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Semiclassical Universe from First Principles

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Abstract

Causal Dynamical Triangulations in four dimensions provide a background-independent definition of the sum over space-time geometries in nonperturbative quantum gravity. We show that the macroscopic four-dimensional world which emerges in the Euclidean sector of this theory is a bounce which satisfies a semi-classical equation. After integrating out all degrees of freedom except for a global scale factor, we obtain the ground state wave function of the universe as a function of this scale factor.

1. Introduction

One important application of any theory of quantum gravity is a description of the quantum evolution of the very early universe. This is also the realm of quantum cosmology, which tries to capture the essence of the gravitational dynamics by quantizing only a finite number of degrees of freedom characterizing the universe as a whole. The path integral formulation of quantum cosmology came to prominence with the work of the Cambridge group and others on Euclidean quantum gravity [1], and in particular that of Hartle and Hawking [2]. Central in this and related approaches is the construction of a "wave function of the universe", either as a solution of the Wheeler-DeWitt equation or a propagator for the theory (see, for example, [3, 4, 5, 6, 7]). In attempting to do this, a variety of technical and conceptual issues has to be addressed, including the choice of boundary conditions for the wave function, the unboundedness of the gravitational action and ensuing divergence of the Euclidean cosmological path integral, the appropriateness of the minisuperspace and/or semiclassical approximations, and the physical interpretation of the construction (see [8] for a recent concise review).

One could hope that a nonperturbative path integral formulation which does not impose any a priori symmetry restrictions on the geometry of the universe would help resolve some of these issues. "Causal Dynamical Triangulations" provide exactly such a background-independent, nonperturbative definition of quantum gravity, in which the sum over all space-time geometries is constructively defined and the causal (Lorentzian) structure of space-time plays a crucial role [9, 10, 11, 12, 13]. It can be viewed as a realization of an idea of Teitelboim's, who argued that in a (continuum) proper-time formulation of the Lorentzian gravitational path integral one should integrate over positive lapse functions only, thereby building a notion of causality into the quantum dynamics [14].

In the context of quantum cosmology it has been argued [6, 15] that a tunneling wave function à la Vilenkin, where a universe tunnels from a vanishing to an extended three-geometry, is a special case of Teitelboim's causal propagator between two three-geometries. In the present work, where the wave function of the universe will be constructed from first principles, we will indeed observe a similar phenomenon, although our interpretation in the end will be somewhat different.

In [13] we reported that the approach of Causal Dynamical Triangulations, despite its background independence, generates a four-dimensional universe around which (small) quantum fluctuations take place. The purpose of this letter is to identify the effective action which determines the shape of this macroscopic 4d world. Rather surprisingly we find that the effective action which describes the infrared, long-distance part of the universe is closely related to a simple minisuperspace action frequently considered in quantum cosmology. The only differences in our full quantum treatment are that (i) the unboundedness problem of the conformal mode in the Euclidean sector is cured, and (ii) the ultraviolet, short-distance part of the effective action is such that the solution to the Euclidean action describes a bounce from a universe of no spatial extension to one of finite spatial size. While resembling Vilenkin's picture of a "universe from nothing" [3], the interpretation in the present context is rather in terms of the ground state wave function of the universe with everything but the scale factor integrated out. We describe how one can determine this wave function from first principles.

It should be emphasized that unlike in standard minisuperspace models for cosmology, we do not assume homogeneity or isotropy, nor do we impose any other a priori symmetry conditions on the gravitational degrees of freedom. We perform the full path integral and determine the effective Lagrangian which describes the dynamics of the global scale factor, as well as the ground state wave function of the universe as a function of this scale parameter.

The rest of this article is organized as follows: in the next section we recall some salient features of the Causal Dynamical Triangulations approach, including the set-up of the numerical simulations and recent numerical results in four dimensions. We then demonstrate in Sec. 3 that the numerical data are perfectly described by a simple minisuperspace action. Sec. 4 outlines how this result relates to the ground state wave function of the universe, and the final Sec. 5 is devoted to a discussion.

