Light-in-flight recording. 3: Compensation for optical relativistic effects

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Light-in-flight recording by holography and other ultrahigh-speed recording methods are used to study objects at speeds that are a large fraction of that of light. At these velocities, usually referred to as relativistic, image distortions caused by the limited speed of light can no longer be neglected. A slightly modified holodiagram is introduced which can be used graphically to restore the images of objects that appear distorted because of a constant relativistic velocity. The diagram compensates automatically for all known distortions of time and space and should thus be especially useful for the evaluation of ultrahigh-speed motion pictures. The derivation of the diagram is simple and can be of value as an educational tool in the optics of special relativity.

I. Introduction

Recordings of phenomena moving at ultrahigh-speed velocities produce a distorted image because of the limited speed of light. Methods are therefore needed to restore the true shape of a moving object from its apparent shape.

Important tools for this work are the equations of Einstein's special theory of relativity.[1] However, by using the ellipsoids of the holodiagram we have found a graphical method to compensate for all the distortions caused by the flight time of the information-carrying light. This method, which includes the accepted formula for relativistic optical effects, appears to be easy to use and to understand.

Thus it is our impression that not only can the theory of relativity assist in the evaluation of ultrahigh-speed holograms, but that holography can also assist in the understanding of relativity. Einstein's special theory of relativity was physically based on the experiments by Michelson and Morley, who used a 1-D interferometer. Holography represents 3-D interferometry, and it is therefore plausible that the rules and the equations today found in the field of hologram interferomety could be used to further understand the principles of relativity.

Light-in-flight recording by holography is a method of recording phenomena that move with the speed of light or with a speed approaching that of light. Such velocities are usually referred to as relativistic because they cause effects that are explained by Einstein's theory of relativity. In Ref. 2 it is shown that spherical wave fronts appear elliptical if the point of emission of a light pulse (A) is separated from the point of observation (B). It was also demonstrated that the ellipsoids of the holodigram can be used to explain these distortions of the wave fronts.

In this paper we shall show that the conventional holodiagram can be used to explain relativistic optical effects if the separation of A and B represents the distance traveled during the time between pulse emittance and observation. By a slight modification of the holodiagram it becomes a practical device for visualizing the apparent distortions of time and space in all directions around a traveling observer or object. In a simple graphical way we derive the following:

The aberration of light rays and lines of sight; the longitudinal and angular (transverse) magnification; the apparent changes in size along the line of travel of which the Lorentz contraction is a special (transverse) case; the apparent change in the speed of time (clock) of which the relativistic time dilation is a special (transverse) case; the apparent change in wavelength of which the transverse Doppler shift is a special case; and the apparent distortion of rigid bodies of which the relativistic rotation is a special case of more general bending of straight lines into curves. Straight lines that are parallel to the direction of travel are in this case an exception as they always stay straight, while those passing through the point of observation stay straight only as long as the velocity is constant. Finally an example is given of a graphical restoration of the true shape from its relativistically distorted image.

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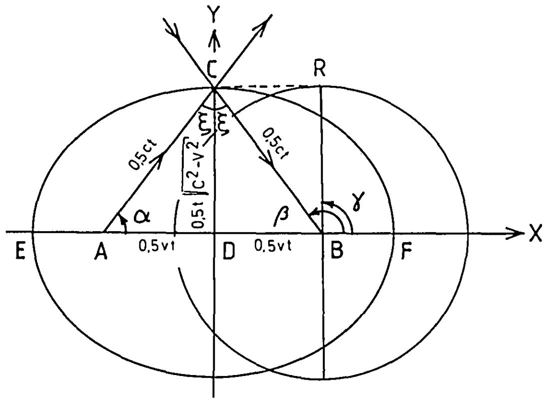

Fig. 1. Experimenter travels from left to right at a speed that is 0.6 times that of light. At A he emits a picosecond light pulse, at B he makes a picosecond observation. The ellipsoid of the holodiagram with A and B as focal points is then the spherical wave around A as seen in a scattering medium from B. Thus the sphere of observation around the observer at B is elongated into this ellipsoid, and objects in the stationary world appear to him to be contracted in the x direction. At C the Lorentz contraction is l/k, the transverse Doppler, ratio is k, and the time dilation is l/k, where (as in the conventional holodiagram) $k = l/\cos\xi = l/\sqrt{l - (v/c)^2}$. Light rays emitted at the angle γ are aberrated by velocity to the angle α, while the lines of sight are aberrated from the angle γ to β.

II. Illumination and Observation

A. Static Separation of Illumination and Observation

A man is making experiments based on gated viewing, which means that a short pulse of light (picosecond pulse) is emitted and, after a short period of time, e.g., 20 nsec, he makes a high-speed recording (picosecond exposure time). If the illumination point (A) and the observation point (B) are close together, the experimenter will find himself surrounded by a luminous spherical shell of ~3-m radius. This spherical shell can be seen only if something scatters the light, e.g., if the experiment is performed in a smoke-filled or dusty space. If there are large objects in the space, he will see these objects illuminated only in those places where they are intersected by the sphere.

