A Direct Test

of the

Lorentz Length Contraction

Curt Renshaw
Tele-Consultants, Inc.

ABSTRACT

One of the most basic tenets of special relativity is the concept of length contraction as seen by an observer in motion. Yet this aspect of relativity has never been tested directly, due to the negligible size of the effect when applied to most situations. However, as the earth orbits the sun, any two stars located out of the plane of the ecliptic will appear to change their angle of separation as viewed during three-month intervals. This is due to the fact that at one instant the motion of the earth lies in the same direction as a line joining the two stars, while three months later the earth's motion is perpendicular to that line. At a velocity of 30 km/sec, the expected length contraction would be approximately 18 micro-arcseconds (μas) per degree of separation. The Space Interferometry Mission (SIM) promises a resolution of ± 1 μas in a field of view of one degree. Special relativity claims that this level of precision has no meaning, or only limited operational meaning, as it is smaller than the anticipated seasonal Lorentz contraction effects. Either, as the author believes, this level of precision is attainable, and special relativity is completely invalid, or the promised sensitivity level cannot be achieved without compensating for length contraction as well as aberration in published star charts.

INTRODUCTION

According to the Lorentz transformations as applied to special relativity, there is time dilation and length contraction in any system moving with respect to the observer. This transformation represents an actual change in the dimensions of length and of time, not just a shortening of rulers or a slowing of clocks. The basis for these transformations is the assumed constant velocity of light from any given source as measured from all inertial frames of reference, Einstein's second postulate. In order to support this hypothesis, and maintain a mathematically consistent world-view, the Lorentz transformations must be invoked.

It would be impossible to test the invariance of the velocity of light directly, for the simple fact that no two observers can absorb the same photon. Though we know the velocity as measured with respect to an observer that detects a particular photon, we can never know the velocity of that particular photon with respect to any other observer. The same is true of any photon detected by any observer in any reference frame – there is a unique detection by one and only one observer of any given photon. As a result, we are left with testing for the implications of the second postulate. The most basic secondary effects to test for would, of course, be length contraction and time dilation.

As simple as this seems, there has never been any direct test of either effect. First consider the case of time dilation. Special relativity predicts that time runs slowly in any inertial frame moving with respect to the frame of the observer. Testing this effect would require two identically constructed and calibrated clocks. Each clock would need to be constructed and calibrated in the appropriate reference frame for the test. Thus each clock's reference frame of rest must already be in motion with respect to the other clock's reference frame during the construction process. I have demonstrated in another paper that any two such clocks, constructed and calibrated by the same methodology in relatively moving inertial frames will maintain synchronous time. [1] However, such a test, including the problem of finding two inertial frames, is extremely difficult and has never been performed.

Instead, we have relied on constructing identical synchronous clocks in the same reference frame. One of these clocks is then moved into another reference frame, and the rates of the clocks are compared, either directly or by comparing the accumulated time on each. The process of moving the clock out of one reference frame and into another of necessity involves the application of acceleration, and thus energy, to the clock system, whether it be a muon or an atomic clock. These clocks do indeed

Author's Current Address:
680 America's Cup Cove, Alpharetta, Georgia 30005 USA.

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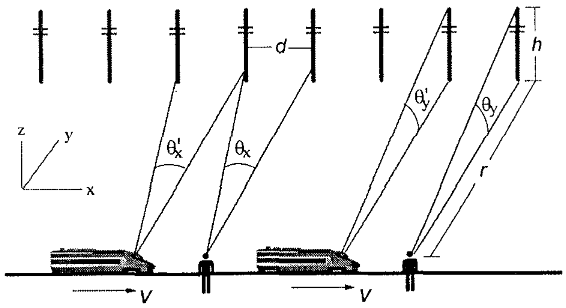

Fig. 1.

slow down, but it is impossible to determine between any of several possible reasons for this effect. The result may be due to the application of energy to the system (as this author has demonstrated is probable), it may be due to an empirical change in time recording mechanisms between reference frames, or perhaps the dimension of time is skewed in a manner predicted by the Lorentz transform.