2. Observing the bounce

The idea to construct a quantum theory of gravity by using Causal Dynamical Triangulations was motivated by the desire to formulate a quantum gravity theory with the correct Lorentzian signature and causal properties [14], and to have a path integral formulation which may be closely related to attempts to quantize the theory canonically. For the purposes of this letter, we will only summarize the main properties of this approach; more details on the rationale and techniques can be found elsewhere [9, 10, 11, 12].

We insist that only causally well-behaved geometries appear in the path integral, which is regularized by summing over a particular class of triangulated, piecewise flat (i.e. piecewise Minkowskian) geometries. All causal simplicial spacetimes contributing to the path integral are foliated by a version of "proper time" t, and each geometry can be obtained by gluing together four-simplices in a way that respects this foliation. Each four-simplex has time-like links of negative length-squared $-a_t^2$ and space-like links of positive length-squared a_s^2, with all of the latter located in spatial slices of constant (integer in lattice units) proper time t. These slices consist of purely space-like tetrahedra, forming a three-dimensional piecewise flat manifold, whose topology we choose for simplicity to be that of a three-sphere S^3. A necessary condition for obtaining a well-defined continuum limit from this regularized setting is that the lattice spacing $a \propto a_t \propto a_s$ goes to zero while the number N_4 of four-simplices goes to infinity in such a way that the continuum four-volume $V_4 := a^4 N_4$ stays fixed. Let us emphasize that the parameter a therefore does not play the role of a fundamental discrete length. A further property of our explicit construction is that each configuration can be rotated to Euclidean signature, a necessary prerequisite for discussing the convergence properties of the sum over geometries, as well as for using Monte Carlo techniques.

The partition function for quantum gravity is

$$Z(\Lambda, G) = \int \mathcal{D}[g]\, e^{iS[g]}, \quad S[g] = \frac{1}{G} \int d^4x \sqrt{|\det g|}\,(R - 2\Lambda), \tag{1}$$

where $S[g]$ is the Einstein-Hilbert action including a cosmological-constant term Λ, and G the gravitational constant[1]. Using our simplicial regularization this becomes

$$Z(\Lambda, G)_{CDT} = \sum_T \frac{1}{C_T}\, e^{iS[T]}, \tag{2}$$

where the integration over Lorentzian geometries is replaced by a sum over causal triangulations, and C_T is a symmetry factor of the triangulation T, the order of its automorphism group. The action $S[T]$ is Regge's version of the Einstein-Hilbert action, appropriate for piecewise linear geometries (see [12] for details). In the remainder of this article we will for simplicity use a continuum notation, but it should be understood that whenever computer simulations are mentioned the implementation of the path integral is in terms of piecewise flat geometries.

The substitution $-a_t^2 \to a_t^2$ turns all time-like into space-like edges and rotates all configurations g to Euclidean space-times g_E, replacing at the same time $iS[g] \to -S_E[g_E]$, where S_E denotes the Euclidean Einstein-Hilbert action. In computer simulations it is often convenient to work with universes of constant four-volume V_4. The Euclidean counterpart of the partition function (1) can be decomposed as

$$Z_E(\Lambda, G) = \int_0^\infty dV_4\, e^{-\frac{1}{G}\Lambda V_4} \tilde{Z}_E(V_4, G), \tag{3}$$

where the partition function $\tilde{Z}_E(V_4, G)$ for fixed four-volume is defined as

$$\tilde{Z}_E(V_4, G) = \int \mathcal{D}[g]\, e^{-\tilde{S}_E[g]}\, \delta\Big(\int d^4x \sqrt{\det g} - V_4\Big), \quad \tilde{S}_E[g] = -\frac{1}{G} \int d^4x \sqrt{\det g}\, R. \tag{4}$$

[1] We ignore a numerical constant multiplying G in (1).

Whenever V_4 is kept fixed we will use $\tilde{Z}_E(V_4, G)$ as our partition function. It is related to $Z_E(\Lambda, G)$ by the Laplace transformation (3).

