>27a</sup>

(3) Moreover, an excitation of a fission fragment in the course of the fission process to an

<sup>27</sup> H. L. Anderson, E. Fermi and H. B. Hanstein, Phys. Rev. **55**, 797 (1939); L. Szilard and W. H. Zinn, Phys. Rev. **55**, 799 (1939); H. von Halban, Jr., F. Joliot and L. Kowarski, Nature **143**, 680 (1939).

<sup>27a</sup> N. Feather, Nature **143**, 597 (1939).

energy sufficient for the subsequent evaporation of a neutron cannot be responsible for the *delayed* neutrons, since even by radiation alone such an excitation will disappear in a time of the order of  $10^{-13}$  to  $10^{-15}$  sec.

(4) The possibility that gamma-rays associated with the beta-ray transformations following fission might produce any appreciable number of photoneutrons in the source has been excluded by an experiment reported by Roberts, Hafstad, Meyer and Wang.<sup>28</sup>

(5) The energy release on beta-transformation is however in a number of cases sufficiently great to excite the product nucleus to a point where it can send out a neutron, as has been already pointed out in connection with the estimates in Table III. Typical values for the release are shown on the arrows in Fig. 8. The product nucleus will moreover have of the order of  $10^4$  to  $10^5$  levels to which beta-transformations can lead in this way, so that it will also be overwhelmingly probable that the product nucleus shall be highly excited.

We therefore conclude that the delayed emission of neutrons indeed arises as a result of nuclear excitation following the beta-decay of the nuclear fragments.

The actual probability of the occurrence of a nuclear excitation sufficient to make possible neutron emission will depend upon the comparative values of the matrix elements for the beta-ray transformation from the ground state of the original nucleus to the various excited states of the product nucleus. The simplest assumption we can make is that the matrix elements in question do not show any systematic variation with the energy of the final state. Then, according to the Fermi theory of beta-decay, the probability of a given beta-ray transition will be approximately proportional to the fifth power of the energy release.<sup>29</sup> If there are  $\rho(E)dE$  excitation levels of the product nucleus in the range  $E$  to  $E+dE$ , it will follow from our assumptions that the probability of an excitation in the same energy interval will be given by

$$w(E)dE = \text{constant} (E_0 - E)^5 \rho(E)dE, \quad (66)$$

<sup>28</sup> R. B. Roberts, L. R. Hafstad, R. C. Meyer and P. Wang, Phys. Rev. **55**, 664 (1939).

<sup>29</sup> L. W. Nordheim and F. L. Yost, Phys. Rev. **51**, 942 (1937).

where  $E_0$  is the total available energy. According to (66) the probability  $w(E)$  of a transition to the excited levels in a unit energy range at  $E$  reaches its maximum value for the energy  $E = E_{\max}$  given by

$$E_{\max} = E_0 - 5 / (d \ln \rho / dE)_{E_{\max}} = E_0 - 5T, \quad (67)$$

where  $T$  is the temperature (in energy units) to which the product nucleus must be heated to have on the average the excitation energy  $E_{\max}$ . Thus the most probable energy release on beta-transformation may be said to be five times the temperature of the product nucleus. According to our general information about the nuclei in question, an excitation of 4 Mev will correspond to a temperature of the order of 0.6 Mev. Therefore, on the basis of our assumptions, to realize an average excitation of 4 Mev by beta-transformation we shall require a total energy release of the order of  $4 + 5 \times 0.6 = 7$  Mev.

The spacing of the lowest nuclear levels is of the order of 100 kev for elements of medium atomic weight, decreases to something of the order of 10 ev for excitations of the order of 8 Mev, and can, according to considerations of Weisskopf, be represented in terms of a nuclear level density varying approximately exponentially as the square root of the excitation energy.<sup>23</sup> Using such an expression for  $\rho(E)$  in Eq. (66), we obtain the curve shown in Fig. 9 for the distribution function  $w(E)$  giving the probability that an excitation  $E$  will result from the beta-decay of a

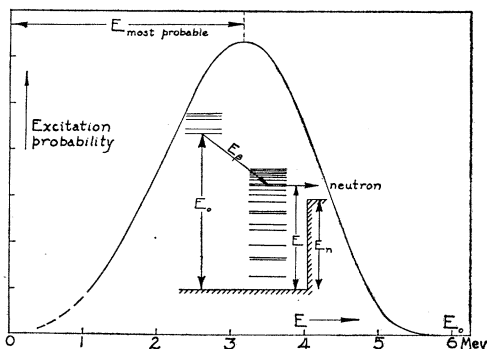


FIG. 9. The distribution in excitation of the product nuclei following beta-decay of fission fragments is estimated on the assumption of comparable matrix elements for the transformations to all excited levels. With sufficient available energy  $E_0$  and a small enough neutron binding  $E_n$  it is seen that there will be an appreciable number of delayed neutrons. The quantity plotted is probability per unit range of excitation energy.

