$$\text{yield}=\frac{n_f}{1.4\times 10^{23}\ \text{kT}^{-1}}=13\ \text{kT}. \quad (5)$$

DISCUSSION OF RESULTS

How does our result compare with the accepted value? Non-nuclear measurements (seismographic, air pressure, earth displacement, etc.) performed at Trinity gave yield estimates which ranged from 5–15 kT.[10] As previously stated Anderson's radiochemical analysis produced a figure of 18.6 kT. The currently accepted value is from 20–22 kT.[5] We have done quite well, coming to within 35–40 % of the accepted value. We attribute this to two opposing sources of error. It is very difficult to be quantitative in assigning uncertainties to our assumptions—uniform distribution of fission products, and the deposition of 1% of the total ^{137}Cs activity within the 1200 yard radius of Ground Zero. The first, uniform distribution almost certainly overestimates the yield. Although we do not know its precise distance from ground zero, the sample had to lie somewhere within the roughly 400 yard "trinitite radius." A distribution decreasing with distance implies that the sample contains a larger fraction of the total ^{137}Cs activity than we have calculated—we should have used a larger "effective sample area" in our calculation. On the other hand, the assumption of a 1200 yard cutoff almost certainly underestimates the yield—the actual area over which the fission products were distributed is larger, and the sample then represents a smaller fraction of the total activity. The errors introduced here, then, tend to cancel. (The 1% figure for the fraction of ^{137}Cs left "in the vicinity of the blast" could vary either way, but it probably is good to within ±0.5%. Its effect on our estimate cannot be determined.)

ACKNOWLEDGMENT

A belated thank you is due Los Alamos National Laboratory for supplying the trinitite sample.

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[2]It is no longer permitted to remove trinitite from the site. In those prelitigious days, we suppose, no one had yet thought to say "kids, don't try this at home."

[3]Richard Rhodes, *The Making of the Atomic Bomb* (Simon and Schuster, New York, 1986).

[4]*Alpha-, Beta-, and Gamma-Ray Spectroscopy,* edited by Kai Siegbahn (North-Holland, Amsterdam, 1965).

[5]Ferene Szasz, *The Day the Sun Rose Twice* (University of New Mexico Press, Albuquerque, 1984).

[6]Lillian Hoddeson, *Critical Assembly* (Cambridge University Press, Cambridge, 1993).

[7]*The Effects of Nuclear Weapons*, 3rd edition, edited by Samuel Glasstone and Philip J. Dolan (Energy Research and Development Administration, Washington D.C., 1977).

[8]Seymour Katcoff, "Fission-product yields from U, Th, and Pu," Nucleonics **16**(4), 78–85 (1958).

[9]6.6% is the total, cumulative yield of ^{137}Cs produced by the entire mass-137 decay chain.

[10]Kenneth T. Bainbridge, "Trinity," Los Alamos National Laboratory Technical Report LA-6300-H, May 1976.

Lorentz contraction: A real change of shape

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A simple model of a solid is created by use of a few charged point particles interacting only with electromagnetic forces. Such a model cannot fix the size of the solid since the particles will be in unstable equilibrium, but the shape will be determined by the requirement of equilibrium. It is then easy to show that when this *solid* is in motion, it must Lorentz contract in the direction of the motion (as compared with the transverse dimensions) in order for the charges to remain in equilibrium.

INTRODUCTION

From the earliest popular expositions[1] to the present time, students get the idea that rapidly moving bicycles and streetcars "look" shorter. The Lorentz *contraction* is usually discussed in terms of the Lorentz *transformation* and not in terms of atoms pulling and pushing on each other to create the contraction. In fact, after the work by Terrell[2] students (and professors) correctly believe that objects do not even look shorter, but rather look as if they are rotated a bit. Thus the connection between the contraction and the appearance of an object is not so simple.

In an interesting paper by J. S. Bell on "How to teach special relativity"[3] it is suggested that students would find relativity a less radical change from classical physics and more easily understandable if the Lorentz contraction was treated as a real physical effect due to motion dependent forces acting on the body.

This is the paper in which Bell poses a little relativity quiz supposedly failed by most physicists in the CERN lunchroom. Two identical rocket ships, initially at rest one in front of the other and connected by a string, accelerate with the same acceleration program and thus retain the same front to back separation that they started with (as seen from the original rest frame). As they speed up, the string (as seen from the original rest frame) wants to Lorentz contract. Since it is not strong enough to pull the rocket ships closer together, the string breaks. True or false? The description is true: the moving string atoms pull on each other until the stress tension breaks the string.

Bell then treats the shape change of a moving atom modeled by classical motion under electromagnetic forces. This part of the paper gets cumbersome and not very convincing.