The experiment described can be used to map the space around the experimenter and it is identical to well-known radar methods (radio detection and ranging). By changing the delay between emission and recording, intersections of different radius spheres can be studied. In this way the outside world is mapped in polar coordinates. The experimenter should keep in mind that, at the moment when he observes a 3-m radius sphere, its true radius is 6 m.

If the illumination point (A) and the observation point (B) are separated, the situation will be different. As the luminous sphere around A grows, the observer will see nothing until the true sphere reaches B. Then he will find himself inside an ellipsoidal luminous shell. One focal point of the ellipsoid will be A, the other B. By changing the delay between emission and recording, intersections of ellipsoids with different sizes (but identical focal points) can be studied. In this way the space around the experimenter can be mapped (in bipolar coordinates). The experimenter should know the separation of A and B so that his mapping will be correct. If he erroneously believes the separation is zero, he will misjudge the ellipsoids as spheres and make errors, e.g., in the measurement of lengths parallel to the line AB. He will also make angular errors because of the angular differences between the spherical and the ellipsoidal shells.

The ellipsoids described will in the following be referred to as the ellipsoids of the holodiagram. This diagram has been published as a practical device for making and evaluating holograms. The method of producing 3-D picosecond recordings of waved fronts (pulse fronts) has been published in Ref. 2. In Ref. 3 methods are described to compensate for errors caused by studying the apparent ellipsoidal wave fronts instead of the wanted true spherical ones. Finally Ref. 4 describes how a combination of the holodiagrams and the method of light-in-flight recording produces results that can be used to explain the Lorentz contraction and other relativistic effects.

In the following we shall make a closer study of the possibilities of using the concept of the holodiagram to visualize special relativity more generally. Our goal is to find a simple graphical way to restore the true shape of an object from its relativistically distorted ultrahigh-speed recording.

B. Dynamic Separation of Illumination and Observation

We have already described that, if the illumination point (A) is separated from the observation point (B) the gated viewing system produces recordings of intersections of ellipsoids having A and B as focal points. Now, let me, the author, and you, the reader, be stationary in a stationary space and study what a traveling experimenter will see of our stationary world when he travels past at relativistic velocity using light-in-flight recordings.

The traveling experimenter (referred to as the traveler) emits one single light pulse when he is at A and makes one single recording when he is at B. (The points A and B are both fixed in stationary space.) If his velocity (v) is close to the speed of light (c) he will travel a measurable distance (vt) during the time delay (t) between illumination and observation. Thus his observation sphere has, because of his velocity, been transformed into an observation ellipsoid with the focal separation AB equal to $v \cdot t$ (Fig. 1).

Perhaps the traveler does not know that he is traveling or perhaps he does not know that his velocity has the influence described on his sphere of observation. In any case he will, because of the eccentricity of the ellipsoid, observe more of the stationary world along his line of travel. Thus the stationary world will appear to him to be contracted along that line. This apparent contraction is in relativistic literature referred to as the Lorentz contraction.

Let us study Fig. 1. An experimenter travels along the positive x axis, he emits a light pulse at A, and makes a picosecond observation at B. The speed of the

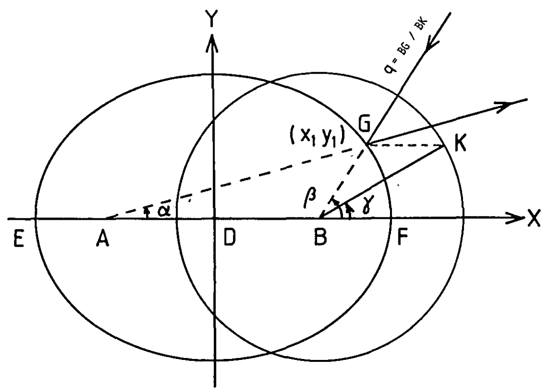

Fig. 2. Situation is identical to that of Fig. 1 but now we study how an arbitrary point G of the stationary world appears to the traveling observer at B. Because of his velocity the traveler's sphere of observation is transformed into an ellipsoid of observation. To the traveler G appears to exist at K which is found by drawing a line of constant y = value from G to the sphere. Light rays emitted by the traveler at the angle γ are aberrated by his velocity to the angle α, while his lines of sight are aberrated from the angle γ to β. The wavelength of light appears to the traveler to be changed in the ratio $BG/BK = q$, while all radial lengths appear to be changed in the ratio l/q. The speed of time (clock) also appears to be changed in the ratio l/q independent of whether there is an expansion.

experimenter (traveler) is v and the time delay between illumination and observation is t. Let us study the light rays passing ACB.