typical fission fragment. It is seen that there will be appreciable probability for neutron emission if the neutron binding is somewhat less than the total energy available for the beta-ray transformation. We can of course draw only general conclusions because of the uncertainty in our original assumption that the matrix elements for the various possible transitions show no systematic trend with energy. Still, it is clear that the above considerations provide us with a reasonable qualitative account of the observation of Booth, Dunning and Slack that there is a chance of the order of 1 in 60 that a nuclear fission will result in the delayed emission of a neutron.<sup>30</sup>

Another consequence of the high probability of transitions to excited levels will be to give a beta-ray spectrum which is the superposition of a very large number of elementary spectra. According to Bethe, Hoyle and Peierls, the observations on the beta-ray spectra of light elements point to the Fermi distribution in energy in the elementary spectra.<sup>31</sup> Adopting this result, and using the assumption of equal matrix elements discussed above, we obtain the curve of Fig. 10 for the qualitative type of intensity distribution to be expected for the electrons emitted in the beta-decay of a typical fission fragment. As is seen from the curve, we have to expect that the great majority of electrons will have energies much smaller in value than the actual transformation energy which is available. This is in accord with the failure of various observers to find any appreciable number of very high energy electrons following fission.<sup>32</sup>

The half-life for emission of a beta-ray of 8 Mev energy in an elementary transition will be something of the order of 1 to 1/10 sec., according to the empirical relation between lifetime and energy given by the first Sargent curve. Since we have to deal in the case of the nuclear fragments with transitions to  $10^4$  or  $10^5$  excited levels, we should therefore at first sight expect an extremely short lifetime with respect to electron emission. However, the existence of a

<sup>30</sup> E. T. Booth, J. R. Dunning and F. G. Slack, *Phys. Rev.* **55**, 876 (1939).

<sup>31</sup> H. A. Bethe, F. Hoyle and R. Peierls, *Nature* **143**, 200 (1939).

<sup>32</sup> H. H. Barschall, W. T. Harris, M. H. Kanner and L. A. Turner, *Phys. Rev.* **55**, 989 (1939).

sum rule for the matrix elements of the transitions in question has as a consequence that the individual matrix elements will actually be very much smaller than those involved in beta-ray transitions from which the Sargent curve is deduced. Consequently, there seems to be no difficulty in principle in understanding lifetimes of the order of seconds such as have been reported for typical beta-decay processes of the fission fragments.

In addition to the delayed neutrons discussed above there have been observed neutrons following within a very short time (within a time of the order of at most a second) after fission.<sup>27</sup> The corresponding yield has been reported as from two to three neutrons per fission.<sup>33</sup> To account for so many neutrons by the above considered mechanism of nuclear excitation following beta-ray transitions would require us to revise drastically the comparative estimates of beta-transformation energies and neutron binding made in Section I. As the estimates in question were based on indirect though simple arguments, it is in fact possible that they give misleading results. If however they are reasonably correct, we shall have to conclude that the neutrons arise either from the compound nucleus at the moment of fission or by evaporation from the fragments as a result of excitation imparted to them as they separate. In the latter case the time required for neutron emission will be  $10^{-13}$  sec. or less (see Fig. 5). The time required to bring to rest a fragment with 100 Mev kinetic energy, on the other hand, will be at least the time required for a particle with average velocity  $10^9$  cm/sec. to traverse a distance of the order of  $10^{-3}$  cm. Therefore the neutron will be evaporated before the fragment has lost much of its translational energy. The kinetic energy per particle in the fragment being about 1 Mev, a neutron evaporated in nearly the forward direction will thus have an energy which is certainly greater than 1 Mev, as has been emphasized by Szilard.<sup>34</sup> The observations so far published neither prove nor disprove the possibility of such an evaporation following fission.

<sup>33</sup> Anderson, Fermi and Hanstein, reference 27. Szilard and Zinn, reference 27. H. von Halban, Jr., F. Joliot and L. Kowarski, *Nature* **143** 680 (1939).

<sup>34</sup> Discussions, Washington meeting of American Physical Society, April 28, 1939.

