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The Magnetic Pole, A Useless Concept

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The description of magnetism in terms of moving electric charges rather than in terms of magnetic poles reduces both the number of experimental laws upon which the theory is based and the number of new quantities defined. Certain sources of confusion are discussed and an elementary description is developed which permits a clearer visualization of the process of magnetization than does the pole theory and which is, in certain respects, less confusing. Emphasis is placed on the distinctions between the electric fields, E and D, and the magnetic fields, B and H, and their relative importance. The definitions are chosen so as to give each of these four quantities simple physical meaning.

THE magnetic pole, as an entity, as a unit of magnetism from which 4π "lines of force" emanate, is showing signs of vanishing from the theory of electricity and magnetism. No free poles have ever been found. Swann[1] some years ago classed the pole as obnoxious and unnecessary and Mason and Weaver[2] consider it non-existent. The most accurate determinations of magnetic quantities are now made in terms of magnetic forces of currents.[3] In most textbooks, however, the pole still clings tenaciously to a place of fundamental importance. It appears as the new and as the all-important quantity in Coulomb's law of attraction and repulsion of poles, upon which the theory of magnetism is based. When the magnetic field strength is defined as the force on a pole, it is inevitable that the student accept the pole as very fundamental and he can give up the pole later only with considerable reluctance and confusion. Developing a subject in historical order has advantages but when such development fixes in the mind of the student that the pole, which is non-existent, is the fundamental quantity of magnetism, the method loses its value. This fiction is avoidable and it seems opportune that an improvement of presentation should filter down into intermediate and elementary teaching.

In this paper an endeavor is made to show that an elementary description of magnetic phenomena entirely in terms of currents composed of moving electrons can be made more simple and less confusing than the discussions involving poles. Instead of three laws, (*a*) Coulomb's law for charges, (*b*) the corresponding expression for magnetic poles and (*c*) the interaction of poles and currents, we can have two fundamental expressions: (*a*) Coulomb's law for charges and (*b*) the corresponding law for the magnetic effect of moving charges, Eq. (8), while the properties of magnetic substances can be expressed in terms of current whirls. This procedure treats magnetism with the introduc-

[1] Swann, *Bulletin of the National Research Council* 4, Part 6, No. 24, 12, Dec., 1922.

[2] Mason and Weaver, *The Electromagnetic Field*, University of Chicago Press, 1929.

[3] In his address before the American Physical Society at Chicago in June, 1933, Dr. H. L. Curtis stated that the ampere is now determined most accurately by means of the current balance.

tion of no new quantities. The Gaussian system of units is used throughout.[4]

The first difficulty with the pole theory appears in Coulomb's law itself. As usually expressed, the law, $f = mm'/\mu r^2$, implies that the poles are permanent quantities unchanged by the introduction of magnetic material of permeability μ and it states clearly that the force of attraction between the "north" end of a long magnet and the nearby "south" end of another long magnet is decreased upon the introduction of soft iron ($\mu > 1$) between them. This is contrary to fact and, rather than consider a revision of the law, one usually employs devious explanations in this, the most evident case of the effect of the magnetic medium. Why resort to consideration of the "induced" poles in the soft iron, when the permeability is introduced expressly for the purpose of describing the behavior of the medium?

The "demagnetizing effect of the induced poles" is another source of difficulty. The student learns to believe that by the use of the "demagnetizing field" he has explained quite simply the experimental fact that the so-called "magnetic induction" B, in a short piece of a given specimen of soft iron in a given field H, is much less than B in a long piece in the same field H. Yet when he attempts to follow out his explanation more in detail, his induced poles fade away and he is forced to consider the interaction between the atoms. Indeed, when one expresses the magnetic moment of the atom in terms of Amperian currents and motions of electrons, he has no language in which to describe the imaginary stuff at the ends of the iron, and such a demagnetizing field does not exist. All the atoms in the iron may be divided into two classes: those whose fields at the point in question oppose the applied field H and those whose fields have components in the direction of the applied field. Letting A (Fig. 1) be the point at which the field is desired, and letting C be an atom whose field at A is normal to H, we can see that

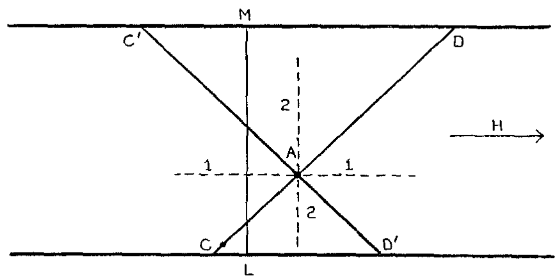

FIG. 1. Atoms in regions 2 contribute a demagnetizing field at A and those in regions 1, a self-magnetizing field.

