Čerenkov effect and the Lorentz contraction

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The power spectral formula of Čerenkov radiation of the system with two equal charges is derived in the framework of the source theory. The distance between charges is supposed to be relativistically contracted, which manifests in the spectral formula. The knowledge of the spectral formula then can be used to verify the Lorentz contraction of the relativistic length. The radiative corrections to the Čerenkov effect are also considered. A feasible experiment for the verification of the Lorentz contraction is suggested. [S1050-2947(97)09202-0]

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I. INTRODUCTION

A fast moving charged particle in a medium when its speed is faster than the speed of light in this medium produces electromagnetic radiation, which is called Čerenkov radiation. This radiation was first observed experimentally by Čerenkov [1] and theoretically interpreted by Tamm and Frank [2] in the framework of classical electrodynamics. A theoretical description of this effect was given by Schwinger, Tsai, and Erber [3] at the zero-temperature regime and the classical spectral formula was generalized to the finite-temperature situation in the framework of the source theory by Pardy [4].

The question arises: What is the relation of the Čerenkov radiation to the relativistic length? The relativistic length can be formed by the system of charges of the linear chain or only by the two charges of the rest distance l. The problem of the radiation of the composed systems of charges is not new and it was defined in the pioneering work of Ginzburg [5]. Later, Frank [6] gave the solution of the problem of Čerenkov radiation of the electrical and magnetical dipoles oriented parallel and perpendicular to the direction of motion. While the parallel orientation gives no surprising result the situation with the perpendicular orientation gives a special anomaly that has been frequently discussed in the literature. In 1952 an article was published discussing Čerenkov radiation of the arbitrary electrical and magnetical multipoles [7]. A review of the problems of Čerenkov radiation of the magnetic and electrical multipoles was given by Frank [8]. Extensive work concerning radiation by uniformly moving sources is involved in Ref. [9]. However, the problem of testing the Lorentz contraction by the Čerenkov effect is considered here.

While the original articles on the Čerenkov radiation involve only determination of spectral formulas, there is interest in the question of the relationship between the spectral formula and Lorentz contraction of the length of some linear object. The specific situation forms the system of two equal or opposite charges of the rest distance l. Then we can expect that the spectral formula of the Čerenkov radiation involves the Lorentz contraction which follows immediately from the Lorentz transformation for coordinates between systems S' and S:

$$x' = \gamma(x - vt), \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{1}$$

where x are coordinates in the system S and x' are corresponding coordinates in the system S', which is moving with velocity v relative to the system S. If the left and right points of the moving rod are x_1, x_2 in the system S and x_1' and x_2' in the system S', then from Eq. (1) we have

$$x_2' - x_1' = \gamma(x_2 - x_1), \tag{2}$$

which can be transcribed in the form

$$a = l\sqrt{1 - v^2/c^2}, \tag{3}$$

where l is the length of the rest rod and a is the length of the moving rod.

The formula (3) is well known and since the formulation of the special theory of relativity by Einstein it is generally believed that the so-called Lorentz contraction (3) should be visible to the eye. Also Lorentz stated in 1922 that the contraction could be photographed. Similar statements appear in other references concerning the special theory of relativity. However, the special theory of relativity predicts that the contraction can be observed by a suitable experiment with the nuance that there is a distinction between observing and seeing. The situation was analyzed, for instance, by Terrell [10] and Weisskopf [11], who proved that the photograph obtained by an observer depends only on the place and time in which the picture was taken and is independent of the relative motion of the observer and object photographed.

In other words, an observation of the shape of a fast moving object involves simultaneous measurement of the position of a number of points on the object. If done by means of light, all quanta should leave the surface simultaneously, as determined in the observer position at different times. In such an observation the data received must be corrected for the finite velocity of light, using measured distances to various points of the moving object. In seeing the object, on the other hand, or photographing it, all the light quanta arrive

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simultaneously at the eye having departed from the object at various earlier times. In such a way this should make a difference between contracted shape, which is in principle observable, and the actual visual appearance of a fast-moving object.