It is the purpose of this note to demonstrate by a simple model that the Lorentz contraction can be thought of as being due to required changes in the internal forces arising from the motion. Although this point of view is not given much attention today in most widely used textbooks, it was the view of Lorentz himself prior to Einstein's work.[4]

MODEL OF A SOLID SHAPE

In elementary physics courses a solid is often modeled by a set of connected springs. This is not useful for the present discussion since it would be necessary to postulate how the spring forces and thus the shape would change with velocity. Only for electromagnetic forces is their correct velocity dependence well known. The difficulty in making a model involving only electromagnetic forces acting on point charges is the fact that there is no *stable* equilibrium state for such a configuration. True stable systems require electromagnetism and quantum mechanics, and a relativistic treatment would be far beyond the comprehension of undergraduates.

However, there are configurations of *static* equilibrium for charges under Coulomb forces, which can define a shape in space. Two charges can never be in equilibrium, but three charges can. If two equal (unit) charges are placed with a third charge q midway between them, all three charges will feel zero force if $q=-0.25$. The central charge feels no force by symmetry and the attractive and repulsive forces on each end unit charge exactly cancel. Three particles of charges $q=1,\ 1,\ -0.25$ will be in equilibrium only if they are in a straight line with the negative charge symmetrically in the middle. Although the shape (symmetrical straight line) is defined by requiring equilibrium, the size is not. The equilibrium is clearly *unstable* as the energy would be lowered if one of the unit charges moves close to the negative charge and the other unit charge moves off to large distance.

For four or more particles more interesting shapes can be defined by requiring equilibrium. For example, n unit charges on the n vertices of a regular polygon can have their repulsive Coulomb forces exactly balanced by an appropriate negative charge located at the center. If $n=3$ the required charge at the center of the equilateral triangle is $q=-1/\sqrt{3}$; for $n=4$, the required central charge is $q=-(1/4+1/\sqrt{2})$. We will model a solid as a square of four unit charges with the fifth central charge $q=-(1/4+1/\sqrt{2})$.

If the four unit charges are constrained to a circle of fixed radius, the square configuration has stable equilibrium. With no constraint, a change of size of the square involves no energy change (neutral equilibrium) just like the case of the three charges on a line. The square is clearly unstable since the energy can be lowered, e.g., by bringing one of the unit charges close to the negative central charge with the other three going out to large distance.

For five point charges with $q=1,\ 1,\ 1,\ 1,\ -(1/4+1/\sqrt{2})$, the only static equilibrium shape is the square and *not* a rectangle or any other shape.

THE MODEL SOLID IN MOTION

The shape of the model solid is defined by the condition of equilibrium, that the force on each of the five point charges vanishes. For the charges at rest, the force on each point is the vector sum of the Coulomb forces due to the other four charges, where

$$\mathbf{F}=(qq'4\pi\epsilon_0)\mathbf{r}/r^3. \qquad (1)$$

This leads to the central q value above required for equilibrium.

For the same five charges in uniform motion with velocity v in the x direction the forces will be different. At the location of each charge there will be an **E** field due to the modified Coulomb interaction with each of the other charges, and a **B** field due to their currents since the charges are moving. Both fields will produce forces on the charge since it is also moving. The form of these fields and forces was first calculated in the 19th century and can be understood by an undergraduate physics student. A new derivation has been presented recently.[5]

The standard derivation as presented in the widely used textbook by Resnick[6] uses the transformation of the fields in a moving coordinate system to show that the Coulomb field of a moving charge is

$$\mathbf{E}=q/4\pi\epsilon_0\times(1-\beta^2)(1-\beta^2\sin^2\theta)^{3/2}\times\mathbf{r}/r^3, \qquad (2)$$

where θ is the angle between $\mathbf{r}$ and the velocity $\mathbf{v}$, and $\beta=v/c$. The charge and field points are taken at the same (not retarded) time. This field is radial but not spherically symmetric, being modified from the usual Coulomb force by the factors of β and $\sin\theta$.

The magnetic field is

$$\mathbf{B}=\epsilon_0\mathbf{v}\times\mathbf{E}. \qquad (3)$$

The Lorentz force on each moving charge q' is given by

$$\mathbf{F}=q'(\mathbf{E}+\mathbf{v}\times\mathbf{B}). \qquad (4)$$

an expression valid in any coordinate system.

THE LORENTZ CONTRACTION SHAPE CHANGE

First repeat the force calculation for the case that the five charges form a square as before, with two of the sides parallel to the velocity. The **E** field at the center due to the charges on the corners of the square will vanish as before by symmetry. However, at each corner the **E** field contributions due to the charge $-q$ at the center and due to the three other corner charges will *not* add to zero as they did in the stationary case. It is the factor $\sin^2\theta$ in Eq. (2) that causes the problem. This produces a net force on each of the corner charges that is not canceled out by the magnetic force which is also present. Thus the square shape is not in static equilibrium and the shape would change with time and cannot model a solid.

