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RELATIVITY, ROTATION AND RIGIDITY

ABSTRACT. Much of Essler's work has been devoted to bringing science and philosophy together for the purpose of conceptual clarification. One particularly interesting area for such cooperation between science and philosophy has been relativity theory. In this paper I will consider one instance of such interplay: the transformation that our notions of rotation and rigidity have undergone in general relativity and what this process can teach us. I will start by saying a little about the physics of the situation and then go on to some philosophical observations about meaning and theory.

1. MALAMENT ON ROTATION

My remarks are inspired by David Malament, who recently has presented a short and simple argument to show that our ordinary notion of rotation is falling apart if we try to apply it within relativity theory.[1]

Within Newtonian mechanics there is a well-defined vector that can be used to define the angular movement and velocity of one object relative to another. However, Malament points out:

> . . . the situation is more delicate in relativity theory. Here no such simple interpretation of "relative rotation" is available, and some work is required to make sense of the notion at all. (It seems to me unfortunate that this is often overlooked by parties on both sides when it is debated whether relativity theory supports a "relativist" conception of rotation.)[2]

Malament considers two criteria of "non-rotation". The first, the "angular momentum criterion", exploits the "Sagnac effect": one makes use of two light signals that are sent in opposite directions along a ring to see which signal comes back first. The second criterion, the "compass of inertia on the axis" criterion, uses a telescope, together with a gyroscope, or a water bucket, to test whether a ring is rotating.

While these two criteria obviously agree in flat spacetime, Malament shows that they do not agree in general, in particular, they do not agree in the important Kerr solution to the field equations of general relativity. Malament then formulates two adequacy conditions on criteria of non-rotation. To these two criteria we shall now devote special attention.

The first adequacy condition is the "**relative rotation**" condition: Given two rings R_1 and R_2 with the same axis, if R_1 is "non-rotating", and if R_2

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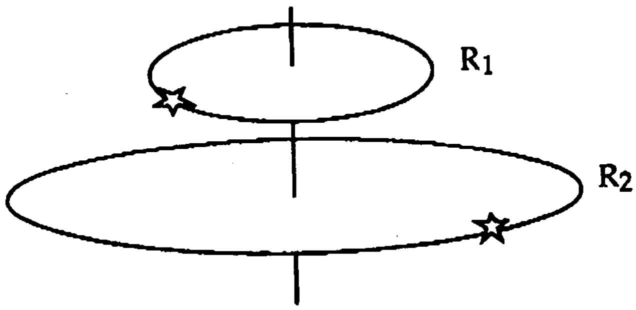

Figure 1.

is non-rotating relative to R_1 (in the sense that the distance between any point on R_1 and any point on R_2 is constant), then R_2 is "non-rotating".

Figure 1, from Malament, illustrates the condition.

The second adequacy condition is the "**limit for small rings**" condition: Given a sequence of "non-rotating" rings (with the same axis) whose radii go to 0, the angular velocities of the rings, as measured relative to the compass of inertia on the axis, go to 0.

Malament shows easily that neither of the two criteria of non-rotation, in general, satisfies the "relative rotation" adequacy condition. In particular, neither does so in the Kerr solution. Both criteria of non-rotation satisfy the "limit for small rings" condition. Malament then finally proves, as his main result, that in the case of the Kerr solution, there is no generalized criterion of non-rotation that satisfies both adequacy conditions.

Malament's conclusions are incontestable, so are his arguments, which, as he himself points out, are simple and straightforward. However, they raise some physical and philosophical issues, that I shall now discuss.

2. RIGIDITY AND ROTATION IN RELATIVITY THEORY

First, some remarks on the physics of the situation. Malament's two adequacy conditions have in my opinion very different standings in relativity theory. The second condition, the "limit for small rings" condition, is mandatory if relativity theory is going to have Newtonian physics as a limit case. Any usable notion of rotation would have to satisfy such a limit constraint. The first condition, however, seems wholly inappropriate for relativity theory. The idea of two rings that are situated such that the distance between any point on one of them and any point on the other is constant is on a par with the classical idea of a rigid object, which is known to crumble in relativity theory. In classical mechanics a body is defined as being rigid if and only if the distance between any two points within the body remains unchanged. This condition is not Lorentz invariant, so the classical rigid body is excluded in relativity theory. From 1907 on there was a lively discussion concerning a suitable definition of rigidity, in which many prominent physicists took part, among them Einstein, Born, Ehrenfest, Sommerfeld, Herglotz, Fritz Noether, and von Ignatowski.[3] There is a whole cluster of notions where our classical definitions do not carry over: rigidity, rotation, angular momentum, etc. The discussion and clarification of these basic notions has still not reached an end.

