UCI-TR-2006-1

Stable Models of Super-acceleration

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(Dated: January 24, 2006)

We show that in models where dark energy is coupled to matter, there is a generic instability. This instability may be cured in models that predict an equation of state for dark energy that is smaller than -1, i.e., super-acceleration. These models are no more fine-tuned than quintessence, and they do not exhibit acausal behavior or contain ghosts. We also explore other ways to avoid this instability.

MOTIVATION

Observations of distant Type Ia supernovae [1, 2] and the cosmic microwave background [3] together strongly prefer an accelerated expansion of the universe in the recent past. In the standard cosmological model this is accommodated by introducing "dark energy", a component which is usually taken to interact with the rest of the universe only gravitationally and which has a significantly negative pressure causing the acceleration of the universe.

In order to accelerate the universe, the dark energy component must have an energy density which decreases (if at all) much more slowly than matter density as the universe expands. Current data favor a dark energy density which is almost constant or even increasing with time [4, 5, 6, 7, 8, 9, 10, 11, 12] and exciting results can be expected in the future [13, 14, 15, 16]. We label the phase when the *effective* dark energy density is increasing with the expansion of the universe as super-acceleration [17]. In such a phase, the apparent equation of state (pressure over energy density) $w_{\rm DE}$ is less than -1. However this does not imply that the theory contains ghosts or that it violates causality as discussed below.

Scalar field models with canonical kinetic term always produce $w_{\rm DE} > -1$. Effective models with the opposite sign kinetic term [4] imply $w_{\rm DE} < -1$ but are unstable [18] unless more than one scalar field is considered [19, 20]. Models with higher derivative terms or scalar-tensor theories can give rise to an apparent $w_{\rm DE} < -1$, but are extremely constrained [21, 22]. Interpreting a non-GR gravity theory in the context of GR can also lead to super-acceleration [23, 24]. In our view, the simplest way to obtain super-acceleration is to consider a model where the dark energy is coupled to all of matter. One naturally expects such interactions if the dark energy is described by scalar degrees of freedom [25]. The time variation of the comoving matter density (resulting from the coupling) can then produce an apparent equation of state which is less than -1 [26].

The coupling of dark energy to matter could be such that the total matter density decreases more slowly than $1/a^3$ where a is the scale factor of the universe. When we interpret observations in such a universe with a canonical matter density term (that decreases with expansion as $1/a^3$) and dark energy, we would infer an equation of state for dark energy more negative than it truly is. There is no physical reason why this inferred equation of state cannot be below -1. Note that the total energy density of the universe is always decreasing as the universe expands in this model.

Super-acceleration is a property of the dark energy density defined as $3m_{\rm pl}^2(H^2(a) - \Omega_M H_0^2/a^3)$ [17] where H_0 is the present expansion rate of the universe and $m_{\rm pl} = 1/(8\pi G)$. It *does necessarily not* imply that the speed of sound is larger than the speed of light (i.e., acausal behavior) or signal the presence of ghosts. We mention another equally harmless model [27] to emphasize this point: conversion of photons to axions in a universe dominated by cosmological constant (or quintessence).

In this paper, we analyze models in which dark energy is coupled to non-relativistic matter. We find that these models suffer from a generic instability, which we label Z-instability. This instability was first pointed out in the context of mass-varying neutrinos (MaVaN) [28]. We show that Z-instability may occur very generally in models of dark energy coupled to matter, such as the MaVaN scenario [29], the Chameleon dark energy scenario [30] and the Cardassian expansion scenario [31].

We then show how one may build models in which this instability is avoided. We find that successful models generically lead to super-acceleration, i.e., the *apparent* dark energy density increases as the universe expands.

Future SNIa and CMB observations have the potential to detect super-acceleration [17]. No other combination has been shown to robustly detect the signature of super-acceleration, although combining SNIa and baryon oscillation [9] or weak lensing data set seem promising. Note that a measurement of just the average equation of state [32] is not sufficient for this purpose [33]. This was made explicit recently [34] using a simple single scalar field model.

