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Geometric absorption of electromagnetic angular momentum

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Abstract

Circularly polarized electromagnetic fields carry both energy and angular momentum. We investigate the conditions under which a circularly polarized wave field transfers angular momentum to a perfectly conducting macroscopic object, using exact electromagnetic wave theory in a steady-state calculation. We find that axisymmetric perfect conductors cannot absorb or radiate angular momentum when illuminated. However, any asymmetry allows absorption. A rigorous, steady-state solution of the boundary value problem for the reflection from a perfectly conducting infinite wedge shows that waves convey angular momentum at the edges of asymmetries. Conductors can also radiate angular momentum, so their geometric absorption coefficient for angular momentum can be negative. Such absorption or radiation depends solely on the specific geometry of the conductor. The geometric absorption coefficient can be as high as 0.8, and the coefficient for radiation can be −0.4, larger than typical material absorption coefficients. We apply the results to recent experiments which spun roof-shaped aluminum sheets with polarized microwave beams. Applications of geometric, instead of material, absorption can be quite varied. Though experiments testing these ideas will be simpler at microwavelengths, the ideas work for optical ones as well.

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A polarized classical electromagnetic wave carries angular momentum as the sum of many photon spins. Much work has been devoted to the flow of energy in classical electrodynamic systems, but how classical wave angular momentum interacts with macroscopic bodies seems largely unexplored [1–4]. Recently, new interest in handling of small objects electromagnetically has stimulated ideas for using polarized optical beams [5–7] and other uses further afield [8].

Boundary effects can enter into angular momentum coupling, but there seems no study of how the geometry of the target object affects this. Particularly, we shall find that a perfect conductor can

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still absorb angular momentum geometrically from a circularly polarized beam, with no material absorption whatever. We calculate model problems to determine how angular momentum is conveyed to solid bodies classically and compare with recent experiments that measured such effects for the first time.

A plane wave encountering an obstruction necessarily deforms in passing by it (see Fig. 1). The wave of electric field strength $\vec{E}$ can acquire a component $E_\parallel$ parallel to the wave vector $\vec{k}$ depending on the specific obstacle shape. Acquiring a parallel component $E_\parallel$ results in a perpendicular component of the energy flow $S_\perp$ that conveys angular momentum to the obstacle.

If the wavelength λ exceeds the obstacle size d, the electric field created parallel to $\vec{k}$ is roughly of magnitude [1]

$$E_\parallel \sim \frac{\lambda}{d} E. \qquad (1)$$

Seen quantum mechanically, wave angular momentum L acts to produce a torque through an effective arm of a wavelength, so each photon conveys a quantum $\hbar$ to the object,

$$L = N\hbar \qquad (2)$$

with N the photon number. The wave energy is $E = Nh\nu$, so the ratio of L/E imparted by a wave is $L/E \sim 1/\nu$. Therefore, longer wavelengths are more efficient in producing spin. The ratio of angular to linear momentum $P = h/\lambda$ is

$$L/P \sim \lambda^2/d \qquad (3)$$

since d is also the transverse scale of the beam. This scaling with λ/d shows why lasers cannot usefully spin objects, since for them this ratio is $\ll 1$, but microwaves can.

The total angular momentum of a wave field is defined by [2]

$$\vec{L} \equiv \frac{1}{c^2} \int \vec{r} \times \vec{S}\, \mathrm{d}^3x \qquad (4)$$

with $\vec{S} = \frac{c}{8\pi} \Re(\vec{E} \times \vec{H}^*)$ the Poynting vector. This can be broken into independent spin and orbital terms, but the distinction between the two parts is somewhat unhandy since neither the orbital angular momentum nor the spin are gauge invariant. Only the total angular momentum (4) is gauge invariant and can therefore be assigned to a physical quantity. Here we only consider the total