The specific (Euclidean) partition function we will consider is the so-called quantum-gravitational proper-time propagator defined by

$$G^E_{\Lambda,G}(g_3(0), g_3(t)) = \int \mathcal{D}[g_E]\; e^{-S_E[g_E]}. \tag{5}$$

where the integration is over all four-dimensional (Euclidean) geometries g_E of topology $S^3 \times [0,1]$, each with proper time running from 0 to t, and with spatial boundary geometries $g_3(0)$ and $g_3(t)$ at proper times 0 and t.

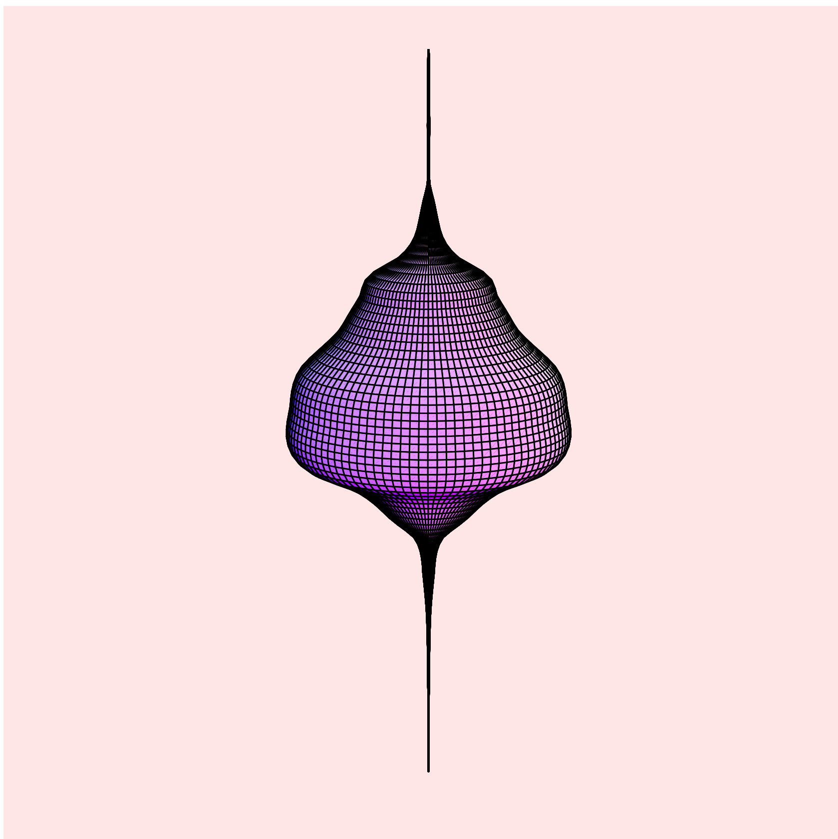

Figure 1: Monte Carlo snapshot of a "typical universe" of discrete volume 91.100 four-simplices and total time extent (vertical direction) $t = 40$. The circumference at integer proper time s is proportional to the spatial three-volume $V_3(s)$. The surface represents an interpolation between adjacent spatial volumes, without capturing the actual 4d connectivity between neighbouring spatial slices.

While it may be difficult to find an explicit analytic expression for the full propagator (5) of the four-dimensional theory, Monte Carlo simulations are readily available, using standard techniques from Euclidean dynamically triangulated quantum gravity [16]. For convenience of the computer simulations, we keep the total four-volume V_4 of space-time fixed and also often use periodic rather than fixed boundary conditions, i.e. sum over space-times with topology $S^3 \times S^1$ rather than $S^3 \times [0,1]$. This periodicity does not affect the results reported below, as is illustrated by Fig.1. This shows the typical "shape" (spatial three-volume $V_3(s)$ as a function of proper time s) of a space-time configuration generated by the computer.[2] For given space-time volume V_4, as long as t is chosen sufficiently large, the configuration will develop a thin stalk like the one shown in Fig.1. It will then not matter for the analysis of the large-scale geometry whether or not time is periodically identified.

