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# KALUZA-KLEIN THEORY IN PERSPECTIVE <sup>1</sup>

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## ABSTRACT

The Kaluza-Klein idea of extra spacetime dimensions continues to pervade current attempts to unify the fundamental forces, but in ways somewhat different from that originally envisaged. We present a modern perspective on the role of internal dimensions in physics, focussing in particular on superstring theory. A novel result is the interpretation of Kaluza-Klein string states as extreme black holes.

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# 1 The Kaluza idea

Cast your minds back to 1919. Maxwell's theory of electromagnetism was well established and Einstein had recently formulated his General Theory of Relativity. By contrast, the strong and weak interactions were not well understood. In searching for a unified theory of the fundamental forces, therefore, it was natural to attempt to merge gravity with electromagnetism. This Kaluza [1] was able to do through the remarkable device of postulating an extra fifth dimension for spacetime. Consider Einstein's theory of pure gravity in five spacetime dimensions with signature  $(-, +, +, +, +)$ . The line element is given by

$$d\hat{s}^2 = \hat{g}_{\hat{\mu}\hat{\nu}} d\hat{x}^{\hat{\mu}} d\hat{x}^{\hat{\nu}} \quad (1)$$

where  $\hat{\mu} = 0, 1, 2, 3, 4$  and all hatted quantities are five-dimensional. Kaluza then made the  $4 + 1$  split

$$\hat{g}_{\hat{\mu}\hat{\nu}} = e^{\phi/\sqrt{3}} \begin{pmatrix} g_{\mu\nu} + e^{-\sqrt{3}\phi} A_\mu A_\nu & e^{-\sqrt{3}\phi} A_\mu \\ e^{-\sqrt{3}\phi} A_\nu & e^{-\sqrt{3}\phi} \end{pmatrix} \quad (2)$$

where  $\hat{x}^{\hat{\mu}} = (x^\mu, y)$ ,  $\mu = 0, 1, 2, 3$ , and all unhatted quantities are four-dimensional. Thus the fields  $g_{\mu\nu}(x)$ ,  $A_\mu(x)$  and  $\phi(x)$  transform respectively as a tensor, a vector and a scalar under four-dimensional general coordinate transformations. All this was at the classical level, of course, but in the modern parlance of quantum field theory, they would be described as the spin 2 graviton, the spin 1 photon and the spin 0 dilaton<sup>3</sup>. Of course it is not

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<sup>3</sup>This was considered an embarrassment in 1919, and was (inconsistently) set equal to zero. However, it was later revived by Jordan [4] and Thiry [5] and subsequently stimulated

enough to call  $A_\mu$  by the name photon, one must demonstrate that it satisfies Maxwell's equations and here we see the Kaluza miracle at work. After making the same  $4 + 1$  split of the five-dimensional Einstein equations  $\hat{R}_{\hat{\mu}\hat{\nu}} = 0$ , we correctly recover the not only the Einstein equations for  $g_{\mu\nu}(x)$  but also the Maxwell equation for  $A_\mu(x)$  and the massless Klein-Gordon equation for  $\phi(x)$ . Thus Maxwell's theory of electromagnetism is an inevitable consequence of Einstein's general theory of relativity, given that one is willing to buy the idea of a fifth dimension.

## 2 The Klein idea

Attractive though Kaluza's idea was, it suffered from two obvious drawbacks. First, although the indices were allowed to range over  $0, 1, 2, 3, 4$ , for no very good reason the dependence on the extra coordinate  $y$  was suppressed. Secondly, if there is a fifth dimension why haven't we seen it? The resolution of both these problems was supplied by Oskar Klein [2] in 1926. Klein insisted on treating the extra dimension seriously but assumed the fifth dimension to have circular topology so that the coordinate  $y$  is periodic,  $0 \leq my \leq 2\pi$ , where  $m$  is the inverse radius of the circle  $S^1$ . Thus the space has topology  $R^4 \times S^1$ . It is difficult to envisage a spacetime with this topology but a simpler analogy is provided by a hosepipe: at large distances it looks like a line  $R^1$  but closer inspection reveals that at every point on the line there is

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Brans-Dicke [6] theories of gravity. As we shall see, the dilaton also plays a crucial role in superstring theory

a little circle, and the topology is  $R^1 \times S^1$ . So it was that Klein suggested that there is a little circle at each point in four-dimensional spacetime.