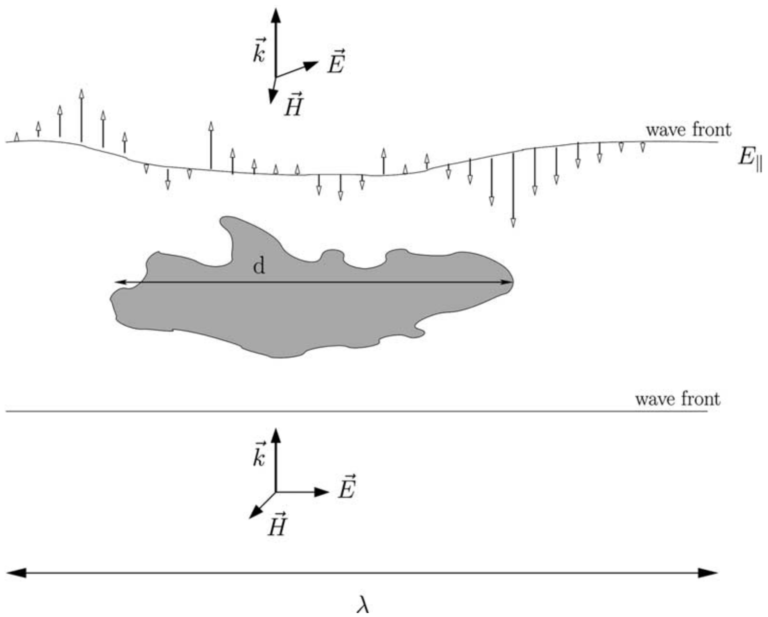

Fig. 1. A plane wave passing an obstacle of extension d acquires field components $E_\parallel$ and $H_\parallel$ parallel to the wave vector $\vec{k}$.

angular momentum $\vec{L}$ and its transfer to macroscopic objects.

First, we state a theorem: a perfectly conducting body of revolution with a piecewise smooth surface around the axis of symmetry (for instance a cone, a disk, a cylinder, or a sphere) absorbs *no* angular momentum $\vec{L}$ from an axisymmetric electromagnetic wave field.

Our discussion is physical, not mathematical. The effect occurs because the magnetic field lies in the surface of the obstacle. The same holds true for the surface current density $\vec{j}$ which of course cannot have a component perpendicular to an infinitely thin surface. As a second boundary condition, the electric field has no tangential component on the surface of a perfect conductor.

Therefore the electromagnetic force density $\vec{F} = \rho_s\vec{E} + (1/c)\vec{j} \times \vec{H}$ with surface charge density ρ_s is normal to the surface. Since the obstacle is a body of revolution this implies that $\vec{F}$ has no azimuthal component, so the torque $\vec{T} = \vec{r} \times \vec{F}$ at any point of the surface can therefore have no component along the z-axis which would make the obstacle spin around the axis of symmetry. Other components of the torque cancel out on opposite sides of the surface since the value of the force F is independent of the angle θ for an axisymmetric wave field. This does not hold if we shift the object away from the wave axis of symmetry.

Now consider the parts of the surface $\mathscr{F}_e$ which do not have tangential planes like edges, rims, or kinks. The surface current density still lies inside the surface, but there is no surface normal anymore, so the former boundary condition for the magnetic field is not true. But all *symmetric* edges, rims or kinks are one-dimensional circles around the axis of symmetry. Therefore, the current density only has an azimuthal component j_θ while the tangential component of the electric field E_θ vanishes. Again, the force $\vec{F}$ does not have an azimuthal component that would give rise to a torque around the z-axis. Other components of the torque cancel out on opposite sides of the object because of the symmetry of the problem. Field or geometric asymmetries destroy this theorem.

What can be said about more complex surfaces? Consider the scattering problem for the 'wedge' in Fig. 2 whose rigorous solution [3,4] may be applied to recent experiments with aluminum 'roofs' spun by microwave beams.

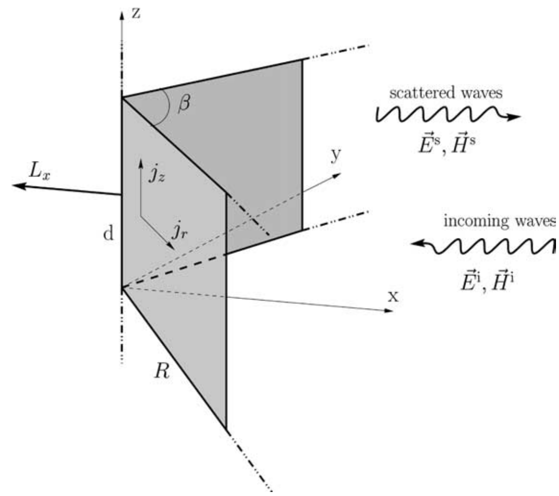

Fig. 2. Incoming plane waves $\vec{E}^i$ and $\vec{H}^i$ are reflected by an infinitely extended, perfectly conducting wedge, which gains an angular momentum L_x.

