Light-in-flight recording. 4: Visualizing optical relativistic phenomena

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The modified holodiagram is used to solve and visualize in a graphical way a number of problems that are important for the evaluation of ultrahigh-speed recordings. A simplified diagram is introduced to explain the focusing effect of fast-moving light sources or observers. The diagram is used to show the distortion of an orthogonal coordinated system to simplify the study of apparent deformations of arbitrary shaped rigid objects. These distortions are compared with those of pulse fronts or wave fronts of light observed with holographic light-in-flight recordings.

I. Introduction

Einstein's theory of relativity is physically based on 1-D measurements using a conventional interferometer (the Michelson-Morley experiment). Now that holographic interferometry enables us to make 3-D measurements new possibilities exist to solve evaluation problems in connection with relativistic distortions of time and space. Certainly much of the knowledge now applied to fringe interpretation could be used to interpret image distortions caused by ultrahigh velocities of the recorded object. The advent of picosecond and femtosecond laser pulses will revolutionize high-speed photography and make compensations for relativistic effects necessary. This is especially the case when light-in-flight recordings by holography are used because they represent four dimensions: the three dimensions found in ordinary holograms plus the time domain.

II. Relativistic Holodiagram

The diagram of Fig. 1 (Fig. 4 in Ref. 1) shows an experimenter (the traveler) who travels from left to right at a constant velocity (v) which is 0.6 that of light (c). As he passes through stationary space filled with particles that scatter light, the traveler emits picosecond light pulses at A_1, A_2, A_3, and A_4. The time separation (t) of the pulses is constant. At time t after the last light pulse (at A_4), he makes a short (picosecond) observation at B. The traveler then finds himself surrounded by four luminous spheres all with himself in the center. We, who are stationary, understand that the traveler is surrounded, not by four observation spheres but by four observation ellipsoids all with one common focal point at B, the other focal points being A_1, A_2, A_3, and A_4.

The ellipsoids are identical in shape (the same eccentricity), only the scale factor differs. Straight lines radiating from B intersect all ellipsoids at identical angles and the distances between adjacent intersections are also constant along these lines. The distances separating A_1, A_2, A_3, A_4, and B are $t \cdot v$. The diameters (d) of the spheres are

$$d = t \cdot c\sqrt{1 - \left(\frac{v}{c}\right)^2}. \qquad (1)$$

The minor axis of each ellipsoid is identical to those of the spheres while the major axis is $t \cdot c$.

The straight lines, in the following referred to as q lines, radiating from the point of observation (B), represent the traveler's lines of sight aberrated by his velocity. On each q line the angular direction prior to aberration is printed, that is, the direction in which the traveler is looking, e.g., the angle of the axis of a telescope he uses for his observation (angle γ of Fig. 2).

In Ref. 1 we showed how the "relativistic holodiagram" could be used to visualize all the relativistic optical phenomena described by Einstein in his 1905 paper. Let us now study how our concept can be used to understand and visualize the answers to relativistic questions that have been raised in later years.

III. "Headlight Beam"

One phenomena pointed out by Weisskopf[2] is that spherically emitted light from a moving source is con-

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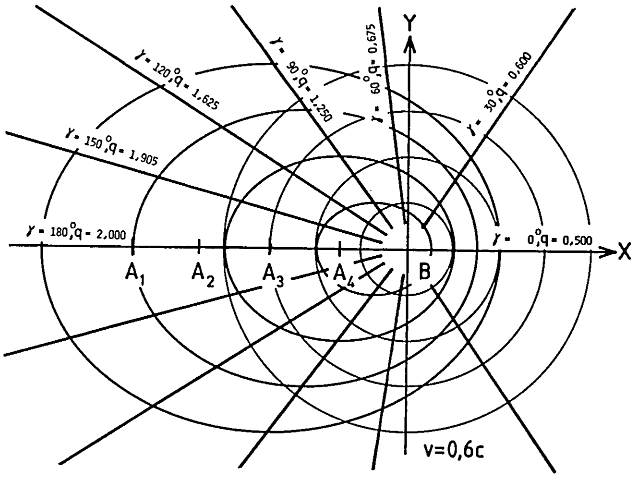

Fig. 1. A traveler moves at constant speed (0.6c) to the right. His lines of sight are aberrated from angle γ to the angles drawn in the diagram (the q lines). Along each q line the separation of the intersections by the ellipsoids of observation have a constant value, the q value. Doppler ratio, apparent speed of time, and apparent angular and longitudinal magnification are all functions of q, while transversal Doppler shift, time dilation, and Lorentz contraction depend only on the q line representing $\gamma = 90°$.