In thinking of a scalar degree of freedom coupled to matter (e.g., [35, 36, 37, 38, 39, 40, 41]) with the scalar field acting as dark energy, we will assume that the mass of this scalar degree of freedom is much larger than the expansion rate (for example, the MaVaN scenario [29]). In this regard interacting dark energy models are much less fine-tuned than quintessence models that have a mass of order or smaller than H_0. Also, in this regard, the models studied in the present work are *different* from interacting quintessence models wherein (for the same reasons), it is possible to obtain super-acceleration [26].

Z-INSTABILITY

In this section we will assume that the dark energy density is coupled to the non-relativistic matter density in some unspecified manner. For an example of how this could occur, suppose that non-relativistic matter particles are coupled to a scalar field. Thus the local density of the matter particles can influence the vacuum expectation value (vev) of the scalar field. The change in the potential of the scalar then affects the dark energy, thus coupling dark matter and dark energy.

More generally, define $\rho_{\rm DEM} = \rho_{\rm DE} + \rho_{\rm M}$ to be the combined density of the dark energy (DE) and matter (M) fluids. We take $\rho_{\rm DEM} = f(\rho_{\rm M})$. We note that this is the same form as the Cardassian model [31]. We now implicitly assume that the underlying micro-physics responds to changes in $\rho_{\rm M}$ on time-scales much shorter than $1/H(a)$. We wish to consider perturbations of this system and analyze its stability.

On length scales much larger than m_χ^{-1}, the evolution of the system is adiabatic and hence the sound speed is

$$c_a^2 = \frac{\dot{P}_{\rm DEM}}{\dot{\rho}_{\rm DEM}}\,. \tag{1}$$

Here the pressure $P_{\rm DEM} = f'\rho_{\rm M} - f$ is defined by the energy conservation equation $\dot{\rho}_{\rm DEM} = -3H(\rho_{\rm DEM}+P_{\rm DEM})$. If c_a^2 is negative, it indicates that the system is unstable on sub-horizon scales much larger than $1/m_\chi$.

In this model, there are two ways to define the equation of state. One may simply define the equation of state to be $w_{\rm DEM} = P_{\rm DEM}/\rho_{\rm DEM}$ using the above definitions. However, in cases where one considers the matter component coupled to DE to be the bulk of the dark matter in the universe, it is more useful to have the observationally motivated definition of the effective $w_{\rm DE} \equiv P_{\rm DEM}/(\rho_{\rm DEM} - \rho_{\rm M})$.

The adiabatic sound speed in this theory can then be expressed as

$$c_a^2 = \frac{\rho_{\rm M} w'_{\rm DE} + w_{\rm DE}(1+w_{\rm DE})}{1 + w_{\rm DE} + \rho_{\rm M}/(f-\rho_{\rm M})}\,, \tag{2}$$

$$= \frac{\rho_{\rm M} w'_{\rm DEM} + w_{\rm DEM}(1+w_{\rm DEM})}{1 + w_{\rm DEM}}\,. \tag{3}$$

For an accelerating universe $w_{\rm DEM} < 0$. Now $\rho_{\rm M} w'_{\rm DEM}$ cannot be large and positive over a long period of time, since this will drive $w_{\rm DEM}$ to positive values in the past. If we assume that the $w'_{\rm DEM}$ term is sub-dominant, then we have $c_a^2 \approx w_{\rm DEM} < 0$, and the system is unstable.

We dub this the Z-instability. This instability was first noted in the context of the MaVaN scenario [28].

For cases where the coupled matter is all the dark matter in the universe, it is more useful to consider Eq. 2. First, consider the case where $w_{\rm DE} > -1$: the denominator is positive and if the $w'_{\rm DE}$ term is sub-dominant, then Z-instability sets in. Note that this instability will likely set in well before the current epoch because at early times $\rho_{\rm M}/(f-\rho_{\rm M}) \gg 1$. We also note that this instability may not be present in models with $w_{\rm DE} < -1$. This point will be discussed in more detail below.

Once the instability sets in, linear perturbation theory becomes invalid and a full non-perturbative calculation is required to track the behavior of the coupled dark energy and matter system. We do not speculate here on the outcome of the instability *except* to assume that the instability is such that we can no longer have an accelerating expansion solution. This is reasonable because the instability arose from the requirement that the dark energy cause the expansion of the universe to accelerate.

