The block is now pulled down and held while in its new position, as shown in Fig. 2.
The teacher asks the class to predict what will happen in this real situation when the block is released, and to give a reason supporting and prediction.
Which of the following gives the best prediction and supporting reason?
(A) The block will move down because the gravitational force on the block will be greater at this lower level.
(B) The block will remain stationary because there will be no resultant force on the block.
(C) The block will move up and return to its original position because this will conserve potential energy.
(D) The block will move up and return to its original position because that was its equilibrium position.

Question D: The bouncing ball problem

A steel ball of mass m is dropped on to a steel-topped table from a height of 1 m. The ball is in contact with the table for 0.01 s and rebounds with only a very small loss of kinetic energy.
Which of the following statements best describes the average force exerted on the ball by the table during the collision?
(A) It equals the normal reaction force mg.
(B) It is slightly less than mg because the collision is not quite elastic.
(C) It is greater than mg.
(D) It is greater than the force exerted by the ball on the table, in order to make the ball rebound.

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Bell's theorem: Does quantum mechanics contradict relativity?

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(Received 14 April 1986; accepted for publication 9 October 1986)

Special relativity demands a locality principle (no instantaneous action at a distance); locality implies Bell's theorem; quantum mechanics violates Bell's inequality, therefore, quantum mechanics contradicts relativity! Or so it would seem. It is shown, however, that the locality principle needed for Bell's theorem is stronger than the simple locality that is needed to satisfy the demands of relativity and that quantum mechanics satisfies the latter. The stronger locality principle is equivalent to the conjunction of simple locality and predictive completeness, and it is the latter principle that fails. The notion of predictive completeness is weaker than, and is implied by, the completeness criterion of Einstein, Podolsky, and Rosen. But the quantum mechanical state description is not only incomplete but incompletable, for any local complete state description would satisfy Bell's inequality and disagree with experiment.

I. INTRODUCTION

It is common knowledge that special relativity forbids instantaneous action at a distance and, more generally, that it forbids the propagation of energy or information at speeds exceeding the speed of light. This assertion has occasionally been subject to controversy, but after reviewing the relevant analysis in Sec. II, we shall conclude that in this case "common knowledge" is correct. We shall refer to this prohibition of nonlocal causality as the principle of *relativistic locality*.

An apparently similar locality principle is used to derive Bell's theorem,[1] which is an inequality restricting correlations between the results of causally independent measurements on separated systems that may be correlated only because of their past history. The profound surprise is that certain quantum mechanical correlations violate Bell's inequality, revealing a previously unsuspected contradiction between quantum mechanics and the locality principle that is used to derive Bell's theorem. Hence, the question in our title, "Does quantum mechanics contradict relativity?"

Such a contradiction, if indeed one exists, between two fundamental and exceedingly well-verified theories would constitute a major crisis in theoretical physics.[2] We shall argue in this article that, in fact, Bell's theorem does not establish a contradiction between special relativity and quantum mechanics. This is the case because the proof of Bell's theorem requires a stronger form of locality (which we call *strong locality*) than the *simple locality* principle entailed by special relativity. We shall show that the principle of strong locality is logically equivalent to the conjunction of simple locality and a form of predictive completeness of the state description. It is the failure of this completeness condition, rather than any failure of simple locality, that is responsible for the violation of Bell's inequality by quantum mechanics. Finally, we discuss the relation of our conclusions to those of Einstein, Podolsky, and Rosen.

II. LOCALITY IN SPECIAL RELATIVITY

Consider two space-time points (or events) whose coordinates are $(\mathbf{r}_1,t_1)$ and $(\mathbf{r}_2,t_2)$, and whose space-time separation is $(\mathbf{r},t) = (\mathbf{r}_2 - \mathbf{r}_1, t_2 - t_1)$. The quantity $\sigma = (ct)^2 - \mathbf{r}^2$ is invariant under Lorentz transformations, which means that it has the same value in all uniformly moving frames of reference. The separation between events 1 and 2 is described as timelike, lightlike, or spacelike according to whether σ is positive, zero, or negative. A signal passing from point 1 to point 2 will travel at a speed less than, equal to, or greater than the speed of light c in these three cases.

