Visualization in the Einstein Year 2005: A Case Study on Explanatory and Illustrative Visualization of Relativity and Astrophysics

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ABSTRACT

In this application paper, we report on over fifteen years of experience with relativistic and astrophysical visualization, which has been culminating in a substantial engagement for visualization in the Einstein Year 2005—the 100th anniversary of Einstein's publications on special relativity, the photoelectric effect, and Brownian motion. This paper focuses on explanatory and illustrative visualizations used to communicate aspects of the difficult theories of special and general relativity, their geometric structure, and of the related fields of cosmology and astrophysics. We discuss visualization strategies, motivated by physics education and didactics of mathematics, and describe what kind of visualization methods have proven to be useful for different types of media, such as still images in popular-science magazines, film contributions to TV shows, oral presentations, or interactive museum installations. Although our visualization tools build upon existing methods and implementations, these techniques have been improved by several novel technical contributions like image-based special relativistic rendering on GPUs, an extension of general relativistic ray tracing to manifolds described by multiple charts, GPU-based interactive visualization of gravitational light deflection, as well as planetary terrain rendering. The usefulness and effectiveness of our visualizations are demonstrated by reporting on experiences with, and feedback from, recipients of visualizations and collaborators.

CR Categories: I.3.3 [Computer Graphics]: Picture/Image Generation; I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture; I.3.8 [Computer Graphics]: Applications; J.2 [Computer Applications]: Physical Sciences and Engineering

Keywords: Visualization, explanatory computer graphics, illustrative visualization, special relativity, general relativity, astrophysics, visualization of mathematics, terrain rendering

1 INTRODUCTION

Albert Einstein (1879–1955) was the first truly international pop star of science, and his popularity has never been matched by any other scientist since. In part, his popularity is certainly due to his extraordinary personality, appearance, and political engagement. Even more importantly, though, special and general relativity are concerned with concepts that everybody knows from daily life, such as space, time, and light—at the same time engendering an aura of scientific complexity and paradoxical effects. Therefore, most people are both attracted and appalled by Einstein's theories, which show that properties of space, time, and light in relativistic physics are dramatically different from those of our familiar environment governed by classical physics.

A major and typical problem in explaining special and general relativity to non-physicists is a lack of mathematical background, especially in differential geometry. We strongly believe that visualization can be used to address this problem because it is an excellent means of conveying important aspects of Einstein's theories without the need for mathematical formalism. Our goal is to develop visualizations that are explanatory, illustrative, and pedagogical in nature. Our approach does not target data exploration but the communication of ideas, theories, and phenomena to others. Although data and information exploration is the focus of most research efforts in the visualization community, we think that visual communication is an equally important aspect of visualization. Relativistic and astrophysical visualization is heavily based on mathematics, physics, and computer graphics and, therefore, is rooted in the tradition of scientific visualization.

In this application paper, we report on over fifteen years of experience with relativistic and astrophysical visualization, our group having started related research at the end of the 1980s [12, 35]. Our long-term commitment has been culminating in a strong engagement for visualization in the Einstein Year 2005. This year, the 100th anniversary of Einstein's *annus mirabilis*, in which he published his seminal articles on Brownian motion, the photoelectric effect, and special relativity, is celebrated. This anniversary is the motivation for numerous world-wide activities for popular-science presentations, and 2005 has also been declared "World Year of Physics" by the UNESCO.

We have been working on various visualization projects in the context of the Einstein Year: accompanying visualizations for several popular-science articles in magazines or book chapters; animated visualizations for TV shows; technical and scientific contents for major exhibitions in Ulm (March 2004–August 2004), Stuttgart (June 2004), Bern (June 2005–April 2006), and others. Different types of visualization require different methods, equipment, workflows, and know-how. Therefore, we are an interdisciplinary team of 15–20 people with expertise in computer graphics, relativistic physics, physics education, visual perception, user interfaces, computer-based modeling and animation, and museum design. Moreover, this technical team collaborates with journalists and writers in the context of popular-science publications.

This case study discusses our strategies for explanatory visualization as well as technical issues of algorithms, implementations, and workflows. We present how and why we employ different methods and tools, and we describe technical contributions in the form of some extended or novel visualization methods for image-based special relativistic rendering on GPUs (graphics processing units), general relativistic ray tracing for spacetimes with non-trivial topology, GPU-based interactive visualization of gravitational light deflection, as well as realistic planetary terrain rendering.

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2 VISUALIZATION STRATEGIES

In general, visualizations contain both explicit and implicit information [8, 25]. A mismatch between facts implicit in the visualization and those implicit in the theory or the data results in two types of problems. The visualization is not *complete* if not all facts implied by the theory are also implied by the visualization. Incomplete visualizations do not need to be incorrect. In fact, incompleteness is a valid means of reducing complexity and is frequently used in our visualizations in order to decrease the cognitive load for the user. Conversely, a visualization is not *sound* if it implies facts that are not valid consequences of the theory or data. Unfortunately, many popular-science presentations of relativity contain visualizations that are misleading, not sound, or even completely wrong.

We address the issue of soundness by relying on a visualization metaphor that is easy to explain and that avoids unnecessary implications: A virtual experiment is conducted under the influence of relativistic effects and the images taken by a virtual camera are the basis for the visualization, i.e., an egocentric view is adopted in a visually enriched thought experiment [3]. For example, special relativistic effects can be demonstrated by virtual flights at a velocity close to the speed of light, or general relativity can be illustrated by viewing a virtual galaxy behind a black hole that acts as source for gravitational light bending. The idea is to construct instructive, interesting, and compelling scenarios, e.g., with high-speed travel, black holes, wormholes, or the large scales covered by cosmology.