A convenient "observable" is the spatial volume-volume correlator defined by

$$C_{V_4}(\Delta) \equiv \langle V_3(0) V_3(\Delta) \rangle_{V_4} = \frac{1}{t^2} \int_0^t ds \; \langle V_3(s) V_3(s+\Delta) \rangle_{V_4}, \tag{6}$$

where we have identified the spatial boundary geometries at times 0 and t. We have measured this correlator for various four-volumes V_4 and plotted the result as a function of the scaled variable $x = \Delta / V_4^{1/d}$, after normalizing the correlator for a given V_4 to have an integral equal to 1 (in lattice units). The variable d is then determined from the condition that the overlap between the correlators of different V_4 be maximal.[3] Fig.2 illustrates the almost perfect overlap obtained for $d = 4$. We take this as strong evidence that our "macroscopic" space-times (we are using up to 360.000 four-simplices) are genuinely four-dimensional.[4] As emphasized in [13], this is a highly non-trivial result and does *not* follow from the fact that the individual building blocks at the cut-off scale a are four-dimensional. Additional evidence for a macroscopically four-dimensional universe was presented in [13], and an extended analysis will appear in due course [18].

Our goal here will be to understand the precise analytical form of the volume-volume correlator $C_{V_4}(\Delta)$. To this end, let us consider the distribution of differences in the spatial volumes V_3 of successive spatial slices at proper times s and $s + \delta$, where δ is infinitesimal, i.e. $\delta = 1$ in lattice proper time units. We have measured the probability distribution $P_{V_3}(z)$ of the variable

$$z = \frac{V_3(s+\delta) - V_3(s)}{V_3^{1/2}}, \qquad V_3 = V_3(s) + V_3(s+\delta). \tag{7}$$

[2] "Typical" in this context is used to characterize the geometric characteristics that will be shared with probability 1 by a randomly chosen member of the ensemble of four-geometries.

[3] We are employing standard finite size scaling methods from the theory of critical phenomena in statistical mechanics, see, for example [17].

[4] Similar data, but with a smaller maximal four-volume, were already published in [13].

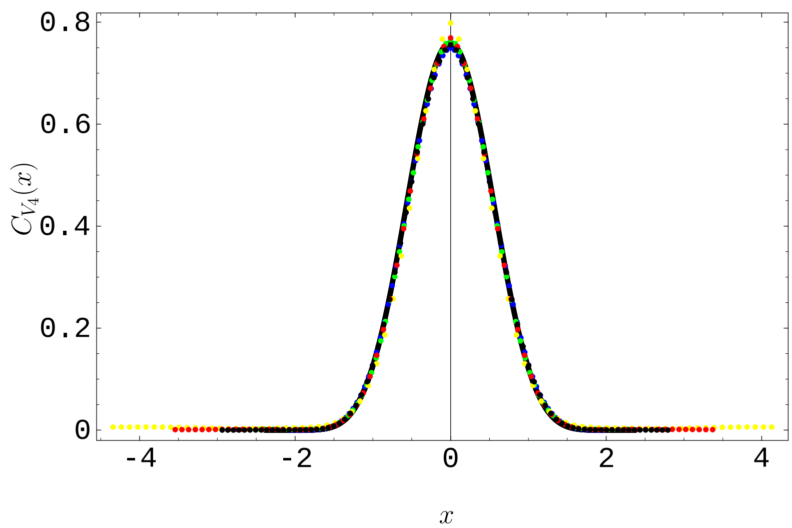

Figure 2: Measurement of spatial volume-volume correlator for space-times with 22.250, 45.500, 91.000, 181.000 and 362.000 four-simplices, plotted as function of the scaled variable $x = \Delta/V_4^{1/4}$.