In the case of a timelike separation $(\sigma > 0)$, one can find a frame of reference such that the spatial separation r vanishes. This is the rest frame of a particle moving uniformly from point 1 to point 2. In the case of a spacelike separation $(\sigma < 0)$ it is possible to find a frame of reference in which the two events are simultaneous $(t_2 - t_1 = 0)$. Thus a prohibition of instantaneous action at a distance in all frames of reference is equivalent to a prohibition of signal propagation at any speed greater than c.

Lorentz invariance by itself does not rule out the possibility of objects (called tachyons) that move faster than light, but there are other difficulties with this notion. Suppose events 1 and 2 are the emission and absorption of a tachyon. Because the separation is spacelike there exist frames of reference in which $t_2 > t_1$ and others in which $t_2 < t_1$, thus the principle that cause must preceed effect is violated. Even more serious is the possibility of using a tachyon relay to send a message to one's own past. This leads to very strange consequences: one could arrange to kill one's mother before one was born, and thus have a situation in which the tachyon message is sent if and only if it is not sent. For reasons such as these, it is now agreed that the possibility of traveling or signaling at superluminal speeds is strictly incompatible with special relativity.[3] Since the prohibition of superluminal communication is equivalent to the prohibition of instantaneous nonlocal action, we shall refer to it as the principle of *relativistic locality*.

III. LOCALITY IN SPIN CORRELATION EXPERIMENTS

The practical tests of Bell's theorem involve the measurement of a spin component on each of two correlated particles. (If the particles are photons the analog of a spin component is polarization.) In this section we analyze the implications of relativistic locality for such an experiment.

Since Bell's theorem does not presuppose quantum mechanics, we shall need a notation that is more general than the notation of quantum states and observables. We use the labels "L" and "R" to refer to the left-hand member and right-hand member of the pair of particles, and of the pair of measuring devices. Let $\mathbf{d}_L$ and $\mathbf{d}_R$ be unit vectors that define the component of spin measured by the L device and R device, respectively. Let x_L and x_R denote the results of such measurements. (We shall have $x = \pm 1$ in most cases.) The probability[4] of obtaining the particular results x_L and x_R is of the form $P(x_L, x_R | \mathbf{d}_L, \mathbf{d}_R, S_L, S_R, \lambda)$. Here S_L and S_R denote the premeasurement states of the L device and R device. [Strictly speaking, S_i is a set of parameters that includes $\mathbf{d}_i$ $(i = L,R)$, but the vectors $\mathbf{d}_i$ are so important that we write them explicitly.] The state of the two-particle system is denoted by λ, and it may be very general. Here λ may be a quantum state, a quantum state plus arbitrary hidden variables, or some nonquantum-mechanical form of state description. It need not separate into parts that describe the two particles individually.

We denote by $Q_L(x_L | \mathbf{d}_L, S_L, S_R^0, \lambda)$ the probability that the L device obtains the result x_L while the R device performs no measurement (represented by the R-device state S_R^0), and $Q_R(x_R | \mathbf{d}_R, S_R, S_L^0, \lambda)$ is similarly the probability that the R device obtains the result x_R while the L device does no measurement.

We now suppose that the measurements on particle L and particle R can be carried out at spacelike separation from each other and we assert that the necessary and sufficient condition for the spin-correlation measurement to obey the relativistic locality principle is that the following conditions be obeyed:

$$Q_L(x_L | \mathbf{d}_L, S_L, S_R^0, \lambda) = \sum_{x_R} P(x_L, x_R | \mathbf{d}_L, \mathbf{d}_R, S_L, S_R, \lambda), \tag{1a}$$

$$Q_R(x_R | \mathbf{d}_R, S_R, S_L^0, \lambda) = \sum_{x_L} P(x_L, x_R | \mathbf{d}_L, \mathbf{d}_R, S_L, S_R, \lambda). \tag{1b}$$

We shall call these conditions *simple locality*. Note that these conditions imply that the value of (1a) must be independent of the direction of $\mathbf{d}_R$ and the value of (1b) must be independent of the direction of $\mathbf{d}_L$.