Our approach has several advantages. First, the underlying scenario can be easily described to a layperson. Second, the concept of a real or virtual camera is well-known from cinema, TV, or computer games. Third, image generation corresponds to a physical experiment that simulates light propagation. Therefore, the fundamental issue of coordinate system dependency, inherent to general relativity, is automatically addressed [44]. We apply the egocentric strategy for special relativity (Section 3), general relativity (Sections 4 and 5), and cosmological flights (Section 7). The strategy also lends itself to the intentional use of incompleteness: Separate relativistic effects on light propagation can be selectively switched on or off, e.g., color changes due to Doppler shift may be switched off to focus on geometric effects only.

An alternative strategy is based on the tradition of mathematical visualization. In general, this strategy tends to rely on more abstract metaphors that are more difficult to explain and that may be misleading. Therefore, an exocentric mathematical visualization of relativity has to be carefully designed from a didactical point of view. Section 6 describes our approach for an exocentric illustration of curved geometry.

3 SPECIAL RELATIVISTIC RENDERING

The mathematical foundation of special and general relativity is built on the concept of 4D spacetime, i.e., the combination of 3D space and 1D time. Special relativity is able to describe the kinematics of photons, which have vanishing rest mass, and massive objects alike. General relativity is only required when gravitation needs to be included (see Section 4). Special relativistic effects become noticeable at velocities comparable to the speed of light. Therefore, our approach to special relativistic visualization is based on virtual motion at very high speed. This section briefly reviews two methods for special relativistic rendering, and evaluates them in the context of our visualization applications.

Special relativity is based on flat spacetime, described by the Minkowski metric [28, 30]. Points in spacetime, so-called events, are transformed between reference frames moving at different velocities by the Lorentz transformation. The Lorentz transformation of light emission events is the basis for an object-space approach to special relativistic rendering [19, 45]. For relativistic motion in a static scene, object-space rendering boils down to a non-linear transformation of spatial positions from the static scene to spatial positions as seen by the fast moving camera. For interactive rendering, this transformation is first applied to the vertices of the scene objects, and then the transformed geometry is rendered by usual methods of non-relativistic computer graphics.

More recently, we have developed image-based special relativistic rendering [46] as an alternative rendering method that only requires computations in 2D image space. It builds upon the concept of the plenoptic function $P(\mathbf{x}, \theta, \phi, \lambda)$, which describes the radiance of the light depending on direction (θ, ϕ) in spherical coordinates, spacetime position $\mathbf{x}$, and wavelength λ [1]. The basic idea is to first record the plenoptic function within a static scene and for a static camera, and then to transform the plenoptic function into the frame of a moving camera. Afterwards, non-relativistic rendering methods can be applied to construct the final image.

The Lorentz transformation of the plenoptic function is determined by three relativistic effects: relativistic aberration of light, Doppler effect, and searchlight effect. The relativistic aberration of light causes a modification of the direction of light and is able to describe the apparent geometry seen by a fast moving camera. The Doppler effect accounts for the transformation of wavelength and causes a change in color. The searchlight effect transforms radiance and, e.g., increases the brightness of objects ahead when the observer is approaching these objects at high velocity.

Let us consider two inertial frames of reference, S and S', with S' moving with velocity v along the z axis of S. A light ray is described by direction (θ, ϕ) and wavelength λ in frame S, and by (θ', ϕ') and λ' in frame S'. Then, the Lorentz transformation of the plenoptic function from S to S' is [46]

$$P'(\theta', \phi', \lambda') = D^{-5} P\left(\arccos\frac{\cos\theta' + \beta}{1 + \beta\cos\theta'}, \phi', \frac{\lambda'}{D}\right), \qquad (1)$$

with the Doppler factor $D = \gamma(1 + \beta\cos\theta')$, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = v/c$, and the speed of light c. The plenoptic functions P and P' are located at corresponding spacetime positions, which are no longer explicitly shown in the mathematical expressions.

A direct application of image-based special relativistic rendering is used to transform real-world images [46]. Real images are most useful in illustrating relativistic effects in our familiar environment: They provide a "before-and-after" effect applied to our everyday world and therefore facilitate an easy recognition of relativistic effects affecting well-known scenes. An important technical problem is that aberration leads to severe distortions in image space that require the acquisition of high-resolution input images with a very large field of view—usually even a full spherical panorama (i.e., with 4π solid angle) is needed. Data acquisition is time-consuming for a single panorama and extremely difficult for flights based on a series of several hundred panoramas. Even if a collection of panorama images was available, storage and real-time processing of the data would be challenging, making an interactive rendering of relativistic motion very difficult. Therefore, we use real-world image-based rendering only for pre-computed illustrations—either for still images in magazines, or in films for exhibitions and TV shows.