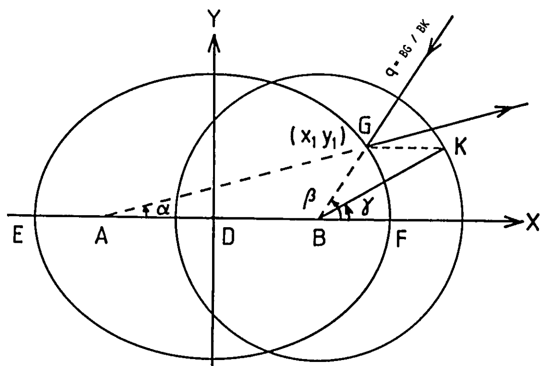

Fig. 2. To the traveler an arbitrary point G of the stationary world appears to exist at K which is found by drawing a line of constant Y value from G to the sphere. Light rays emitted by the traveler at angle γ are aberrated by his velocity to angle α, while his lines of sight are aberrated from angle γ to angle β.

centrated like a "headlight beam" in the forward direction. This fact was stated in Sec. III.A and Fig. 2 in Ref. 1 but because our goal then was to study image distortions we discontinued our work on the emitted rays and concentrated on the lines of sight.

Let us now study the emitted light rays in more detail and assume that the traveler who is moving from left to right directs a laser in the direction BK of Fig. 2. The direction of the beam in relation to the stationary world will then be AG. Point G is found by drawing a line parallel to the line of travel (the x axis) from the point (K) on the sphere of observation to the corresponding point (G) on the ellipsoid of observation. As the point of observation is identical in space and time in the two systems the center of the spheres should coincide with the focal point of the ellipsoid of observation as in Fig. 1.

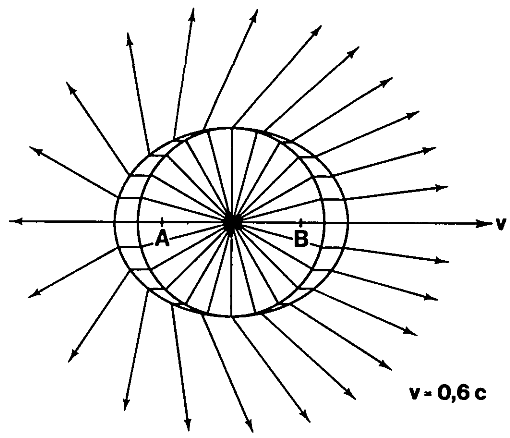

Fig. 3. The traveler in the center of the circle is moving to the right. The radii inside the circle represent directions of the traveler emitted light rays as seen by himself, while the lines leaving A represent those outgoing rays after they have been aberrated (as seen by the rester). Thus, light thrown in the forward direction appears focused as if by a positive lens, while light thrown backward appears diverged as if by a negative lens.

However, to simplify the drawing and the visualization we make a new diagram (Fig. 3) where the center of the sphere is in the center of the ellipsoid. The graphical result is identical to that of Fig. 2 but the diagram is less clustered by lines and the aberration is easier to follow. Thus the radii in the circle represent directions of the emitted light rays as seen by the traveler, while the lines leaving A represent outgoing rays after they have been aberrated, as seen by a stationary observer (the rester).

From Fig. 3 it is easy to see that the emitted rays are concentrated forward as if there was a focusing effect by a positive lens. This phenomenon, which was already pointed out by Einstein, results in the light energy from a moving source appearing to be concentrated forward, for two reasons: the light frequency is increased by the Doppler effect and the light rays are aberrated forward. This explains why the electron–synchrotron radiation appears in intensity sharply peaked in the forward tangential direction of motion of the electrons as described by Weisskopf in Ref. 2. In the backward direction we have the opposite effect. The light is defocused as if by a negative lens.

In Fig. 4 the lines of sight are seen aberrated backward in the direction found by drawing a line through G from focal point B of Fig. 2. The traveler is still moving to the right and his direction of observation (telescope axis) is in the direction BK. Thus the stationary world around the traveler appears concentrated in the forward direction (as demagnified by a negative lens). In the backward direction we have the opposite effect. The stationary world appears magnified as if by a positive lens.