Simple locality asserts that the probability of obtaining the result x_L by means of a joint measurement of x_L and x_R in which the x_R value is ignored [right-hand side of (1a)] is the same as the probability of obtaining x_L when the R device is not operating [left-hand side of (1a)]. Locality requires that the probability for the outcome x_L of a measurement of particle L be determined only by the two-particle state λ and the state of the L device. It does *not* exclude the possibility of obtaining information about particle L by performing a measurement on the distant particle R. That sort of information will be contained in λ if the two particles are correlated (presumably because of some interaction in the past). But it does exclude the possibility that the preparation of the distant R device in some state $(S_R, \mathbf{d}_R)$ can exert a causal influence on the probabilities for the possible outcomes of the L measurement.

We now prove our assertion that a violation of conditions (1) would provide, at least in principle, the means for superluminal signal transmission.[5] Let us suppose, without loss of generality, that (1a) is violated; i.e., we assume

$$Q_L(x_L|\mathbf{d}_L, S_L, S_R^0, \lambda) \neq \sum_{x_R} P(x_L, x_R|\mathbf{d}_L, \mathbf{d}_R, S_L, S_R, \lambda) \tag{1a'}$$

for some specified λ, $x_L, \mathbf{d}_L, S_L, \mathbf{d}_R, S_R$, and S_R^0. Suppose further that an ensemble of pairs of particles is prepared in the state[6] λ and let E_L (E_R) be an experimenter prepared to perform measurements on the $L(R)$ particles of the ensemble in space–time region Γ_L (Γ_R), where the separation between the two regions Γ_L and Γ_R is spacelike.

Let it be arranged between E_L and E_R in advance that (i) E_L will measure the $\mathbf{d}_L$ component of the spin of each of the L particles by preparing his ensemble of L devices in state S_L and (ii) E_R may choose *either* to measure the $\mathbf{d}_R$ component of the spin of each of the R particles by preparing his ensemble of R devices in the state S_R *or* by preparing his R devices in state S_R^0, to perform no measurements at all on the R particles.[7] Moreover, E_R is to delay his decision until the decision "event" lies in Γ_R.

In accordance with (1a'), the statistics E_L compiles from his measurement outcomes will depend on E_R's choice. In this way, by using sufficiently large ensembles, E_R can inform E_L of his decision within an arbitrarily short time interval, with probability approaching unity.[8] By suitably correlating messages of interest with decisions to perform or not to perform measurements, violations of (1) can thus be exploited, at least in principle, for superluminal communication.[9] (It should be noted here that the human experimenters are not essential in this scheme. A switching mechanism of some sort, playing the role of E_R, could just as well make the "choice" which, via the results of the automated measurements in Γ_L, triggers some corresponding physical response at the distant site.)

Finally, we demonstrate that quantum mechanics satisfies the locality conditions (1). Let ρ_{LR} be the statistical operator for the two-particle state. Let $|x_L\rangle$ be an eigenvector of $\boldsymbol{\sigma}_L \cdot \mathbf{d}_L$ and $|x_R\rangle$ be an eigenvector of $\boldsymbol{\sigma}_R \cdot \mathbf{d}_R$, where $\boldsymbol{\sigma}_L$ and $\boldsymbol{\sigma}_R$ are the spin operators of particles L and R. The probability that the pair of measurements will yield the pair of results x_L and x_R is (omitting the conditional state parameters for notational simplicity)

$$\begin{aligned} P(x_L, x_R) &= \mathrm{Tr}(|x_L, x_R\rangle\langle x_L, x_R|\rho_{LR}) \\ &= \langle x_L, x_R|\rho_{LR}|x_L, x_R\rangle, \end{aligned}$$

where the vector $|x_L, x_R\rangle$ is the tensor product $|x_L\rangle|x_R\rangle$. The probability that a single measurement on particle L will yield the result x_L is

