A number of sources of aluminum cylinders to be used as the rotors were considered. Because of the ease of acquiring beer or soft-drink cans and because of their very light weight, these were selected, as can be seen by Fig. 2. Whatever the source of the cylinders, their height should not be much more than about 4 in., otherwise the torque caused by the cross wind can easily become large enough to tilt the glider so that one of its lower edges rubs against the air rail. The particular drive motor used turns the cans at about 1500 rpm. The tops of the cans are cut off squarely (as described below) and the open end is gently pushed onto a thin aluminum disk turned in a lathe with a shoulder to hold the axis of the can parallel to the axis of the motor. A hole in the center of the disk fits onto the motor shaft and is held in place by a set screw. In order to cut the cans, they are mounted inside a hollow wooden cylinder, also turned in a lathe. The cylinder is slit down the side to make inserting the cans easy. When chucked up in a lathe, the wooden cylinder holds the cans firmly and without wobbling. Cutting of the can is done by means of a razor bladelike tool made from a piece of drill rod or an old drill which can be rigidly held in the tool holder of the lathe.

The "cross wind" which propels the ship best is generated by the exhaust port of a vacuum cleaner, but a source of compressed air will also work if the volume of air is sufficiently large. An electric fan can be used to show the effect, but the one tried here did not work as well. The vacuum cleaner exhaust provides the greatest mass of air per second concentrated within a cross-sectional area which matches the dimensions of the can.

Fig. 2. A model of the rotor ship using an air track glider.

[1]R. M. Sutton, *Demonstration Experiments in Physics* (McGraw-Hill, New York, 1938), p. 117.

[2]J. Walker, *The Flying Circus of Physics* (Wiley, New York, 1975), p. 85.

[3]R. A. R. Tricker, *Bores, Breakers, Waves, and Wakes* (American Elsevier, New York, 1967), pp. 105–106.

[4]*Encyclopedia Britannica* (Encyclopaedia Brittanica, Inc., William Benton, Chicago, IL, 1968), Vol. 19—under "Rotor Ship."

Consistency of minimum time and length intervals with special relativity

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This note addresses the consistency of the existence of minimum or "quantized" time and length intervals, respectively, with special relativity. By the existence of shortest time and length intervals, we mean that no proper time or proper length intervals exist that are smaller, respectively, than these minimum intervals.[1]

A historical account of the possible existence of minimum time and length intervals is given by V. L. Ginzburg in his informative work, *Key Problems of Physics and Astrophysics.*[2] These notions are also mentioned in an equally informative article by J. A. Wheeler.[3]

At first thought, it would appear that so "absolute" a concept as minimum intervals is alien to special relativity. Suppose, for example, that a shortest length interval exists which we label L_0. A rod of this length is fixed in a frame S_0 which moves with uniform velocity with respect to the frame S. Performing a Lorentz transformation, we find that the corresponding measured length in S has the value $L = L_0/\gamma \ll L_0$, which appears to contradict our starting premise. In the preceding expression $\gamma = (1-\beta^2)^{-1/2}$, $\beta = v/c$.

Let us examine this situation more carefully. Let all frames be equipped with length-measuring devices calibrated to the smallest interval L_0. Again, let a rod of length L_0 be fixed in S_0 which in turn moves with constant velocity relative the frame S. Measurement of this length in S would find a value less than L_0, then we may conclude that this length is immeasurable in S as such lengths cannot be resolved by a measuring device whose minimum calibration is L_0. So, it is consistent with relativity to say that lengths $L < L_0$ are not observed in any frame.

A minimum time interval is more readily shown to be consistent with relativity. We assume a minimum time interval and call it t_0. Let clocks in all frames be calibrated to this minimum interval. Consider a clock that is stationary in a frame S_0 which, in turn, moves with constant velocity relative to the frame S. An observer in S measures the time $t = \gamma t_0 > t_0$, corresponding to the interval t_0 in S_0. (The clock in S has ticked off more than one t_0 interval.) We may conclude that all observers measure time intervals $t \geqslant t_0$.