The Lorentz transformation of the plenoptic function can also be applied to synthetic images. The basic idea is to construct a panorama by rendering a virtual scene and then to transform this panorama. Graphics hardware has tremendously improved in performance and functionality since the development of image-based special relativistic rendering in 2000 [46]. First, cube maps, which are effective in storing a panorama, have become widely available. Second, cube maps can be efficiently constructed by render-to-texture functionality. Third and most importantly, the Lorentz transformation from Eq. (1) can today be implemented by GPU

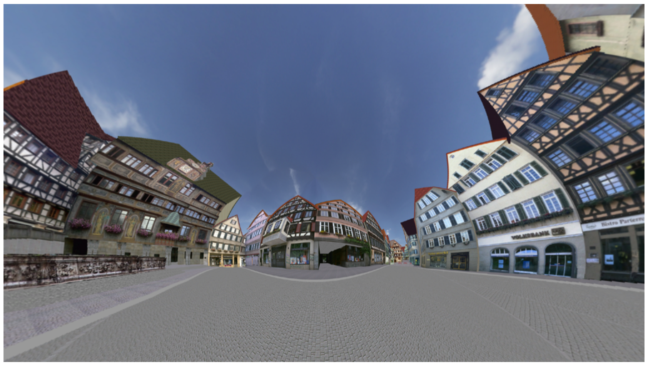

Figure 1: Special relativistic visualization of apparent geometry at $\beta = 0.95$.

fragment programs. Therefore, an extremely fast and per-pixel accurate computation has become possible.

Our interactive application is implemented in C++ and OpenGL. Aberration of light is realized with a dependent texture read from the synthetic cube map, implemented by a few fragment program instructions. Depending on the simulated velocity and the resulting distortion, different combinations of supersampling and filtering are used to achieve high image quality. The Doppler and searchlight effects are implemented using pre-computed textures as lookup tables for the transformed colors of different materials in the scene.

Based on our experience, we have decided to exclusively use image-based rendering as basis for interactive special relativistic visualization. The advantages of image-based rendering are: First, almost the same rendering pipeline can be used as for non-relativistic rendering. Unmodified non-relativistic rendering is used to fill the panorama cube maps; the subsequent Lorentz transformation is just one additional rendering step. In particular, image-based rendering does not interfere with the core rendering routines and the scene representation, while object-space rendering needs a fine and ideally view-dependent re-tessellation of the scene to avoid artifacts from non-linear transformations applied to vertex positions. Second, illumination computations, based on the Doppler and searchlight effects, can be readily included. Third, image-based rendering is per-pixel accurate, both for geometric and illumination effects. Figure 1 shows an example for the image-based visualization of geometric effects for an observer traveling with 95 percent of the speed of light. For this scene, our GPU implementation achieves some 90–125 fps (frames per second) for the visualization of apparent geometry and some 60 fps for the visualization of geometric and illumination effects on an ATI Radeon X800 XT GPU.

As basis for navigation and camera control, the relativistic-vehicle-control metaphor [45] is adopted—an extension of the non-relativistic flying-vehicle or virtual-camera metaphors. The user can control the direction and velocity of motion with a joystick (for smaller installations or desktop environments) or a bicycle interface, which is useful for large-screen installations. Figure 2 shows an example from the exhibition in the "Stadthaus" in Ulm. From experience, the bicycle interface is very intuitive for (untrained) visitors of exhibitions because it provides a good and expected mapping between controls and their effects: This interface exploits functional mimicry because relativistic navigation imitates real-world navigation on a bicycle [36]. Our installation for Ulm allowed the user to navigate through a highly detailed 3D model of Tübingen that was originally designed for the Virtual Tübingen project [41] by the Max Planck Institute for Biocybernetics, Tübingen. The exhibition in Bern uses a 3D model of Bern, showing Einstein's commuting route to his workplace at the patent office. The Bern model was specifically designed for the exhibition by our collaborators at the "Historisches Museum" Bern.

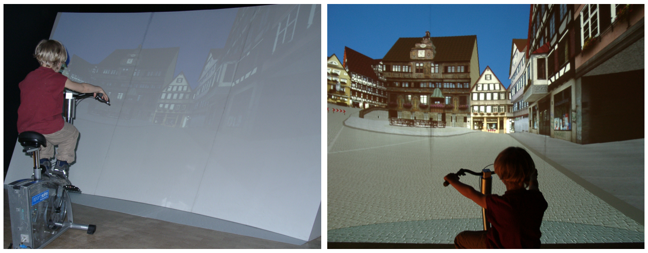

Figure 2: Interactive special relativistic visualization. Users can control their motion by a bicycle interface.

4 GENERAL RELATIVISTIC RAY TRACING

General relativity extends special relativity to include gravitation. Through gravitational sources, the flat Minkowski spacetime of special relativity becomes curved. Concepts from differential geometry are employed to describe curved spacetimes [28, 43]. A basic concept of differential geometry is the line element $\mathrm{d}s^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}(\mathbf{x})\,\mathrm{d}x^\mu \mathrm{d}x^\nu$, where $g_{\mu\nu}(\mathbf{x})$ is an element of the 4×4 metric tensor at spacetime position $\mathbf{x}$, and $\mathrm{d}x^\mu$ is an infinitesimal distance in the μ direction of the coordinate system. Light travels along geodesics—the analogues to straight lines in curved spacetime. Geodesic lines can be computed as solutions to the geodesic equation,

$$\frac{\mathrm{d}^2 x^\mu(\lambda)}{\mathrm{d}\lambda^2} + \sum_{\nu,\rho=0}^{3} \Gamma^\mu{}_{\nu\rho}(\mathbf{x}) \frac{\mathrm{d}x^\nu(\lambda)}{\mathrm{d}\lambda} \frac{\mathrm{d}x^\rho(\lambda)}{\mathrm{d}\lambda} = 0\,, \tag{2}$$

where λ is an affine parameter for the geodesic line. The Christoffel symbols $\Gamma^\mu{}_{\nu\rho}$ are computed from the metric:

$$\Gamma^\mu{}_{\nu\rho}(\mathbf{x}) = \frac{1}{2} \sum_{\alpha=0}^{3} g^{\mu\alpha}(\mathbf{x}) \left(\frac{\partial g_{\alpha\nu}(\mathbf{x})}{\partial x^\rho} + \frac{\partial g_{\alpha\rho}(\mathbf{x})}{\partial x^\nu} - \frac{\partial g_{\nu\rho}(\mathbf{x})}{\partial x^\alpha} \right)\,,$$

where $g^{\mu\alpha}(\mathbf{x})$ is the inverse of $g_{\mu\alpha}(\mathbf{x})$.