Obviously, Čerenkov radiation of the charged two-particle system involves the Lorentz contraction of their rest distance. We will consider the system of two equal charges e, which have the mutual rest distance l. The Lorentz contraction will be involved in the power spectral formula for this linear system.

In this paper we evaluate in source theory the power spectral formula of the Čerenkov radiation of the two-charge system moving with velocity v in the dielectrical medium. Radiative corrections to two-body Čerenkov radiation are considered too. In conclusion, a feasible experiment is suggested for the verification of the Lorentz contraction.

II. THE SOURCE THEORY FORMULATION OF THE PROBLEM

Source theory [3,12,13] is the theoretical construction that uses quantum-mechanical particle language. Initially it was constructed for a description of the particle physics situations occurring in high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity where the interactions are mediated by the photon or graviton, respectively.

The basic formula in the source theory is the vacuum to vacuum amplitude [3]

$$\langle 0_+|0_-\rangle = e^{(i/\hbar)W(S)}, \tag{4}$$

where the minus and plus signs on the vacuum symbol are causal labels, referring to any time before and after the space-time region where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements, which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add [12,13].

The electromagnetic field is described by the amplitude (4) with the action

$$W(J) = \frac{1}{2c^2}\int (dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{5}$$

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is the charge and current densities. The symbol $D_{+\mu\nu}(x-x')$ is the photon propagator and its explicit form will be determined later.

It may be easy to show that the probability of the persistence of vacuum is given by the following formula [3]:

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{-\frac{2}{\hbar}\mathrm{Im}W\right\} \overset{d}{=} \exp\left\{-\int dt\,d\omega\,\frac{P(\omega,t)}{\hbar\omega}\right\}, \tag{6}$$

where we have introduced the so-called power spectral function [3] $P(\omega,t)$. In order to extract this spectral function from ImW, it is necessary to know the explicit form of the photon propagator $D_{+\mu\nu}(x-x')$.

The electromagnetic field is described by the four-potentials $A^\mu(\phi,\mathbf{A})$ and it is generated by the four-current $J^\mu(c\varrho,\mathbf{J})$ according to the differential equation [3]

$$\left(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right)A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)J_\nu \tag{7}$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_+^{\mu\nu} = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right)D_+(x-x'), \tag{8}$$

where $\eta^\mu \equiv (1,\mathbf{0})$, μ is the magnetic permeability of the dielectric medium with the dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_+(x-x')$ was derived by Schwinger, Tsai, and Erber [3] in the following form:

$$D_+(x-x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega\,\frac{\sin(n\omega/c)|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{9}$$

Using formulas (5), (6), (8), and (9), we get for the power spectral formula the following expression [3]:

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\,\frac{\sin(n\omega/c)|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\times\cos[\omega(t-t')]\times\left\{\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')\right\}. \tag{10}$$

Now, we are prepared to apply the last formula to the situations of the two equal charges moving in the dielectric medium.

III. ČERENKOV RADIATION OF THE TWO-CHARGE SYSTEM

It is usually assumed that Čerenkov radiation in electrodynamics is produced by uniformly moving charge with constant velocity. Here we consider the system of two equal charges e with the constant mutual distance $a=|\mathbf{a}|$ moving with velocity $\mathbf{v}$ in the dielectric medium. In this situation the charge and the current densities for this system are given by the following equations:

$$\varrho = e[\delta(\mathbf{x}-\mathbf{v}t) + \delta(\mathbf{x}-\mathbf{a}-\mathbf{v}t)], \tag{11}$$

$$\mathbf{J} = e\mathbf{v}[\delta(\mathbf{x}-\mathbf{v}t) + \delta(\mathbf{x}-\mathbf{a}-\mathbf{v}t)], \tag{12}$$

where $\mathbf{a}$ is the vector going from the left charge to the right charge with the length of $a=|\mathbf{a}|$ in the system S.