Next we look at length contraction. While the tests of time dilation are weak and inconclusive, there has never been any direct test of the Lorentz length contraction. The reason for this is quite simple. In order to test directly for length contraction, we need a combination of very high speeds and very precise measurements. Even as the twentieth century draws to a close, there is no way to test directly for the Lorentz length contraction. But such a test will soon be possible.

In 2005, the National Aeronautics and Space Administration (NASA) will launch the Space Interferometry Mission (SIM). This satellite uses a highly stable platform combined with a very precise interferometer to measure angular separation of stars and galaxies. This project promises unprecedented accuracy in terms of astrometric grid calculations and star mapping. The sensitivity of the mission is ± 1 microarcsecond (μas) in a field of view of one degree, and ± 4 μas in a field of view of fifteen degrees. This resolution is less than the width of your finger as seen from the moon. The next best star mapping ever performed has been by the HIPPARCOS satellite, at a precision of only milliarcseconds (μas). To understand the importance of this increased sensitivity, we must first look at an effective approach to measuring length contraction.

MEASURING LENGTH BY SUBTENDED ANGLES

In the spirit of Einstein, in Figure 1, we see a highspeed train and another observer stationary on the embankment. In the extreme distance is a uniformly spaced set of equal height telephone poles. The distance from the tracks to the poles is very large, and measured in advance to be a certain value *r*. Under special relativity, this distance will not change for the train observer, as it is always normal to the direction of the train's motion. The Lorentz transformations are such that the effect on length occurs only in the direction of motion. Thus, if we assume the train is moving along the x axis, then the distance to the poles lies along the y axis, and the height of the poles lies along the z axis. Neither of these dimensions will change. However, the distance between each set of poles is measured along the x axis and will change according to the Lorentz length contraction formula.

If the poles are far enough in the distance, they will not appear to move in the field of view of the train rider. This observer can make very precise measurements of the angle of separation between any two poles. Since the value of *r* was determined in advance, the observer can calculate a value for the distance between the poles as follows:

$$\sin\theta_x = d/r \qquad (1)$$

$$d = r\sin\theta_x \qquad (2)$$

The train rider can also determine the height of the poles by a similar method, such that:

$$h = r\sin\theta_y \qquad (3)$$

The observer on the embankment can make similar angle measurements and thus determine the separation distance and height as seen from the embankment frame of reference. Clearly, any change in *d* produces a proportional change in θ_x. Now we can introduce the effects predicted by special relativity.

ABERRATION

The first effect to consider is aberration. Aberration is a well documented phenomenon, and is not unique to the theory of special relativity. For the moving train rider, all angles measured along the x axis will be precessed by the aberration factor. The y axis itself will be moved forward through an angle such that:

$$\sin\theta_a = v/c \qquad (4)$$

This skewing of angles due to aberration results in a change in the apparent angular separation between any two distant objects lying apparently parallel to the x axis. As can be seen from Figure 2, the apparent distance, *d'*, between any two objects is reduced as below:

$$d' = d\sqrt{1-\frac{v^2}{c^2}} = d\gamma^{-1} \qquad (5)$$

Substituting (5) into equation (1) we get:

$$\sin\theta_x' = \frac{d'}{r} = \frac{d}{r}\gamma^{-1} = (\sin\theta_x)\gamma^{-1} \qquad (6)$$

For small angles, the sin of the angle changes in direct proportion to the angle itself, and we can approximate:

$$\theta_x' \approx \theta_x \gamma^{-1} \tag{7}$$

While the above approximation is useful for conceptual studies, in any real observations the actual angles would need to be computed directly from the sin values, or vice versa to determine the true effects.