Let us consider Klein's proposal from a modern perspective. We start with pure gravity in five dimensions described by the action

$$\hat{S} = \frac{1}{2\hat{\kappa}^2} \int d^5\hat{x} \sqrt{-\hat{g}} \hat{R} \quad (3)$$

$\hat{S}$  is invariant under the five-dimensional general coordinate transformations

$$\delta \hat{g}_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} \hat{\xi}^{\hat{\rho}} \hat{g}_{\hat{\rho}\hat{\nu}} + \partial_{\hat{\nu}} \hat{\xi}^{\hat{\rho}} \hat{g}_{\hat{\rho}\hat{\mu}} + \hat{\xi}^{\hat{\rho}} \partial_{\hat{\rho}} \hat{g}_{\hat{\mu}\hat{\nu}} \quad (4)$$

The periodicity in  $y$  means that the fields  $g_{\mu\nu}(x, y)$ ,  $A_\mu(x, y)$  and  $\phi(x, y)$  may be expanded in the form

$$\begin{aligned} g_{\mu\nu}(x, y) &= \sum_{n=-\infty}^{n=\infty} g_{\mu\nu n}(x) e^{inmy}, \\ A_\mu(x, y) &= \sum_{n=-\infty}^{n=\infty} A_{\mu n}(x) e^{inmy}, \\ \phi(x, y) &= \sum_{n=-\infty}^{n=\infty} \phi_n e^{inmy} \end{aligned} \quad (5)$$

with

$$g^*_{\mu\nu n}(x) = g_{\mu\nu -n}(x) \quad (6)$$

etc. So (as one now finds in all the textbooks) a Kaluza-Klein theory describes an infinite number of four-dimensional *fields*. However (as one finds in none of the textbooks) it also describes an infinite number of four-dimensional *symmetries* since we may also Fourier expand the general coordinate parameter

$\hat{\xi}^{\hat{\mu}}(x, y)$  as follows

$$\begin{aligned}\hat{\xi}^{\mu}(x, y) &= \sum_{n=-\infty}^{n=\infty} \xi^{\mu}_n(x) e^{inmy} \\ \hat{\xi}^4(x, y) &= \sum_{n=-\infty}^{n=\infty} \xi^4_n(x) e^{inmy}\end{aligned}\tag{7}$$

with  $\hat{\xi}^{*\hat{\mu}}_n = \hat{\xi}^{\hat{\mu}}_{-n}$ .

Let us first focus on the  $n = 0$  modes in (5) which are just Kaluza's graviton, photon and dilaton. Substituting (2) and (5) in the action (3), integrating over  $y$  and retaining just the  $n = 0$  terms we obtain (dropping the 0 subscripts)

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{4} e^{-\sqrt{3}\phi} F_{\mu\nu} F^{\mu\nu}] \tag{8}$$

where  $2\pi\kappa^2 = m\hat{\kappa}^2$  and  $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ . From (4), this action is invariant under general coordinate transformations with parameter  $\xi^{\mu}_0$ , i.e (again dropping the 0 subscripts)

$$\begin{aligned}\delta g_{\mu\nu} &= \partial_{\mu} \xi^{\rho} g_{\rho\nu} + \partial_{\nu} \xi^{\rho} g_{\mu\rho} + \xi^{\rho} \partial_{\rho} g_{\mu\nu} \\ \delta A_{\mu} &= \partial_{\mu} \xi^{\rho} A_{\rho} + \xi^{\rho} \partial_{\rho} A_{\mu} \\ \delta \phi &= \xi^{\rho} \partial_{\rho} \phi,\end{aligned}\tag{9}$$

local gauge transformations with parameter  $\xi^4_0$

$$\delta A_{\mu} = \partial_{\mu} \xi^4 \tag{10}$$

and global scale transformations with parameter  $\lambda$

$$\delta A_{\mu} = \lambda A_{\mu}, \quad \delta \phi = -2\lambda/\sqrt{3} \tag{11}$$

The symmetry of the vacuum, determined by the VEVs

$$\langle g_{\mu\nu} \rangle = \eta_{\mu\nu}, \quad \langle A_\mu \rangle = 0, \quad \langle \phi \rangle = \phi_0 \quad (12)$$

is the four-dimensional Poincare group  $\times R$ . Thus, the masslessness of the graviton is due to general covariance, the masslessness of the photon to gauge invariance, but the dilaton is massless because it is the Goldstone boson associated with the spontaneous breakdown of the global scale invariance. Note that the gauge group is  $R$  rather than  $U(1)$  because this truncated  $n = 0$  theory has lost all memory of the periodicity in  $y$ .