Images, as seen by a virtual camera in a general relativistic setting, can be generated by non-linear 4D ray tracing [13, 35, 44, 47]. Starting point is standard 3D Euclidean ray tracing, which needs three major extensions to incorporate general relativistic rendering. First, straight light rays in three dimensions have to be replaced by geodesic light rays in four dimensions, which can be approximated by polygonal lines. Second, the ray projector that generates a light ray corresponding to a pixel on the image plane has to be modified to compute light propagation governed by the geodesic Equation (2). The initial value problem for this system of ordinary differential equations can be solved by numerical integration, e.g., an adaptive fourth-order Runge-Kutta method. Initial values are determined by the position, orientation, and field of view of the observer's camera and by the coordinates of the corresponding pixel on the image plane. The initial values are first computed in the local frame of the camera (a local Minkowski system) and then transformed into the global coordinate system. The third extension concerns the intersection between light rays and scene objects that has to take into account a fourth, temporal coordinate. Figure 3 shows an example of general relativistic ray tracing: It displays a spherical surface located in Kerr spacetime, which describes the metric of a rotating massive object. The surface texture of Earth is applied to visualize the distortions due to light deflection.

So far, we have assumed a single coordinate system in which light rays are computed. In general, however, the geometry of a spacetime has a non-trivial topology that can only be represented by an atlas containing several charts, i.e., several coordinate systems. A teapot with handle and pot is an example of a 2D manifold with non-trivial topology. The implementation of an atlas leads to an

Figure 3: Visualization of Kerr spacetime. © 2005 Th. Müller

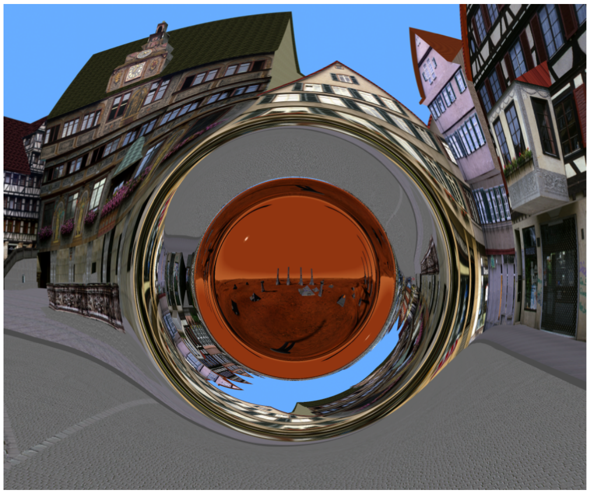

Figure 4: Ray tracing in non-trivial topology: A wormhole between the market place of Tübingen and a fictitious Mars station. © 2005 M. Borchers and Th. Müller

extension of the data structures for ray tracing. First, a light ray now consists of 5D points with four spacetime coordinates and one chart number (an ID). A light ray is decomposed in different segments that belong to different charts. In each chart, the light ray segment is determined by the geodesic Equation (2), based on the associated spacetime metric. When the light ray exits one chart and enters another chart, the position and direction of the previous segment are transformed from the previous chart to the new one, yielding the initial values for the following ray segment. The concept of an atlas also affects the representation of scene objects. Each object is associated with a single chart, and ray–object intersections are computed on a chart-by-chart basis. As a further extension, we represent scene objects with respect to a local reference frame. In this way, moving objects can be described, similarly to the motion of a camera described above. The position and velocity of a free-falling object is determined by the geodesic Equation (2), whereas the vectors of the local frame have to satisfy the equation of parallel transport [28].

An interesting spacetime with non-trivial topology is a wormhole connecting two far away regions of spacetime [31]. As detailed by one of us, Th. Müller [32], the ray-tracing visualization of wormholes is a good tool for teaching general relativity. Figure 4 shows an example of a wormhole between the market place of Tübingen and a fictitious Mars station. Wormholes are typically visualized by drawings like Figure 5, which could be considered an “industry standard” since they appear in practically every popular article on the subject. In fact, this kind of drawing probably gave wormholes their very name. While such an illustration is scientifically correct, it is nevertheless not sound because it gives the impression that a wormhole is a tube-like structure. In fact, a wormhole is not a tube but a spherical object. Our visualization of the wormhole on the Tübingen market place (Figure 4) comes much closer to giving this impression, especially if several pictures from several direction are presented, or if an animated sequence can be shown. The unsound visualization of Figure 5 does not show a wormhole in a (3+1)-D spacetime, but rather a (2+1)-D one, embedded in Euclidean space. Only the surface seen in the drawing comprises the wormhole, all the rest of the 3D space is not part of it. However, it involves a major step of mathematical abstraction to fully grasp this fact, even if it is carefully laid out in some accompanying text.