In the above we have seen that the aberration effect produces an apparent length contraction in the direction of motion of the moving observer. The value of this apparent contraction is even the same as that predicted by Lorentz contraction, namely y^{-1}. However, this effect is not Lorentz length contraction. If you hold a ruler at arm's length, it will appear to have a certain length, or subtended angle, in your field of view. If you now rotate that ruler through some small angle, the apparent length, or subtended angle,

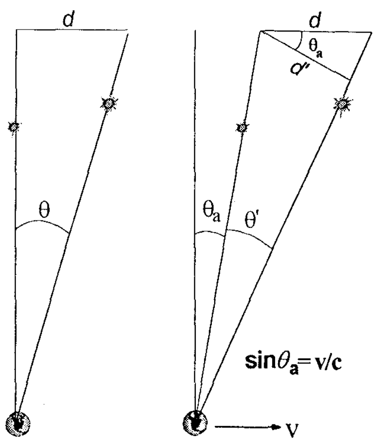

Fig. 2.

will become smaller. This foreshortening of objects rotated through an angle is the effect we see in aberration. Its value is coincidentally the same as that predicted by Lorentz contraction, and in fact, it combines with Lorentz contraction to produce an even greater overall observed effect. As with the case of length contraction, referring again to Figure 1, we see that aberration will affect the apparent distance between the poles, but will have no effect on the apparent height of the poles. Note also that aberration does not always work in such a manner as to reduce the apparent angular separation. Velocity and angle combinations that aberrate the stars to a position closer to θ equals zero will actually serve to increase the apparent angular separation. Also, any initial angle other than θ equals 0 will produce an aberration factor different from the Lorentz contraction factor.

LORENTZ LENGTH CONTRACTION

We can measure the length of a train from an embankment by the following procedure. We use several observers stationed along the embankment to mark the location adjacent to the front of the train and the end of the train at some particular time measured in the reference frame of the embankment observers. After the train has moved on, we measure the distance between these two marks with a meter stick. According to special relativity, this measured distance will be less than the length of the train measured by a rider on the train with a similar meter stick. The train's length measured by the embankment observer will be shortened by the factor y^{-1} as in Eq. (5). Lorentz contraction is, however, completely independent of aberration.

According to the Lorentz transformations, both observers in Figure 1 will determine the same value for the height, h, of the poles, independent of the effects of Lorentz contraction or aberration. However, ignoring aberration for the moment, the train rider should see the distance between the poles shortened by the Lorentz contraction. This observer's value compared with that of the embankment observer is exactly as expressed in Eq. (5).

Using the same reasoning as we did for aberration, we see that the apparent subtended angle due to length contraction alone is reduced to the same degree as we saw for aberration. Thus, still ignoring aberration, the relation between the subtended angle between poles seen by the train rider, θ_x, and the angle seen by the embankment observer, θ_x, is as given in Eq. (7).

ABERRATION AND LORENTZ CONTRACTION IN ASTROMETRIC OBSERVATIONS

We can now determine the entire effect of Lorentz contraction combined with aberration, by consulting Figure 3. In Figure 3, on next page, with the earth moving in the same direction as the apparent line joining the two objects, θ represents the apparent change in subtended angle due to aberration, while θ' represents the combined effect of aberration and Lorentz contraction. From the above analysis, we have:

$$\sin\theta_x'' = (\sin\theta_x')\gamma^{-1} = (\sin\theta_x)\gamma^{-2} \tag{8}$$

or, for conceptual approximation purposes:

$$\theta_x'' \approx \theta_x \gamma^{-2} \tag{9}$$

Consider a set of stars or galaxies appearing almost overhead from the plane of the earth's orbit about the sun. As the earth orbits the sun, its velocity along a line joining any two such objects will change from a minimum of 0 km/sec to a maximum of about 30 km/sec every three months. The aberration angle along this line will also change due to the combined effects of aberration and Lorentz contraction as in Eq. (9). The change in subtended angle will vary from a maximum to a minimum every three months as well. We can quite easily calculate the magnitude of this effect for the velocity of the earth:

As the earth orbits the sun, the angular separation of stars and galaxies should vary by ±36 μas per degree of field of view. This variation, equal to only 1 part in 10^{-8} is very small, well below conventional means for detection. But the means will soon be available.