Now, however, let us include the  $n \neq 0$  modes. An important observation is that the assumed topology of the ground state, namely  $R^4 \times S^1$  restricts us to general coordinate transformations periodic in  $y$ . Whereas the general covariance (9) and local gauge invariance (10) simply correspond to the  $n = 0$  modes of (4) respectively, the global scale invariance is no longer a symmetry because it corresponds to a rescaling

$$\delta \hat{g}_{\hat{\mu}\hat{\nu}} = -\frac{1}{2} \lambda \hat{g}_{\hat{\mu}\hat{\nu}} \quad (13)$$

combined with a general coordinate transformation

$$\xi^4 = -\lambda y / m \quad (14)$$

which is now forbidden by the periodicity requirement. The field  $\phi_0$  is therefore merely a pseudo-Goldstone boson.

Just as ordinary general covariance may be regarded as the local gauge symmetry corresponding to the global Poincare algebra and local gauge invariance as the gauge symmetry corresponding to the global abelian algebra, so the infinite parameter local transformations (7) correspond to an infinite-parameter global algebra with generators

$$\begin{aligned}
P^\mu_n &= e^{inmy} \partial^\mu \\
M^{\mu\nu}_n &= e^{inmy} (x^\mu \partial^\nu - x^\nu \partial^\mu) \\
Q_n &= ie^{inmy} \partial / \partial(my)
\end{aligned} \tag{15}$$

It is in fact a Kac-Moody-Virasoro generalization of the Poincare/gauge algebra [7]. Although this larger algebra describes a symmetry of the four-dimensional theory, the symmetry of the vacuum determined by (12) is only Poincare  $\times U(1)$ . Thus the gauge parameters  $\xi^\mu_n$  and  $\xi^4_n$  with  $n \neq 0$  each correspond to spontaneously broken generators, and it follows that for  $n \neq 0$  the fields  $A_{\mu n}$  and  $\phi_n$  are the corresponding Goldstone boson fields. The gauge fields  $g_{\mu\nu n}$ , with two degrees of freedom, will then each acquire a mass by absorbing the the two degrees of freedom of each vector Goldstone boson  $A_{\mu n}$  and the one degree of freedom of each scalar Goldstone boson  $\phi_n$  to yield a pure spin 2 massive particle with five degrees of freedom. This accords with the observation that the massive spectrum is pure spin two [8]. Thus we find an infinite tower of charged, massive spin 2 particles with charges  $e_n$  and masses  $m_n$  given by

$$e_n = n\sqrt{2}\kappa m, \quad m_n = |n|m \tag{16}$$

Thus Klein explained (for the first time) the quantization of electric charge [3]. (Note also that charge conjugation is just parity transformation  $y \rightarrow -y$  in the fifth dimension.) Of course, if we identify the fundamental unit of charge  $e = \sqrt{2}\kappa m$  with the charge on the electron, then we are forced to take  $m$  to be very large: the Planck mass  $10^{19} \text{ GeV}$ , way beyond the range of any current or foreseeable accelerator. This answers the second question left unanswered by Kaluza because with  $m$  very large, the radius of the circle must be very small: the Planck size  $10^{-35} \text{ meters}$ , which satisfactorily accords with our everyday experience of living in four spacetime dimensions.

It is interesting to note that, despite the inconsistency problems [9] that arise in coupling a finite number of massive spin two particles to gravity and/or electromagnetism, Kaluza-Klein theory is consistent by virtue of having an *infinite* tower of such states. Any attempt to truncate to a finite non-zero number of massive modes would reintroduce the inconsistency [10]. We also note, however, that these massive Kaluza-Klein modes have the unusual gyromagnetic ratio  $g = 1$  [11], which seems to lead to unacceptable high-energy behaviour for Compton scattering [12]. Moreover, as we shall see in section (8), where we embed the theory in a superstring theory, these Kaluza-Klein states will persist as a subset of the full string spectrum. However, string theory comes to the rescue and ensures correct high-energy behaviour.

In summary, it seems that a five-dimensional world with one of its di-



mensions compactified on a circle is operationally indistinguishable from a four dimensional world with a very particular (albeit infinite) mass spectrum. From this perspective, therefore, it seems that one could kick the ladder away and forget about the fifth dimension.