To take rigidity first, there have been numerous attempts to give a modified definition of the notion of rigidity, suitable for general relativity. In special relativity theory a notion that is widely used is Born-rigidity, proposed by Max Born in 1909 and in a revised version the following year. It preserves some of the features for which rigidity was introduced, while, of course, other important features are missing. In general relativity theory there is no natural plausible candidate so far; there is no one notion that captures what we might want to capture. The notion splits up in different directions, depending on what we are after.[4]

The situation with regard to angular momentum is somewhat similar, but not quite. Here, too, the classical notion comes apart. However, there are certain features of the classical notion that are particularly important, notably the possibility to record angular momentum and changes in angular momentum so that one can calculate the amount of angular momentum that has been gained or lost during a certain period of time. Here peculiarities of relativity theory restrict what one can hope for. However, over the years considerable progress has been made in trying to find a notion that allows for this. Anthony Rizzi from Princeton has recently succeeded in solving this problem, by formulating a mathematically precise definition of angular momentum that allows one to record the "total" angular momentum is spacetime as seen from future null infinity for any given moment of retarded time, and to record the change in this total angular momentum and so calculate the amount of angular momentum the total interior of the spacetime has gained or lost during any given span of retarded time.[5]

3. PRERELATIVISTIC DISCUSSIONS OF RIGIDITY

Long before the advent of relativity theory it was well known that the notion of a rigid body, defined in terms of constancyof distances within the body, was in trouble in universes whose scalar curvature is not constant. Such universes were considered an open possibility by Riemann in his Habilitation lecture "Ueber die Hypothesen, welch der Geometrie zu Grunde liegen" (1854). When this lecture was published posthumously in 1867, it stimulated Helmholtz's ongoing interest in the connections between rigidity and space, and led him to respond with "Ueber die Tatsachen, welche der Geometrie zu Grunde liegen" (1868) and other important work on the foundations of geometry, which was continued by Sophus Lie. It could be tempting for me to go into Sophus Lie's contribution, since he was a countryman and also because I worked for two years as an assistant to Professor Carl Størmer, who (long before) had produced the official survey of a part of Sophus Lie's 20,000 page Nachlass.[6] However, a superb survey of the Riemann–Helmholtz–Lie development has been written by Howard Stein.[7] I shall therefore mention only a few pointsthat are pertinent to the philosophical observations I want to make.

First, while Riemann had a rather holistic understanding of the interplay between curved space, light and empirical measurements, Helmholtz started from his influential studies of the physiology of visual perception, in particular of the physiological problem of the localization of objects in the field of vision. These studies led him to the view that all our knowledge of space, such as our notions of congruence, distance, etc., comes from our observation of the properties of rigid bodies. From this starting-point, Helmholtz derived four axioms about space and argued that they were satisfied only by spaces with uniform curvature. Lie replaced Hemholtz's notion of mobility in space by the mathematical notion of transformation between two coordinate systems. He noticed a gap in Helmholtz's proof and strengthened the axioms so as to enable him to prove the consequential Helmholtz–Lie Theorem, from which it follows that there are three and only three possible kinds of spaces in which rigid bodies can move freely: Euclidean, hyperbolic and elliptic.[8]

This illustrates how, as Howard Stein has pointed out, even in the case of as trailblazing a theory as the theory of relativity, we should think of the development of science more as evolution than as revolution. Second, and this is a main philosophical moral I want to draw from this story, the terms we use to describe our surroundings get their meaning from our interaction with our surroundings, not only our observations of them, but also, as Helmholtz pointed out, our practical dealings with them.

4. THE MEANING OF SCIENTIFIC TERMS

Malament shows in a neat and simple way how concepts and definitions that serve well in one theory may fall apart when they are transferred to another theory, the reason being that features that went together in the original theory do not go together in the new theory. Rotation could, for example, be observed and measured in many different ways according to the first theory, while the different methods of measurement would yield different results in the new theory, and some kinds of measurement no longer make sense at all.