Our statements are based on the fact that distances measured by the speed of light will always produce the same results in all directions (if acceleration and gravity gradients are zero). The well-known experiment by Michelson and Morley has proved this fact. Further, we accept the statement that a measurement of the speed of light (c) always must involve information sent in a closed loop, a fact that has been fully discussed by Einstein. The result of such a measurement always produces the same value of (c) independent of any constant velocity of the light source or observer.[1]

We who are stationary (the resters) understand that the traveler's observation ellipsoid has its focal points at A and B and that light with the speed of c travels ACB. The traveler, on the other hand, believes his observation sphere to be centered at B and that the light simply has traveled with the speed of c in the path BRB. Thus everything in the stationary world parallel to the x axis will appear to the traveler contracted in the ratio:

$$\frac{l_T}{l_R} = \frac{2RB}{EF} = \frac{t\sqrt{c^2 - v^2}}{ct} = \sqrt{1 - \left(\frac{v}{c}\right)^2}, \tag{1}$$

where $EF = ACB$,

l_T = the length as measured by the traveler, and

l_R = the true length (as measured by the rester).

This result is identical to the accepted value of the Lorentz contraction.

In the conventional holodiagram the value $l/\cos\xi = l/\sqrt{l - (v/c)^2} = k$. Thus $l_T/l_R = l/k$. We, the resters, see a certain number of lightwaves along the path ACB. The traveler sees the same number of waves along BRB. Thus

$$\frac{\lambda_R}{\lambda_T} = \frac{ACB}{BRB} = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \tag{2}$$

where λ_R refers to the wavelength measured by the rester and λ_T refers to the wavelength emitted by the traveler.

Because of the symmetry of Fig. 1,

$$\frac{AC}{BR} = \frac{CB}{RB}. \tag{3}$$

Thus, the rester and the traveler observe an identical transverse red shift of each other's light.

Now, referring to the conventional holodiagram, $\lambda_R/\lambda_T = k$, it is interesting to note that in conventional uses of the holodiagram the factor k is used exactly as if it had the meaning of a red shift.

This result is identical to the accepted value of the relativistic transverse Doppler ratio. The result is also identical to the inverted value of the accepted time dilation. The reason is that the longer wavelength, and thus lower frequency, is explained as caused by a slower passing of time for the emitter (the traveler). Another way to say the same thing is that time (clock) must run slower for the traveler otherwise he would not accept that the light has only passed BRB when in reality it has passed the whole distance ACB. Thus

$$\frac{t_T}{t_R} = \sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{1}{k}. \tag{4}$$

The traveler does not observe that the angles α and β differ from 90°; he believes the light is simply traveling BRB. The reason can be described in the following way:

Let the traveler direct a telescope in the Y direction. As he passes the point A he emits a light pulse that leaves the telescope like a cannonball in the direction AC. The reason the light rays travel in the direction α toward C is that the telescope functions like a cannon that is moving sideways and therefore gives the cannonball a motion component in the direction of the positive x axis.

The light pulse bounces like a cannonball at C and returns in the direction toward B. As the light pulse enters the telescope it moves parallel to the moving telescope axis so that the traveling observer at B thinks it arrives in the direction of the Y axis, while a stationary observer would say it arrives at the angle β. Thus the traveler is in every way unaware that he is traveling.

The deviation of the light caused by the velocity of the traveler is calculated from Fig. 1 in the following way:

$$\tan\alpha = \frac{0.5t\sqrt{c^2 - v^2}}{0.5vt} = \frac{\sqrt{1 - \left(\frac{v}{c}\right)^2}}{\frac{v}{c}} = \frac{k}{\frac{v}{c}}, \tag{5}$$

$$\tan\beta = -\tan\alpha.$$

These two results are identical to the values from accepted equations for relativistic aberration. Thus, outgoing light (emitted by the traveler) is bent at the angle $\gamma - \alpha$ in the forward direction. Incoming light (the line of sight of the traveler) is bent at the angle $\beta - \gamma$ in the backward direction. These two angles have identical numerical values but different signs for $\gamma = 90°$. In the following we shall see if the concept of the ellipsoids of the holodiagram is also useful in the general cases when γ might have any value.

III. General Cases

A. Relativistic Aberration

Let us study Fig. 2 which is identical to Fig. 1 except for the studied point G which is at an arbitrary position on the ellipsoid. The traveler believes he is in the center (B) of his observation sphere and looks at one point K of the outside (stationary) world at the angle γ. However, the point K does not exist in the sphere but instead its true position is at G on the ellipsoid. The point G is found by drawing a line parallel to the x axis from K until it intersects the ellipsoid. Thus the aberrated line of sight is BG at the angle β and the aberrated emitted light tray is AG at the angle α.