### 3 The Kaluza-Klein black hole

The equations which follow from (8) admit electrically charged black hole solutions [13, 14, 15, 16]:

$$\begin{aligned}
 ds^2 &= -\Delta_+ \Delta_-^{-1/2} dt^2 + \Delta_+^{-1} \Delta_-^{1/2} dr^2 + r^2 \Delta_-^{-3/2} d\Omega^2 \\
 e^{2\phi} &= \Delta_-^{\sqrt{3}} \\
 e^{-\sqrt{3}\phi} F &= (r_+ r_-)^{1/2} \epsilon_2
 \end{aligned} \tag{17}$$

where  $\Delta_{\pm} = 1 - r_{\pm}/r$  and  $\epsilon_2$  is the volume form on  $S^2$ . The electric charge  $e$  and ADM mass  $m$  are related to  $r_{\pm}$  by

$$\begin{aligned}
 \sqrt{2}\kappa e/4\pi &= (r_+ r_-)^{1/2} \\
 2\kappa^2 m/4\pi &= 2r_+ - r_-
 \end{aligned} \tag{18}$$

The existence of an event horizon,  $r_+ \geq r_-$ , thus implies the bound

$$\sqrt{2}\kappa m \geq e \tag{19}$$

In the extreme limit,  $r_+ = r_-$ , the line element reduces to

$$ds^2 = -\Delta_-^{1/2} dt^2 + \Delta_-^{-1/2} dr^2 + r^2 \Delta_-^{-3/2} d\Omega^2 \tag{20}$$

and the bound (19) is saturated. Note that this yields exactly the same charge to mass ratio (16) as the massive Kaluza-Klein states. As we shall show in section (8), this is no coincidence: the massive states *are* extreme black holes!

The same equations also admit the magnetically charged black hole solution with the same metric but with

$$e^{-2\phi} = \Delta_-^{\sqrt{3}}$$

$$F = (r_+ r_-)^{1/2} \epsilon_2 \quad (21)$$

and with magnetic charge  $g$  given by

$$\sqrt{2}\kappa g/4\pi = (r_+ r_-)^{1/2} \quad (22)$$

In the extreme limit,  $r_+ = r_-$ , this is the Kaluza-Klein monopole [15, 17, 18].

We note that the four-dimensional monopole metric  $g_{\mu\nu}$  of (20) exhibits a curvature singularity at  $r = r_-$ , even though the five-dimensional metric  $\hat{g}_{\hat{\mu}\hat{\nu}}$  of (2) from which it is descended is perfectly regular! This appears to contradict the impression gained at the end of the last section that the five-dimensional perspective is an unnecessary luxury. However, consider the Weyl rescaled metric  $\tilde{g}_{\mu\nu} = e^{\sqrt{3}\phi} g_{\mu\nu}$ . The magnetic monopole line element is now

$$d\tilde{s}^2 = -\Delta_-^{-1} dt^2 + \Delta_-^{-2} dr^2 + r^2 d\Omega^2 \quad (23)$$

and the curvature singularity at  $r = r_-$  has disappeared! The physical significance of this metric is that it is the one that couples to the worldline

of an *electrically* charged point particle [19, 20]. We shall return to this in section (8).

## 4 The humble torus

In  $D > 5$  dimensions the pure gravity field equations  $\hat{R}_{\hat{\mu}\hat{\nu}} = 0$  are consistent with the ground state  $M_4 \times T^k$  where  $T^k$  is the metrically flat  $k$ -torus  $T^k = S^1 \times S^1 \times \dots S^1$ . The gauge group is now  $G = [U(1)]^k$ . The count of massless modes (degrees of freedom) is: 1 spin 2 (2);  $k$  spin 1 ( $2k$ );  $k(k+1)/2$  spin 0 ( $k(k+1)/2$ ). Note that the total number of degrees of freedom is  $(4+k)(1+k)/2$  which matches the degrees of freedom of a graviton in  $D = 4 + k$  dimensions. The number of scalars is given by the moduli of  $T^k$  and they parameterize the non-linear  $\sigma$ -model  $GL(k, R)/SO(k)$  [21]. Thus even the *humble torus* (in the words of Abdus Salam), the simplest of extra-dimensional geometries one could envisage, gives rise to a non-trivial four-dimensional world.