The concept of the lines of sight of Fig. 4, which are identical to the q lines of Fig. 1, appears to demonstrate an advantage of our approach compared with other techniques. This idea is derived from the concept of "wavefronts of observation" which was introduced for

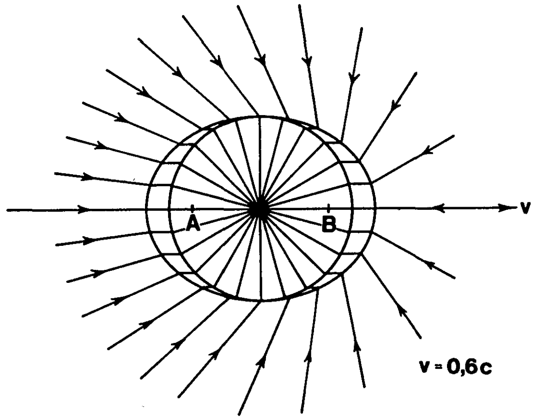

Fig. 4. Same situation as in Fig. 3. The radii inside the circle, however, now represent the traveler's line of sight (direction of his telescope) instead of outgoing light rays. The lines leaving B represent the traveler's aberrated lines of sight. Thus, to the traveler the stationary world appears in the forward direction demagnified as if by a negative lens while in the backward direction it appears magnified as if by a positive lens.

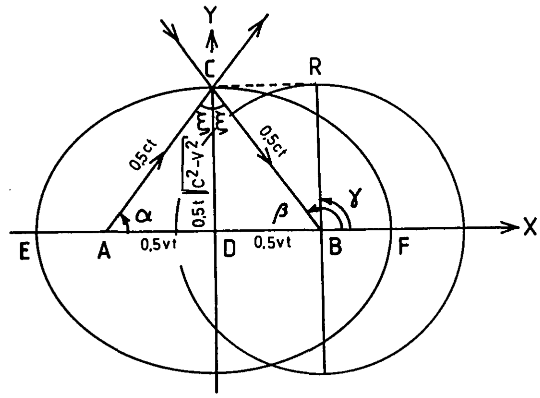

Fig. 5. The traveler looks in the direction (γ) perpendicular to his direction of travel which is to the right. His line of sight is aberrated backward to angle β. He sees a stationary object at C as if it existed at R, while the viewing angle in the image is unchanged. The object therefore appears rotated at angle γ. When it approaches, its back side is seen even before the object seems to be at the closest range.

the evaluation of holographic interference fringes by the use of a conventional holodiagram (Ref. 3, p. 32).

IV. Changes of Speed

If I look at a star that is one light-year away its light will immediately become Doppler shifted when I nod my head. But if the star suddenly changes its direction it will take one year before the Doppler-shifted light reaches my eyes.

Thus the diagram of Fig. 1 is correct when point B represents an observer traveling at constant velocity from left to right through objects of the stationary world. It is correct even immediately after B makes a change of velocity or direction.

The diagram is also correct when point B represents a stationary observer that is passed by objects moving at constant velocity from right to left. It is, however, not correct when these objects make any changes of velocity or direction. It will become correct first when the object velocity again has been constant for enough time for the light to pass from object to observer. This important restriction is valid not only for our diagram but for special relativity as a whole. Thus, when B represents a stationary observer who is studying a fast-moving object, the relativistic holodiagram of Fig. 1 changes with the speed of light as the object changes its direction of travel. However, if the observer at B changes his direction of travel in a stationary world, our diagram changes with infinite speed (faster than light).

This concept also explains why acceleration but not velocity can be detected in a closed system. If observer and object move as a unit, the apparent red shift and blue shift will compensate each other, but after a change in velocity there will be a delay before the compensation again takes place.

Now after visiting the border of general relativity, let us go back to the holodiagram used within the limits of special relativity and study how it, in a general way, can visualize the apparent relativistic distortion of objects moving at high velocities.

V. Relativistic Rotation

In 1959 Terrell published a paper,[4] in which he states that a camera photographing a fast-moving object will not record any Lorentz contraction; instead it will record an apparent rotation of the object. In the following we study whether this statement agrees with our diagram.