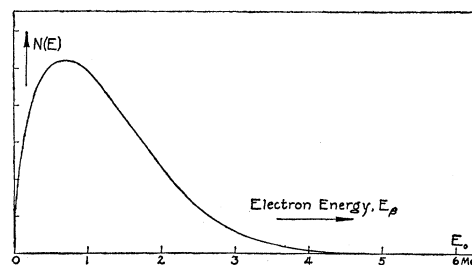


FIG. 10. The superposition of the beta-ray spectra corresponding to all the elementary transformations indicated in Fig. 9 gives a composite spectrum of a general type similar to that shown here, which is based on the assumption of comparable matrix elements and simple Fermi distributions for all transitions. The dependent variable is number of electrons per unit energy range.

We consider briefly the third possibility that the neutrons in question are produced during the fission process itself. In this connection attention may be called to observations on the manner in which a fluid mass of unstable form divides into two smaller masses of greater stability; it is found that tiny droplets are generally formed in the space where the original enveloping surface was torn apart. Although a detailed dynamical account of the division process will be even more complicated for a nucleus than for a fluid mass, the liquid drop model of the nucleus suggests that it is not unreasonable to expect at the moment of fission a production of neutrons from the nucleus analogous to the creation of the droplets from the fluid.

The statistical distribution in size of the fission fragments, like the possible production of neutrons at the moment of division, is essentially a problem of the dynamics of the fission process, rather than of the statistical mechanics of the critical state considered in Section II. Only after the deformation of the nucleus has exceeded the critical value, in fact, will there occur that rapid conversion of potential energy of distortion into energy of internal excitation and kinetic energy of separation which leads to the actual process of division.

For a classical liquid drop the course of the reaction in question will be completely determined by specifying the position and velocity in configuration space of the representative point of the system at the instant when it passes over the potential barrier in the direction of fission. If the energy of the original system is only

infinitesimally greater than the critical energy, the representative point of the system must cross the barrier very near the saddle point and with a very small velocity. Still, the wide range of directions available for the velocity vector in this multidimensional space, as suggested schematically in Fig. 3, indicates that production of a considerable variety of fragment sizes may be expected even at energies very close to the threshold for the division process. When the excitation energy increases above the critical fission energy, however, it follows from the statistical arguments in Section III that the representative point of the system will in general pass over the fission barrier at some distance from the saddle point. With general displacements of the representative point along the ridge of the barrier away from the saddle point there are associated asymmetrical deformations from the critical form, and we therefore have to anticipate a somewhat larger difference in size of the fission fragments as more energy is made available to the nucleus in the transition state. Moreover, as an influence of the finer details of nuclear binding, it will also be expected that the relative probability of observing fission fragments of odd mass number will be less when we have to do with the division of a compound nucleus of even charge and mass than one with even charge and odd mass.<sup>35</sup>

## VI. FISSION PRODUCED BY DEUTERONS AND PROTONS AND BY IRRADIATION

Regardless of what excitation process is used, it is clear that an appreciable yield of nuclear fissions will be obtained provided that the excitation energy is well above the critical energy for fission and that the probability of division of the compound nucleus is comparable with the probability of other processes leading to the break up of the system. Neutron escape being the most important process competing with fission, the latter condition will be satisfied if the fission energy does not much exceed the neutron binding, which is in fact the case, as we have seen, for the heaviest nuclei. Thus we have

<sup>35</sup> S. Flugge and G. v. Droste also have raised the question of the possible influence of finer details of nuclear binding on the statistical distribution in size of the fission fragments, *Zeits. f. physik. Chemie* **B42** 274 (1939).

to expect for these nuclei that not only neutrons but also sufficiently energetic deuterons, protons, and gamma-rays will give rise to observable fission.

### A. Fission produced by deuteron and proton bombardment

Oppenheimer and Phillips have pointed out that nuclei of high charge react with deuterons of not too great energy by a mechanism of polarization and dissociation of the neutron-proton binding in the field of the nucleus, the neutron being absorbed and the proton repulsed.<sup>36</sup> The excitation energy  $E$  of the newly formed nucleus is given by the kinetic energy  $E_d$  of the deuteron diminished by its dissociation energy  $I$  and the kinetic energy  $K$  of the lost proton, all increased by the binding energy  $E_n$  of the neutron in the product nucleus:

$$E = E_d - I - K + E_n. \quad (68)$$

The kinetic energy of the proton cannot exceed  $E_d + E_n - I$ , nor on the other hand will it fall below the potential energy which the proton will have in the Coulomb field at the greatest possible distance from the nucleus consistent with the deuteron reaction taking place with appreciable probability. This distance and the corresponding kinetic energy  $K_{\min}$  have been calculated by Bethe.<sup>37</sup> For very low values of the bombarding energy  $E_D$ , he finds  $K_{\min} \sim 1$  Mev; when  $E_d$  rises to equality with the dissociation energy  $I = 2.2$  Mev he obtains  $K_{\min} \sim E_d$ ; and even when the bombarding potential reaches a value corresponding to the height of the electrostatic barrier,  $K_{\min}$  still continues to be of order  $E_d$ , although beyond this point increase of  $E_d$  produces no further rise in  $K_{\min}$ . Since the barrier height for single charged particles will be of the order of 10 Mev for the heaviest nuclei, we can therefore assume  $K_{\min} \sim E_d$  for the ordinarily employed values of the deuteron bombarding energy. We conclude that the excitation energy of the product nucleus will have only a very small probability of exceeding the value

$$E_{\max} \sim E_n - I. \quad (69)$$

Since this figure is considerably less than the

<sup>36</sup> R. Oppenheimer and M. Phillips, *Phys. Rev.* **48**, 500 (1935).

<sup>37</sup> H. A. Bethe, *Phys. Rev.* **53**, 39 (1938).

estimated values of the fission barriers in thorium and uranium, we have to expect that Oppenheimer-Phillips processes of the type discussed will be followed in general by radiation rather than fission, unless the kinetic energy of the deuteron is greater than 10 Mev.

We must still consider, particularly when the energy of the deuteron approaches 10 Mev, the possibility of processes in which the deuteron as a whole is captured, leading to the formation of a compound nucleus with excitation of the order of

$$E_d + 2E_n - I \sim E_d + 10 \text{ Mev.} \quad (70)$$

There will then ensue a competition between the possibilities of fission and neutron emission, the outcome of which will be determined by the comparative values of  $\Gamma_f$  and  $\Gamma_n$  (proton emission being negligible because of the height of the electrostatic barrier). The increase of charge associated with the deuteron capture will of course lower the critical energy of fission and increase the probability of fission relative to neutron evaporation compared to what its value would be for the original nucleus at the same excitation. If after the deuteron capture the evaporation of a neutron actually takes place, the fission barrier will again be decreased relative to the binding energy of a neutron. Since the kinetic energy of the evaporated neutron will be only of the order of thermal energies ( $\approx 1$  Mev), the product nucleus has still an excitation of the order of  $E_d + 3$  Mev. Thus, if we are dealing with the capture of 6-Mev deuterons by uranium, we have a good possibility of obtaining fission at either one of two distinct stages of the ensuing nuclear reaction.

The cross section for fission in the double reaction just considered can be estimated by multiplying the corresponding fission cross section (42) for neutrons by a factor allowing for the effect of the electrostatic repulsion of the nucleus in hindering the capture of a deuteron:

$$\sigma_f \sim \pi R^2 e^{-P} \left\{ \frac{\Gamma_f(E')}{\Gamma(E')} + \left[ \frac{\Gamma_n(E')}{\Gamma(E')} \right] \left[ \frac{\Gamma_f(E'')}{\Gamma(E'')} \right] \right\}. \quad (71)$$

Here  $P$  is the new Gamow penetration exponent for a deuteron of energy  $E$  and velocity  $v$ :<sup>38</sup>

$$P = (4Ze^2/\hbar v) \left\{ \arccos x^{\frac{1}{2}} - x^{\frac{1}{2}}(1-x)^{\frac{1}{2}} \right\}, \quad (72)$$

<sup>38</sup>H. A. Bethe, Rev. Mod. Phys. 9, 163 (1937).

with  $x = (ER/Ze^2)$ .  $\pi R^2$  is the projected area of the nucleus.  $E'$  is the excitation of the compound nucleus, and  $E''$  the average excitation of the residual nucleus formed by neutron emission. For deuteron bombardment of  $U^{238}$  at 6 Mev we estimate a fission cross section of the order of

$$\pi(9 \times 10^{-13})^2 \exp(-12.9) \sim 10^{-29} \text{ cm}^2 \quad (73)$$

if we make the reasonable assumption that the probability of fission following capture is of the order of magnitude unity. Observations are not yet available for comparison with our estimate.

Protons will be more efficient than deuterons for the same bombarding energy, since from (72)  $P$  will be smaller by the factor  $2^{\frac{1}{2}}$  for the lighter particles. Thus for 6-Mev protons we estimate a cross section for production of fission in uranium of the order

$$\pi(9 \times 10^{-13})^2 \exp(-12.9/2^{\frac{1}{2}}) (\Gamma_f/\Gamma) \sim 10^{-28} \text{ cm}^2,$$

which should be observable.