From Figs. 1 and 2 the following calculations are derived:

$$\begin{cases} \tan\alpha = \dfrac{y_1}{x_1 + 0.5vt}, \\ \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, \\ \quad a = 0.5EF = 0.5ct, \\ \quad b = CD = 0.5\sqrt{(ct)^2 - (vt)^2}, \\ \quad y_1 = BK \sin\gamma = b \sin\gamma. \end{cases}$$

Result:

Outgoing light (by traveler-emitted light rays):

$$\tan\alpha = \frac{\sin\gamma \sqrt{1 - \left(\frac{v}{c}\right)^2}}{\frac{v}{c} + \cos\gamma}. \tag{6}$$

Incoming light (the traveler's lines of sight):

$$\tan\beta = \frac{\sin\gamma \sqrt{1 - \left(\frac{v}{c}\right)^2}}{-\frac{v}{c} + \cos\gamma}. \tag{7}$$

These two equations, solely derived from Figs. 1 and 2, are identical to accepted relativistic equations (e.g., Ref. 5, p. 49).

B. Relativistic Doppler Effect

Let us again study Figs. 1 and 2 and calculate the Doppler ratio. The traveler observes the wavelength λ_T as he measures the true wavelength λ_R from the stationary world:

$$\begin{cases} \dfrac{\lambda_T}{\lambda_R} = \dfrac{BG}{BK}, \\ \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, \\ \quad BK = b = 0.5\sqrt{(ct)^2 - (vt)^2}, \\ \quad BG = \sqrt{y_1^2 + (x_1 - 0.5vt)^2}, \\ \quad y_1 = b \sin\gamma = 0.5 \sin\gamma \sqrt{(ct)^2 - (vt)^2}, \\ \quad x_1 = 0.5ct \cos\gamma. \end{cases}$$

Result

Incoming light (the wavelength λ_T which the traveler observes when he studies the true wavelength λ_R from the stationary world):

$$\frac{\lambda_T}{\lambda_R} = \frac{1 - \frac{v}{c}\cos\gamma}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}. \tag{8}$$

This equation, derived solely from Figs. 1 and 2, is identical to accepted relativistic equations (e.g., Ref. 5, p. 47).

Outgoing light (the wavelength λ_R which the rester observes when he studies the true wavelength λ_T emitted by the traveler):

$$\frac{\lambda_R}{\lambda_T} = \frac{1 - \frac{v}{c}\cos\alpha}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}. \tag{9}$$

Because of the constant speed of light the apparent speed of time (t_t) to the traveler will be

$$\frac{t_T}{t_R} = \frac{\sqrt{1 - \left(\frac{v}{c}\right)^2}}{1 - \frac{v}{c}\cos\alpha}. \tag{10}$$

C. Zero Doppler Shift

Let us study Fig. 3 and seek the directions (γ) in which the traveler should look to experience zero Doppler shift. He should not look backward because in that direction there is a red shift. Nor should he look directly sideways because even then there is a red shift (the transverse red shift). Forward is the blue shift, so apparently he should look slightly forward. The way to find the zero Doppler shift for incoming light (γ of Fig. 3) and outgoing light (α) is as follows:

$G(x_1y_1)$ is the point where the traveler's sphere of observation (as seen by the traveler) and the ellipsoid of observation (as seen by the rester) intersect. Draw a line parallel to the x axis from G to the corresponding point on the sphere (K). Thus BK at the angle γ is the direction the traveler should look to see zero Doppler shift. BG at the angle β represents that direction after the line of sight has been relativistically aberrated.

The line AL, at the angle γ, represents the light rays that are emitted by the traveler and which the rester experiences as having zero Doppler shift. From Fig. 3

let us calculate the angle γ of zero Doppler shift from the intersection of the ellipsoid and the spheres:

$$\begin{cases} \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1, \\ x = 0.5vt - R\cos\gamma, \\ y = R\sin\gamma, \\ R = b = 0.5\sqrt{(ct)^2-(vt)^2}, \\ a = 0.5ct. \end{cases}$$

Result

Incoming light:

$$\cos\gamma = \frac{c}{v}\left(1-\sqrt{1-\left(\frac{v}{c}\right)^2}\right). \tag{11}$$

Outgoing light:

$$\cos\alpha = \frac{c}{v}\left(1-\sqrt{1-\left(\frac{v}{c}\right)^2}\right). \tag{12}$$

Exactly the same result is found by using Eq. (8):

$$\frac{\lambda_T}{\lambda_R} = \frac{1-\dfrac{v}{c}\cos\gamma}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}} = 1.$$

Thus Fig. 3 produces a result that is identical to that of accepted relativistic equations.