This situation would be as seen in Fig. 5. The observer is moving from left to right which in this case is identical to the object moving from right to left. The stationary camera at B observes an apparent object rotation of ξ at the moment when its distance appears to be at a minimum at R. This is the situation we study in the following and this is the situation in which the two theories agree completely. There is no doubt that the relativistic rotation at large becomes easy to understand and almost trivial when we study Fig. 5 and simply compare the view of an object at C with that seen from either D or B.

A. Lorentz Contraction

Let us start with Terrell's first statement that "in contrast to what has been previously expected, the Lorentz contraction is invisible when recorded two-dimensionally by using a camera."

The apparent distortion of a fast object, as stated by the special theory of relativity, results in the front and the back surfaces of, e.g., a passing car appearing tilted. The sides that are parallel to the motion appear, however, to stay this way independent of the velocity. Thus the car appears to be sheared, not rotated. Terrell, however, states that an observer using a camera cannot decide from the photo whether the object is sheared or simply rotated. The reason behind this statement is that the sides that are parallel to the travel are shortened by the Lorentz contraction so that their visual solid

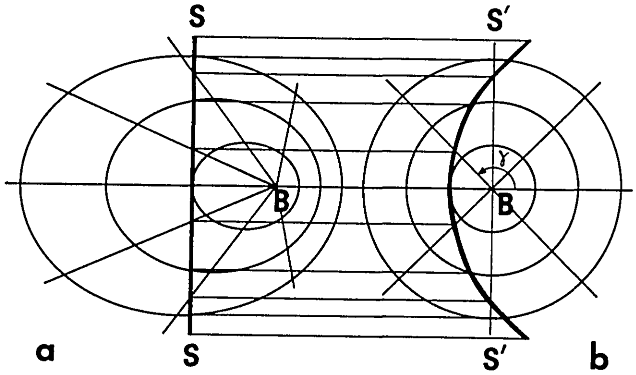

Fig. 6. A flat surface (s–s) perpendicular to the direction of travel will appear distorted to the traveler (B) into the curved surface s'–s', which is constructed in the following way: Draw lines parallel to the x axis from the points where s–s intersects the ellipsoids of observation until they reach corresponding points on the spheres of observation. The same can be done going from the intersections with the aberrated lines of sight (q lines) to the original lines of sight (at angle γ). Connecting these points results in the traveler's impression s'–s' of the stationary surface s–s. (a) Ellipsoids of observation and aberrated lines of sight, (b) Spheres of observation and original lines of sight.

angle is decreased as if the total object had been rotated without any distortion at all. He proves that the Lorentz contraction has exactly the value to make this interpretation possible.

Let us now see whether the statement about the "invisibility of the Lorentz contraction" agrees with our derivation of the Lorentz contraction from the holodiagram.

Equation (1) in Ref. 1 states that the Lorentz contraction is represented by

$$\frac{l_T}{l_R} = \sqrt{l - (v/c)^2} = l/k, \tag{2}$$

where l_T = the length as measured by the traveler,
l_R = the true length (as measured by the rester), and
$k = l/\cos\xi$ (Fig. 5).

The resulting equation,

$$\frac{l_T}{l_R} = \cos\xi, \tag{3}$$

simply states that l_T appears shortened as if it were seen from the angle ξ to the normal. Thus we find full agreement with Terrell's first statement. Looking at our Fig. 5 it is almost trivial to understand that the observer at B sees the object C (traveling from right to left in relation to the observer) almost 45° from behind. When he looks in direction BR he sees along the aberrated line of sight BC. Further discussion of this phenomena is given in Sec. VI and Fig. 7.

B. Distortion of a Sphere

Terrell's second statement is that the Lorentz contraction is also invisible in the case of a passing sphere. One would expect the sphere to appear shortened along its line of travel so that it is transformed into an ellipsoid, but according to Terrell no such effect is seen in a photographic recording. Let us see whether this statement also agrees with our derivation of the relativistic effects based on the holodiagram.

The sphere differs from the studied car in that a rotation of a rotational symmetric object is invisible. Therefore it is sufficient for us to study whether there is any change in the solid angle of view.