the surfaces, of which CD and $C'D'$ are traces, containing all the atoms C, divide the iron into two parts. In regions 2 the atoms contribute a demagnetizing field at A and in regions 1 the atoms contribute a partial field in the direction of H, the self-magnetizing field. One should not consider the demagnetizing field (regions 2) without at the same time considering the self-magnetizing field (regions 1). If the iron be cut in two at LM and the left end removed so that A is near the end, we see that the field at A is reduced, not because of a demagnetizing field but because of a reduced region 1 supplying a self-magnetizing field. In a short piece of a paramagnetic material, in which there is no appreciable interaction between the atoms, the field B should be less than in the long piece, simply because there are not so many atoms of the material contributing to B. These two causes, the fields of neighboring atoms together with the effect of the number of atoms, are ample to describe qualitatively the experimental facts in iron. The foregoing demagnetizing and self-magnetizing fields are effects of the applied field H as well as causes of B. B is always the combined effect of the applied field H and the contribution of the atomic orbital currents. It is customary to designate a field H' within the iron which is the combined effect of the applied field H and the usual "demagnetizing" field. This classical "magnetic field within iron," H', serves no useful purpose. The numerical difference between B and H is the contribution of the magnetic material. The permeability μ, introduced as the factor describing the medium, should represent the fractional increase of field upon addition of the magnetic medium; hence H inside the iron should be the applied field alone. This means that the ratio B/H in the short piece of iron

[4] Since, in electrostatics, the Gaussian system is identical with the electrostatic system and, in the case of magnetic quantities, identical with the electromagnetic system, it can readily be used in elementary teaching. At the same time it is equally as satisfactory in theoretical developments as the other rational system.

differs from that in the long piece and also varies from point to point in iron which is nonuniformly magnetized. For quantitative computations, rather than use tables of "demagnetization factors" for correction for H', we should use the corresponding tables for B.

The elimination of the "demagnetizing field of the induced poles" is in harmony with the useful approximate relation between flux, magnetomotive force and reluctance in the magnetic circuit. Such a circuit is composed of comparatively short lengths of magnetic media of different permeabilities, yet it has been customary here not to introduce the "demagnetizing" fields but rather to consider "ampere turns" as the sole cause of magnetization, and to add the correction in terms of flux leakage.

Electrostatics

The electric field, defined as the force on a charge, $E=f/q'$, takes within a parallel plate condenser the form,

$$E=4\pi\sigma, \tag{1}$$

where σ is the surface density of charge on either plate. When a dielectric material is introduced between the plates of the condenser, the charge q being maintained on the plates, it is convenient to compare the (weaker) field E in the dielectric medium with the field which was there with the same charge q in vacuum. The latter field is the so-called "electric induction" D,

$$D=4\pi\sigma. \tag{1a}$$

Each electron of the medium experiences a force directed toward the positive plate equal to $E\times 4.77\times 10^{-10}$ dynes. The resulting displacement of the electron depends on this force and on the atomic restoring forces. The displaced electron together with the equal positively charged atom constitutes a dipole. The electric moment of the dipole may be defined either as the product of the charge and its displacement, or as the ratio of the maximum torque on a rigid dipole to the field E,

$$es=L_{\max}/E. \tag{2}$$

The electric moment of an atom is the sum of the products of the charge on each electron and its displacement. The sum of the electric moments of all the atoms divided by the volume is a quantity which is a measure of the state of the dielectric medium and is called the polarization P,

$$P=\Sigma es/v. \tag{3}$$

Polarization is a volume effect; it is a shift in position of electrons throughout the material. In any cubic centimeter within the material there are as many negative charges as positive charges, while there is a surface charge of negative electricity at the boundary of the dielectric medium near the positive plate and a positive charge at the negative plate. In order to see that the surface density of this charge is equal to P, we have only to observe that q_pl, the product of this charge and the distance l between q_p and $-q_p$, is equal to the total electric moment, Σes. Solving for q_p and dividing by the area, we see that the surface density of the induced charge is $q_p/A=\Sigma es/Al=\Sigma es/v=P$, the polarization. The net charge at the plates is thus reduced from q to $q-q_p$, q remaining constant, and the net surface density of charge is $\sigma-P$; hence the field is reduced from

$$D=4\pi\sigma \tag{1a}$$

to

$$E=4\pi(\sigma-P); \tag{4}$$

thus

$$E=D-4\pi P. \tag{5}$$

The potential difference between the plates is given by

$$V=W/q'=\int Eds, \tag{6}$$

and in vacuum $V_0=\int Dds$.