As we shall now show, the torus becomes even more non-trivial in the context of supergravity theory. Here, in addition to the metric  $\hat{g}_{\hat{\mu}\hat{\nu}}$ , the  $D = 10$  supergravity multiplet contains a 2-form  $\hat{B}_{\hat{\mu}\hat{\nu}}$  and a dilaton  $\hat{\Phi}$ . After compactification to  $D = 4$  on a torus, the bosonic degrees of freedom count now is: 1 spin 2; 12 spin 1; and 38 scalars composed of 36 moduli parameterizing  $SO(6, 6)/SO(6) \times SO(6)$  and an axion and dilaton parameterizing  $SL(2, R)/U(1)$ . There will also be an equal number of fermion degrees of

freedom: 4 spin 3/2 and 28 spin 1/2. If we include the Yang-Mills multiplet we obtain a further 16 spin 1; 64 spin 1/2 and 96 spin 0 so that the moduli coset is then  $SO(6, 22)/SO(6) \times SO(22)$ . Since this theory is the field theory limit of the heterotic string compactified on a generic torus let us consider the action in more detail [26, 27]. Its bosonic sector is given by:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-G} e^{-\Phi} [R_G + G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{12} G^{\mu\lambda} G^{\nu\tau} G^{\rho\sigma} H_{\mu\nu\rho} H_{\lambda\tau\sigma} - \frac{1}{4} G^{\mu\lambda} G^{\nu\tau} F_{\mu\nu}{}^a (LML)_{ab} F_{\lambda\tau}{}^b + \frac{1}{8} G^{\mu\nu} Tr(\partial_\mu ML \partial_\nu ML)] \quad (24)$$

where  $F_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a$  and  $H_{\mu\nu\rho} = (\partial_\mu B_{\nu\rho} + 2A_\mu{}^a L_{ab} F_{\nu\rho}{}^b) + \text{permutations}$ . Here  $\Phi$  is the  $D = 4$  dilaton,  $R_G$  is the scalar curvature formed from the string metric  $G_{\mu\nu}$ , related to the canonical metric  $g_{\mu\nu}$  by  $G_{\mu\nu} \equiv e^\Phi g_{\mu\nu}$ .  $B_{\mu\nu}$  is the 2-form which couples to the string worldsheet and  $A_\mu{}^a$  ( $a = 1, \dots, 28$ ) are the abelian gauge fields.  $M$  is a symmetric  $28 \times 28$  dimensional matrix of scalar fields satisfying  $MLM = L$  where  $L$  is the invariant metric on  $O(6, 22)$ :

$$L = \begin{pmatrix} 0 & I_6 & 0 \\ I_6 & 0 & 0 \\ 0 & 0 & -I_{16} \end{pmatrix}. \quad (25)$$

The action is invariant under the  $O(6, 22)$  transformations  $M \rightarrow \Omega M \Omega^T$ ,  $A_\mu{}^a \rightarrow \Omega^a{}_b A_\mu{}^b$ ,  $G_{\mu\nu} \rightarrow G_{\mu\nu}$ ,  $B_{\mu\nu} \rightarrow B_{\mu\nu}$ ,  $\Phi \rightarrow \Phi$ , where  $\Omega$  is an  $O(6, 22)$  matrix satisfying  $\Omega^T L \Omega = L$ . *T-duality* corresponds to the  $O(6, 22; \mathbb{Z})$  subgroup and is known to be an exact symmetry of the full string theory. The equations of motion, though not the action, are also invariant under the  $SL(2, \mathbb{R})$  transformations:  $\mathcal{M} \rightarrow \omega \mathcal{M} \omega^T$ ,  $\mathcal{F}_{\mu\nu}{}^{a\alpha} \rightarrow \omega^\alpha{}_\beta \mathcal{F}_{\mu\nu}{}^{a\beta}$ ,  $g_{\mu\nu} \rightarrow g_{\mu\nu}$ ,  $M \rightarrow M$

where  $\alpha = 1, 2$  with  $\mathcal{F}_{\mu\nu}{}^{a1} = F_{\mu\nu}{}^a$  and  $\mathcal{F}_{\mu\nu}{}^{a2} = (\lambda_2(ML)^a{}_b \tilde{F}_{\mu\nu}{}^b + \lambda_1 F_{\mu\nu}{}^a)$ , where  $\omega$  is an  $SL(2, R)$  matrix satisfying  $\omega^T \mathcal{L} \omega = \mathcal{L}$  and where

$$\mathcal{M} = \frac{1}{\lambda_2} \begin{pmatrix} 1 & \lambda_1 \\ \lambda_1 & |\lambda|^2 \end{pmatrix}, \quad \mathcal{L} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (26)$$