Is there a shape for these five charges that is in equilibrium when all are moving with velocity **v**? Of course the answer is yes. Instead of a square let the four unit charges be arranged in a rectangular shape with the side transverse to the motion of unit length and the longitudinal side of length $(1-\beta^2)^{1/2}$. That is, let it be a Lorentz contracted square with the negative charge still at the center.

The **E** field at the center of the rectangle will vanish as before because of the symmetry of the rectangular figure and of Eq. (2). The calculation of the **E** field at each corner of the rectangle with the help of Eq. (2) is completely elementary. Use rectangular, x, y components and calculate the field at the lower right-hand corner of the square. The field due to the adjacent corner in the direction of motion has $\sin\theta=0$, $r^2=1-\beta^2$, the Lorentz contracted length, and $q=1$. That **E** vector has an x component only. The field due to the adjacent corner transverse to the motion has $\sin\theta=1$, $r^2=1$, $q=1$ and only a y component.

The **E** field at the corner due to the diagonally opposite corner charge has $r^2=d^2$ where d is the length of the diagonal of the rectangle, $d^2=2-\beta^2$. Also $\sin^2\theta=1/d^2$ and again $q=1$. This **E** field is in the direction of the diagonal and must be resolved into x and y components with the factors $\cos\theta$ and $\sin\theta$, respectively. The **E** field components due to the charge at the center are of the opposite sign since the central charge is $q=-(1/4+1/\sqrt{2})$. The r from the center is half that from the diagonally opposite corner and θ is the same.

When the **E** fields at a corner due to the four other charges are added together, the result is $\mathbf{E}=0$. Thus the moving rectangle of point charges has no electric force acting at any of the charges. Then from Eq. (3) it is seen that there are no **B** fields acting either and thus there is no force on any part of the moving rectangle of charges; they are in static equilibrium and will all continue to move with constant velocity **v**.

To be in equilibrium and thus act like a solid (of fixed shape) the five charges must have this rectangular shape, shortened in the dimension of the direction of motion by the Lorentz contraction as compared to the transverse direction. The forces change, due to the motion, in just such a way that the only stress free state for the moving square is one in which it is Lorentz contracted into a rectangle.

This is no surprise. The same result would follow from application of the Lorentz transformation to the positions (and times) of the five charges and to the strength and directions of the electromagnetic fields.

CONCLUSIONS

The simple *static* equilibrium model for a solid discussed here defines the shape, but not the size of the object. Thus, the elementary calculation presented does not show that the string in Bell's quiz must try to get shorter and thus must break—from this little model it might stay the same length and just get a little fatter instead. However, the model does give a simple physical explanation why a moving solid must change its shape due to the Lorentz contraction. The internal electromagnetic forces acting upon it change with motion in just the right way to push and pull the object into the new shape.

This calculation makes a suitable problem for an undergraduate relativity course.

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[3] J. S. Bell, "Collected papers on quantum philosophy," *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University, Cambridge, 1987), pp. 67–80.

[4] D. Bohm, *The Special Theory of Relativity* (Benjamin, New York, 1965), pp. 23–25.

[5] O. D. Jefimenko, "Direct calculation of the electric and magnetic fields of an electric point charge moving with constant velocity," Am. J. Phys. **62**, 79–85 (1994).

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Entangled quantum systems and the Schmidt decomposition

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Quantum systems comprised of interacting subsystems become highly correlated and their individual identities become entangled. This entanglement can be described using the Schmidt decomposition, in which a pair of preferred orthonormal bases can be constructed to emphasize the tight correlations between two quantum subsystems. Examples are given of how the Schmidt basis can be exploited to shed new light on entangled subsystems in quantum optics, paying special attention to two-mode squeezed states and to atom–field interaction. © *1995 American Association of Physics Teachers.*

I. ENTANGLEMENT

A composite quantum system, i.e., one that includes several quantum objects which are denoted subsystems, can be prepared in a so-called *entangled* state.[1] The consequences of this fact are anything but trivial and lead to various formulations of Bell's theorem.[2] In this paper we are not concerned with the widely publicized philosophical implications of Bell's theorem and concentrate, instead, on a mathematical formulation known as the Schmidt decomposition, which apart from being a convenient mathematical tool also provides additional insights into the nature of quantum entanglement.[3]

Imagine two subsystems $\mathcal{U}$ and $\mathcal{V}$, with which the state spaces $\mathcal{H}_u$ and $\mathcal{H}_v$ are associated. As usual, we associate the