With constant q,

$$\sigma/(\sigma-P)=q/(q-q_p)=D/E=V_0/V=C/C_0=\epsilon, \tag{7}$$

where C and C_0 are the capacitances of the condenser with and without the dielectric. The dielectric constant ϵ is an indicator of the effect of the medium. It gives the field E within the medium for any applied field D.

In many cases of practical importance the potential difference between the plates of the condenser, rather than the charge q, is kept constant or externally controlled, when the dielectric medium is introduced. The charge on the plates q_0 then increases to q until the field E is equal to the original field E_0. Hence

$$E_0=4\pi\sigma_0,$$

$$E=E_0=4\pi(\sigma-P),$$

and therefore $\sigma=\sigma_0+P$, while $D=4\pi\sigma$ and $E=D-4\pi P$ as before. Thus

$$\sigma/\sigma_0=q/q_0=D/E=C/C_0=\epsilon. \tag{7a}$$

Since, in practice, electric fields are usually set up between charges on metallic conductors the potentials of which are externally controlled, it is V and E which are of physical importance. Were the charge q externally controlled, D and V_0 would assume greater importance. In both cases D is the field due to the "free" charge q, whereas E is the field due to the combined effect of q and the induced charge q_p.

Magnetostatics

Magnetism is an effect of moving charges. Two currents i and i' exhibit an attraction when flowing in the same direction and a repulsion when flowing in the opposite direction. These magnetic forces are proportional to the magnitudes of the currents i and i' and to their lengths s and s' and vary inversely as the square of the distance between them; that is

$$f = kisi's'/c^2r^2, \tag{8}$$

provided s and s' are short compared to r and s' is a part of a closed circuit. The constant k is dimensionless, a function of the angles between i and i', i and r, i' and r. Eq. (8) is the fundamental "law" of magnetism. The justification for designating this expression as fundamental is that on it can be built the theory necessary for describing all phenomena of magnetostatics. If i is to be expressed in the (electrostatic) units used in Eqs. (1) to (7), c must have the dimension of a speed. Experimentally c is found to equal within the limits of error, the speed of light,[5] $c = 2.99796\times10^{10}$ cm·sec.$^{-1}$.

Since is and $i's'$ are directed quantities it is found convenient to use vector notation and to define the magnetic field as the region around one closed circuit made up of current elements $i's'$, in which another current element is experiences a force. The field strength H, in gauss, is equal in magnitude to the force in dynes acting on c electrostatic units of current (1 cgsm) 1 cm in length and its direction is given by the well-known right-hand rule. That is,

$$H = f/(is/c), \tag{9}$$

when i is perpendicular both to f and to H. The force on a wire carrying current making an angle α with the field H is, in view of Eq. (9), given by

$$f = H(i/c)s \sin \alpha. \tag{9a}$$

The force on a single charge moving with speed v perpendicular to a magnetic field is $f = Hqv/c$.

By combining Eq. (9a) with Eq. (8), the remaining part of k becomes $\sin\theta$, where θ is the angle between i' and r, and

$$H = \Sigma(i's'/cr^2) \sin \theta, \tag{10}$$

where the direction of H is given by the proper right-hand rule. This holds only when s' is part of a closed circuit. The field H due to an extended wire carrying current is found by adding up the fields due to the short lengths of current s' in Eq. (10). In the notation of the calculus,

$$H = \int (i'ds'/cr^2) \sin \theta, \tag{10a}$$

the integral sign indicating a vector addition. At the center of a long solenoid, H is uniform and

$$H = 4\pi ni'/cl, \tag{11}$$

where l is the length of the solenoid.

