SYMMETRIES & CONSERVATIONS LAWS

Homework

If you have problems, do not hesitate to contact me:

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Homework Set 1 – Lectures 1 & 2

Q 1.1) By considering the first few terms of the expansions, prove that

$$\exp(A)\exp(B) \neq \exp(A+B)$$

if A and B do not commute.
However, show that the equality holds if A and B do commute.

Q 1.2) Find an expression for $\exp(i\alpha A)$ where $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Q 1.3) If [A,B]=B, find an expression for $\exp(i\alpha A)B\exp(-i\alpha A)$

Q 1.4) Consider which of the following are groups:

- Integers under Addition
- Integers under Subtraction
- Integers under Multiplication
- Reals under Multiplication

Q 1.5) Demonstrate that there is only one group combination table for 3 distinct objects, i.e. all groups for 3 objects have the same form (are isomorphic) to Z_3.

Q 1.6) Show that the set of Lorentz Transformations:

$$g(\beta) \quad \begin{cases} x' = \gamma(x - \beta t) \\ t' = \gamma(t - \beta x) \\ \gamma = 1/\sqrt{1-\beta^2} \end{cases} \quad |\beta| < 1$$

form an Abelian Lie group under the operation "follows".

Hint: start by combining two boosts: $g(\beta_2)\ g(\beta_1)$ and showing that these correspond to a third boost.

Homework Set 2 – Lectures 2 & 3

Q 2.1) Show that U(n) and SU(n) are groups.

Q 2.2) Consider rotations in 3D about the x-, y- and z-axes – SO(3).
Identify generators appropriate to

a) Scalar wavefunctions $\psi(x)$ – we have done this in the Lectures; you can just write down the QM operators (do not write lots of blah, just write down operators)
b) Real vectors in 3D space – consider infinitesimal rotation matrices; the generators will be 3×3 matrices (give the rotation matrices, consider small angles and identify generators)

In both cases, find the structure constants. (Don't work out every single possibility, but appeal to symmetry.)

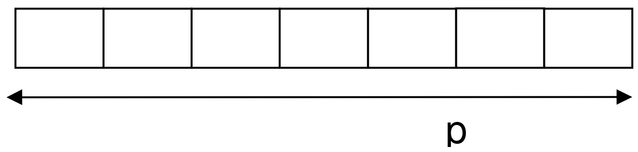
Q 2.3) For the generators $\{L_x,L_y,L_z\}$ in question 2.2, part (b), find the <u>simultaneous</u> eigenvectors of L^2 and L_z (i.e. eigenvectors of both operators).

Q 2.4) Find the adjoint matrices for the generators in question 2.2, part (b).
In this case, it is obvious that they satisfy the Lie algebra.

Q 2.5) Verify the form of $R_y(\theta)$ for spin-1 given in Lecture 3.

Homework Set 3 – Lectures 4 & 5

Q 3.1) Using the Young Tableaux rules, write down the multiplicity for p particles of SU(n) in a totally symmetric state, namely a row of p boxes:



Now consider examples of how the corresponding states could be labelled by supplying quantum numbers {1,2,…,n}.

1	1	1	1	1	1	1

1	1	1	1	1	1	2

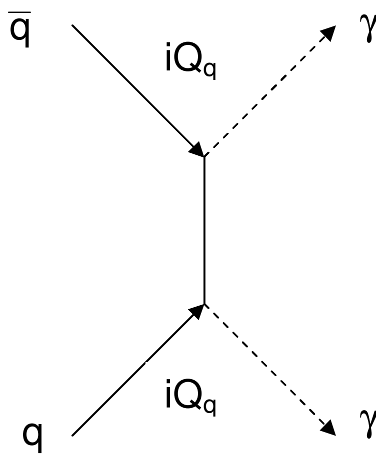
1	1	1	1	1	2	2

By considering the possible configurations, verify the multiplicity.
Hint: This is *so* trivial, that it requires *no* algebra, but you have to spot the trick!
The trick is to consider the number of ways of listing p boxes with (n–1) transitions of state label.

Q 3.2) Using the Young Tableaux rules, verify that the multiplicity of a general multiplet in SU(2) is (a+1) and in SU(3) is ½ (a+1)(b+1)(a+b+2).

Q 3.3) Considering only flavour, find the ratio of matrix elements for $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$.
Do this for a general case of mixing angle θ_p, and then choose θ_p such that the η has no strange-quark content.
Is the –ve sign in the π^0 wavefunction meaningful ?

How to proceed:
Label the scattering operator S and the meson state $|M>$.
What you need is $<\gamma\gamma|S|M> = \sum_q <\gamma\gamma|S|q\bar{q}><q\bar{q}|M>$



$<\gamma\gamma|S|q\bar{q}> \sim Q_q^{\,2}$

And

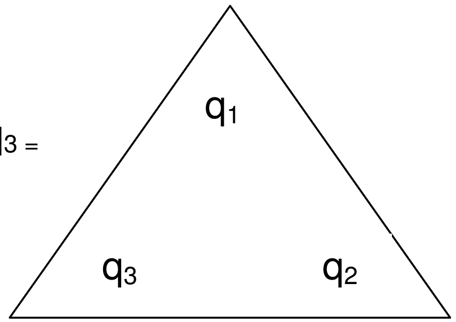
$|\eta> = \cos\theta_p\,|\eta_8> - \sin\theta_p\,|\eta_1>$
$|\pi^0> = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ etc

Q 3.4) Express the flavour-spin wavefunction for the Λ baryon in an explicitly symmetric way, as we did for the proton.

$$\phi_{M,S} = \tfrac{1}{2}\{(sd+ds)u-(su+us)d\} \quad \text{and} \quad \phi_{M,A} = \tfrac{1}{\sqrt{12}}\{(sd-ds)u-(su-us)d-2(du-ud)s\}$$

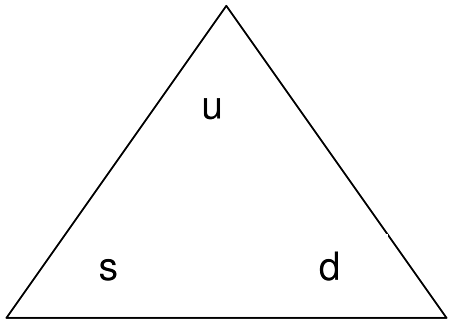
It is tricky to really make the wavefunction look symmetric due to the limitations of notation and writing on a page.
To help, I devised a simple notation:

$q_1q_2q_3$ =



so

uds =



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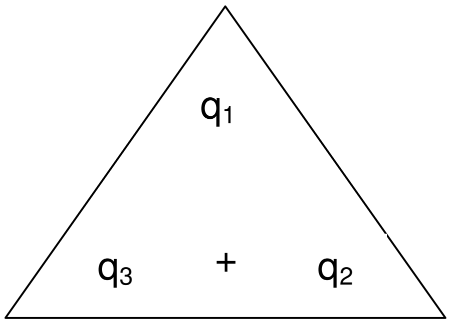

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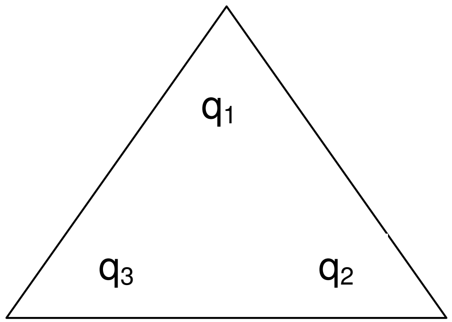

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