3D/4D equivalence, the twins paradox and absolute time

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1. *Introduction*

A lot has been written recently on the subject of whether the physical world is made up of three-dimensional objects which endure through time, or four-dimensional objects which are extended in time. The ordinary, everyday view is that a rabbit, say, is a 3D continuant which changes yet retains its identity from one moment to the next. Opposed to this is the theory that a rabbit is a 4D object composed of, or divisible into, temporal segments or parts. A number of considerations favour the second view, among them the fact that it is impossible to understand or come to terms with puzzles such as the train/tunnel and the twins paradoxes without recourse to four-dimensional geometry. Is a choice then forced upon us between 3D and 4D ontology, between common sense and science? Not in our opinion. The position of this paper is that the 3D and the 4D descriptions of the world are equivalent, in a clear and precise sense to be explained, and that it is not a question of one being true and the other false. In describing a material object in 3D or in 4D terms we are giving alternative descriptions of one and the same thing, and to see the world aright is to realize this.

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Our second purpose is to examine what theory of time sits best with the thesis of 3D/4D equivalence, and to this end a study of the twins paradox yields valuable insights. When twin *S* steps off her spaceship she is markedly younger than twin *T*, and their difference in age is not something that can be made to disappear by a Lorentz transformation. The principal key to understanding the paradox is, as in the case of the train/tunnel paradox, four-dimensional geometry. But it is unclear that four-dimensional geometry alone explains differential ageing – the wrinkles on the face of one twin and the smooth skin of the other.

Clocks measure temporal length to be sure, but we shall argue that they also do something else, namely measure the A-theory phenomenon of temporal passage. Since the two twins are, in a perfectly good sense, 3D 'clocks' which keep time with their heartbeats, and since they agree at the start of the journey and disagree at the end of it, it would appear that time flows not 'absolutely' or 'globally', but at different rates in different reference frames. This will in fact be the overall conclusion of the paper. If we lived in a world that was exclusively 4D, then a B-theory of time based on relations of 'earlier' and 'later' would be sufficient. But since we live in a world that is also (and equivalently) 3D, and since each twin manifestly undergoes temporal passage and ageing in the particular frame or frames she occupies, A-theory conceptions are also indispensable. There is, in the physical world, differential frame-dependent time flow, rather than a universal 'tide of becoming', and the wrinkles on the twins' faces bear witness to it. Or so we shall argue.

2. *The 3D/4D debate*

Most people take it for granted that familiar 3D objects change, being qualitatively *different* at different times, while also continuing in existence, i.e. remaining numerically *identical* from one moment to the next. The coffee cup I drink out of is cracked; last month it wasn't, yet it is the same cup I have used for the past three years. Is there a problem about this? Aristotle remarks that a man may be pale at one time and dark at another, and in this sense 'receive contraries' (*Categories* 4a10–21). Beyond saying that 'nothing like this is to be seen in any other case', Aristotle does not appear to be troubled by this circumstance, but takes it as part of what it is to be an individual substance *X* that different properties may be predicated of *X* at different times.

Quine (1953) gave a Heraclitean answer to Aristotle's problem about change that has set the tone for all subsequent discussions of enduring objects and identity through time. How is it possible to step twice into the same river, e.g. the river Cayster? Quine's answer is that because a river is an entity continuously in flux, more like a process than a thing, strictly speaking what is stepped into is a river-stage rather than a river. A river-stage is a momentary aggregation of all the water molecules which lie between the river's banks at a certain time. It is one of a huge multiplicity of temporal stages or parts which collectively constitute the 4D object known as River Cayster. Since fresh waters are always flowing upon us, there is no enduring 3D object into which we can step even once, let alone twice. Echoing Hume, Quine asserts that only the relation of resemblance, not that of identity, holds between river-stages. Quine calls this relation 'river-kinship'. He agrees with Hume that in imagining the existence of an enduring 3D river, the resemblance in question causes us to 'substitute the notion of identity, instead of that of related objects' (*Treatise* I, pt 4, sect. 6).

Over the years, the proposal to replace the 3D ontology of enduring objects having no temporal parts, by a 4D ontology of collections of temporal parts which do not endure, has generated a large literature. On the 4D side: Quine 1953, 1960, 1981; Smart 1972; Perry 1972: 466–69; Armstrong 1980; Lewis 1983: 76–77, 1986: 202–4, 2002; Heller 1984, 1990; Sider 1997, 2001; Le Poidevin 2000. Four-dimensionalists are divided into (i) those for whom the basic ontological elements are 4D volumes (the 'worm' ontology), and (ii) those who consider as basic temporal stages, which are either 'thin' 4D objects with nonzero temporal extension, or are instantaneous 3D slices of 4D volumes (the 'stage' ontology). (Instantaneous slices of 4D objects, though three-dimensional, of course differ sharply from enduring 3D objects.) Those who support enduring 3D objects include Geach 1965/1972; Chisholm 1976: appendix A; van Inwagen 1981, 1990; Mellor 1981: 104; Thomson 1983; Lowe 1987, 1998: 114–25; Simons 1987, 2000; Haslanger 1989, 1994. Hawley 2001 surveys the debate.