The example in Figure 3 is a very special case in which the aberration and Lorentz contraction effects are equal and combine to produce the total effect. However, as was noted earlier, sometimes the aberration effect acts in a

$$v \approx 30 km/\sec$$

$$1-\gamma^{-1} \approx 5\mathrm{E}-09$$

$$Aberration: \frac{\Delta\theta_A}{\theta} \approx 5\mathrm{E}-09 \deg/\deg \approx 18\mu\mathrm{as}/\deg$$

$$Lorentz\ Contraction: \frac{\Delta\theta_L}{\theta} \approx 5\mathrm{E}-09 \deg/\deg \approx 18\mu\mathrm{as}/\deg$$

$$Combined\ Effect: \frac{\Delta\theta}{\theta} \approx 36\mu\mathrm{as}/\deg$$

manner to increase the apparent subtended angle. In the example on the cover of this issue, the Lorentz contraction and aberration effects are almost equal and opposite, and the net effect is to leave the observed angle unaffected. There are also cases where only Lorentz contraction appears, and aberration plays almost no role.

If the observer is moving directly toward the two stars of interest, the Lorentz contraction will reduce their effective distance, and thus increase the apparent angle subtended by the stars. In this case, Lorentz contraction, acting alone, actually makes the observed angles larger, not smaller. Thus as with aberration, the Lorentz factor depends on angle of line of sight, and may increase, decrease, or leave unchanged the apparent subtended angle. The important consideration is that the effect is large enough, in certain angles, to be observed directly.

Many papers have been written on the apparent shape of objects viewed by observers in relative motion, as well as the apparent length and size of such objects. Three of these are noted in the references [2, 3, 4] In the current study, the factors of greatest importance are the following: 1) the field of regard or field of view, extending from one degree to fifteen degrees, is not trivial, thus we are not dealing with differential subtended angles; 2) the observer always has some nonzero velocity with respect to the object(s) of view. For example, while motion may be in the same direction as a line joining two stars in one viewing, it will be normal to that line three months later. We are always comparing the image as viewed by one moving observer with that as viewed by another moving observer, and never by an observer stationary with respect to the stars. The combined effect of different viewing periods can be relatively large or almost negligible; 3) contrary to the example given, there can be a change in the z-axis or "height" subtended angle due to motion along the x axis. Except for objects located directly on the y-axis, there will be a component of the x-axis velocity along the line joining the observer and the target object. Length contraction along this line will reduce the distance to the object, thus resulting in a larger "height" or z-axis angle for the observer; and 4) the objects under consideration are far enough away that, for a given velocity, their aberrated angle does not change with time. In other words, if we observe a star on January 1 at twenty degrees, it will basically appear at twenty degreess the next day as well, even though the earth will have moved a distance of 2.5

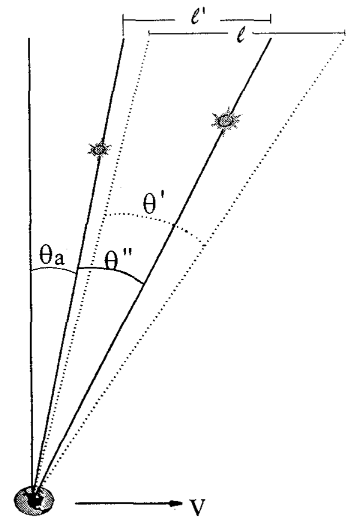

Fig. 3.

million kilometers. Thus the effect of the earth's motion during the path-length delay from two different stars may be ignored in most cases. Each of these points must be considered in reviewing the literature.

NASA'S SPACE INTERFEROMETRY MISSION

In 1886, Michelson and Morley performed a test for the earth's motion through the presumed aether using two orthogonal beams of light and the interference pattern obtained. This device, referred to as a Michelson interferometer, almost single handedly eliminated the concept of an all pervading aether from the minds of most physicists of the day, and led the way for the acceptance of Einstein's special theory of relativity nine years later. The same concept, observing the shift in interference patterns produced by divergent and recombined light paths from the same source, has been gaining popularity in the world of astronomy. From the Very Large Baseline Interferometer (VLBI) array to the recent space-based HIPPARCOS mission, unprecedented resolution in visibility, position and proper motion measurements have been made.