The torque on a coil of n turns carrying a current i in a uniform magnetic field H is found by application of Eq. (9a) to be $L = (niA/c)H \sin\phi$, where ϕ is the angle between the axis of the coil and H, and A is the area of the coil. The ratio of the maximum torque $L_{\max}$ to the field H is called the magnetic moment M (see Eq. (2) for electric moment),

$$M = niA/c = L_{\max}/H. \tag{12}$$

If the orbits of the electrons in an atom lie predominately in any plane, the atom has a magnetic moment normal to that plane and behaves like a coil. The sum of the magnetic moments of all the atoms divided by the volume is a measure of the state of magnetization and is called the intensity of magnetization I,

$$I = \Sigma M/v. \tag{13}$$

If a bar of iron is placed in a solenoid the effect of the field H is to rotate, perhaps indirectly, the orbits of the electrons in the atoms of the iron so that the orbits become lined up in planes perpendicular to H. The electrons are moving around in the atoms in the same direction as the electrons in the copper wire of the solenoid, counterclockwise as one looks along H, and contribute an additional field which can be shown to be equal to $4\pi I$. As in electrostatics, polarization can be described in terms of the induced charge appearing at the surfaces, the charges balancing out within the material; likewise, magnetization can be described in terms of current whirls i_m flowing around the surface of the iron, the currents within the iron balancing out. Let us assume, for the sake of setting up the picture, that the electron orbits are rectangular and that they touch one another (Fig. 2).

[5] If we arbitrarily set $c = 1$ cm·sec.$^{-1}$, then when $s = s' = 1$ cm, $r = 10$ cm and $f = 1/100$ dyne approximately, i and i' have the magnitude of the electromagnetic unit of current and the dimension of the electrostatic unit. If we set $c = 1$ without dimension, then i and i' are measured in electromagnetic units.

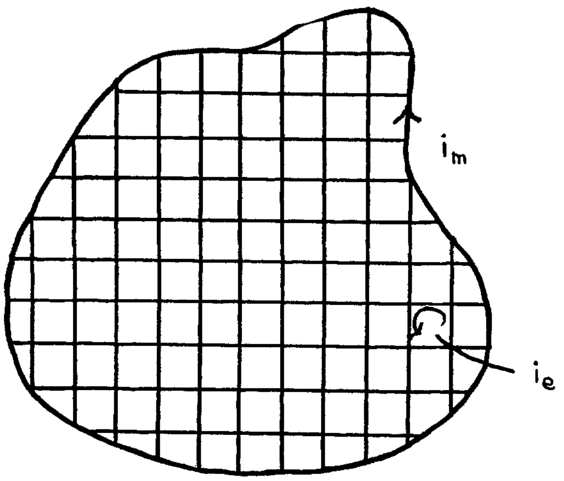

FIG. 2. The orbital currents of the atoms i_e furnish a current i_m around the surface of the iron.

We see that within the material the orbital currents cancel out, the sum of the areas A_e of the orbits being equal to A, the cross section of the iron, and the current i_m around the periphery of one layer of atoms being equal to i_e, the current around each atom. The orbital currents are not, of course, rectangular and the orbits may not touch or may overlap but, in any case, the current at the surface of the iron i_m is of such magnitude that its magnetic moment ni_mA/c is equal to the sum of the magnetic moments of all the atoms $\Sigma i_eA_e/c$. Thus $i_m = (\Sigma i_e/n)(\Sigma A_e/A)$ and the current whirl per unit length of the iron,[6] ni_m/lc, is equal to the intensity of magnetization I, $ni_m/lc = ni_mA/Alc = M/v = I$. The net current around the solenoid and iron is increased from i' to $i'+i_m$, i' remaining constant; hence the field is increased from

$$H = 4\pi ni'/cl \qquad (11), (11a)$$

to

$$B = 4\pi n(i'+i_m)/cl; \qquad (14)$$

thus

$$B = H + 4\pi I. \qquad (15)$$

B, the so-called "magnetic induction," is the force per unit length on a current i, in the presence both of the current i' in the solenoid and of the current i_m around the magnetized iron.

If we were to make the definitions of magnetic fields strictly parallel to those of electric fields we should have to define B, not H, in Eq. (9) as equal to the force on a current i, and replace H by B in subsequent equations excepting Eq. (11a) and Eq. (15). However, to the engineer, H is the field due to conduction currents whether in vacuum or near iron and it is well to define it so. Also, H is of more importance than D because it is H, a function of i', rather than B, a function of $i'+i_m$, which is externally controlled, whereas in electrostatics it is usually E, a function of V and $q-q_p$, rather than D, a function of q, which is externally controlled.