for different values of V_3. As shown in Fig. 3 they fall on a common curve.[5] Furthermore, the distribution $P_{V_3}(z)$ is fitted very well by a Gaussian e^{-cz^2}, with a constant c independent of V_3. From estimating the entropy of spatial geometries, that is, the number of such configurations, one would expect corrections of the form V_3^α, with $0 \leq \alpha < 1$, to the exponent $c\,z^2$ in the distribution $P_{V_3}(z)$. Unfortunately it is impossible to measure these corrections directly in a reliable way. We therefore make a general ansatz for the probability distribution for *large* $V_3(s)$ as

$$\exp\left[-\frac{c_1}{V_3(s)}\left(\frac{dV_3(s)}{ds}\right)^2 - c_2 V_3^\alpha\right], \tag{8}$$

where $0 \leq \alpha < 1$, and c_1 and c_2 are positive constants.

We are thus by "observation" led to the following effective action for large three-volume $V_3(s)$:

$$S_{V_4}^{(eff)} = \int_0^t ds\,\left(\frac{c_1}{V_3(s)}\left(\frac{dV_3(s)}{ds}\right)^2 + c_2 V_3^\alpha(s) - \lambda V_3(s)\right), \tag{9}$$

[5]Again we have applied finite size scaling techniques, starting out with an arbitrary power V_3^α in the denominator in (7), and have determined $\alpha = 1/2$ from the principle of maximal overlap of the distributions for various V_3's.

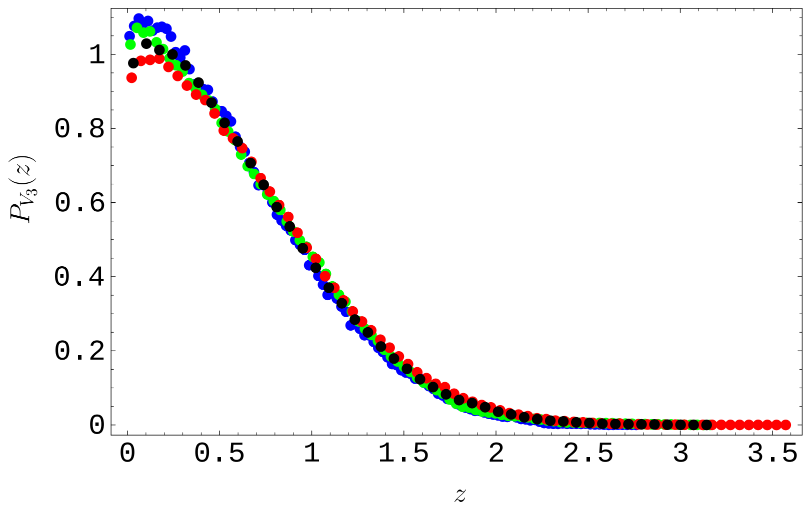

Figure 3: Distribution $P_{V_3}(z)$ of volume differences of adjacent spatial slices, for three-volumes $V_3 = 10.000$, 20.000, 40.000 and 80.000 tetrahedra.

where λ is a Lagrange multiplier to be determined such that

$$\int_0^t ds\, V_3(s) = V_4. \tag{10}$$

From general scaling of the above action it is clear that the only chance to obtain the observed scaling law, expressed in terms of the variable $t/V_4^{1/4}$, is by setting $\alpha = 1/3$. In addition, to reproduce the observed stalk for large times t the function $V_3^{1/3}$ has to be replaced by a function of V_3 whose derivative at 0 goes like V_3^ν, $\nu \geq 0$, for reasons that will become clear in Sec.3 below. A simple modification, which keeps the large-V_3 behaviour intact, is given by

$$V_3^{1/3} \to (1+V_3)^{1/3} - 1, \tag{11}$$

but the detailed form is not important. If we now introduce the (non-negative) *scale factor* $a(s)$ by

$$V_3(s) = a^3(s), \tag{12}$$

we can (after suitable rescaling of s and $a(s)$) write the effective action as

$$S_{V_4}^{eff} = \frac{1}{G}\int_0^t ds\, \left(a(s) \left(\frac{da(s)}{ds}\right)^2 + a(s) - \lambda a^3(s) \right), \tag{13}$$

with the understanding that the linear term should be replaced using (12) and (11) for small $a(s)$. We emphasize again that we have been led to (13) entirely by "observation" and that one can view the small-$a(s)$ behaviour implied by (11) as a result of quantum fluctuations.

