

PROBLEM CLASS

Contents

- Problems taken from previous exams.

Note: the exams are **open-book**.

I tend to award $\frac{1}{2}$ a mark for each key-point.

2002 – η Meson

Explain briefly what is required to go from the singlet description in $SU(2)_{\text{flavour}}$ for the η meson to a complete description. [4/20 marks]

2003 – SU(3) Invariance of Singlet

The singlet $SU(3)_{\text{flavour}}$ state for three quarks is: $uds+dsu+sud-sdu-usd-dus$
Demonstrate that this is invariant under $SU(3)$ transformations.

It will suffice to consider infinitesimal $SU(3)$ transformations: $U = \exp(i\varepsilon\cdot\lambda) \approx 1 + i\varepsilon\cdot\lambda$
[6/20 marks]

Hint: consider the transformations generated by λ_1 , λ_2 and λ_8 separately along with suitable use of “likewise”. Under a transformation U , $q_1q_2q_3 \rightarrow U(q_1) U(q_2) U(q_3)$ – where the operator acts separately on the three individual quark states. The λ matrices are provided.

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ +i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

2004 – Neutron Wavefunction

The proton wavefunction ($S_z = +\frac{1}{2}$) in terms of flavour and spin is:

$$\frac{1}{\sqrt{2}} \frac{1}{\sqrt{6}} \{(ud + du)u - 2uud\} \frac{1}{\sqrt{6}} \{(\uparrow\downarrow + \downarrow\uparrow)\uparrow - 2\uparrow\uparrow\downarrow\} + \frac{1}{\sqrt{2}} \frac{1}{\sqrt{6}} (ud - du)u \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)\uparrow$$

Apply the Isospin lowering operator $I_- = I_-^1 + I_-^2 + I_-^3$ (superscripts refer to 1st, 2nd and 3rd quarks) to this wavefunction. Identify the resultant state and express its wavefunction in a form to make the similarity with that of the proton obvious. [4/20 marks]

2004 – Magnetic Moment

The magnetic moment operator for a single quark is proportional to the product of the charge and the spin operators: $Q\mathbf{S}$. When considering a baryon, since the wavefunction is symmetrised with respect to all three quarks, it will suffice to consider only the third quark (and multiply the result by 3). Find the ratios of the magnetic moments of the proton and neutron, by considering the matrix elements for the operator $Q^3\sigma_z^3$ (the superscript “3” indicates that these operators only act on the third quark).
[7/20 marks]

Hint. Q only operates on the flavour states and σ_z only operates on the spin states. The charge-flavour parts and spin parts of the calculation factorise. Work with the flavour ($\phi_{M,S}$ and $\phi_{M,A}$) and spin ($\chi_{M,S}$ and $\chi_{M,A}$) wavefunctions and avoid evaluating expressions until really necessary. Use the symmetries identified in the previous question.

2004 – Angular Distribution in GUT Decay

In some GUT Models, $p \rightarrow \pi^0 e^+$. Suppose a proton is polarised with its spin upwards, then the angular distribution of the positrons will depend on the amplitudes for the various helicity states. What would be the angular distribution of the right-handed positrons? Various rotation matrices which you can find in the PDG are reproduced below. [2/20 marks]

$$d_{+\frac{1}{2}+\frac{1}{2}}^{\frac{1}{2}}(\theta) = \cos \frac{\theta}{2} \quad d_{+\frac{1}{2}-\frac{1}{2}}^{\frac{1}{2}}(\theta) = -\sin \frac{\theta}{2}$$
$$d_{+1+1}^1(\theta) = \frac{1}{2}(1 + \cos \theta) \quad d_{+10}^1(\theta) = \frac{-1}{\sqrt{2}} \sin \theta \quad d_{+1-1}^1(\theta) = \frac{1}{2}(1 - \cos \theta) \quad d_{00}^1(\theta) = \cos \theta$$

2005 – SO(3)

Explain what is meant by the group $SO(3)$. Give a non-trivial example of one of the members (i.e. not the identity). [2/20 marks]

The members of the group $SO(3)$ can be written in the form $\exp(i\alpha G)$, where α is a real number and G is one of the generators of the group. In the context of a vector-space consisting of real vectors (rather than wavefunctions), derive the properties of the generators.

How many independent generators are there ? Identify a suitable set of independent generators.

[4/20 marks]

By exponentiating one of the generators, demonstrate how a typical member of $SO(3)$ (such as the one you identified in the first part) can be constructed. [3/20 marks]

Give the definition for the structure constants and adjoint of a group.

How many adjoint matrices are there for $SO(3)$.

Obtain one of the adjoint matrices corresponding to your choice of generators (you may assume cyclic symmetry).

Using symmetry, write the other adjoint matrices. [4/20 marks]

2004 – Quark Model

In the $SU(3)_{\text{colour}} \otimes SU(3)_{\text{flavour}} \otimes SU(2)_{\text{spin}}$ quark model, explain the observed meson states and their multiplicities, using Young Tableaux (there is no need to give wavefunctions or quantum numbers).
[5/20 marks]

2004 – Spin Mass-splittings

In the Hamiltonian for the meson masses, there is a term $\kappa \mathbf{S}_1 \cdot \mathbf{S}_2$, where $\mathbf{S}_{1,2}$ are the quark spins. Explain what effect this would have on the meson masses. What other effects are present which can account for the mass splittings ? [2/20 marks]

2005 – Closure in SU(2)

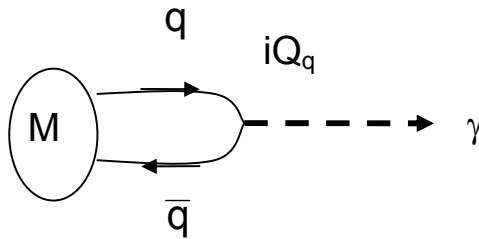
Given that the SU(2) operators can be represented by $\exp(i\alpha\mathbf{n}\cdot\boldsymbol{\sigma})$, where $\boldsymbol{\sigma}$ is the vector of Pauli spin matrices, show that the set of operators form a group under the operation “follows”. It is sufficient to consider infinitesimal transformations. [3/20 marks]

2005 – Multiplets

When the combinations of states of different particles are formed, it is natural to identify multiplets. Give the characteristics of multiplets, both within the group theory context and for physical combinations of quarks and hadrons. [1/20 marks]

2005 – π^0 decay

Consider the decay of a π^0 (member of an SU(2) triplet) and an η (SU(2) singlet, not SU(3) state, so no $s\bar{s}$ contribution) to a single photon through a decay diagram:



Calculate the ratio of the decay amplitudes. [2/20 marks]

Hint. If the decay operator is S , then the amplitude for meson M to decay to a photon γ is

$$\langle \gamma | S | M \rangle = \sum \langle \gamma | S | q\bar{q} \rangle \langle q\bar{q} | M \rangle$$

where the sum is over the possible quark flavours and $\langle \gamma | S | q\bar{q} \rangle \propto Q_q$, the quark charge.

Explain why the decay is unphysical (two reasons). [1/20 marks]