3. *Four-dimensionalism and relativity theory*

The train/tunnel paradox raises the issue of four-dimensionalism in a particularly direct way. A train, when standing in a tunnel, fits exactly inside it. If the tunnel is equipped with doors which open and shut rapidly, the two ends of the train make light contact with the doors when closed. Imagine the train moving rapidly through the tunnel, and let E be the event of the centre of the train coinciding with the tunnel's mid-point. At that moment two observers with stop-watches, one at each end of the tunnel, will observe the two ends of the train safely inside, with a certain margin to spare. Therefore, in the tunnel's rest-frame, the train is shorter than the tunnel. This apparent shortening of the train is not an illusion, but perfectly real and objective. If the tunnel doors work quickly enough, there will be a short interval during which they are closed and the train is inside. A photographer in a balloon, stationed vertically over the mid-point of the tunnel, will be able to take a photo showing both doors shut.

In the train's rest-frame, on the other hand, there is a different story. Two observers at each end of the train, with synchronized stop-watches, will record at a time simultaneous with E that the doors are open and both ends of the train are sticking out of the tunnel. Again this is no illusion. A second photographer in an airplane, flying above the tunnel at the same velocity as the train, will take a photo showing the doors open and the two ends of the train plainly visible. In the train's rest-frame, the train is longer than the tunnel, not shorter than it. If some matter of great importance hung on the question of whether the tunnel or the train was longer, and if you were the judge in a trial at which the balloon-photo and the airplane-photo were produced, which would you choose?

A satisfactory resolution and understanding of the train/tunnel paradox cannot be had without recourse to four-dimensional geometry. Diagrams which represent the train and the tunnel as 4D objects in their respective inertial frames are found in Maudlin 1994: 54 and Balashov 2000: 336. These diagrams show clearly how it comes about that the train is both longer than the tunnel in one frame, and shorter than it in another. The length contraction of the train in the tunnel-frame, and of the tunnel in the train-frame, are best understood as 'perspectival' phenomena produced by the structure of relativistic space-time geometry. Analogously, the foreshortening of distant objects in the human visual field, and the apparent trapezoidal shape of rectangular objects perceived obliquely, are best understood as perspectival features of 3D geometry: witness the perspectival analysis of the variable length of Cyrano de Bergerac's nose in Janssen and Balashov (forthcoming). Since to grasp the train/tunnel paradox and similar relativistic effects requires an understanding of four-dimensional geometry, the paradox supplies a strong argument in favour of a 4D as opposed to a 3D ontology.

This being said, it is noteworthy that the Special Theory of Relativity was originally presented by Einstein in 1905 in three-dimensional style, with 3D measuring rods and clocks suffering contractions and dilations when observed in different reference frames. Only after the appearance of Minkowski's paper in 1910 did Einstein adopt the 4D space-time interpretation of relativity theory. In this paper, he notes, Lorentz transformations are nothing but rotations of the coordinate system in 4D space (Schilpp 1949: 59; Craig 2001: 78–79). Craig remarks that 'the seriousness with which Einstein took [the four-dimensional] conception may be seen in the fact that when his life-long friend Michael Besso died, Einstein sought to comfort his bereaved family by reminding them that for physicists Besso had not ceased to exist but exists tenselessly as a permanent feature of the space-time reality' (79).

4. *The thesis of 3D/4D equivalence*

If common sense recommends an ontology of enduring 3D objects, and if relativity theory is best understood through the medium of 4D objects at rest in different inertial frames, are we forced to choose between the two competing ontologies? No. The thesis of 3D/4D equivalence states that objects of the physical world can be described using either 3-dimensional or 4-dimensional language, and that the descriptions are equivalent in the sense of intertranslatable. Furthermore, there is no 'fact of the matter' in the world which makes one of the descriptions true and the other false. Some examples drawn from different philosophical and mathematical contexts will help make the equivalence thesis clear.

The first example concerns circles. How is the concept of a circle defined? There are two standard ways, 'statically', as the set of all points equidistant from a given point, or 'dynamically', as the locus of a point moving continuously at a fixed distance from a given point. The two definitions are equivalent. One and the same object is described by them. Just as a circle can be defined either 'statically' or 'dynamically', so a physical object like a rabbit can be defined either in 3D language, as an active animal that hops about, or in 4D language, as a set of rabbit-stages. Alternatively, in 4D mode, a rabbit is an elongated volume of several years' temporal extension, early cross-sections of which yield a baby rabbit nursing its mother, and later cross-sections a fully developed adult.

