On the physical meaning of the Levi-Civita Connection in Einstein's General Relativity

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Since Z is not the only one who cannot properly connect the mathematical symbols of general relativity to their actual operational definitions in experimental physics I am taking time to explicate this more fully in my Stargate book. Indeed many eminent theorists who do good mathematical work in the field have not understood what I am about to say. Of course there are exceptions like Kip Thorne, Cliff Will and others.

The key idea that especially engineers get muddled on is the meaning of "acceleration". There are two operationally different definitions of "acceleration" which the Pundits muddle causing confusion.

The local tensor proper acceleration given by the covariant derivative of the first rank tensor 4-velocity of a test particle is DV^u/ds, with ds the proper time differential along the world line of the point test particle in this CLASSICAL LOCAL theory- no quantum theory here as yet.

DV^u/ds is measured locally by a physical material accelerometer clamped to the local rest frame of the test particle.

When the "good" accelerometer pointer reads "0" that local rest frame is "inertial," i.e. LIF, and the center of mass (COM) of the test particle is on a timelike geodesic of the local metric guv field. The presence or absence of local 4th rank tensor curvature is completely irrelevant here. Everything I say here is just as true in globally flat Minkowski spacetime as in a generally curved spacetime. The presence or absence of Tuv sources is irrelevant for these elementary considerations.

On the other hand, when the accelerator pointer reads not-0 then the COM of the test particle is not on a timelike geodesic of the local actual physical metric field - a local objective reality at the classical level. Its rest frame is then LNIF.

DV^u/ds = dV^u/ds - {Levi-Civita Connection}^uvwV^vV^u

dV^u/ds is the relative kinetic acceleration of the test particle where V^u is its relative 4-velocity.

"Relative" to what?

To a “detector” obviously.

This simple obvious truth seems to escape many of the Pundits who are very competent in manipulating the formal symbols of theoretical physics today without giving much thought to their physical meaning.

Both V^u and dV^u/ds are not measured with accelerometers, but with light signals, mirrors and devices like Doppler radars.

The covariant derivative

DV^u/ds = dV^u/ds - {Levi-Civita Connection}^uvwV^vV^u

is a RELATIONSHIP between observer and observed - or, in this case, between a test particle and a detector.

Furthermore, the above formula is an APPROXIMATION depending on what Einstein called “local coincidences” in a famous remark that I repeat several times in Stargate.

Essentially, the spacetime intervals between test particle and detector must be small compared to the local radii of curvature for the above local differential formula to be accurate in describing real measurements.

We now get to the heart of the matter, what is the meaning of the term

- {Levi-Civita Connection}^uvwV^vV^u ?

Clearly, that term describes the local frame of reference of the detector that is bouncing light signals off the test particle as it monitors it making measurements.

In particular, if we want to describe the REST FRAME OF THE DETECTOR then

TEST PARTICLE = DETECTOR

(analog of the Cantor diagonal, Godel self-reference)

V^i = 0 and dV^i/ds = 0 , i = 1,2,3 space axes with 0 as the time axis.

This leaves only

DV^i/ds = - {Levi-Civita Connection}^i00V^0V^0 = - - {Levi-Civita Connection}^i00

If that DETECTOR rest frame is LIF, then obviously

{Levi-Civita Connection}^i00 = 0

If that DETECTOR rest frame is LNIF, then obviously

{Levi-Civita Connection}^i00 =/= 0

Before, I end this note that, in the rest frame of the DETECTOR:

DV^0/ds = dV^0/ds - {Levi-Civita Connection}^0ijV^iV^i - {Levi-Civita Connection}^000V^0^V^0

= dV^0/ds - {Levi-Civita Connection}^000V^0^V^0

If the detector is LIF, then

DV^0/ds = 0

The point here is that even in globally flat Minkowski spacetime one can have non-vanishing Levi-Civita connections describing LNIFs in which we have artificial gravity.

Basically, the above considerations are simply an alternative way to understand Einstein's equivalence principle that the observer is weightless in free fall, and in a complementary way, that Newton's universal gravity field is LOCALLY a completely inertial fictitious pseudo-force contingent artifact of a reference frame with proper tensor acceleration.

In particular, every point on a RIGID spherical Tuv source surface has a radially outward proper acceleration DV^u/ds even though the kinetic acceleration dV^u/ds = 0 because the spacetime is curved not flat.

Furthermore, Mach's intuition was completely wrong even though it took Einstein years to outgrow it.

Proper acceleration is absolute in Einstein's GR because it is a first rank tensor under the T4(x) group of locally gauged translations called "general coordinate transformation."

Imagine a spinning disk in an empty universe Tuv = 0 with only a few test objects whose self gravity fields are tiny.

We do not need any distant stars in the sense of Mach. The local metric field guv

is a solution of Guv = 0. It is a real geometrodynamic field as real as the electromagnetic field Fuv. guv is even curvilinear in LNIFs even though Ruvwl (Weyl-Ricci curvature tensor) = 0 globally.

Small accelerometers clamped to the extended disk will still read not-0 if it is spinning in this empty world.

That said, of course when Tuv =/= 0 then we have things like inertial frame-dragging and in that case Mach's Principle is excess metaphysical baggage bringing no value added to the movable feast.

Furthermore, gravity is completely irrelevant to the understanding of the rest masses m of elementary particles.
Even in Wheeler's geometrodynamics of "mass without mass" geon wormholes, the problem is what are the radii of the mouths of these micro-wormholes? That requires non-gravity fields like the spin 0 Higgs field and the eight internal SU3 spin 1 gluon strong force fields to understand. There is also the issue of the cosmic landscape of the Tegmark Level 1 and Level 2 multiverse in which the m's are completely random.

The role of gravity is to determine the real force-free geodesics of Newton's first law of mechanics.

Gravity has nothing to say about the inertial resistance m to real internal symmetry forces pushing test particle off timelike geodesics.

Therefore, I think the Woodward theory based on Sciama's "electromagnetic" analogy is profoundly wrong with no possibility of working in terms of a practical propellantless propulsion device. That's just my opinion and I would be glad to be proved wrong.

Some general remarks:

All physical properties must be tensors (or spinors that are "square roots" of tensors in the sense of Rindler and Penrose).

Tensors and spinors are relative terms defined with respect to different symmetry Lie groups of "frame transformations."

The global dynamical actions and their local Euler-Lagrange field equations must be invariant and covariant respectively under the elements of these continuous Lie transformation groups. Forget discrete groups for the moment.

All the classical boson fields of interest in physics today are compensating

connection fields of localized gauge groups.

Classical gravity is the local connection field of the space-time symmetry groups.

In the case of Einstein's 1916 GR the "gravity field" is the Levi-Civita connection. This actually corresponds to Newton's conception of "gravity force".

The Weyl-Ricci curvature tensor is the covariant curl of the Levi-Civita connection with itself.

Einstein's 1916 theory corresponds to the localization of the global Poincare group of his 1905 special relativity with the additional ad_hoc constraint of zero torsion.

More general extensions of gravity are immediately found by extending the Poincare group to the de Sitter group with a cosmological dark energy term and to Roger Penrose's conformal twistor group that includes the conformal boosts and the dilation.