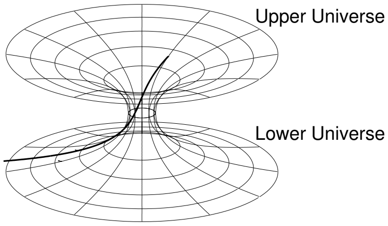

Figure 5: A popular visualization of a wormhole that is scientifically correct, but nevertheless is not sound. © 2004 Th. Müller

General relativistic ray tracing, as used for producing the egocentric visualizations in Figures 3 and 4, tends to be several orders of magnitude slower than non-relativistic ray tracing because of the significantly increased number of computations for constructing geodesic lines and intersecting polygonal rays with spacetime geometry. Therefore, parallelization is an urgent need for general relativistic ray tracing. Domain decomposition can be performed on the image plane because the computation of geodesics and ray–object intersections for one pixel is independent of those for other pixels. Good dynamic load balancing is achieved by choosing an appropriate granularity, which can be as fine as a single pixel. Parallel ray tracing scales well even on distributed memory architectures with slow network connections because only little communication is required between parallel computations. We regularly use a Linux cluster with 128 dual-processor nodes and Myrinet network connection. To give an impression of the rendering performance on our cluster computer: A 1000^2 image of a typical general relativistic scene takes about one to two hours on 28 nodes equipped with dual Pentium III (650 MHz) CPUs. The implementation of general relativistic ray tracing is based on *RayViS* [14], an object-oriented and extensible ray tracing program written in C++. Originally, *RayViS* was designed for non-relativistic standard ray tracing. The aforementioned extensions have been included into the system by extending the functionality by subclassing.

5 INTERACTIVE GRAVITATIONAL LIGHT DEFLECTION

As shown in the previous section, interactive visualization of gravitational light bending is very challenging and impossible with today’s and near-future low-cost hardware. Nevertheless, a few restricted, yet interesting scenarios can be visualized in real time. Several aspects have to be exploited simultaneously to achieve interactive visualization. First, only stationary scenarios in which scene objects are fixed and the metric is time-independent are considered. In this way, the representation of light rays and the intersection between rays and objects is reduced to three spatial dimensions. Second, symmetric spacetimes are used to further decrease the number of independent dimensions. Third, the degrees of freedom for scene objects can be reduced. Fourth, visualization data can be partly precomputed and reused. Fifth, texturing capabilities of GPUs can be used to efficiently perform per-pixel computations.

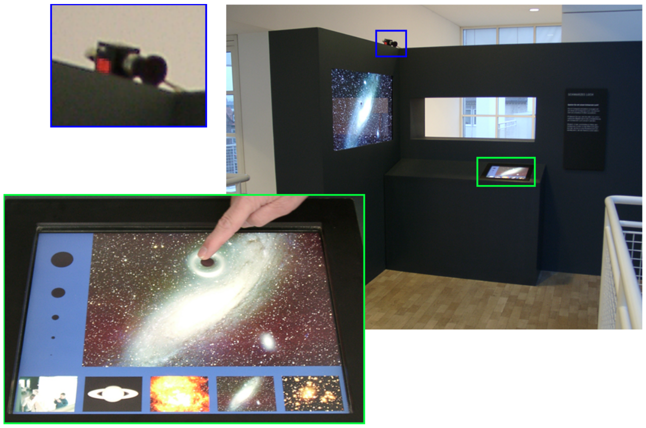

Figure 6: Museum installation for the visualization of gravitational light bending. A black hole serves as gravitational source and can be interactively controlled by direct manipulation on the touch panel. The background image can be chosen from a collection of stored astronomical pictures or from real-time camera input (top-left part of the image).

We combine theses aspects to reduce the visualization problem to computations on a 2D domain, which essentially results in image warping. In this way, efficient image-based general relativistic rendering is possible [21]. So far, we use two different spacetimes that facilitate image-based visualization: the Schwarzschild spacetime [28], which describes non-rotating stars and black holes, and the warp spacetime, which allows for faster-than-light travel [2]. This faster-than-light travel can be visualized as seen from the bridge of the warp spaceship by transforming a spherical panorama of objects that surround the spaceship at a sufficiently large distance, as detailed in our previous work [21].

We have recently implemented the real-time visualization of the Schwarzschild spacetime for interactive museum installations. Figure 6 shows our installation for the exhibition in the "Stadthaus" in Ulm. The scenario contains a black hole, which serves as gravitational source, and a background image, which is distorted due to gravitational light deflection. Because of the spherical symmetry of the Schwarzschild spacetime, the light rays starting at the camera exhibit cylindrical symmetry around an axis defined by the camera and the center of the black hole, i.e., it is sufficient to compute a 1D set of geodesic curves, described by the angle between light ray and symmetry axis. Moreover, the background geometry is assumed to be infinitely far away from the black hole so that, similarly to environment mapping, the direction of deflected light is sufficient to describe the intersection between light rays and background. Therefore, gravitational light bending leads to image deformations with cylindrical symmetry around the black hole. In our implementation, the CPU computes a 1D lookup table with deflection angles, which is used as dependent texture to reconstruct the warping of the background by a GPU fragment program. Our OpenGL GPU implementation on an ATI Radeon X800 XT GPU achieves some 70 fps for the simultaneous visualization on two output screens (1280×768 and 1024×768), processing a 1600×1200 video input stream as background image in real time.