Let us suppose that $\mathbf{v}\|\mathbf{a}\|\mathbf{x}$. Then, after insertion of Eqs. (11) and (12) into Eq. (10), putting $\tau = t'-t$, and $\beta = v/c$, where $v=|\mathbf{v}|$, we get instead of the formula (10) the following relation:

$$P(\omega,t)=2P_1(\omega,t)+P_2(\omega,t)+P_3(\omega,t), \quad (13)$$

where

$$P_1(\omega,t)=\frac{1}{4\pi^2}\frac{e^2\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right]\times\int_{-\infty}^{\infty}d\tau\frac{\sin n\omega\beta\tau}{\tau}\cos\,\omega\tau, \quad (14)$$

$$P_2(\omega,t)=\frac{1}{4\pi^2}\frac{e^2\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right]\times\int_{-\infty}^{\infty}d\tau\frac{\sin n\omega\beta|a/v+\tau|}{|a/v+\tau|}\cos\,\omega\tau, \quad (15)$$

$$P_3(\omega,t)=\frac{1}{4\pi^2}\frac{e^2\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right]\times\int_{-\infty}^{\infty}d\tau\frac{\sin n\omega\beta|a/v-\tau|}{|a/v-\tau|}\cos\,\omega\tau. \quad (16)$$

The formula (14) contains the known integral:

$$J_1=\int_{-\infty}^{\infty}d\tau\frac{\sin n\omega\beta\tau}{\tau}\cos\,\omega\tau=\begin{cases}\pi, & n\beta>1\\ 0, & n\beta<1.\end{cases} \quad (17)$$

Formulas (15) and (16) contain the following integrals:

$$J_2=\int_{-\infty}^{\infty}d\tau\frac{\sin n\omega\beta|a/v+\tau|}{|a/v+\tau|}\cos\,\omega\tau \quad (18)$$

and

$$J_3=\int_{-\infty}^{\infty}d\tau\frac{\sin n\omega\beta|a/v-\tau|}{|a/v-\tau|}\cos\,\omega\tau. \quad (19)$$

Using the integral (17) we finally get the power spectral formula P_1 of the produced photons:

$$P_1(\omega,t)=\frac{e^2}{4\pi}\frac{\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right], \quad n\beta>1 \quad (20)$$

and

$$P_1(\omega,t)=0, \quad n\beta<1. \quad (21)$$

Using transformations

$$\frac{a}{v}+\tau=T, \quad \frac{a}{v}-\tau=T, \quad (22)$$

we get after evaluation of the corresponding integrals J_2,J_3, the corresponding spectral formulas P_2,P_3:

$$P_2(\omega,t)=\frac{e^2}{4\pi}\frac{\mu\omega}{c^2}\cos\left(\frac{\omega a}{v}\right)v\left[1-\frac{1}{n^2\beta^2}\right]=P_3, \quad n\beta>1 \quad (23)$$

and

$$P_2(\omega,t)=P_3(\omega,t)=0, \quad n\beta<1. \quad (24)$$

The sum of the partial spectral formulas forms the total radiation emitted by the Čerenkov mechanism of the two-charge system:

$$P(\omega,t)=2(P_1+P_2)=\cos^2\left(\frac{a\omega}{2v}\right)\frac{e^2}{4\pi}\frac{\mu\omega}{c^2}v\left[1-\frac{1}{n^2\beta^2}\right], \quad n\beta>1 \quad (25)$$

and

$$P(\omega,t)=0, \quad n\beta<1. \quad (26)$$

The zero point of the function $P(\omega,t)$ are as follows:

$$\omega_0=0, \quad \frac{\omega_n a}{2v}=\frac{(2n-1)}{2}\pi, \quad n=1,2,3,\ldots. \quad (27)$$

From the last equation follows

$$a=\frac{(m-n)2\pi v}{(\omega_m-\omega_n)}=l\sqrt{1-\frac{v^2}{c^2}}, \quad (28)$$

or

$$l=\frac{2\pi v}{\sqrt{1-v^2/c^2}}\frac{m-n}{\omega_m-\omega_n}. \quad (29)$$

If we know the nth and mth zero points with the corresponding ω's and velocity of the charges we can exactly determine their rest distance. Then, the rest distance determined by the formula (29) can be compared with the rest distance of the charges obtained by direct measurement and in such a way that we can verify the Lorentz contraction.