3. Minisuperspace

Let us now consider the simplest minisuperspace model for a closed universe in quantum cosmology, as for instance used by Hartle and Hawking in their semiclassical evaluation of the wave function of the universe [2]. In Euclidean signature and proper-time coordinates, the metrics are of the form

$$ds^2 = dt^2 + a^2(t) d\Omega_3^2, \tag{14}$$

where the scale factor $a(t)$ is the only dynamical variable and $d\Omega_3^2$ denotes the metric on the three-sphere. The corresponding Einstein-Hilbert action is

$$S_{eff} = \frac{1}{G} \int dt \left(-a(t) \left(\frac{da(t)}{dt} \right)^2 - a(t) + \lambda a^3(t) \right). \tag{15}$$

If no four-volume constraint is imposed, λ is the cosmological constant. If the four-volume is fixed to V_4, such that the discussion parallels the computer simulations reported above, λ should be viewed as a Lagrange multiplier enforcing a given size of the universe. In the latter case we obtain the same effective action as in (13) *up to an overall sign*, due to the infamous conformal divergence of the classical Einstein action. Let us for the moment ignore this overall minus sign and compare the two potentials relevant for the calculation of semiclassical Euclidean solutions associated with the actions (15) and (13). The "potential"[6] is

$$V(a) = -a + \lambda a^3, \tag{16}$$

and is shown in Fig. 4, without and with small-a modification, for the standard minisuperspace model and our effective model, respectively. The quantum-induced difference for small a is important since the action (13) allows for a classically stable solution $a(t) = 0$ which explains the "stalk" observed in the computer simulations. Moreover, it is appropriate to speak of a Euclidean "bounce" because $a = 0$ is a local maximum. If one therefore *naively* turns the potential upside down when rotating back to Lorentzian signature, the metastable state $a(t) = 0$ can tunnel to a state where $a(t) \sim V_4^{1/4}$, with a probability amplitude per unit time which is (the exponential of) the Euclidean action. We will discuss this further in the next section.

In order to understand how well the semiclassical action (13) can reproduce the Monte Carlo data, that is, the correlator $C_{V_4}(\Delta)$ of Fig.2, we have solved for the semiclassical bounce using (13), and presented the result as the continuous

[6] To obtain a standard potential – without changing "time" – one should first transform to a variable $x = a^{\frac{3}{2}}$ for which the kinetic term in the actions assumes the standard quadratic form. It is the resulting potential $\tilde{V}(x) = -x^{2/3} + \lambda x^2$ which in the case of (13) should be modified for small x such that $\tilde{V}'(0) = 0$.

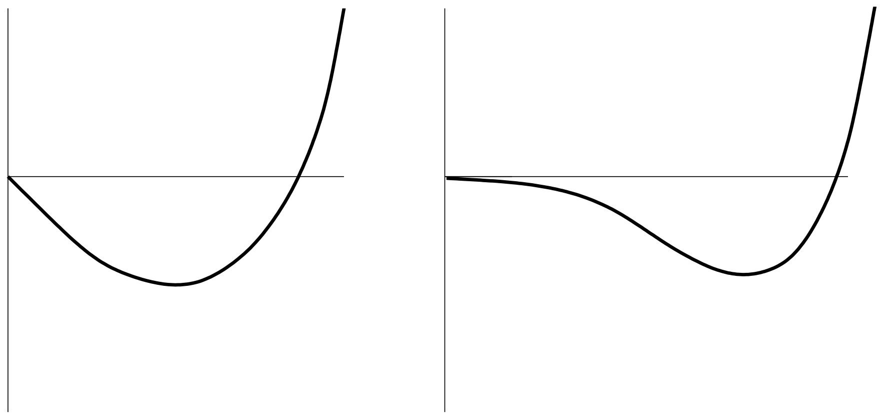

Figure 4: The potential $V(a)$ of (16) underlying the standard minisuperspace dynamics (left) and the analogous potential in the effective action obtained from the full quantum gravity model, with small-a modification due to quantum fluctuations (right).

black curve in Fig.2.[7]. The agreement with the real data generated by the Monte Carlo simulations is clearly perfect.