D. Longitudinal and Angular Magnification

The Doppler effect, or changes in the passing of time, influences our measurements of distances. A shorter wavelength, or a faster passage of time (clock), makes distances appear longer. Thus if λ_R/λ_T is larger than 1, longitudinal stationary distances appear to the traveler longer than their true value. The traveler experiences a longitudinal magnification M_l:

$$M_l = \frac{\lambda_R}{\lambda_T} = \frac{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}{1-\dfrac{v}{c}\cos\gamma}. \tag{13}$$

The relativistic aberration does not only change the direction of sight, its derivative also changes the apparent angular (transverse) size of objects. If $\delta\beta$ is smaller than $\delta\gamma$, there will be an angular magnification M_a.

Let us differentiate Eq. (7):

$$\tan\beta = \frac{\sin\gamma\sqrt{1-\left(\dfrac{v}{c}\right)^2}}{\dfrac{v}{c}+\cos\gamma}.$$

Result:

$$M_a = \frac{\delta\gamma}{\delta\beta} = \frac{1-\dfrac{v}{c}\cos\gamma}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}. \tag{14}$$

Thus $M_a = 1/M_l$ or $M_a \cdot M_l = 1$. The product of longitudinal and angular magnification is equal to unity everywhere. Only at the angle (cone) of zero Doppler shift are they equal; they both have a magnification factor of 1.

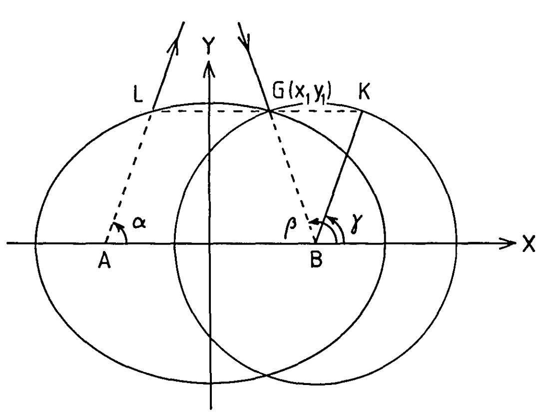

Fig. 3. Zero Doppler shift is observed by the traveler when he looks in the direction γ (where γ is defined in the traveler's world). The angle γ is found by drawing a line of constant Y value from the point of intersection of the sphere with the ellipsoid (G) until it again intersects the sphere (K). Zero Doppler shift of light emitted by the traveler is found at α (where α is defined in the stationary world). The angles α and γ are identical. As γ is <90°, the red shift in our universe is predominant over the blue shift.

To verify our concept of angular magnification let us study if it can be used to derive the Lorentz contraction. If the traveler looks normal to his travel (γ = 90 of Fig. 1), the aberrated line of sight has the angle β.

The traveler studies a stationary object that is situated at the Y axis of Fig. 1 and has the small true length l_0 parallel to the x axis. Because of the oblique observation the length appears shortened into l_B:

$$l_B = l_0\sin\beta = l_0\sqrt{1-\left(\frac{v}{c}\right)^2}.$$

Thus

$$l_{\text{app}} = \underset{\delta\beta}{\frac{l_B}{0.5ct}}\cdot\underset{M_a}{\frac{\delta\gamma}{\delta\beta}}\cdot\underset{CD}{0.5ct\sqrt{1-\left(\frac{v}{c}\right)^2}} = l_0\sqrt{1-\left(\frac{v}{c}\right)^2},$$

where l_{app} = apparent l_0 (to the traveler),
$\delta\beta$ = true angle of view, and
$\delta\gamma$ = the traveler's angle of view.

Thus our calculation based on the concept of an angular magnification (M_a) results in an apparent length which is identical to the Lorentz contracted length, which in turn is a special case of the longitudinal magnification (M_l).

IV. Complete Diagram

Figure 4 shows an experimenter (the traveler) who moves from left to right at a constant velocity (v) which is 0.6 of that of light (c). He emits picosecond light pulses at A_1, A_2, A_3, and A_4. The time separation (t) of the pulses is constant. At time t after the last light pulse (at A_4) he makes a short (picosecond) observation at B. The traveler then finds himself surrounded by four spheres all with himself in the center. We, who are stationary, understand that the traveler is surrounded not by four spheres of observation but by four ellipsoids