IV. RADIATIVE CORRECTIONS

We will investigate for the sake of completeness how the spectrum of the two-charge Čerenkov radiation is modified if we involve the radiation correction in the photon propagator. According to [13–16], the photon propagator involving the polarization of the vacuum is given in the momentum representation in the form

$$\widetilde{D}(k)=D(k)+\delta D(k) \quad (30)$$

or

$$\widetilde{D}(k)=\frac{1}{|\mathbf{k}|^2-n^2(k^0)^2-i\epsilon}+\int_{4m^2}^{\infty}dM^2\frac{a(M^2)}{|\mathbf{k}|^2-n^2(k^0)^2+M^2c^2/\hbar^2-i\epsilon}, \quad (31)$$

where the last term in Eq. (31) is derived on the virtual photon condition

$$|\mathbf{k}|^2-n^2(k^0)^2=-\frac{M^2c^2}{\hbar^2}. \quad (32)$$

The weight function $a(M^2)$ has been derived in the following form [13,14]:

$$a(M^2)=\frac{\alpha}{3\pi}\frac{1}{M^2}\left(1+\frac{2m^2}{M^2}\right)\left(1-\frac{4m^2}{M^2}\right)^{1/2}. \tag{33}$$

The x representation of $\delta D_+(x-x')$ in Eq. (30) is as follows:

$$\delta D_+(x-x')=\frac{i}{c}\frac{1}{4\pi^2}\int_{4m^2}^{\infty}dM^2a(M^2)\times\int d\omega\frac{\sin[n^2\omega^2/c^2-M^2c^2/\hbar^2]^{1/2}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\times e^{-i\omega|t-t'|}. \tag{34}$$

The function (34) differs from the original function D_+ especially by the factor

$$\left(\frac{\omega^2n^2}{c^2}-\frac{M^2c^2}{\hbar^2}\right)^{1/2} \tag{35}$$

and by the additional mass integral, which involves the radiative corrections to the original Čerenkov effect.

The electromagnetic action involving the modified photon propagator is now of the form

$$W(J)=\frac{1}{2c^2}\int(dx)(dx')J^{\mu}(x)\widetilde{D}_{+\mu\nu}(x-x')J^{\nu}(x'), \tag{36}$$

where

$$\widetilde{D}_{+}^{\mu\nu}=\frac{\mu}{c}\left(g^{\mu\nu}+\frac{n^2-1}{n^2}\eta^{\mu}\eta^{\nu}\right)\widetilde{D}_{+}(x-x'). \tag{37}$$

Now, if we insert Eq. (36) into Eq. (6), we get after extracting $\delta P(\omega,t)$ the following general expression for this spectral function:

$$\delta P(\omega,t)=-\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\left[\int_{4m^2}^{\infty}dM^2a(M^2)\frac{\sin[n^2\omega^2/c^2-M^2c^2/\hbar^2]^{1/2}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\right]\times\cos[\omega(t-t')]\left[\varrho(\mathbf{x},t)\varrho(\mathbf{x}',t')-\frac{n^2}{c^2}\mathbf{J}(\mathbf{x},t)\cdot\mathbf{J}(\mathbf{x}',t')\right]. \tag{38}$$

Now, let us apply the formula (38) in order to get Čerenkov radiation with radiative corrections for the system of two charges. Using the same procedures that we applied to the case with no radiative corrections, we have for $\delta P(\omega,t)$

$$\delta P(\omega,t)=\frac{e^2}{4\pi^2}\frac{v\mu\omega}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\cos^2\left(\frac{\omega a}{2v}\right)\times\int_{4m^2}^{\infty}dM^2a(M^2)\times\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\sin\left(\left[\frac{n^2\omega^2}{c^2}-\frac{M^2c^2}{\hbar^2}\right]^{1/2}v\tau\right)\cos\omega\tau, \tag{39}$$