4. The wave function of the universe

The picture emerging from the above for the effective dynamics of the scale factor resembles that of a universe created by tunneling from nothing (see, for example, [3, 4, 5]), although the presence of a preferred notion of time makes our situation closer to conventional quantum mechanics. In the set-up analyzed here, there is apparently a state of vanishing spatial extension which can "tunnel" to a universe of finite linear extension of order $a \sim V_4^{1/4}$. Adopting such a tunneling interpretation, the action of the bounce is

$$S_{V_4}^{eff} \sim \frac{V_4^{1/2}}{G}, \tag{17}$$

and the associated probability per unit proper time for the tunneling given by

$$P(V_4) \sim e^{-S_{V_4}^{eff}}. \tag{18}$$

However, recall that this picture arose from a situation where for computer-technical reasons we imposed a constraint on the four-volume. Since our formu-

[7]More precisely, we solved the classical equation of motion corresponding to the potential shown in Fig.4 on the right, with an energy slightly below zero (the closer to zero the longer the stalk), and used this solution to create an artificial distribution of three-volumes $V_3(s)$ analogous to the one generated by Monte Carlo simulation from first principles. We then treated this artificial distribution precisely as if it had come from real Monte Carlo data.

lation possesses a well-defined Hamiltonian[8] which is bounded below, the ground state wave function can be chosen real and *positive*. In view of this, calling (18) a tunneling probability is misleading, since it would imply an oscillating behaviour for $a \gg V_4^{1/4}$. The correct interpretation of (18) is rather that of the square of the ground state wave function for $a \sim V_4^{1/4}$.

To illustrate what we have in mind, let us consider a quantum-mechanical system with Hamiltonian $H = p^2/2 + V(x)$, where the minimum of the potential V is at $x=0$. The ground state wave function has the path integral representation

$$\Psi_0(a) \sim \int_{x(-\infty)=0}^{x(0)=a} \mathcal{D}x(t) \; e^{-S_E[x(t)]}, \tag{19}$$

where $S_E[x(t)]$ is the classical Euclidean action

$$S_E[x(t)] = \int_{-\infty}^{0} dt \; \left[\frac{1}{2}\dot{x}^2 - (-V(x))\right]. \tag{20}$$

If there is a classical solution $x_{cl}(t)$ which extremizes the Euclidean action (20) with boundary conditions $x_{cl}(-\infty) = 0$ and $x_{cl}(0) = a$, a semiclassical calculation of $\Psi_0(a)$ involves a saddle point expansion around that solution and the leading exponential of the wave function is

$$\Psi_0(a) \sim e^{-S_E[x_{cl}(t)]}, \qquad S_E[x_{cl}(t)] = \int_0^a dx \, \sqrt{2V(x)}. \tag{21}$$

As an example, for the harmonic oscillator we have $x_{cl}(t) = a\, e^{\omega t}$ and (21) is exact. For a general potential the semiclassical approximation will of course not be exact. Nevertheless, we have presented strong evidence that in the case of quantum gravity, integrating over all degrees of freedom except the three-volume and defining $a = V_3^{1/3}$, the semiclassical approximation is excellent. If we assume that it is equally good in the absence of the four-volume constraint, we compute in a straightforward way from (21) that

$$\Psi_0(a) \sim e^{-\frac{c}{\Lambda G}((1+\Lambda a^2)^{3/2}-1)}, \tag{22}$$

with c a constant of order one. Note that Λ in (22) is the real cosmological constant and no longer a Lagrange multiplier. *We have thus calculated the wave function of the universe from first principles* up to prefactors and corrections to the semiclassical approximation.