The user interface relies on direct manipulation. The black hole can be dragged on the touch-screen by using a finger, as shown in Figure 6. Different background images can be chosen from a collection of stored astronomical pictures or from a camera covering the installation area (lower part of the touch-screen). The mass of the black hole can be modified by selecting different sizes of black hole icons (left part of the screen). This restrictive and specialized interaction model is used to shift the flexibility–usability tradeoff, inherent to any interactive system, towards high usability [37].

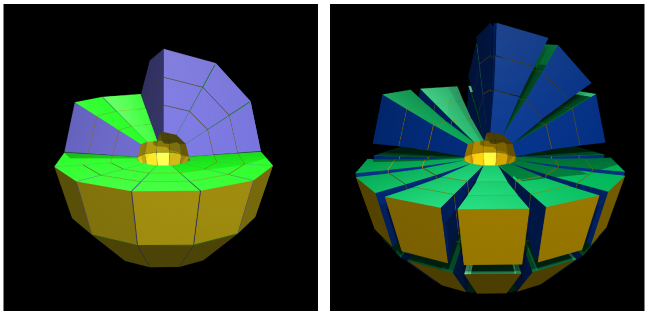

Figure 7: Visualization of flat and curved spaces with building bricks models. Left: flat (Euclidean) space. Right: Curved space surrounding a black hole (inner boundary of the model at 1.25 Schwarzschild radii). © 2005 U. Kraus and C. Zahn

6 GEOMETRIC AND EXOCENTRIC VISUALIZATION

In addition to the egocentric visualization strategy, we selectively also use exocentric approaches that are rooted in the tradition of mathematical visualization. Traditionally, geometry is an important aspect of mathematical visualization [17, 38, 42].

As one example, we illustrate the concept of a curved space in an intuitive way that does not require mathematical formalism [22]. This visualization is based on the principle of the Regge calculus, where a 4D curved spacetime is subdivided into small sections that are each intrinsically flat, similar to the approximation of a curved surface by small flat elements of surface area. Here, we confine ourselves to 3D space (a space-like hypersurface of constant Schwarzschild time) which is subdivided into sections with intrinsic Euclidean geometry. These building bricks are assembled into a model that can be displayed in computer animations and can also be constructed as a paper model (see Figure 7). Such a model is a 3D map of the space, computed to scale, and can be used, e.g., to determine geodesics (straight lines on the map) and the parallel transport of vectors (parallel lines on the map) by drawing on the model instead of solving differential equations. This permits a quantitative treatment of curved space, geodesics, and parallel transport on a high-school or undergraduate level where the analytic description of curved manifolds would be too advanced.

Other examples of exocentric visualization are used to illustrate the propagation of light rays within special and general relativity and can thus explain the structure of egocentric visualizations from a different point of view.

7 PLANETARY AND COSMOLOGICAL RENDERING

General relativity is the accepted theory of gravitation and, thus, the basis for cosmology. Therefore, illustration of astronomical and cosmological aspects nicely comes along with relativistic visualization. Cosmology covers enormous time and length scales, which were, e.g., excellently visualized in the classic film "Powers of Ten" [10]. More recently, Hanson et al. [16] have presented a truly large-scale visualization for cosmology, including the metaphor of the "cosmic clock", which is used for their "Solar Journey" project [15]. Other related work deals with a sophisticated rendering of galaxies or nebulae [26, 33].

In this section, we describe our visualization methods used in a film project for the exhibition in Bern. This film covers a time-lapse virtual journey from Earth through the solar system, further away from our galaxy, and finally to large-scale galaxy clusters. The main goal is to show the enormous length scales involved in astronomy and cosmology and, thus, physical correctness (e.g., concerning the sizes of objects as well as their temporal and spatial relationships) is most important. Another goal is a visually aesthetic rendering that allows us to motivate museum visitors to view further, more complex visualizations of general relativity and cosmology.

The film production is decomposed in separate projects for planetary rendering and galaxy visualization. We start by discussing our tools for a virtual flight through the solar system. There already exist numerous and excellent tools for astronomical visualization and planet rendering (unlike the situation in relativistic visualization). We use “Celestia” [4] as basis for planetary visualization because it is an extensible open-source tool with good visualization quality, excellent interactive camera navigation, and a physically correct modeling of both planetary and star constellations. We have slightly modified “Celestia” in three respects. First, antialiasing and motion blur have been included to improve image quality. Second, camera paths are interpolated with cubic or exponential splines, depending on the traversed length scale. Third, a file-based interface has been added to exchange camera paths with other tools.

Although “Celestia” is suitable for most parts of the journey through the solar systems, it is not appropriate for close-by flights because extremely high-resolution terrain models are not supported. Therefore, we have developed a tool for high-quality and efficient planetary rendering with graphics hardware. Similarly to Cignoni et al. [6], who derive a planet-optimized version of their original BDAM terrain rendering method [5], we extend the terrain rendering software by Röttger et al. [39] for planetary visualization.

The original terrain rendering technique targets the visualization of DEM (digital elevation model) and color texture data defined on a flat uniform grid. Planetary terrain rendering essentially extends the domain from a planar surface to a spherical surface. Elevation data is interpreted as displacements along the normal vector of the spherical domain. Spherical geometry has the problem that a global isometric mapping to a flat 2D texture is not possible. Therefore, the domain is typically split into several tiles that exhibit an almost uniform sampling rate for DEM and texture data. Tiling is realized by triangulating the boundaries between tiles, which may even have different resolutions of DEM and texture data. To save memory and reduce stress on the geometry pipeline of the GPU, an adaptive tessellation of the height field is employed. Following [39], we use a continuous level-of-detail of which the refinement criterion is governed by the distance of the viewer and a quality level. Temporal popping artifacts are eliminated by geomorphing, which smoothly interpolates between two neighboring resolution levels. Aliasing and flickering artifacts that may be caused by sampling of the surface texture are avoided by GPU-supported MIP mapping.