Take an infinitely extended, perfectly conducting wedge of opening angle β with its edge along the z-axis and its two faces each enclosing an angle $\beta/2$ against the x-axis (Fig. 2). An infinitely extended plane wave $\vec{E}^i$, $\vec{H}^i$ with angular frequency ω, incident along the x-axis, reflects from the faces of the wedge as the scattered wave field $\vec{E}^s$, $\vec{H}^s$.

A circularly polarized incoming wave field which is infinitely extended and homogeneous does not carry angular momentum, since the Poynting flux is parallel to the wave vector. However, the reflected wave field does have a finite angular momentum L_x. To conserve angular momentum, the wedge *geometrically* absorbs the negative angular momentum L_x of the reflected wave field.

Using Carslaw's solutions [4] we calculate directly the coupling coefficient observed in experiments. When microwaves reflect from a 'roof', angular momentum constantly transfers from the wave field to the object. The roof undergoes a constant torque $\tau = \mathrm{d}L/\mathrm{d}t \approx L_x/\Delta t$ where L_x is the total angular momentum contained in the volume between the faces of the wedge out to a radius R, taken from the edge of the wedge. We assume that the angular momentum L_x is built up in the 'crossing time' $\Delta t = 2R/c$ in which the microwaves cross the interior of the roof and are scattered back. This is a simple approximation, replacing the tedious work of finding and evaluating a retarded, time-dependent solution of the Helmholtz equation. It implies a steady state compatible with causality.

The equation of motion for the roof is

$$\tau = I\frac{d\omega}{dt} = \frac{\alpha^*}{2\pi\nu}P, \tag{5}$$

where I is the inertia of the roof, ω its angular frequency, ν the microwave frequency, and P the power of the microwave field radiated onto the roof.

In analogy to the material absorption case, we define a *geometric absorption coefficient* α^*. For an absorbing material, we have to replace α^* in (5) by a function $\alpha_T(\alpha, \alpha^*, \delta)$ of the material and the geometric absorption as well as the skin depth δ of the microwaves. The described experiment deals with aluminum, so we can neglect any material absorption.

For the incoming plane wave the constant power is given in terms of the Poynting flux S,

$$P = SA = \frac{c}{8\pi}E_0^2 A, \tag{6}$$

where $A = 2R\sin(\beta/2)d$ is the cross-section of the roof.

By using the 'crossing-time' approximation for the torque we find the geometric absorption coefficient,

$$\alpha^*(\chi_0) = \frac{2\pi}{\sin\frac{\beta}{2}}\widehat{L_x}(\chi_0)\frac{1}{\chi_0^2}, \tag{7}$$

where $\chi_0 = \kappa R$ indicates the size R of the wedge along its faces in terms of the wave number $\kappa = 2\pi/\lambda$.

Fig. 3 shows α^* vs. β and the normalized 'roof size' χ_0. The geometric absorption coefficient takes its maximum value of approximately 0.75 at $R \approx \lambda/2$ and $\beta \approx \pi/2$. For larger R or smaller λ the coupling falls very fast. Notice that α^* also assumes negative values down to ≈ -0.4 and therefore physically differs very much from a material absorption which is restricted to values between 0 and 1. This surprising result means that a perfectly conducting roof can radiate angular momentum at small opening angles.

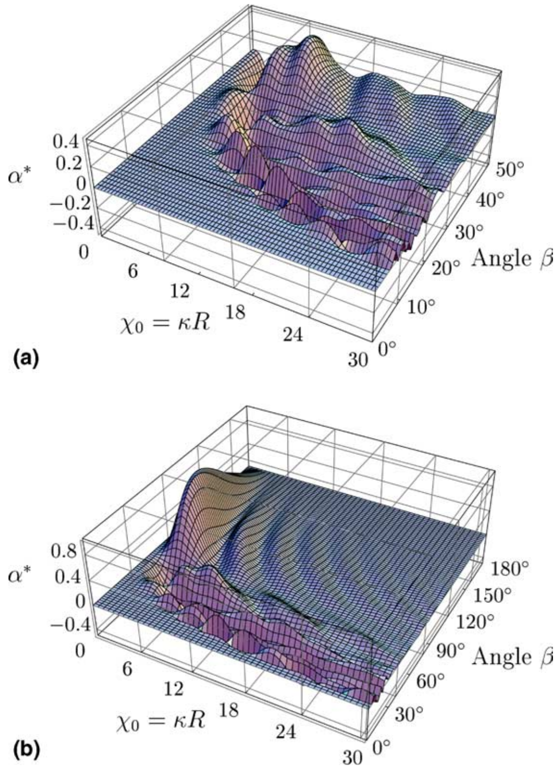

Fig. 3. The geometric absorption coefficient α^* vs. opening angle β and 'roof size' χ_0.