[8]In the framework of Causal Dynamical Triangulations one has a transfer matrix between consecutive proper-time slices whose logarithm gives in principle the full quantum Hamiltonian, see [12] for details.

It is important to understand that the wave function $\Psi_0(a)$ *can* be addressed via computer simulations using a decomposition analogous to (3), namely,

$$\Psi_0(a) = \int dV_4 \; e^{-\frac{\Lambda}{G}V_4} \; \tilde{\Psi}_0(a;V_4), \tag{23}$$

$$\tilde{\Psi}_0(a;V_4) = \int_0^a \mathcal{D}[g] \; e^{\frac{1}{G}\int d^4x \sqrt{\det g} R} \; \delta(\int d^4x \sqrt{\det g} - V_4), \tag{24}$$

where the functional integration in (24) is over four-geometries with $V_3(-\infty) = 0$ and $V_3(t=0) = a^3$. The computer simulations reported here were done for the special case $V_3(t=0) = 0$.[9] Using (23) and (24) one can now check whether the semiclassical approximation reported here is valid for all values of a and V_4. If so, one will be led to (22).

5. Discussion

Causal Dynamical Triangulations constitute a framework for defining quantum gravity nonperturbatively as the continuum limit of a well-defined regularized sum over geometries. We reported recently on the outcome of the first Monte Carlo simulations in four dimensions [13]. Very encouragingly, we observed the dynamical generation of a macroscopic four-dimensional (Euclidean) world, with small quantum fluctuations superimposed. In this letter we showed that the scale factor characterizing the macroscopic shape of this ground state of geometry is well described by an effective action similar to that of the simplest minisuperspace model used in quantum cosmology. However, in our case such a result has for the first time – we believe – been derived from first principles.

The negative sign of the kinetic term in the standard minisuperspace action (15) reflects the well-known unboundedness of the conformal mode in the Euclidean Einstein-Hilbert action. Amazingly, after integrating out all variables except the scale factor (which is simply the global conformal mode), we obtain a *positive* kinetic term in our effective action (13). This is consistent with a continuum formulation of the gravitational path integral in proper-time gauge, where a strong argument was made for the nonperturbative cancellation of the conformal divergence by a measure factor coming from a Faddeev-Popov determinant [19]. Similar ideas were pursued in [20], although in that case matter fields were needed to change the sign of the conformal mode term. The phenomenon of sign change of the conformal kinetic term is also familiar from the 2d Euclidean quantum theory where, due to the conformal anomaly, integrating out unitary matter with central charge $0 \leq c \leq 1$ yields an effective term $-c(\partial\phi)^2$ for the conformal

[9] Since the spatial volume of the universe is not a monotonic function of V_4, the integration range in (24) in this case should be split into two intervals $[0, a_{\max}]$ and $[a_{\max}, 0]$.

factor ϕ in the action, making it unbounded below. Again it is a Faddeev-Popov determinant, arising from the requirement to integrate not over metrics, but only geometries, which adds a $26(\partial\phi)^2$ and ensures that the combined kinetic term is positive.

A number of open issues remain to be addressed, including the details of the renormalization mechanism. Here Causal Dynamical Triangulations gives us the possibility to study Weinberg's scenario of "asymptotic safety" [21] in the context of an explicit quantum-gravitational model. As indicated in earlier work on Causal Dynamical Triangulations in space-time dimension three, the renormalization may be non-standard [22], which in a way would be welcome. This is also supported by the present computer simulations, in the sense that no fine-tuning of the bare gravitational coupling constant seems to be necessary to reach the continuum limit. Details of this will be discussed elsewhere [18].

A most interesting question is of course how the above semiclassical cosmological picture is changed by the inclusion of matter fields. We have now the chance to investigate a number of possible scenarios suggested in quantum cosmology from first principles.

Acknowledgment

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