The final results of the HIPPARCOS mission provided a star table with a median standard error in position and proper motion calculations on the order of 0.8 mas from a defined grid. The effects predicted in the analysis above are much smaller than the resolution of

HIPPARCOS and would not appear in the overall results from that program, the best to date. When fully operational, the US Navy's ground-based Prototype Optical Interferometer (NPOI) promises a resolution of 200 μas. This array of six telescopes is incredibly useful for observations of orbits of double stars and for planet hunting, but still does not possess the resolution required for a direct test of Lorentz contraction.

In 2005, NASA plans to launch the Space Interferometry Mission (SIM) satellite. SIM operates by comparing the change in path length along two arms of an interferometer of light from a test star compared to a baseline or grid star. By measuring this change in path length to an accuracy of 1 nanometer the position of the test star can be obtained within about 4 μas of the grid star. This level of accuracy can be obtained over the entire field of view of the instrument, which is about 15 degrees. In a field of view of 1 degree or less, the predicted accuracy is on the order of 1 μas. The changes predicted by special relativity due to aberration and length contraction, illustrated in an exaggerated manner in Figure 4, amount to almost 540 μas in a field of view of 15 degrees every three months, more than 400 times the proposed resolution capability of the instrument.

Among the requirements currently placed on SIM is the ability to point everywhere on the celestial sphere outside of a fifty-degree sun exclusion area during a thirty-day period. If an observation were to be made of a particular pair of objects at approximate zenith at one point in time, then that same pair of objects should be readily available for viewing again after the desired 90 day interval. Thus the capability, opportunity and required precision exist in SIM to perform the direct-test of Lorentz length contraction proposed in this article.

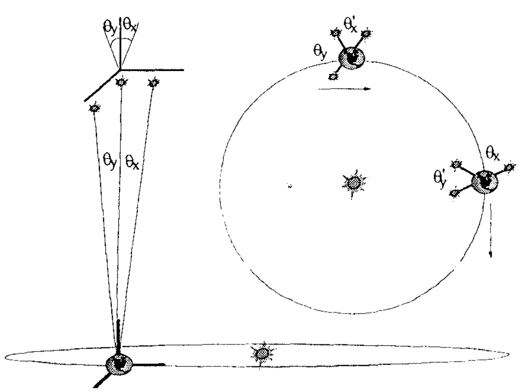

Fig. 4.

CONCLUSIONS

In 1919, Sir Arthur Eddington made observations of the angular displacement of stars during a solar eclipse. Newtonian theory predicted one value, while general relativity predicted exactly twice that value. As is the case today, the effects to be observed were at the extreme limits of available technology. When Eddington's observations appeared to come down on the side of general relativity, a New York Times headline proclaimed: *"New Theory of the Universe. Newtonian Ideas Overthrown."*

Special relativity predicts that if we look at two stars with an angular separation of about 1 degree at time intervals of ninety days, we will see a variation in the observed angle of separation. This variance is made up of two equal components. In the example presented in this article, the first component is a variation of 18 μas due to aberration. The second component is a variation of 18 μas due to Lorentz length contraction. The SIM promises an angular resolution of 1 μas in a field of view of 1 degree, more than an order of magnitude better than the effects predicted. An added benefit of this level of precision is that SIM can determine the angle of aberration itself to an unprecedented degree of accuracy. Using this measured angle of aberration, the angular displacement effects of that aberration may be calculated directly. As in 1919, special relativity predicts a value twice that to be otherwise expected, with the residual due to Lorentz contraction. The author has demonstrated in several previous papers that the Lorentz length contraction likely does not exist, and, therefore, will not be found by SIM. If the residual is found, then for the first time special relativity will have passed a direct test of one of its most fundamental predictions. If the residual is not found, then special relativity will have to be abandoned completely and the New York Times may wish to consider a retraction.

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