In Ref. 1, Eqs. (13) and (14) show that

$$M_a \cdot M_l = 1, \tag{4}$$

where M_a is the angular magnification and M_l is the longitudinal magnification (along the line of sight). Thus the solid angle of view appears to depend inversely on the distance, as we are used to in our stationary world. Accordingly, the sphere appears essentially circular in outline, independent of its velocity. However, this result is restricted to small solid angles of view. This is also true for our derivation of M_a because Eq. (14) in Ref. 1 was produced by differentiation, and represents the only approximation in that paper. This restriction limits the use of the equations but in no way limits the use of our diagram, which clearly shows the apparent relativistic rotation to be only a by-product of the total object distortion, as seen in our Fig. 7 and in Fig. 5 in Ref. 1.

We now understand one advantage of holographic methods over photographic. They include the distance information so that the difference between shear and rotation is seen, thus making the Lorentz contraction visible as described in Sec. VI.

VI. Transformation of an Orthogonal Coordinate System

Now we use the graphic method introduced in Fig. 4 in Ref. 1 to study the transformation of an orthogonal coordinate system (Fig. 6).

The traveler (B) is passing from left to right at a speed of 0.6 c through the stationary world. Figure 6(b) shows the traveler's spheres of observation and his lines of sight. In Fig. 6(a) we see how, in relation to the stationary world, his lines of sight are aberrated and his spheres are transformed into ellipsoids. Let us study one stationary straight line that is perpendicular to the direction of travel and see how it appears distorted to the traveler.

From every point at which the studied stationary line of Fig. 6(a) is intersected by an ellipsoid or by an aberrated line of sight, a horizontal line is drawn to Fig. 6(b) until it intersects the corresponding sphere or the corresponding (unaberrated) line of sight. The curve connecting these intersections in Fig. 6(b) then represents the straight line of Fig. 6(a) as it appears distorted to the traveler.

In Fig. 7(a) a total stationary orthogonal coordinate system is shown and in Fig. 7(b) we seen the corresponding distorted image as observed by the traveler, represented by the small circle (i,O) passing from left to right. The identical transformation would occur if the observer at B were stationary and instead the orthogonal coordinate system were passing him with the constant speed of 0.6 c from right to left. From Fig. 7(b) we find the following:

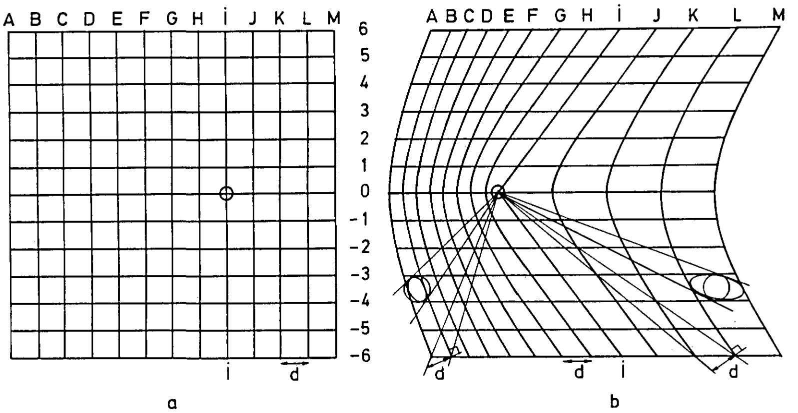

Fig. 7. Orthogonal coordinate system of the stationary world (a) appears to the traveler transformed into that of (b). The traveler exists at the small circle (i,O) and is moving to the right at a speed of 0.6c. This situation is identical to that when the observer is stationary while the coordinate system is moving to the left. From the diagram we find that flat surfaces are transformed into hyperboloids that point like arrows in the direction of travel. The plane (i–i) through the observer is transformed into a cone. The back side can be seen on all objects that have passed this cone. The separation of advancing hyperboloids is increased, while that of those moving away is decreased.

A. Surfaces Parallel to Velocity

All apparent displacements are caused by the fact that different points on the object are studied at different points of time. During this time difference the object has moved in relation to the observer, but only along its line of motion. Thus, flat surfaces parallel to the direction of velocity are not changed in respect to flatness, angle, or separation.

B. Surfaces Orthogonal to Velocity

(1) Flat surfaces moving toward the observer are transformed into hyperboloids that appear convex to the observer.

(2) Flat surfaces moving away from the observer are transformed into similar hyperboloids that appear concave to the observer.