The second example derives from A. N. Whitehead's method of extensive abstraction. Whitehead (1920: ch. 4) defines the notion of a 'point' in space to be an infinitely descending nested set of volumes. An alternative approach in topology is to define a 'volume' as a connected set of points. In giving theoretical foundations for topology or geometry, we may (i) define 'point' in terms of 'volume', or (ii) define 'volume' in terms of 'point'. Since, given Whitehead's insight, the two methods yield the same results, a conceptual structure which takes 'point' as primitive is equivalent to a structure which takes 'volume' as primitive.

In the same way, we propose, the argument over whether physical objects are 'really' three- or four-dimensional is also pointless (no pun intended). Suppose someone were to raise the ontological question of whether space is 'really' composed of points, or 'really' composed of tiny volumes. Before Whitehead's definition, it was perhaps arguable that this question raised a substantive philosophical issue. But if a 'point' can be defined as an infinite set of volumes, and if, equally, a 'volume' can be defined as an infinite set of points, then to argue about which of these items are the real constituents of space is, in a different sense, pointless. In analogous fashion, if physical objects can be considered as 3-dimensional entities existing through time, and if they can equally well be considered as

4-dimensional objects composed of, or divisible into, temporal parts, then the dispute over whether the physical world is really 3D or 4D is similarly empty. It may be both, or it may be neither, or it may be either, depending on which features of the world we wish to focus on. But it will never be one to the exclusion of the other; never the case that the 3D view is vindicated and the 4D view defeated, or vice versa.

A third example underlines the tendency of ontological disputes to disappear under the weight of a 'principle of indifference', according to which it is immaterial which of two underlying theories is true.

Imagine that intelligent life on earth disappears, and that many years later Martians arrive and discover, among other artifacts, quantities of sheet music, LPs and compact discs. Although they have no ears and cannot hear anything, the Martians have a keenly developed aesthetic sense and become very intrigued about what 'music' might be. A lively debate springs up between those who think that music consists of patterns of notes printed on 5-line staffs, and those who think music consists of cyclical irregularities in the grooves of LPs and compact discs. The debate is spirited, with many articles in learned journals, until one day a particularly intelligent participant produces a translation manual which demonstrates the existence of a complete structural isomorphism between the 'sheet music' and the 'groove' interpretations. (A small correction factor to the overall isomorphism correlates minor systematic variations in groove patterns with the occurrence of words like 'piano' and 'violin' on sheet music, but with these corrections the translation is perfect.) Given a sheet music description, a groove pattern description is constructible and vice versa. Once the translational equivalence of the two theories is established, interest in the ontological question of what music really is disappears. Any intelligent Martian knows that whether described in sheet music or in groove pattern terms, Beethoven's Fifth Symphony is a landmark composition which can be studied, loved and appreciated in either of its two ontological manifestations. As with the thesis of sheet music/groove equivalence, so with the thesis of 3D/4D equivalence.

5. *The twins paradox*

Writ large, the twins paradox has twin *S* remaining on earth, while twin *T* flies on a spaceship to Alpha Centauri and back. When she returns she has aged 30 years, while twin *S* has aged 40 years. The difference in elapsed time does not arise from the fact that *T* has suffered accelerations and decelerations, while *S* has remained at rest in something approximating an inertial frame. In the 3-clock version of the paradox, let clocks 1 and 2 be synchronized at space-time point *X*. Suppose that each clock follows a different inertial path, and let clock 3 be synchronized with clock 2 at point

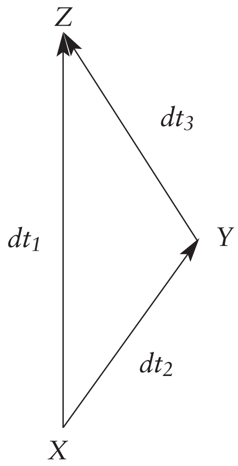

Figure 1

Y (Bondi 1964: 80–87; Marder 1971: 73–8, 112–13). If clock 3 follows a third inertial path, and if this path intersects clock 1's path at *Z*, then the total elapsed time of 2 and 3 will always be less than the elapsed time of 1 (Figure 1).