The so-called magnetomotive force, defined as $MMF = \int H_s ds$, is formally similar to the potential difference or e.m.f. defined in Eq. (6) but more similar to $V_0 = \int D_s ds$. It is useful in describing conditions in a magnetic circuit. $\int B_s ds$ would correspond more to $\int E_s ds$ but is not so useful. There is no necessity for imagining a tunnel cut lengthwise through the iron in order to evaluate the line integral $\int H_s ds$, for H is simply the partial field due to currents i' alone. The average value of the force on an electron due to its motion should be B, the same whether the average is taken along a line along B or across a section perpendicular to B. Taking the line integral just inside the iron and back just outside, $\int B_s ds - \int H_s ds = 4\pi i_m/c$. The line integral $\int H_s ds$ within the iron is equal to that just outside the iron and equal to that with the iron removed, the current i' remaining constant.

The ratio

$$(i'+i_m)/i' = B/H = \mu \qquad (16)$$

indicates the fractional increase of the field upon introduction of the medium and is called the permeability.

Inside iron within a solenoid, H is the magnetizing field due to the solenoid alone. In the short cylinder of iron, the intensity of magnetization I and its contribution to the field vary from point to point. Near the ends this contribution is one-sided, hence I has its maximum value at the center. Both the average and maximum values of B and I are less in the short cylinder of iron than in the long one. As a general expression we may write in place of Eq. (15),

$$B = H + aI, \qquad (17)$$

where $a = 4\pi$ only in the ideal case of an infinitely long, uniformly magnetized bar. Just as in electrostatics the electrons move over until the elastic return force just balances the (reduced) field E, so the orbits line up until the opposing torques, due to local fields and thermal agitation, just balance the torque due to the (increased) B. For a given value of B at a point, there is a definite value of I and a definite ratio I/B, no matter whether B is obtained with a very strong

[6] The same development applies to spinning electrons.

applied field H and a medium of limited size or with a weaker applied H and a larger medium. This ratio may be found from the ideal case of a uniformly magnetized bar (approximated in the endless toroid ring). By solving Eq. (15) for I and combining with Eq. (16) and Eq. (17), we find

$$I=(\mu-1)B/(4\pi\mu)$$
$$=[(\mu-1)/(4\pi\mu-a)(\mu-1)]H. \quad (18)$$

For a uniformly magnetized sphere, the partial field aI is found, by integrating the surface density of i_m over the surface, to be $8\pi I/3$, whence for a sphere, $a=8\pi/3$, and

$$I=[3(\mu-1)/(\mu+2)](H/4\pi);$$

and for $\mu\gg 1$,

$$B=H+4\pi I/3=3H.$$

There is thus no necessity for postulating a "demagnetizing" field.

Just outside the end of the short iron cylinder, B determines the force per unit length on current i (or the force on a moving charge q) due both to solenoid and iron and is equal to the field B just inside the iron but not equal to the average value of B throughout the iron.

In permanent magnets the electron orbits are at least partially lined up in planes perpendicular to the direction of magnetization and i_m is permanent. The field inside the ideal magnet is $4\pi n i_m/cl$, whereas the field outside is found from i_m in the same way that the field outside a slender solenoid is computed; that is, on the axis of a magnet

$$H=(2ni_mA/c)(d/(d^2-l^2/4)^2)$$
$$=2Md/(d^2-l^2/4)^2. \quad (19)$$

The actual permanent magnet is, of course, most strongly magnetized (elementary magnets in best alignment) at its center and l, the length of the equivalent uniformly magnetized magnet, corresponds to the usual "distance between the poles."

To Professor Webster, whose article follows this one, the author wishes to express sincere appreciation for his extraordinary courtesy in reading this manuscript and writing his paper in such a way as to avoid duplication. With Professor Webster's conclusion that it is more logical to define B rather than H as the force on a current, the author agrees. The author has, however, endeavored to strip the classical magnetic field (H') of its fictitious character, allowing the H of this paper to retain as much of the usual meaning as is compatible with Amperian currents. This H has a twofold use: first, it is a very definite part of the total field B, the part most easily computed, that due to conduction currents; second, it is the cause of magnetization in the sense that it is the force (per unit current) which is externally applied. For simplicity, a single sample of iron has been considered. If in any problem it can be ascertained that the fields due to conduction and Amperian currents in other pieces of matter be unaffected by the local magnetization, then these fields may be included in H; otherwise they must be included in the contribution of magnetized matter.

The electric fields E and D and the magnetic fields, B and H, as introduced in this paper, may be shown to lead directly to Maxwell's wave equations (containing only E and H) in a development which further emphasizes the distinctions between these four fields. Lack of space forbids including this derivation.