In addition to high-resolution terrain data, the most significant illumination aspects have to be considered to achieve a convincing and realistic visualization. Planets like Earth or Mars have an atmosphere that greatly affects their appearance. Our starting point for atmospheric rendering is the model by Nishita et al. [34], who take into account Rayleigh and Mie scattering. By splitting the rendering process into a pre-processing and a GPU-based part, an interactive visualization is possible [9]. We adopt this rendering method and modify it in a few ways: The optical length lookup-table is enhanced so that it can be used for light attenuation between sample points and light source as well as between sample points and viewer. Attenuation is determined at runtime and mapped onto just two spheres. Therefore, we avoid lookups for shaded areas and expensive volume rendering for spherical layers.

Planetary rendering can be performed in real time if a low quality level is used. Interactive rendering is most useful for camera path planning. The final film rendering is done at high resolution, high quality level, and with time-consuming supersampling for spatial antialiasing and motion blur. Figure 8 shows an example for Mars rendering, based on the MOLA terrain and texture data provided by NASA [29], with a resolution of $180 \times 360 \times 128^2$ floating-point elevation samples.

The other parts of the virtual cosmological journey require the rendering of objects like gas clouds, stars, or galaxies. Astronomical photographs or other 2D data sources are used as basis to construct 3D particle systems and fluids for Alias “Maya”, a commercial and generic modeling and animation tool, in order to model objects like the Horsehead Nebula, the distribution of cosmic background radiation, or quasars. As another source, we use the animated virtual voyage from the Milky Way to the Virgo cluster that is part of the PBS show “Runaway Universe” [40].

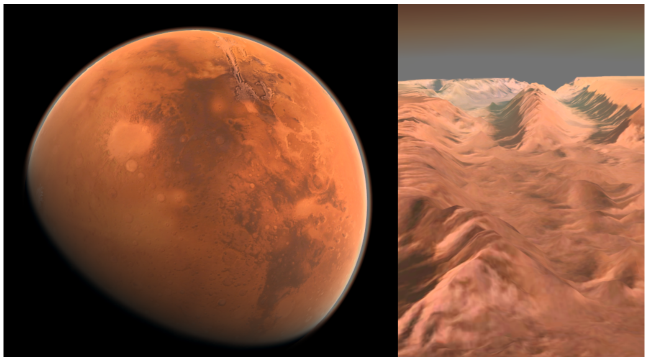

Figure 8: Planetary terrain visualization of Mars, including atmospheric rendering: Outside view (left), close-by flight (right).

The complete film is composed from separate visualization sequences generated by the above tools or included from the aforementioned resources. A consistent and smooth camera path is constructed by exchanging camera positions and orientations between different tools. Video editing is used to construct the final film.

8 VALIDATION AND EFFECTIVENESS

Our visualization activities can be validated from different points of view. A first class of goals can be classified as technical goals. Here, one objective is physical accuracy because our visualization strategy with egocentric views leads to realistic looking images that require accurate visual representation. This accuracy is achieved by applying numerical schemes with explicit error control to solve underlying physical simulations. Another objective is real-time capability for interactive visualizations. We achieve this goal by using efficient GPU implementations along with adaptive rendering methods. Third, stable and error-proof software is required for unsupervised interactive installations. Long-term installations in museum exhibitions, which typically last for several months, have demonstrated the robustness of our software.

Another, even more important class of goals is concerned with the human recipients of visualization. Our objectives are to communicate phenomena of relativistic physics, to explain underlying concepts, and to motivate and inspire. Usually, the effectiveness of visualization is validated by user studies with controlled settings and a thorough statistical evaluation. This type of user study is not feasible for our visualizations because we address large groups of people with whom we have no, or only indirect and loose, contact. Therefore, a direct and controlled evaluation of the effects of visualization is difficult.

Nevertheless, we have considerable experience and manifold evidence for the effectiveness of our approach. One group of users are readers of popular-science publications in magazines or books. We have no immediate contact with readers, but we can report on experiences with journalists, writers, and graphics designers who are responsible for preparing these publications. A general finding is that the egocentric strategy is strongly preferred by print media. Another observation is that familiar scenes are most popular—a large portion of our special relativistic visualizations for magazines show high-speed travel through the Brandenburg Gate in Berlin, towards the Eiffel Tower in Paris, or around Saturn (the level of familiarity, of course, is dependent on cultural background). Both observations show that visualizations with obvious implied information (i.e., natural egocentric visualization, mimicry based on well-known scenes) is most effective for publications that have to be concise because of limited printing space. A strong evidence for the effectiveness of our printed visualizations is a continuing media presence: Several popular-science publications are carrying our visualizations. Our experiences with TV are very similar to those with print media; the same type of egocentric visualization is appropriate for films.

Oral presentations for a general audience are another environment in which visualization plays an important role. One of us, H. Ruder, has extensive experience with invited talks on relativity and astrophysics; only in 2004 and 2005, e.g., he gave or will give more than 70 invited presentations for general audiences. The feedback from the audience is exclusively positive, even enthusiastic. Not only is the number of talks impressive, but also the number of people in the audience: On several occasions, talks were given to far more than a thousand people. H. Ruder was also awarded the most prestigious "Robert-Wichert-Pohl-Preis" by the "Deutsche Physikalische Gesellschaft" (German Physical Society, the world's first and largest physical society) in 2002 for his excellence in communicating physics.