In Minkowski geometry with metric signature +---, in which the length of a line segment *ds* is given by the formula $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$, the proper elapsed time *dt* of a clock at rest in an inertial frame equals the length *ds* of the corresponding line segment. Consequently, if we build a geometry out of inertial lines, i.e. the paths followed by objects at rest in inertial frames, the lengths of which are equal to the objects' proper times, we arrive at a simple inequality. What the clock paradox rests on is that in such a geometry the length dt_1 in Figure 1 is always greater than the sum of the lengths dt_2 and dt_3:

$$dt_1 > dt_2 + dt_3.$$

In Euclidean geometry, triangle inequality operates the other way around: $dt_1 < dt_2 + dt_3$. But in Minkowski inertial line geometry, one side of a triangle is greater, not less, than the sum of the other two sides. In relativistic athletics, the medal for the shortest elapsed time goes not to the runner who runs straight, but to the runner who takes a zigzag path to the finish. What lies at the root of the twins paradox is a reversed law of triangle inequality.

The twins paradox carries important consequences for our everyday notion of time. If *S* stands at the sideboard while *T* walks over to the table and back, *T* will have aged less than *S* when she returns. The way to keep youthful is to keep on the move! As in the train/tunnel paradox, the difference in age between stay-at-home S and traveller T is not an illusion, but is perfectly real. In fact it is even more real than the difference in length of train and tunnel, because the two photographs showing the latter were shot in different inertial frames, whereas the twins' difference in age is witnessed by increased grey hair and wrinkles once reunited. These differences are absolute, Lorentz-invariant, and cannot be transformed away by changing reference frames. We shall argue that the differential ageing of the twins in different frames, when studied within the context of the A-theory of time, requires that the idea of global or absolute becoming be abandoned in favour of *frame-dependent* temporal passage.

Consider in more detail the two individuals S and T who stand together at the sideboard, separate when T walks to the table, and then are reunited. At one level of explanation, the fact that S's age has increased relative to T's is explained by the fact that in Minkowski geometry the unbroken path followed by S is longer than the broken path followed by T. But there is another level of explanation, also relativistic, which concerns not the temporal *lengths* of the sides of the triangle in Figure 1 but the temporal *rates* at which physical processes take place. The distinction between temporal lengths and temporal rates takes us back to 3D/4D equivalence. The triangle of Figure 1 which measures elapsed time is a triangle in 4D geometry, whereas the clock or the individual which follows the 4D trajectory and undergoes temporal passage is necessarily a 3D object. Only for such objects do the notions of 'temporal process' and 'rate of temporal process' make sense. However, as will be seen, 3D/4D equivalence ensures that the A-theory concepts of 'process' and 'rate', and the B-theory concept of 'temporal distance', are two sides of the same coin.

Three-dimensional objects are like little clocks. Their 4D world-lines, divided into equal segments like a ruler, measure quantity of elapsed time. But qua 3D objects they also measure rates of elapsed time, which are A-quantities. For traveller T, who moves to the table and back, time passes more slowly than for S. While T experiences x heartbeats, undergoes t cycles of her neural alpha-rhythm, and hums z bars of *Tosca*, S experiences $x + dx$ heartbeats, $y + dy$ neural cycles, and hums $z + dz$ bars. It would be tempting to say that S has crammed more experiences and events than T into the same time period, but this would be incorrect. The time periods themselves are unequal. Between their initial parting and later reunion, S has more time available than T. Consequently, for him, time moves more quickly, while for T it moves more slowly. Statistically, using counts of heartbeats or neural cycles per week, it should be possible to confirm that office workers who are confined to their desks age more rapidly than mail deliverers and taxi-drivers, who change their inertial frames many times a minute.

This conclusion is admittedly controversial. It violates our instinctive belief that (by definition?) time always moves at the constant rate of one second per second (Prior 1968: 2–3), and that variations in this rate are unthinkable. For this reason it is worth going over once more the argument for variable rates of temporal flow, this time focusing on inertial frames rather than lengths of world lines.

A significant difference between *S* and *T* is that *T* successively occupies (is at rest in) two different inertial frames f_2 and f_3, corresponding to two of the sides of the triangle of Figure 1, while *S* occupies only one frame, f_1. Since a similar elapsed time discrepancy is observed for any two objects, one of which successively occupies two different inertial frames and the other only one, the conclusion is strongly indicated that the time difference is due to differential flow rates in different frames. And this in turn implies that global or absolute time flow, and consequently Global or Absolute Time, is non-existent. Those who argue that nothing in the theory of relativity rules out absolute time (Tooley 1997: 338–54; Craig 2001), cannot reconcile this position with the phenomenon of differential ageing in the twins paradox.

Unlike such Lorentz-invariant quantities as the B-theory notion of spatio-temporal length, the A-concept of temporal passage or flow is inherently frame-dependent, and its rate is particular to each and every inertial frame. Therefore there can be no such thing as Absolute Time. This is admittedly a large conclusion to derive from the small beginnings of the twins paradox, conjoined with 3D/4D equivalence, but we see no alternative to it.

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