$$Q_L(x_L) = \mathrm{Tr}^{(L)}(|x_L\rangle\langle x_L|\rho_L) = \langle x_L|\rho_L|x_L\rangle,$$

with $\rho_L = \mathrm{Tr}^{(R)}\rho_{LR}$ being the reduced statistical operator for particle L. Since $\rho_L = \Sigma_{x_R}\langle x_R|\rho_{LR}|x_R\rangle$, it is clear that we have

$$Q_L(x_L) = \sum_{x_R} P(x_L, x_R),$$

and so (1a) is satisfied. Similarly, we can show that (1b) is satisfied.

IV. STRONG LOCALITY AND BELL'S THEOREM

We have seen in Sec. III that quantum mechanics obeys locality, in the sense that correlation measurements do not permit communication at superluminal speeds. How then does quantum mechanics violate Bell's inequality, which is also derived from a locality condition? The answer is that the derivation of Bell's theorem uses a stronger form of locality postulate than (1). This postulate, which we call *strong locality*, takes the following form for a spin correlation experiment:

$$\begin{aligned} P(x_L, x_R|\mathbf{d}_L, \mathbf{d}_R, S_L, S_R, \lambda) &= Q_L(x_L|\mathbf{d}_L, S_L, S_R^0, \lambda) \\ &\quad \times Q_R(x_R|\mathbf{d}_R, S_R, S_L^0, \lambda). \end{aligned} \tag{2}$$

Several comments on this condition are in order. First, it is obvious that Eq. (2) implies Eqs. (1) but (1) does not imply (2), hence strong locality is indeed a strengthened version of locality, as the names suggest. Second, it is clear that (2) does not hold if the two-particle state λ is taken to be *merely* the quantum state, for that would contradict the possibility of correlations between the values of the two spins x_L and x_R. One must think of λ as some more general (perhaps uncontrollable or unmeasurable) kind of state parameter. The observed distribution of x_L and x_R would then be of the form

$$\begin{aligned} P_{\mathrm{obs}}(x_L, x_R) = \int & Q_L(x_L|\mathbf{d}_L, S_L, S_R^0, \lambda) \\ & \times Q_R(x_R|\mathbf{d}_R, S_R, S_L^0, \lambda)\rho(\lambda)d\lambda, \end{aligned} \tag{3}$$

where $\rho(\lambda)$ is the probability distribution for λ. The interesting content of Bell's theorem is that although state descriptions more detailed than those of quantum mechanics are easy to imagine (and possible to construct), there is no conceivable state description that obeys strong locality (2) and reproduces the quantum mechanical predictions through Eq. (3).

Different derivations of Bell-type inequalities employ and justify Eqs. (2) and (3), or their equivalents, in various ways. If an underlying determinism is assumed,[10] that is if the specification of λ is assumed to determine the actual result of a measurement, then the "probability" $P(x_L, x_R|\mathbf{d}_L, \mathbf{d}_R, S_L, S_R, \lambda)$ is equal to 1 for one particular pair of values (x_L, x_R) and is equal to 0 for all other values. The sums in Eqs. (1a) and (1b) will contain at most one nonvanishing term, and (1) will be equivalent to (2) in this case. Thus if determinism holds, there is no distinction to be made between simple locality and strong locality. The violation of Bell's inequality by quantum mechanics and, apparently,[11] by nature, may therefore be taken as evidence against the existence of a deterministic substratum.[12]

The derivations of Bell-type inequalities without the assumption of determinism are not much more complicated mathematically, but the arguments by which they justify the use of Eqs. (2) and (3), or their equivalents,[13] may be more subtle. The reader should be warned that some writers give the impression that Eqs. (2) and (3) are merely the embodiment of relativistic locality for these experiments, a view that we regard as mistaken. Our own interpretation of the meaning and significance of strong locality is the subject of Sec. V.