The figure reveals a structure of wave interference, with maxima and minima $\sim \lambda/2$ apart for $\beta > \pi/3$. This is not a strict periodicity, due to the widening of the wedge for larger radii. For $\beta < \pi/3$, interference produces parallel valleys, saddles, and shoulders. At $\beta = \pi/n$ with n an odd integer, $\alpha^* = 0$ and the wedge reverses wave polarization. A spatial plot shows that the angular momentum is mostly concentrated at the faces of the wedge while it is close to zero in the middle region. The transfer of angular momentum to the wedge clearly is a *boundary effect*!

Fig. 4 shows the geometric absorption coefficient α^* as a function of the opening angle β for the 'quasi-radius' $\chi_0 \approx 3.8$ used in the experiments. Two data points [9] are also shown taken for roofs with opening angles of approximately 25° and 90°. At the enclosed angle of 90° the measured α^* is about half the theoretical prediction. At 25°, there is rough agreement. This confirms the qualitative

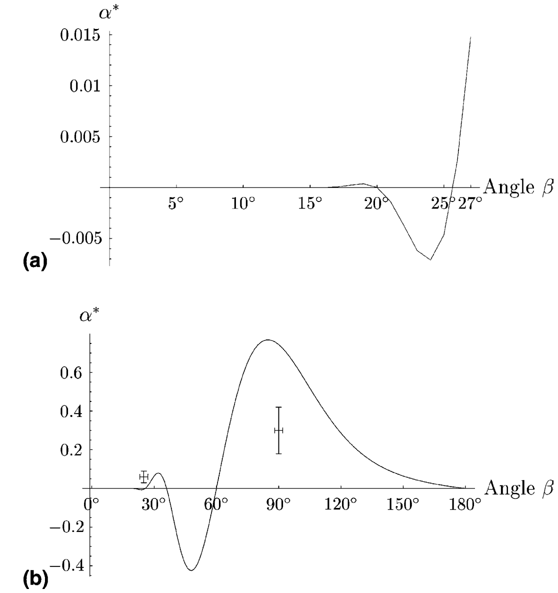

Fig. 4. Geometric absorption coefficient α^* vs. opening angle β at $\chi_0 = \kappa R = 3.8$; data points shown for experiments with aluminum roofs.

features of our idealized model for the wedge. Considering that we took an infinite shape along z, equivalent to a very broad 'roof', and in experiment this was 2.5 cm, comparable to the wavelength, one cannot expect quantitative agreement. As well, the exact solution holds for infinite radius, and again this was only a few wavelengths in experiment. We hope to soon consider less ideal solutions.

The central qualitative conclusion of J. Benford et al. was that, within experimental error, intact disks and cones did not rotate. This conforms with our theorem. Carbon disks and cones did spin readily, since they had an absorption coefficient $\alpha \approx 0.1$ for the frequency used (7.17 Ghz).

Electromagnetic waves not only carry energy but transport angular momentum as well if they are circularly polarized and finite. While the intensity of a wave is a large issue in many interference problems, the angular momentum remains mainly unexplored.

There are 11 exact solutions for an electromagnetic wave reflecting and diffracting around obstacles, since there are 11 coordinate systems in which the wave equation is separable. Of these, most are figures of revolution and by our theorem will not absorb angular momentum – spheres, disks, oblate spheroids, etc. Of the others, the shapes of a wire, a narrow strip, a wedge, square, circular cylinder, elliptical cylinder, and half-plane allow for a calculation of the coupling coefficients for the angular momentum.

The angular momentum density of a wave field physically differs very much from its energy density. Both are subject to constructive or destructive interference when a wave is diffracted or reflected by a finite obstacle. But while energy can only be absorbed or reflected, angular momentum can also be radiated. In fact, an object can act as a polarizer when irradiated by an electromagnetic wave, just by reflection, if it has a specific geometry. We have to distinguish between an inherent material absorption coefficient $\alpha \geqslant 0$ and a geometric absorption coefficient α^* determined by the shape of the object. Generally, α^* may be negative, and the object will rotate in the opposite sense than if it were an absorber. Uses of geometric absorption can be many. Small objects in, for example, biological media can be shaped to either respond to an electromagnetic momentum-carrying wave (i.e., be asymmetric) or not. We shall soon present further theory and experiment showing the basic physics.

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