(3) The flat surface passing through the observer (i–i) is transformed into a cone.

(4) The observer can see the back side of all objects that have passed through the surface of the cone.

(5) The spacing of the surfaces moving toward the observer is increased.

(6) The spacing of the surfaces moving away from the observer is decreased.

(7) Let the original spacing (d) rotate so that it is kept normal to the hyperboloid surface. Then it will always occupy the same angle of view as the spacing of the hyperboloids. This fact, which confirms Terrell's statement, is true only for infinitesimal angles of view.

We have compared results from Fig. 7 with those of several other workers and found good agreement. Bhandari[5] states that a vertical line moving at high speed assumes the shape of a hyperbola. Mathews and Lakshmanan[6] criticize the concept of relativistic rotation and introduce "the train paradox" as follows: When a fast-moving train is studied should one imagine each boxcar to be rotated or the train as a whole rotated? What happens to the stationary rails? Finally they conclude that the rotated appearance is not self-consistent. We agree with this statement, and the train is easily visualized in our Fig. 7 as being one of the horizontal rows of deformed squares, from which it is obvious that the distortion of the total train cannot be explained as caused solely by rotation.

However, the statements by Terrell that objects appear rotated but nondistorted (Sec. V) are verified in Fig. 7 within the assumptions that the studied objects subtend sufficiently small visual angles and that changes in distances are neglected. It is seen from our diagram that the solid angle of sight of the separation of the hyperboloids varies as if the original separation (d) had rotated to keep it perpendicular to the hyperboloids. Thus each infinitesimal original square which has been distorted into a rhomblike shape might, to the observer, appear to be rotated. Further, small spheres are transformed into ellipsoidlike shapes that, however, cover approximately the same solid angle of view as do the original spheres. It is interesting to note that these two statements are equally true whether the object appears Lorentz contracted or expanded as seen in the diagram in Fig. 7.

Even if our diagram in a limited way verifies Terrell's statements, it does so only when the observation is made by a camera. As soon as holography is included, it is easy to study variations in distances and in this way distinguish between shear and rotation so that the large scale distortions as presented in Fig. 7 are observed.

Finally, in the paper by Scott and van Driel,[7] the celestial sphere is studied, and it is shown that at increased speed stars appear moved toward the point of travel. It is also pointed out "though a sphere remains circular in outline, the apparent cross section may be grossly distorted and in some conditions the outside surface of the sphere appears concave." The last statement is verified by our Fig. 7 where a flat surface that has passed the observer appears concave.

Thus we have shown that our diagram (Fig. 7) in a concentrated form produces results that agree with those found in different publications based on the Einstein equations.

C. Comparison to Wave Fronts

Let Fig. 7(b) represent an orthogonal coordinate system moving from right to left at the speed of 0.6 c past the stationary observer at B. As the velocity is increased and approaches that of light, the cone angle mentioned in Sec. VI.B(3) approaches zero. Thus all approaching objects appear rotated through 90° so that their back side [as mentioned in Sec. VI.B(4)] can be

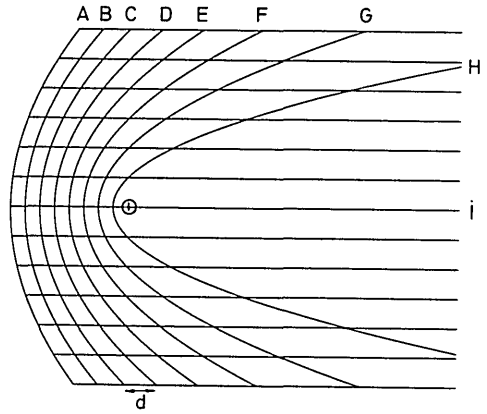

Fig. 8. Orthogonal coordinate system identical to that in Fig. 7(a) is made up of light using horizontal light rays and vertical pulse fronts (wave fronts) that pass from right to left. An observer makes a light-in-flight recording and finds that the flat pulse fronts appear transformed into a set of paraboloids with himself at the common focal point. The back side is seen on all pulse fronts.

seen. The separation of the advancing hyperboloids [mentioned in Sec. VI.B(5)] approaches infinity. At the same time the hyperboloids moving away [mentioned in Sec. VI.B(2)] are transformed into paraboloids and their separation along the line through the observer (O) approaches zero.