where we have also put $\tau=t'-t,\beta=v/c$. In case of $\delta P(\omega,t)$ the evaluation of the τ integral is more complex than in the case with the evaluation of the τ integral in P. It may be easy to show that

$$\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\sin\left(\left[\frac{n^2\omega^2}{c^2}-\frac{c^2M^2}{\hbar^2}\right]^{1/2}v\tau\right)\cos\omega\tau=\begin{cases}\pi, & 0<M^2<(\hbar^2\omega^2/c^2v^2)(n^2\beta^2-1)\\ 0, & M^2>(\hbar^2\omega^2/c^2v^2)(n^2\beta^2-1).\end{cases} \tag{40}$$

From Eq. (40) it immediately follows that $M^2>0$ implies the Čerenkov threshold $n\beta>1$. From Eqs. (39) and (40) we get that the radiative corrections to the original spectral formula of Čerenkov radiation are given by the formula

$$\delta P=\frac{e^2}{4\pi}\frac{v\omega\mu}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\cos^2\left(\frac{\omega a}{2v}\right)\int_{M_1^2}^{M_2^2}dM^2a(M^2), \tag{41}$$

where

$$M_1^2=4m^2,\quad M_2^2=(n^2\beta^2-1)\frac{\hbar^2\omega^2}{c^2v^2}. \tag{42}$$

By the substitution

$$t=\left(1-\frac{4m^2}{M^2}\right)^{1/2}; \tag{43}$$

and after some elementary integration, we get the radiative contribution to the two-charge Cerenkov effect in the following form:

$$\delta P(\omega,t)=\alpha\frac{e^2}{4\pi^2}\frac{v\mu\omega}{c^2}\left(1-\frac{1}{n^2\beta^2}\right)\times\cos^2\left(\frac{\omega a}{2v}\right)\left(\frac{s^2}{9}-\frac{2}{3}s+\frac{1}{3}\ln\left|\frac{1+s}{1-s}\right|\right),\quad n\beta>1, \tag{44}$$

where

$$s=\left(1-\frac{4m^2c^2v^2}{(n^2\beta^2-1)\hbar^2\omega^2}\right)^{1/2},\quad s\geqslant 0. \tag{45}$$

The condition $s \geqslant 0$ in Eq. (45) implies the existence of the radiation corrections to the original Frank-Tamm formula for

$$\omega^2 > \frac{4m^2c^2v^2}{\hbar^2(n^2\beta^2-1)}. \tag{46}$$

For $n=\sqrt{2}$ and $v \approx c$, we get from Eq. (46) $\hbar\omega \approx 2mc^2$, which can be interpreted as the condition for creation of the electron-positron pair by the γ quantum.

V. A FEASIBLE EXPERIMENT

With regard to the situation in laboratories where the great accelerator works, for instance, in Grenoble, DESY, CERN, and SLAC we can suggest a feasible experiment for the verification of the Lorentz contraction. The experiment must be based on the definition of the length. Instead of two electrons we can consider two bunches with 10^{10} electrons in volume 300 μm$\times$40 μm$\times$0.01 m with the rest distance $l=1$ m. After acceleration of the considered bunches the distance of the two bunches is the relativistic length a and it can be determined by the Čerenkov spectrum derived in our article. However, during acceleration the motion of particles in storage rings is influenced by various kinds of perturbations. It is necessary to consider phenomena such as the ground motion, power supply ripple, noise caused by the quantum emission of synchrotron radiation, and noise in the radio-frequency (rf) system, and so on. Therefore it is necessary to include the stochasticity caused by these effects in the calculation of the beam dynamics. The stochastic forces can change the distance of the bunches. So instead of the determination of the rest length at the beginning of the experiment, it is more suitable to determine the rest length immediately after the determination of the Čerenkov spectrum.

We can slow down the velocity of bunches by the simultaneous deceleration of every bunch in order to get the final nonrelativistic velocity v_f instead of the relativistic velocity v in the spectral formula. It can be performed by switching the electric field or by a sufficiently intensive laser field of photons moving in the opposite direction of motion of the bunches. Simultaneity is a necessary inevitable condition in order to conserve the length during deceleration.