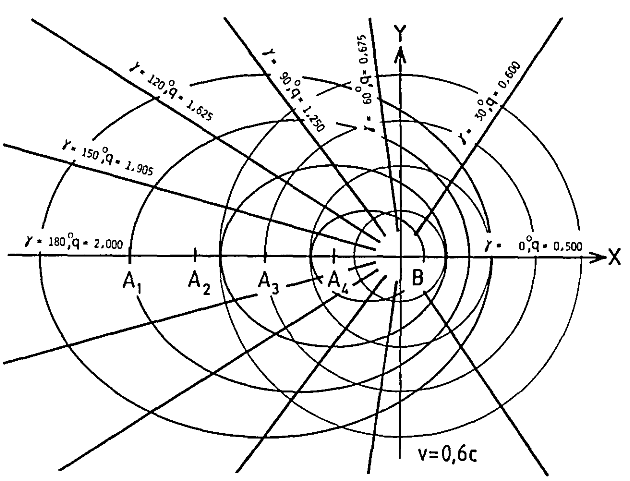

Fig. 4. With constant time separation the traveler emits light pulses at A_1, A_2, A_3, and A_4; finally he makes one observation at B. His concentric spheres of observation around B are transformed by his velocity into a set of ellipsoids that has one common focal point at B while the other focal points are A_1, A_2, A_3, and A_4. The traveler's lines of sight (q lines) are aberrated backward from γ to β as if there was a negative lens in front of the traveler and a positive lens behind him. The separation of the ellipsoids divided by the separation of the spheres along each q line defines its q value. Variation in the spacing of the ellipsoids results in that a longitudinal distance appears magnified in front of the traveler and demagnified behind him. The apparent speed of time (clock) is inverted proportional to the q value.

of observation all with one common focal point at B, the other focal points being A_1, A_2, A_3, and A_4. These ellipsoids are identical in shape (the same eccentricity); only the scale factor differs. Straight lines radiating from B intersect all ellipsoids at identical angles and the distances between adjacent intersections are also constant along these lines.

A. Relativistic Aberration

The straight lines, in the following referred to as q lines, radiating from the observation point (B) represent the traveler's lines of sight aberrated by his velocity. On each q line is printed the angular direction prior to aberration of the direction in which the traveler is looking, e.g., the angle of the axis of the telescope that he uses for his observation (angle γ of Fig. 2).

Thus, when the traveler looks 60° from the forward direction the aberration bends his line of sight so that in the stationary world it is directed slightly backward. When the traveler thinks he is looking perpendicular to the line of travel ($\gamma = 90°$) he is actually looking at the stationary world at a point almost 40° behind that direction ($\beta = 127°$). No wonder there exists a relativistic transverse Doppler shift toward longer wavelengths.

The traveler is moving to the right and all his lines of sight are rotated to the left as blown backward by a wind caused by his velocity. This picture is quite pleasant and easy to visualize. (The effect on the light rays emitted by the traveler is exactly opposite.) The aberration of the line of sight also results in objects in front of him appearing smaller while those behind him appear larger. There is an angular demagnification as by a negative lens in front of him and a magnification as by a positive lens behind him. The magnification factor is equal to the inverted value of the Doppler ratio (λ_T/λ_R) which is printed at each q line. λ_T/λ_R in the following is referred to as the q value or q.

B. Deformations of Space and Time

When the traveler is looking at the stationary world in front of him it appears as if time is going faster. Moving toward a clock results in its hands appearing to rotate faster, because the time of flight of the information-carrying light decreases as the distance decreases. This speeding up of time is represented by the shorter distances between the ellipsoids in front of the traveler.

The distances between intersections with the ellipsoids are constant along the lines of sight from B. As calculated in Eq. (8) the apparent wavelength is a function of this distance compared with the corresponding distance between the intersections of these lines with the spheres. Thus the Doppler shift is constant along the q lines. The apparent time shift is also constant along these lines, so is the longitudinal and the angular magnifications.

Where the separation of the ellipsoids along the q lines is short, time appears to move fast; where the separation is long, time appears to move slow. Thus there are different speeds of time in all different directions. The relativistic time dilation representing the transverse red shift is only a special (transverse) case of the different apparent speeds of time.

Where the separation of the ellipsoids along the q lines is equal to that of the spheres, time appears to be normal, or rather, clocks behave as if stationary in relation to the traveler. Our best clocks are light emitting atoms and therefore the slowing of time is equal to the Doppler red shift and accelerated time is equal to the blue shift. Along the q lines (Fig. 4) the Doppler shift is constant and the q value, the Doppler ratio (λ_T/λ_R), is printed. Thus the q line marked 30° represents the traveler's aberrated line of sight when he looks in the direction $\gamma = 30°$ (Fig. 4). The Doppler ratio he observes in that direction is $q = 0.6$, which means that he observes a wavelength (λ_T) that is 0.6 of the wavelength of the stationary world (λ_R). Thus the traveler sees a blue shift.

The direction of zero time shift and zero red shift is slightly forward ($\gamma \approx 70°$ as seen in Fig. 3). Thus more than 50% of the stationary world appears to the traveler to be red shifted. No wonder that the red shift is predominant in our universe. (A calculation from Fig. 3 where $v = 0.6c$ results in a ratio of red shift to blue shift of 2:1 even if the directions of motions were totally random.)