V. PREDICTIVE COMPLETENESS

A state description is said to be *predictively complete* with respect to a measurement (or set of measurements) M if the results of measurements other than M provide no information relevant to predicting the result of M that is not already contained in that state description. The information provided by non-M measurements is either irrelevant or redundant.

A deterministic state description is always complete because it prescribes the results of all possible measurements, M and non-M, and so the information provided by any particular measurement will be redundant. However, determinism is not necessary for completeness. In particular, this definition does not *by itself* rule out the possibility of quantum mechanics being "complete." For example, suppose a spin one-half particle is prepared in the spin-up state by means of a suitable apparatus. Let M be the set of possible subsequent spin-component measurements. For one such measurement along a direction at an angle Θ from the vertical, the state description predicts a positive result with probability $[\cos(\Theta/2)]^2$. According to quantum theory, the results of any spin measurements that were performed before the operation of state preparation (non-M measurements) are irrelevant for predicting the result of M. Thus the quantum state description is predictively complete with respect to the set M of post-preparation measurements. (This property of rendering previous information irrelevant could well be regarded as part of the definition of a state preparation.) Neither of these examples is directly relevant to Bell's theorem, and they are mentioned only to illustrate the concept of predictive completeness.

To apply this concept to the spin correlation experiment, we take M to be a spin-component measurement on one of the particles. The class of non-M measurements includes all spin measurements on the other particle at a spacelike separation from the M measurement. (The non-M class may be larger than this, but we shall not need to consider any other members.) In order for predictive completeness to hold, the following condition is necessary:

$$P(x_L,x_R\,|\,\mathbf{d}_L,\mathbf{d}_R,S_L,S_R,\lambda) = \sum_{x'_R} P(x_L,x'_R\,|\,\mathbf{d}_L,\mathbf{d}_R,S_L,S_R,\lambda) \times \sum_{x'_L} P(x'_L,x_R\,|\,\mathbf{d}_L,\mathbf{d}_R,S_L,S_R,\lambda). \tag{4}$$

This states that, after specification of the state parameters $(\mathbf{d}_L,\mathbf{d}_R,S_L,S_R,\lambda)$ for the apparatuses and the two-particle system, the R-measurement result x_R gives no further information about x_L and vice versa. The factoring of the joint probability distribution for (x_L,x_R) into a product of a function of x_L and a function of x_R is the mathematical expression of the absence of predictively relevant information. It may indeed be possible that the outcome x_R of an R-measurement may enable one to make inferences about the properties of particle L. But this information is already contained in the state description if it is predictively complete.[14]

Notice the difference between strong locality (2) and predictive completeness (4). If one were to ignore the conditional state parameters, then both equations would merely assert that the joint distribution $p(x_L,x_R)$ is equal to the product of the marginal distributions of x_L and x_R separately. But the conditional parameters are essential. The right-hand side of (2) possesses a locality property: one factor depends upon $\mathbf{d}_L$ but not $\mathbf{d}_R$, while the other factor depends upon $\mathbf{d}_L$ but not $\mathbf{d}_R$. But in Eq. (4) each factor may depend upon *both* $\mathbf{d}_L$ and $\mathbf{d}_R$; the orientation of the R device is allowed to influence the probability of a particular outcome of an L measurement. So predictive completeness, as embodied in Eq. (4), does not imply either strong locality or simple locality.

Strong locality is logically equivalent to the conjunction of simple locality and predictive completeness. It is obvious, by substituting Eq. (2) into (1) and then (2) into (4), that strong locality implies both simple locality and predictive completeness in the context of our spin correlation experiment. Conversely, if we postulate (1), it is obvious that (4) can be rewritten as (2), hence simple locality plus predictive completeness implies strong locality.

Since simple locality (1) is a requirement of special relativity, which is supported by an impressive body of experimental evidence, it is natural to infer that the violation of Bell-type inequalities (by quantum mechanics and in actual experiments) is due to the failure of predictive completeness.