If a particle is accelerated in the system S by the constant acceleration w, then the law of its motion with the initial conditions $x(0)=0, v(0)=0$ is as follows:

$$x_1(t) = \frac{c^2}{w}\left[\sqrt{1+\left(\frac{wt}{c}\right)^2}-1\right] \tag{47}$$

and in case of the initial condition $x(0)=l, v(0)=0$, we have for the law of its motion

$$x_2(t) = x_1(t) + l. \tag{48}$$

So, in the case of the acceleration of the free two-body system we get

$$x_2(t) - x_1(t) = l \tag{49}$$

and the observer in the system S observes that the distance of the electrons is equal to l. In the case where acceleration is replaced by deceleration, the final result is the same. Or, the observer in the system S finds that the distance of the two electrons or bunches does not change during deceleration. In the case of application of the laser field the simultaneity is broken with the difference $l/c \approx 10^{-9}$ s in the system of bunches, for the distance $l = 1$ m. However, such deviation from the simultaneity is sufficiently small in order not to influence substantially the result of experiment. It is evident that in order for the experiment to be meaningful it will be necessary to respect the law of deceleration motion from which Eq. (49) follows.

Our situation does not represent the rigid motion considered by Rindler [18]. He shows that for the so-called rigid motion at every instant $t=$const the two points are separated by a coordinate distance dx inversely proportional to their γ factor, and consequently the element bounded by these points ''moves rigidly.''

The two bunches impinge into the detector with the time difference $\Delta t = l/v_f$. This time difference can be determined by the scintillation detector with the sufficient time resolution. The scintillation detectors or counters consist of scintillating materials, usually a doped plastic, which emit light in response to molecular excitation by the passage of a charged particle. The scintillation light can be detected with photomultipliers or photodiodes. The light yield in a plastic scintillator is usually sufficiently large. The scintillation counters range in size from very small to very large—a few square meters. The important feature of scintillation counters is their speed, which is in the nanosecond range. So they can accurately measure the time of arrival of a charged particle and therefore the speed of a particle [17,18].

The rest length measured by the scintillation detector $l = \Delta t v_f$ can be compared with the formula (29) in order to verify the Lorentz contraction. For velocity $v_f = 10^4$ m, we have the $\Delta t \approx 10^{-4}$ s with the assumption that the Lorentz contraction corresponding to this velocity can be neglected. To our knowledge the detectors have better time resolution than the calculated Δt. So, the verification of the Lorentz formula is in principle possible.

VI. DISCUSSION

We have demonstrated that in the case of the system of two equal charges, the Lorentz contraction can be determined from the spectral formula of the Čerenkov radiation. Obviously this effect can be involved in the group of classical relativistic effects. In the case of the system of opposite charges, or, in other words, of the dipole we have instead of $\cos(\omega a/2v)$ the function $\sin(\omega a/2v)$ in the final formulas. To our knowledge the determination of the Lorentz contraction using the Čerenkov effect was not considered in theory and experiment. After performing the experiment with Čerenkov radiation of the system with two charges the Lorentz contraction will be definitely confirmed.

While the simultaneous acceleration of the system with two equal charges can be performed immediately in every laboratory with the circle accelerator, the simultaneous acceleration of the system with two opposite charges can be performed only with the laser accelerator. In this equipment the opposite charges are accelerated at the same acceleration as a result of the Compton effect.

We have considered for the sake of completeness the radiative corrections (44) to the original power spectral formula of the Čerenkov radiation. The condition (46) concerns the gamma photons rather than the optical ones. The possibility of the existence of the gamma Čerenkov radiation is discussed by Ion and Stocker [19] in nuclear physics.

The experiment suggested by us is feasible in the sense that the bunches of charges are prepared in every circle accelerator and therefore it is not necessary to prepare substantially the arrangement of the equipment for verification of the Lorentz contraction. We hope that eventually such an experiment will be performed.

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