## B. Photo-fission

According to the dispersion theory of nuclear reactions, the cross section presented by a nucleus for fission by a gamma-ray of wavelength  $2\pi\lambda$  and energy  $E = \hbar\omega$  will be given by

$$\sigma_f = \pi\lambda^2 (2J+1)/2(2i+1) \frac{\Gamma_r \Gamma_f}{(E-E_0)^2 + (\Gamma/2)^2} \quad (74)$$

if we have to do with an isolated absorption line of natural frequency  $E_0/\hbar$ . Here  $\Gamma_r/\hbar$  is the probability per unit time that the nucleus in the excited state will lose its entire excitation by emission of a single gamma-ray.

The situation of most interest, however, is that in which the excitation provided by the incident radiation is sufficient to carry the nucleus into the region of overlapping levels. On summing (74) over many levels, with average level spacing  $d$ , we obtain

$$\sigma_f = \pi\lambda^2 [(2J_{av}+1)/2(2i+1)] (2\pi/d) \Gamma_r \Gamma_f / \Gamma. \quad (75)$$

Without entering into a detailed discussion of the orders of magnitude of the various quantities involved in (75), we can form an estimate of the cross section for photo-fission by comparison with the yields of photoneutrons reported by various observers. The ratio of the cross sections



in question will be just  $\Gamma_f/\Gamma_n$ , so that

$$\sigma_f = (\Gamma_f/\Gamma_n)\sigma_n. \quad (76)$$

The observed values of  $\sigma_n$  for 12 to 17 Mev gamma-rays are  $\sim 10^{-26}$  cm<sup>2</sup> for heavy elements.<sup>39</sup> In view of the comparative values of  $\Gamma_f$  and  $\Gamma_n$  arrived at in Section IV, it will therefore be reasonable to expect values of the order of  $10^{-27}$  cm<sup>2</sup> for photo-fission of U<sup>233</sup>, and  $10^{-28}$  cm<sup>2</sup> for division of Th<sup>233</sup>. Actually no radiative fission was found by Roberts, Meyer and Hafstad using the gamma-rays from 3 micro-amperes of 1-Mev protons bombarding either lithium or fluorine.<sup>40</sup> The former target gives the greater yield, about 7 quanta per  $10^{10}$  protons, or  $8 \times 10^5$  quanta/min. altogether. Under the most favorable circumstances, all these gamma-rays would have passed through that thickness,  $\sim 6$  mg/cm<sup>2</sup>, of a sheet of uranium from which the fission particles are able to emerge. Even then, adopting the cross section we have estimated, we should expect an effect of

$$8 \times 10^5 \times 10^{-27} \times 6 \times 10^{-3} \times 6.06 \\ \times 10^{23} / 238 \sim 1 \text{ count/80 min}; \quad (77)$$

which is too small to have been observed. Consequently, we have as yet no test of the estimated theoretical cross section.

#### CONCLUSION

The detailed account which we can give on the basis of the liquid drop model of the nucleus, not only for the possibility of fission, but also for the dependence of fission cross section on energy and the variation of the critical energy from nucleus to nucleus, appears to be verified in its major features by the comparison carried out above between the predictions and observations. In the present stage of nuclear theory we are not able to predict accurately such detailed quantities as the nuclear level density and the ratio in the nucleus between surface energy and electrostatic energy; but if one is content to make approximate estimates for them on the basis of the observations, as we have done above, then the other details fit together in a reasonable way to give a satisfactory picture of the mechanism of nuclear fission.

<sup>39</sup> W. Bothe and W. Gentner, *Zeits. f. Physik* **112**, 45 (1939).

<sup>40</sup> R. B. Roberts, R. C. Meyer and L. R. Hafstad, *Phys. Rev.* **55**, 417 (1939).

## On the Behavior of Matter at Extremely High Temperatures and Pressures

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After some critical remarks on the current notions of stellar neutron cores the suggestion is set forth that an assembly of neutrons can form, under specified circumstances, two different phases by reason of the attracting forces between neutrons. The hypothetical transition from the dilute to the condensed neutron phase affords a concrete physical basis for the idea advocated by Zwicky that the supernovae originate from the sudden transition of an ordinary star into a centrally condensed one.

HUND<sup>1</sup> has analyzed in some detail the general behavior of matter at very high temperatures and pressures. This is a new field of theoretical speculation, and at present it appears impossible to arrive at definite conclu-

sions on this subject, especially because of the extremely poor knowledge that we have today regarding the real nature of internuclear forces and the mechanism of the nuclear chain reactions.

We begin by making a few critical remarks on Hund's theory which is based on the assumption that the nuclear reactions satisfy the following

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<sup>1</sup> F. Hund, *Ergebn. d. exakt. Naturwiss.* **15**, 189 (1936).