VI. EINSTEIN, PODOLSKY, AND ROSEN

Anyone familiar with the famous paper, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," by Einstein, Podolsky, and Rosen, (EPR),[15] cannot fail to notice a similarity between our conclusion (relativistic locality holds but predictive completeness fails) and theirs [locality implies that the quantum mechanical (QM) state description is incomplete]. However, one must look beyond the mere words "locality" and "completeness" in order to assess the relationship between their conclusion and ours, since the definitions of those terms may be different in different contexts.

EPR propose a necessary condition for a theory to be *complete*:

> "every element of the physical reality must have a counterpart in the physical theory."

They give a sufficient condition for identifying an *element of* reality:

> "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

Then by applying a form of the locality principle to a correlated two-particle system, they deduce that there are elements of reality corresponding to both of two noncommuting observables. Since no quantum state can describe exact values for noncommuting observables, their argument demonstrates a contradiction between locality and completeness. Believing that their locality condition was unassailable, EPR concluded that the quantum mechanical description of physical reality is not complete.

Because Bell's theorem has cast doubt on locality (more precisely, on strong locality), the EPR argument loses much of its force. To save it requires, at the very least, an analysis of their form of the locality principle, in order to determine whether it is equivalent to strong locality, relativistic locality, or perhaps neither.

We shall not undertake such an analysis here. Instead we shall show, independently from the EPR argument, that their conclusion is valid: QM is not complete in the sense defined by EPR. To do this we shall show that in the context of the spin-correlation experiments the EPR completeness condition implies the weaker condition of predictive completeness. Since the latter has been shown to fail, so must the former.

To show that EPR completeness implies predictive completeness, we apply the EPR definitions to our spin-correlation experiment. Let the pair of spin one-half particles be in the singlet state of zero total spin. If the orientations of the measuring devices are the same, $\mathbf{d}_L = \mathbf{d}_R$, then the two spins will exhibit perfect anticorrelation.[16] If we measure $\boldsymbol{\sigma}_R \cdot \mathbf{d}_R$ to be $+1$, for example, we may predict with probability unity that $\boldsymbol{\sigma}_L \cdot \mathbf{d}_L$ must have the value -1. But this prediction was made without in any way disturbing particle L, since the R device is at a spacelike separation from it, therefore $\boldsymbol{\sigma}_L \cdot \mathbf{d}_L$ is an *element of reality* according to the EPR criterion. Since we could just as well have chosen a different orientation, $\mathbf{d}'_R$, for the R device, we infer that $\boldsymbol{\sigma}_L \cdot \mathbf{d}_L$ is an element of reality for all orientations of $\mathbf{d}_L$. By interchanging the roles of L and R, we deduce similarly that $\boldsymbol{\sigma}_R \cdot \mathbf{d}_R$ is an element of reality for all orientations of $\mathbf{d}_R$. A state description that was *EPR complete* would have to predict a value for each of these elements of reality, i.e., for every spin component of particles L and R. Because these are all of the measurable variables of our system, such a state description would be deterministic.[17] We have already noted that a deterministic state description necessarily possesses predictive completeness, therefore, we have shown that EPR completeness implies predictive completeness for our experiment. It is also apparent that predictive completeness is a much weaker condition than EPR completeness, since Eq. (4) may hold for a purely probabilistic state description which does not prescribe definite values for any elements of reality.

We have thus vindicated the conclusion of EPR that the QM state description is not complete by showing that it fails to satisfy the weaker and more general property of predictive completeness. However, whereas EPR believed that a complete state description was possible, the implication of Bell's theorem and the analysis in this paper is just the opposite. For we have shown that any conceivable state description (denoted by λ in the preceding sections) that satisfies simple locality and predictive completeness will obey Bell-type inequalities which are violated by both QM and experimental results. Thus the "incompleteness" is, in some sense, a property of nature.

VII. CONCLUSION

We have shown that strong locality, which is the form of locality used to derive Bell's theorem and its generalizations, is logically equivalent to the conjunction of simple locality and predictive completeness of the state description. Simple locality is the condition that a spin correlation measurement must obey in order to satisfy the relativistic prohibition of superluminal communication. Quantum mechanics obeys simple locality, so there is no contradiction between quantum mechanics and special relativity. The violation of Bell-type inequalities by quantum mechanics is due to the failure of predictive completeness.