EXAMPLE OF A Z-UNSTABLE MODEL

We now consider an example of a class of models that are Z-unstable. In this class of models, the matter fields are coupled to a scalar field χ, the dark energy, through Yukawa like couplings. In this case, one may write

$$\rho_{\rm DEM} = V_0(\chi) + g(\chi)\rho_{\rm M}\,. \tag{4}$$

We will assume that m_χ, the mass of the scalar field about its vev χ_0, is much larger than the expansion rate of the universe H. (Note that this is unlike the quintessence scenario where the mass of the scalar field is fine-tuned to be $\lesssim H_0$.)

For small deviations away from the minimum, the scalar field will re-adjust on time-scales of order $1/m_\chi$. Thus the cosmological evolution of the field χ is simply that it adiabatically tracks the minimum of the effective potential as the minimum evolves on time scales of order H. The equation for the minimum of the potential then determines the evolution of χ as the universe expands.

The equation of motion for χ is given by

$$V_0'(\chi_0) + g'(\chi_0)\rho_{\rm M} = 0\,. \tag{5}$$

Thus χ_0 (value of the scalar field at its minimum) is a function of $\rho_{\rm M}$. Now consider small deviations in $\rho_{\rm M}$. The vev of the scalar field shifts to account for this change in $\rho_{\rm M}$. Taking a further derivative, we find

$$m_\chi^2 \frac{\partial \chi_0}{\partial \rho_{\rm M}} + g'(\chi_0) = 0\,. \tag{6}$$

where we have introduced the mass of the scalar field at the minimum $m_\chi^2 = V_0''(\chi_0) + g''(\chi_0)\rho_{\rm M}$.

We can now evaluate the sound speed. We find

$$c_a^2 = -\frac{[g'(\chi_0)]^2 \rho_{\rm M}}{m_\chi^2 g(\chi_0)} \,. \qquad (7)$$

We note that $g(\chi_0) > 0$ since it is the mass term for the matter. It is immediately seen that c_a^2 is always negative; therefore such models are *always* unstable.

We stress that this result is only valid for the cases when the scalar field is much heavier than the expansion rate of the universe and on scales where the gradient term can be neglected. A second point we would like to stress is that this result is valid at any epoch and not just at the present time.

We also note that the instability dies out in the limit of infinite mass because in that limit perturbations don't propagate and we have $c_a^2 = 0$. (c_a^2 scales inversely with the squared mass of the scalar field.) The mass of the scalar field also dictates the scale at which the transition to adiabatic evolution of perturbations occurs. On length scales larger than $1/m_\chi$, the gradient terms become unimportant and the response to perturbations is dictated by the adiabatic sound speed. It is in this regime that the DEM fluid is unstable.

We have seen that with one scalar field coupling linearly to matter, we do not have a stable model. We could consider more than one scalar field. Let these n scalar fields have arbitrary self-interactions and interactions amongst themselves. We next assume that they all couple to the matter fields the same way as χ above with arbitrary coupling constants (that could be zero). Then a similar analysis to that given above shows that this configuration is unstable as long as all the eigenvalues of the $n \times n$ mass-squared matrix are positive. Note that this is a stringent requirement and not necessary for successful model building. In principle one could imagine a small negative mass-squared eigenvalue such that instability develops along the corresponding direction only recently.

AVOIDING THE Z-INSTABILITY

There are several ways in which interacting dark matter may avoid the Z-instability.

Referring to Eq. 2, we see that a model with $w_{\rm DE} > -1$ could be stable if $w'_{\rm DE} > 0$ and if the variation of $w_{\rm DE}$ is sufficiently strong to dominate the $w_{\rm DE}(1 + w_{\rm DE})$ term. This is not easy to achieve as a large positive $w'_{\rm DE}$ would result in a larger dark energy density in the past that may be inconsistent with data. If $\rho_{\rm M} w'_{\rm DE} \ll 1$ in the past, then again we encounter the instability if $w_{\rm DE} < 0$ in the past.

A more interesting way to avoid this instability is to appeal to finite temperature effects. Finite temperature effects will provide a positive contribution to c_a^2 [28] and cure the instability. This is not easy to arrange for [42], especially if matter is cold and weakly interacting.

Thirdly, the scalar field could have additional interactions that keep the sound speed squared from becoming negative.