In addition, visualization is an important element in exhibitions. We have contributed to exhibitions for the "Stadthaus" Ulm (March 2004–August 2004), "Highlights der Physik" in Stuttgart (June 2004), and the "Historisches Museum" in Bern (June 2005–April 2006). Visualization for museum exhibitions is distinct from those for popular-science publications and invited talks. First, more in-depth information can be communicated to museum visitors, who usually take more time for a museum visit than readers for a popular-science article. In an exhibition, manifold information is presented, which may range from historic background to most recent physical theories. Therefore, we are able to include not only egocentric visualizations but also exocentric, mathematical visualizations. These types of visualization are rather complex and need additional explanations, which are facilitated by another benefit of an exhibition: Several kinds of information can be shown simultaneously, e.g., an animated visualization can be displayed on a screen side-by-side with an accompanying explanation on a text panel. A third difference is that interactive applications are possible. Interactive exploration leads to a better understanding than a fixed visualization because the user is actively involved. With large-screen displays and an appropriate user interface, even an impression of immersion can be achieved.

Finally, our illustrative visualizations have proven to be valuable teaching material. They provide a highly motivating introduction into the study of the theory of relativity. Apart from the fun aspect, they seriously assist teaching and learning. In teaching the theory of relativity one must do without classroom experiments. Visualization offers a substitute: "experiments" can be performed with interactive visualization tools and "measurements" can be taken on paper model spacetimes. We make visualizations available to students and teachers (both high school and university teachers) by contributing regularly to physics teachers' continuing education seminars and to conferences on physics education. We have presented these projects in contributions to various teachers' journals and we also maintain a highly frequented website[1] on which we show images, movies, and explanatory texts on a level that is suitable for teaching at high school and introductory university level.

We refer to our project website[2] for a detailed documentation of our visualization activities, including videos, further images, and extensive lists of diverse references.

9 CONCLUSION AND FUTURE WORK

Our most important message is that visualization is a very useful tool for communicating and explaining complicated facts. We have successfully employed methods rooted in the tradition of scientific visualization in order to convey elements of the difficult theories of special and general relativity. We believe that illustrative visualization and visual communication should receive more attention by the scientific visualization community. Computer-based visualization has more to offer than exploration of data sets—it is capable of communicating physical phenomena or theoretical and mathematical concepts. After all, explanatory and illustrative visualization has a huge market of potential "customers"; e.g., we have reached several million recipients with our images and films. We understand the term illustration in its broad and original sense[3], which includes the application of artistic drawing styles or traditional design (as in [11, 18, 24]), but also covers visual explanations.

From our experience, we think that the following aspects play an important role in explanatory visualization of relativity, but could also be valid for other applications.

First, domain knowledge is indispensable, which is in accordance with a long-standing demand in scientific visualization [20, 27]. We have included domain knowledge to a very large extent by teaming up experts in visualization, computer graphics, relativistic physics, physics education, modeling, and museum design.

Second, the facts implied in a visualization have to be taken into account. A good strategy is to reduce the amount and complexity of implied information by using simple and natural metaphors. We think that the approach of egocentric visual experiments, which can be regarded as a modification of *Gedankenexperiments* (thought experiments) [3] frequently used by Einstein, is very successful. For interactive applications, in addition, the issue of the flexibility–usability tradeoff should be considered. We recommend to use highly specialized user interfaces with only a minimal choice of parameters. If more abstract visualizations are employed, they should be combined with additional (textual) information that explicitly states the connection between visual representation and displayed information.

Third, the aesthetic-usability effect [23] is especially important for visual communication because attractive designs promote creative thinking and problem solving. Therefore, we use realistic looking, carefully designed models (e.g., Virtual Tübingen, 3D model of Bern), highly accurate measurements (e.g., MOLA Mars data), or image-based rendering with real-world data.

Fourth, the visualization workflow can usually be built upon a mix between standard off-the-shelf tools and individual developments. Standard tools have the obvious advantage of saving resources. For interactive visualization, however, custom-made software is often necessary. Our visualization methods benefit from the significant improvements graphics hardware has recently made. For example, we have presented new GPU methods for image-based special relativistic rendering and interactive visualization of gravitational light deflection. Another reason for specific software development is that specialized visual mapping methods cannot be handled by existing tools. Examples are our extensions for general relativistic ray tracing with multiple charts or planetary-sized terrain rendering.

Fifth, the development of visualization contents is time-consuming because a good design typically requires many iterations. Thus, large visualization projects need good planning, a realistic timeline, and enough resources. For example, the exhibitions in Ulm and Bern required some two years of preparatory work.

As detailed in this paper, our current visualization approach is primarily based on egocentric presentations. In future work, it

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[1] `www.spacetimetravel.org` (in English)
`www.tempolimit-lichtgeschwindigkeit.de` (in German)

[2] `www.vis.uni-stuttgart.de/relativity`

[3] Illustrate: to explain or decorate (a book, text, etc.) with pictures [from Collins English Dictionary [7]].

could be interesting to investigate abstract, exocentric visualizations in more detail and to compare or, possibly, combine them with the egocentric approach. On more fundamental grounds, it is a grand goal to find some kind of (formalized) metric to assess the effectiveness, soundness, and completeness of educational visualizations. On a technical side, the performance of general relativistic visualization could be improved, e.g., by making use of intentional incompleteness or better numerical methods.

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