The notion of completeness introduced by Einstein, Podolsky, and Rosen is stronger than predictive completeness and implies it. Thus we have vindicated the EPR conclusion that the quantum mechanical description of reality is not complete, and have done so independently of the details of their argument. But contrary to the belief of EPR, it is not merely the quantum mechanical state description that is "incomplete" (in their sense, and in our more general sense). Rather it is the case that *any* state description that yields agreement with the statistical predictions of QM, in particular those that violate Bell's inequalities, must be "incomplete." Since the violation of Bell's inequalities has been confirmed by experiment, this "incompleteness" is, in some sense, a property of nature.

[1]J. S. Bell, Physics **1**, 195 (1964).

[2]One may wonder why so few physicists are apparently concerned about (or even aware of) such a crisis. It is certainly not because of any simple or well-known resolution of the apparent contradiction. A more likely explanation is to be found in the fact that Bell's theorem was originally presented as a contradiction between quantum mechanics and a certain class of hidden-variable theories. Although more general derivations of Bell's theorem have freed it from any assumption of hidden-variable models, the more profound significance of Bell's theorem has not yet become common knowledge.

[3]Some papers that argue this point in greater detail are W. B. Rolnick, Phys. Rev. **183**, 1105 (1969); G. A. Benford, D. L Book, and W. A. Newcomb, Phys. Rev. D **2**, 263 (1970); D. J. Thouless, Nature **224**, 506 (1971). A bibliography about tachyons is given by L. M. Feldman, Am. J. Phys. **42**, 179 (1974).

[4]A referee has pointed out that the probability for the outcome of two measurements can depend upon the order in which they are performed, but our notation does not take this into account. This defect can be remedied by regarding the device state S_L as including the specification of the time at which the L measurement takes place, and similarly for S_R. (The L and R devices could be preprogrammed to automatically operate at those times. If the L and R devices are far enough apart the preprogramming of them could, in principle, be separated by a spacelike interval.) Our argument then proceeds without modification.

[5]For a more detailed proof see J. P. Jarrett, Noûs **18**, 569 (1984), where the converse of this theorem is also proved.

[6]Here and in what follows, while it is assumed that experimenters are able to prepare systems in the desired states, the inability to do so in practice does not undermine the result to be shown, which has to do with what would be possible "in principle."

[7]It might be emphasized here that E_R cannot perform measurements on some of the R particles in the ensemble and decline to do so for the rest. His choice is to perform a $\mathbf{d}_R$ component spin measurement on all of the R particles or to perform no measurements on any of them.

[8]If we were to require that E_L be able to infer E_R's decision with absolute certainty, then this argument would break down; but surely such a demand is unreasonable. Either E_R performs his measurements or he does not, and the probability for the given outcome of E_L's measurement depends on which of these is the case. That one might never have conclusive experimental evidence (i.e., zero uncertainty) for the value of a theoretical probability should not be taken as a challenge to the relativistic basis of simple locality.

[9]One further qualification ought to be mentioned here. Since the backward light cones of the relevant events do overlap, it is at least possible that some unknown, but nevertheless perfectly "local," mechanism produces precisely the same correlation between E_R's decision and E_L's measurement outcomes as would have occurred had there actually been a superluminal physical disturbance. In the absence of any positive grounds for taking such "conspiratorial" possibilities seriously, however, we do not take them to compromise the argument given here in any significant way. We do acknowledge, however, that because the viola-

tion of (1) does not actually *entail* the superluminal transport of matter energy, such a violation *need* not correspond to a violation of relativity theory. In this case, however, it is not clear that the violation of simple locality could be understood at all.

[10]The sense of the word "determinism" used here is that the state description at any instant of time should determine the results of all measurements that may be performed at that time. This notion of a *deterministic state* is distinct from the question of *evolutionary determinism* (the state at one time determining the states at future times). Bell's original paper (Ref. 1) and the pedagogically simple derivations by N. D. Mermin, Am. J. Phys. **49**, 940 (1981); Phys. Today **38** (4), 38 (1985) in effect assume underlying deterministic states.