Where the ellipsoids of observation are closely spaced, the stationary world to the traveler appears elongated in depth. Thus the velocity with which the stationary world appears to run toward the traveler is increased for two reasons: its clocks run faster and it is elongated along its apparent line of travel.

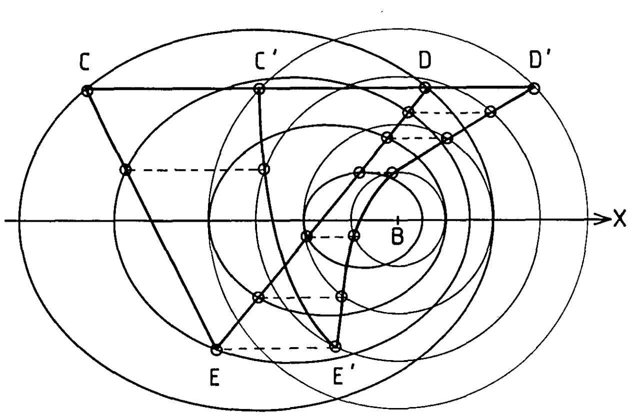

Fig. 5. Graphic visualization of the apparent distortion of the rigid stationary triangle *CDE* as seen by the observer at *B* who is traveling to the right at a velocity of 0.6*c*. From each intersection of the triangle by the ellipsoids a line is drawn parallel to the *X* axis until it reaches the corresponding sphere. The resulting deformed triangle *C′D′E* then represents the stationary triangle *CDE* seen by the traveling observer at *B*. It represents just as well a triangle *CDE* traveling to the left with 0.6*c* velocity as seen by a stationary observer at *B*. To restore the distorted triangle to its true shape the intersections are moved from the spheres to the ellipsoids. The true speed of time (clock) at, e.g., *D* is the observed speed of time (at *D′*) multiplied by *BD*/*BD′*.

The stationary world also appears much distorted as it, at the same time, is longitudinally magnified and angularly demagnified. When the traveler studies the stationary world behind him everything looks just the opposite. Clocks move slow and objects are longitudinally demagnified and angularly magnified.

C. Example of Graphic Evaluation

We have demonstrated that the ellipsoid of the holodiagram produces results that agree with those of accepted relativistic equations. The reason we have made all these comparisons is that it appears to be the best way, apart from experiments, to prove the correctness of our approach.

One advantage of our new approach to relativity is that it uses a method already accepted for conventional optics and therefore makes the conceptual step to relativistic optics rather small. Another advantage is that it visualizes phenomena in special relativity in a simple way and makes possible a graphic compensation for the measuring errors that otherwise occur in ultrahigh-speed recordings. In the following we shall demonstrate how the diagram can be used for practical evaluation of the true shape of rigid bodies, the images of which are relativistically distorted. The true shape is defined as the shape seen by an observer at rest in relation to the studied object.

Again, let a traveling experimenter at high velocity ($v = 0.6c$) pass through the stationary space (Fig. 5). He emits four picosecond light pulses with a constant time separation of t. After another time delay of t he makes one single picosecond observation (at B).

In the stationary world the triangle *CDE* exists. How will it appear to the traveler? Draw a line parallel to the x axis from the corner C on the ellipsoid until it reaches C' at the corresponding sphere. C' now represents the apparent position (to the traveler) of C. Make the same transformation of the corner D to D' and E to E'. Also make similar transformations of all the intersections of the sides of the triangle with the ellipsoids. The deformed triangle $C'D'E'$ then represents the traveler's view of the stationary triangle *CDE*.

The side $C'D'$ still is a straight line because it is parallel to the x axis. However $C'D'$ is shortened in relation to *CD* corresponding to the Lorentz contraction. (From the diagram of Fig. 5 we understand that the Lorentz contraction is a special transverse case of apparent length changes parallel to the x axis.)

The angle of D' is different from that at D because of the relativistic rotation of the segment of the side $D'E'$ that is closest to D'. However, as we study the whole side $D'E'$ we see it is not just rotated, it is relativistically distorted into a curve. The reason is that, in contrast to what happens along *CD*, the angle between ellipsoids and spheres varies along *DE*. (From the diagram of Fig. 5 we understand that the relativistic rotation is only a special case of relativistic distortions.)

The side $C'E'$ and the angles of C' and E' have changed similarly to $D'E'$. The sides are deformed as blown by a wind caused by the speed of the traveler.

The corner C has moved toward C' which is closer to the traveling observer at B. This result could be expected from the well-known Lorentz contraction. But the corner D has moved to D' which is further away from B. These results agree with our earlier statements that longitudinal distances are magnified in front of the traveler and demagnified behind him.

The changes in the directions of the lines of sight toward the displaced corners of the triangle also agree with our earlier statement that there is an angular (transverse) demagnification in front of the traveler and a magnification behind him.