There is hence an interdependence between our concepts and our conception of what regularities there are in the world. (Also our individuation of our surroundings into objects and natural kinds enters into this interdependence, but I will not go into this here.)[9] A radical change of theory will normally lead to a revamping of our stock of concepts. This could be used as a criterion of radical theory change. The introduction of relativity theory gives us many such examples. Some notions carry over fairly well, while others disintegrate. Whether we want to preserve the old labels is partly a matter of taste. If some of the important regularities that were connected with the old notion carry over to the new theory, this might be a reason for preserving the label and say that we are speaking of the same thing, as we do in the parallel case of objects. The notion of angular momentum seems to me to be a case of the latter kind, while rotation does not carry over very well. The two criteria of "non-rotation" Malament gives, illustrate this. While the "angular momentum criterion" and the "compass of inertia" agree in flat spacetime, they split apart within general relativity. And the first of Malament's adequacy conditions, the "relative rotation" condition, is, of course, completely useless in general relativity, dependent as it is on the classical idea of a rigid body.

Note that I use the term "concept" in a non-traditional way here, and that in order to avoid confusion I shun notions like "conceptual change" and "conceptual analysis". According to the traditional views on concepts, some of the regularities connected with a notion are conceptual, or definitional, others are empirical. Thus, "vixens are female foxes" and "bachelors are men who have never been married" are regarded as true by definition. They are called conceptual truths, while other regularities involving vixens or never-married men are reckoned as empirical. According to that view, whether the notions of angular momentum or rotation carry over from classical physics to relativity theory, depends on whether the conceptual properties of these notions carry over. However, following Quine, I would instead look for the "meaning" of a term like "rotation" or "angular momentum" in an encyclopedia, not in a dictionary. The meaning of a term consists in the complex set of features that we expect to find in objects that the word apply to. My discussion above of when to preserve labels is founded on this view.[10]

5. THE EMERGENCE OF CONCEPTS

Finally, some words on how concepts, or "meanings" of words emerge. It seems reasonable to assume that the meanings of our words stem from our observations and interactions with our surrounding world. Given that we live in a part of the universe where many important regularities are associated with rigid bodies, such bodies play an important part in our lives. Helmholtz, as we noted, argued that all our knowledge of space, such as our notions of congruence, distance, etc., comes from our observation of the properties of rigid bodies, and Poincaré held that our motion of our own bodies gives rise to the idea of a Lie group of free motions.[11] If we were jellyfish-like creatures living our lives in open sea, the notion of rigidity would probably not have arisen. And if we were living in a more curved part of the universe, we would probably have developed other notions.

The regularities we observe in our neighborhood and the concepts we find useful to organize our experience need not be the same in other parts of the universe. The theory of relativity specifies quite different local regularities in different parts of the universe, depending on the local constellations of factors that influence what happens. This interaction between propensities that may lead to quite different local regularities is a general feature in all realms of science.[12] That it also leads to different concepts is well illustrated by the examples of rotation and rigidity in relativity theory.

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NOTES

1 In Malament (forthcoming), and in a lecture at Stanford, 19 November 1999.

2 Malament, op. cit. (p. 1).

3 For a brief survey of this discussion, see Miller (1981, pp. 236– 253).

4 Some recent attempts are Barreda and Olivert (1996) (see also their sequel to this article in the subsequent volume of the same journal, pp. 771–784); Bel et al. (1994), Bona (1983).

5 See Rizzi (1998).

6 See Størmer (1902). The survey was completed by Guldberg (1913).

7 See Stein (1977), esp. pp. 21–23 and the long footnote 29 on pp. 36–39, where Stein also surveys briefly the later fate of certain assumptions Lie made about differentiability.

Hilbert in 1900 posed as the fifth of his famous twenty-three problems the problem to what extent assumptions of differentiability can be dispensed with in the theory of Lie groups. In 1952 the problem was solved, it was shown that the differentiability conditions can be dispensed with entirely. An overview of Helmholtz and Lie's work on mobility and its bearing on the study of visual spaces is given in Suppes et al. (1989, esp. Chap. 12).

Eric Curiel has informed me that Howard Stein has translated into English Riemann's Habilitation lecture, as well as Helmholtz's paper 'On the Origin and Significance of the Geometric Axioms', which are two main sources of the above discussion. These translations will give these important texts a much wider readership, and Stein's historical and mathematical commentary will be invaluable to everybody working in this area.

[8] For more on this, and also on Poincaré's contributions, see Friedman (1999, esp. pp. 52–55, 74–84).

[9] A fuller discussion may be found in Føllesdal (1994).

[10] More on this, and also on the interplay between reference and sense may be found in Føllesdal (1986).

[11] Michael Friedman, op. cit. (p. 76).

[12] See Føllesdal (1979). Similar observations have been made by Trygve Haavelmo and Nancy Cartwright.

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