[11]The most recent experiment is by A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).

[12]Strictly speaking, there are two logical possibilities: (a) an underlying determinism does not exist; (b) deterministic hidden variables exist and violate relativistic locality, but we are unable to measure or manipulate them and so cannot employ them to transmit superluminal signals. But (b) is based on an unreasonably narrow interpretation of special relativity (SR). It is generally believed that SR governs nature itself and not merely our (temporary and variable) knowledge of nature. For example, speculations about the unobserved (and perhaps unmeasurable) interactions between quarks invariably assume that SR applies to any such interactions. One may question whether it is meaningful to suppose the existence of something that is *in principle* unobservable, in which case one is led back to (a).

[13]Some examples of papers using the equivalent of our strong locality condition are J. S. Bell, in *Foundations of Quantum Mechanics, Proceedings of the International School of Physics 'Enrico Fermi'*, edited by B. d'Espagnat (Academic, New York, 1971), Course 49, p. 178, Eq. (4.2); J. F. Clauser and M. A. Horne, Phys. Rev. D **10**, 526 (1974), Eqs. (2) and (2a). J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978) give careful discussions of several derivations of Bell's theorem. None of these references distinguish between strong locality and the requirements of special relativity. However, a distinction between two kinds of locality is made by H. P. Stapp, Found. Phys. **15**, 973 (1985).

[14]Although the derivation of Bell's theorem and of the various results in this paper do not require consideration of the time dependence, if any, of the state parameter λ, the interpretation of Eq. (4) and its intuitive plausibility may require further elaboration if λ is time dependent. Such matters are discussed in detail by J. P. Jarrett, Ann. NY Acad. Sci. **480**, 428 (1986).

[15]A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[16]We are assuming here the correctness of the quantum theoretical predictions of correlations that follow from conservation of angular momentum.

[17]To forestall the resurrection of an old misinterpretation, we emphasize that the EPR argument does *not* postulate determinism. It is only because of the simple character of our model that *all* of its measurable variables are identifiable as EPR elements of reality. Thus an EPR complete state description for this model must be deterministic, in the sense that the state at time t determines the results of any measurements that may be performed at that time. Of course nothing is said about the deterministic or indeterministic character of evolution (i.e., whether or not the state at one time determines the states at future times).

Available work from a finite source and sink: How effective is a Maxwell's demon?

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(Received 19 May 1986; accepted for publication 11 September 1986)

The maximum work obtainable from a finite heat source and finite heat sink, initially at respective temperatures T_+ and T_-, is determined as a function of the temperature ratio $\tau = T_-/T_+$ and the heat capacities of the source and sink. The thermal efficiency with which this work is delivered is found to be well approximated by $\eta^* = 1 - \tau^{1/2}$ for $\tau \geqslant 0.1$, independent of the source and sink heat capacities. It is noted that η^* occurs in other contexts for which work or power output is optimized, and is a surprisingly "universal" efficiency. A reversible polycycle that delivers the maximum work using an ideal gas working fluid is found to exist only if the heat capacity of the heat sink exceeds that of the working fluid. An example of a finite source/sink combination from which work can be generated is an enclosed gas, divided in half by a partition with a small, controllable trap door operated by a Maxwell's demon. If the demon opens and closes the door selectively, so as to achieve a temperature difference across the partition, the analysis here enables an estimate of the subsequent maximum work that can be generated and the efficiency of this generation. Numerical estimates show that, as might be expected, such a demon is a rather ineffective work producer.

I. INTRODUCTION

How much mechanical work is "available" for delivery via the transfer of heat from a finite heat source and the rejection of a portion of that heat to a sink at a lower temperature? This question has been addressed by a variety of authors under the assumption that the sink is a heat reservoir with an *infinite* heat capacity.[1] Such studies help explain, among other things, why the available work in a flame, which has finite heat capacity, is only a fraction of