At, e.g., the corner D' the longitudinal magnification is BD'/BD, and the angular (transverse) magnification is BD/BD'; the Doppler ratio is also BD/BD', while the speed of time (clock) is advanced in the ratio BD'/BD.

V. Conclusion

All the results described refer to the observations made by the traveler at B as he travels past the stationary true triangle *CDE*. The reasons we have studied this situation exclusively are twofold.

The first reason is that the ellipsoids of the holodiagram are already used in conventional 3-D optics, e.g., for evaluation of holographic interferograms. The conceptual step is small and one passes easily from this situation to the situation where the separation of the focal points of the ellipsoids is caused by the velocity of the observer.

The second reason is that it is convenient to visualize ourselves, you the reader and I the author, as stationary. When we are stationary we find it to be a simple task to measure the true shape of a stationary object; we simply use optical instruments or measuring rods or any other conventional measuring principle. We believe that we make no fundamental mistakes and thus accept our measurements as representing the true shape. No doubt the traveler has a much more difficult task and thus we do not trust his results but define them as representing the apparent shapes. However we accept the postulate as stated by Einstein:

The laws by which the states of physical systems undergo change are not affected whether these changes of state be referred to the one or the other of two systems of coordinates in uniform translatory motion.[1]

Thus all our statements including the diagram of Fig. 5 could just as well represent a stationary observer at B studying an object which at relativistic velocity travels toward the left. In that case Fig. 5 represents a triangle with the true shape CDE that passes by the stationary observer at B, who then sees the apparent shape $C'D'E'$.

Thus the method of Fig. 5 can be used to restore the true shape of any object that moves at relativistic velocity and is observed by any technique based on the emission and receiving of short pulses of radiation. Will that same restoration also produce the true shape for ordinary observations where the illumination is continuous? The answer is in the affirmative, as explained in the following.

The holodiagram in Fig. 4 is overdetermined (as is the conventional holodiagram). The aberrated lines of sight, the q lines, represent information that is identical to that of the ellipsoids. Thus the graphic transformation of Fig. 5 could just as well have been made using no ellipses and no circles, using only the q lines of Fig. 4.

Each corner of the undistorted triangle CDE is moved parallel to the x axis from the aberrated q lines (at angle β) to the unaberrated lines of sight, at angle γ. The triangle $C'D'E'$ then again represents the distorted image as it appears to the observer at B. At, e.g., the corner D' the longitudinal magnification is l/q, the angular magnification is q, the Doppler ratio is also q, while the speed of time (clock) is advanced in the ratio l/q. The q value is found at the q line through D (not through D').

Only the aberration of lines of sight has been used for this transformation of the triangle and therefore I believe that our method, even if it is based on the ellipsoids of the holodiagram, applies to the restoration of the true shape of an object independent of what optical method is used for its recording.

Our method describes the apparent distortions of objects caused by the elongation of the observation sphere into an ellipsoid. The knowledge of these distortions is important for evaluation of true shape and true passage of time when studying movie recordings of objects at relativistic velocities. This paper does not discuss any deeper meaning of the words apparent or observation sphere. It is, however, interesting to observe that, while a fast object appears shortened by Lorentz contraction, its observation sphere becomes elongated until, at the speed of light, it includes the total length of travel even for a pointlike object.

References

1. A. Einstein *et al.*, *The Principle of Relativity* (Dover, New York, 1952).
2. N. Abramson, "Light-in-Flight Recording: High-Speed Holographic Motion Picture of Ultrafast Phenomena," Appl. Opt. **22**, 215 (1983).
3. N. Abramson, "Light-in-Flight Recording. 2: Compensation for the Limited Speed of the Light Used for Observation," Appl. Opt. **23**, 1481 (1984).
4. N. Abramson, "Light-in-Flight Recording: Compensation for Relativistic Effects," presented at 16th International Congress of High-Speed Photography and Photonics, Strasbourg (1984) 491.172.
5. W. Rindler, *Special Relativity* (Wiley, New York, 1960).

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two-space dimensions. Other chapters discuss spectral transforms for matrix equations (Chaps. 15 and 16) and for partial differential operators (Chap. 14). The last two chapters lead back to the physics of nonlinear waves. Chapter 17 presents and discusses several examples of nonlinear wave systems which exhibit recurrence phenomena. The final chapter (Chap. 18) is devoted to the statistical mechanics of the sine Gordon field. Action-angle variables obtained from IST are used to calculate both the classical and quantum distribution functions, and apparent discrepancies are analyzed.

The articles in this volume are mostly fairly self-contained. The treatment is in most cases either implemented by a review and/or restricted to simple models and techniques demonstrating the essential points. The book will hence serve as a useful introduction to the topic and some aspects of current research. It should not be mistaken, however, for a comprehensive treatment of this vast and expanding subject.

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