It is interesting to compare these results with the apparent shape of the only known flat surfaces that move with the speed of light, namely, flat wave fronts or flat sheets of light that are studied by its scattered light (Fig. 8). The cone angle of Sec. VI.B(3) is zero and the back side of all visible wave fronts is seen. The separation of all approaching surfaces is infinite. As the wave fronts pass by they are tilted through 45° instead of approaching 90°. The wave fronts that move away from the observer are transformed into paraboloids instead of hyperboloids. Their spacing appears to be half of their value instead of zero.

The main difference between a set of flat solid surfaces moving perpendicular to their surfaces and flat wave fronts or flat sheets of light is that the former experience Lorentz contraction which the light surfaces do not. This fact has already been discussed in Sec. VIII.E in Ref. 1. To this could be added that the Doppler shift is a sign and a measure of relativistic transformations. The light reflected from a moving solid surface is Doppler shifted, but scattered light from a moving sheet of light is not, as the wavelength is independent of where it is observed.

The apparent length of the cars of a passing train is equal to the true length only at the angle of sight of zero Doppler shift [close to G–H in Fig. 7(b)]. The apparent length of light pulses, on the other hand, is equal to the true length only at an angle of sight that is perpendicular to light propagation (C–D in Fig. 8).

The rules for the apparent distortion of wave fronts or pulse fronts are much simpler than those of solid objects moving at relativistic velocities. In the following we give three simple rules concerning the practical use of light-in-flight recordings for the study of the shape of wave fronts or the shape of stationary objects.

The curvature of a wave front appears transformed into the curvature of a mirror surface so shaped that it would focus the total wave front into the point of observation. The reason is that a focusing mirror reflects light in such a way that the total wave front arrives to the focal point at one point of time. Thus a small flat wave front that passes by will appear tilted 45°. A larger flat wave front will reveal that it does not only appear tilted, it is also transformed into a paraboloid whose focal point is the point of observation. A spherical wave front appears transformed into an ellipsoid, one focal point of which is the point source of light (A) while the other is the point of observation (B). This configuration represents one of the ellipsoids of the holodiagram (Fig. 5 in Ref. 8).

Light-in-flight recording by holography can be used to reveal the intersection of a light slice and a scattering surface. By positioning this surface in a special way it is possible to produce a cross section of the apparent wave front which is identical to a cross section of the true wave front. Such an undistorted view of the true wave front is formed if the scattering surface is a part of *a sphere the center of which is the point of observation.* A good approximation of this configuration is a flat surface a large distance away with its normal directed toward the observer.

If the apparent wave front is flat it can be used to produce a flat undistorted cross section of a studied object of unknown shape. There are two ways *to produce a flat apparent wave front: R_A and R_B should either be infinite or equal but of different sign.* In practice the latter represents a convergent illumination beam [Eq. (20) in Ref. 8].

VII. Conclusion

In a new graphical way we have solved optical problems found in the literature about special relativity. To arrive at these results we have used the accepted concept of the constancy of the speed of light and the following tools: a string and two nails for making ellipses; a ruler, pen, and paper to draw the diagrams.

No further mathematical knowledge and no preknowledge about relativity is needed to make and use the diagram. The technique is based on a slightly refined diagram, the holodiagram, which was initially designed for holography and conventional interferometry. Simplicity and visualization are the main advantages compared with the uses of the conventional equations presented by Einstein. The result is that each velocity presented in a concentrated form in one diagram shows the distribution in space of the following phenomena: The Doppler shift of which the transverse red shift is found to be one special case. The aberration of light rays and lines of sight. The apparent rotation which is found to be a part of the more general object distortion. The Lorentz contraction which is found to be a special case of apparent expansions and contractions. The time dilation which is found to be a special case of more general apparent speeding up and slowing down of the speed of time.

Further, we have found that the diagram can be used directly after the observer has changed his velocity but not when the object studied changes its velocity. From this we conclude that the ellipses (ellipsoids) and the q lines (cones) of the diagram move at infinite velocity with the observer but only with the speed of light with the observed object.

Finally, we have shown that our method can be used to predict the relativistically distorted image or to restore the true image from it. The apparent distortion of a flat surface that approaches the speed of light is compared with the apparent distortion of a flat wave front observed, e.g., by holographic light-in-flight recording. It is found that these distortions are not identical but their main features are similar.

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