SUPER-ACCELERATION PROVIDES STABILITY

The most interesting way to avoid Z-instability is to look at models with super-acceleration. To see this, go back to Eq. 2. Let us assume that $w'_{\rm DE} < 0$, since that makes the effects of the dark energy sub-dominant in the past. Then c_a^2 can still be positive, as long as $1 + w_{\rm DE} < 0$, implying $w_{\rm DE} < -1$ (super-acceleration) at present. Note that regardless of the sign of $w'_{\rm DE}$, if $\rho_{\rm M} w'_{\rm DE} \ll 1$ then our arguments above demand that $w_{\rm DE} < -1$.

Let us now look at an explicit model. Take the total density to be $f(\rho_{\rm M}) = V_0 + c\rho_{\rm M}^n + \rho_{\rm M}$. We find

$$c_a^2 = \frac{cn(n-1)\rho_{\rm M}^n}{cn\rho_{\rm M}^n + \rho_{\rm M}} \,, \qquad (8)$$

$$1 + w_{\rm DE} = \frac{cn\rho_{\rm M}^n}{V_0 + c\rho_{\rm M}^n} = \frac{nc_a^2\rho}{c_a^2\rho_{\rm M} + V_0(n - 1 - c_a^2)} \,, \qquad (9)$$

where V_0 and c are constants.

We can then obtain stable models of super-acceleration by taking $c > 0, n < 0$ as long as $cn\rho_{\rm M}^{n-1} + 1 < 0$. This last condition can be made to hold over all past times by taking c sufficiently small.

Larger classes of such models can be constructed by coupling matter to scalar fields through more general interactions. We assume as before that all scalar fields are massive enough that they always track the minimum of the potentials. In this situation, the total energy density is a function of the matter density and the minimum of the scalar fields $\rho_{\rm DEM} = g(\rho_{\rm M}, \chi_0)$. Repeating the analysis, we find that in these models

$$c_a^2 = \frac{g''(\rho_{\rm M}, \chi_0) - m_\chi^2 (\chi'_0)^2}{g'(\rho_{\rm M}, \chi_0) + 1} \,, \qquad (10)$$

$$1 + w_{\rm DE} = \frac{g'(\rho_{\rm M}, \chi_0)\rho - \rho}{g(\rho_{\rm M}, \chi_0) - \rho} \,, \qquad (11)$$

where all primes indicate partial derivatives with respect to $\rho_{\rm M}$ at fixed χ. We have written the formulae for the case when matter is coupled to just one scalar field for the sake of clarity. They can be easily generalized to the case of more than one scalar field. It is clear that for a stable system, we need $g''(\rho_{\rm M}, \chi_0) \geq m_\chi^2 (\chi'_0)^2$. In the Z-unstable model discussed above, $g''(\rho_{\rm M}, \chi_0)$ was zero, so this condition could not be satisfied.

CONCLUSIONS

The evidence for an accelerating expansion of the universe has grown over the last decade. The cause of this acceleration, namely dark energy, remains mysterious.

One of the intriguing possibilities that has been explored in the past is that dark energy may interact with matter. Such a hypothesis is natural if the explanation for dark energy requires extra scalar degrees of freedom. We have shown here that these models suffer from a generic instability.

We discussed a few ways in which this instability could be avoided. Most interestingly, we found that this instability can be naturally cured in models where the equation of state of the effective dark energy density is smaller than -1, i.e., super-acceleration.

In these models, super-acceleration does not imply acausal behavior (the speed of sound is not larger than c) or signal the presence of ghosts in the theory. Super-acceleration results from the interaction due to which the matter density decreases more slowly with the expansion of the universe. Therefore, if we fit our observations using a canonical matter density term and dark energy then the apparent equation of state for dark energy will be more negative.

There is a theoretical prejudice against models of $w_{\rm DE} < -1$ due to their apparent theoretical problems. The observational data certainly do not disfavor $w_{\rm DE} < -1$. Indeed a large region of the parameter space allowed by SNIa observations correspond to a constant $w_{\rm DE} < -1$. Here we have shown that it is straightforward to construct stable models with $w_{\rm DE} < -1$ without encountering ghosts or acausal behavior. These models are no more fine-tuned than quintessence models. Thus theoretical bias against $w_{\rm DE} < -1$ should be treated with circumspection, and not be given any weight when interpreting observational data.

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