We have considered the consistency of the notions of smallest time and length intervals with special relativity. Working with simple Lorentz transformations, it was concluded that the existence of shortest length and time intervals, L_0 and t_0, respectively, infers that all length measurements find $L \geqslant L_0$ and that all time interval measurements find $t \geqslant t_0$.

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[1]The proper time of a clock is the time observed in a frame in which the clock is at rest. Proper length of a rod is the length observed in a frame in which the rod is at rest.

[2]V. L. Ginzburg, *Key Problems of Physics and Astrophysics* (Mir, Moscow, 1978), 2nd ed.

[3]J. A. Wheeler, Am. Sci. **74**, 366 (1986).

Diagram for head-on collisions

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A collision may be regarded as the simplest interaction mechanism between two particles (or bodies) since the interaction takes place only for a very short time inverval when the particles are in very close proximity, i.e., in "contact." The quantities directly accessible to measurement are the masses m_j of the particles $j = 1,2$ and their velocities $\mathbf{v}_j$ and $\mathbf{v}'_j$ before and after collision, respectively. However, even for the simplest case of an elastic central collinear ("head-on") collision the well-known relation[1,2] between final and initial velocities

$$\mathbf{v}'_j = \frac{2m_i}{m_i + m_j}\mathbf{v}_i + \frac{m_j - m_i}{m_i + m_j}\mathbf{v}_j, \quad i \neq j, \tag{1}$$

appears by no means trivial.

Of course, formula (1) is necessary for calculations. Yet, for a conceptual grasp of the situation, this expression is already too complicated. Therefore, introductory texts[1-3] commonly take recourse to discussing special cases like equal masses ($m_1 = m_2$), or one particle at rest ($v_2 = 0$), or one particle much more massive than the other ($m_1 \ll m_2$), all of which simplify Eq. (1). On an advanced level, simplification is achieved through transformations to the laboratory frame (where $v_2 = 0$) or to the center-of-mass frame.[4-7] Naturally, the situation becomes considerably more complicated when the collision is inelastic.

On the other hand, formulation of a collision is easy in terms of dynamic variables because of conservation of momentum

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2 \tag{2}$$

and conservation of energy

$$\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{p_1'^2}{2m_1} + \frac{p_2'^2}{2m_2} + \Delta U \tag{3}$$

inherent in the process. Here $\mathbf{p}_j$ and $\mathbf{p}'_j$ denote the particles' momenta just before and after the collision, respectively. The change of internal energy of the system is given by ΔU, which is zero for an elastic collision and maximum, ΔU_c, for a completely inelastic collision.

Even with the elegant formulation of a collision process through conservation laws, some conceptual difficulties remain arising from the fact that the dynamic variables, momentum and energy, are composed rather than directly measurable quantities. In this note we present a diagram which shows, by intersections of two parabolas, (1) both conservation laws simultaneously, (2) the dynamical variables, i.e., initial and final momenta and the energy distribution, (3) the directly observable quantities, i.e., particle masses and (initial and final) velocities, and (4) the collision impulse for elastic as well as inelastic head-on collisions.

In the collision diagram (Fig. 1) we draw the vector $\mathbf{p}_1 = m_1\mathbf{v}_1$ with its tail beneath the origin O_1 of the bottom

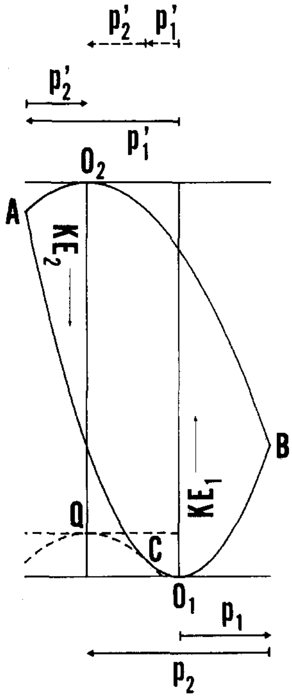

Fig. 1. Diagram for head-on collisions representing momenta and energies before the collision (B), after an elastic collision (A), and after a completely inelastic collision (C). Particle masses are shown by the parabolas' radii of curvature and velocities by slopes at A, B, C. Here the mass ratio is $m_2/m_1 = 2$ and the initial velocites are $\mathbf{v}_2 = -\mathbf{v}_1$.