$\lambda$  is given by  $\lambda = \Psi + ie^{-\Phi} \equiv \lambda_1 + i\lambda_2$ . The axion  $\Psi$  is defined through the relation  $\sqrt{-g}H^{\mu\nu\rho} = -e^{2\Phi}\epsilon^{\mu\nu\rho\sigma}\partial_\sigma\Psi$ . *S-duality* corresponds to the  $SL(2, Z)$  subgroup and, as discussed in section (8) there is now a good deal of evidence [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] in favor of its also being an exact symmetry of the full string theory. It generalizes earlier conjectures in global Yang-Mills theories [33, 34, 35, 36]

## 5 Non-abelian generalization

The arrival of Yang-Mills gauge theories in 1954 presented an altogether different challenge to higher-dimensional gravity theories: could they also account for *non-abelian* gauge bosons? Curiously enough, Oskar Klein [37] came close<sup>4</sup> to discovering non-abelian gauge fields in 1939 while investigating  $D = 5$  geometries, but their significance was never fully articulated. The first concrete attempt seems to be that of De Witt [39] in 1963 and this was followed by work of Rayski [40], Kerner [41], Trautmann [42], Cho [43], Cho and Freund [44], Cho and Jang [45] and others. The breakthrough was the realization that the gauge group  $G$  obtained in  $D = 4$  was connected to the *isometry group* of the extra dimensions which, in analogy with  $S^1$ , were

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<sup>4</sup>See the article by Gross [38].

taken to be compact to ensure the compactness of  $G$ . Thus, it was argued,  $SU(2)$  gauge bosons arose from taking three extra dimensions and assigning to them the geometry of a three-sphere which was, after all, the  $SU(2)$  group manifold.

With the wisdom of hindsight, we can now identify several shortcomings of these non-abelian developments. First, little attention was paid to the question of why the extra dimensions were compactified and whether this was consistent with the higher-dimensional field equations. It was usually a completely ad hoc procedure. This was remedied by the idea of *spontaneous compactification*. Here one looks for stable *ground state* solutions of the field equations for which the metric describes a product manifold  $M_4 \times M_k$  where  $M_4$  is four-dimensional spacetime with the usual signature and  $M_k$  is a compact "internal" space with Euclidean signature. As shown by Cremmer *et al* [46] it was necessary to augment pure gravity with matter fields in order to achieve a satisfactory compactification. A second shortcoming was the failure to realize that the extra-dimensional manifold need not correspond to a group space  $G$  in order to obtain Yang-Mills gauge fields with  $G$  as their gauge group. Now we know that any  $M_k$  with  $G$  as its isometry group will do, i.e., any metric admitting the Killing vectors of  $G$ . This could be a homogeneous space. In this case the group  $G$  acts transitively and we may write the manifold as the coset space  $M_k = G/H$  where  $H$  is the isotropy subgroup of the isometry group  $G$ . The use of such homogeneous spaces in

Kaluza-Klein theories was discussed by Luciani [47]. Since  $k = \dim G - \dim H$ , one was no longer obliged to have only one gauge boson for each extra dimension, as had previously been assumed. Indeed, the isometry group of a group manifold can be as large as  $G \times G$  if we use the bi-invariant metric, so  $S^3$  can give  $SU(2) \times SU(2)$  gauge bosons and not merely  $SU(2)$ .

Deriving Yang-Mills fields from gravity is perhaps the most beautiful aspect of Kaluza-Klein theories so let us examine how it works. Let us make the  $4 + k$  split  $\hat{x}^{\hat{\mu}} = (x^\mu, y^n)$ ,  $\hat{\mu} = 0, 1, \dots, D - 1$ ,  $\mu = 0, 1, 2, 3$  and  $n = 4, 5, \dots, D - 1$ . Consider the  $D$ -dimensional metric  $\hat{g}_{\hat{\mu}\hat{\nu}}(x, y)$  and in particular, the off diagonal component  $\hat{g}_{\mu n}(x, y)$ . In its Fourier expansion, the lowest term looks like

$$\hat{g}_{\mu n}(x, y) = A_\mu{}^i(x) K_n{}^i(y) + \dots \quad (27)$$

where  $K^i = K^{ni} \partial_n$  is a Killing vector obeying the Lie algebra of  $G$

$$[K^i, K^j] = f^{ij}{}_k K^k \quad (28)$$

and  $f^i{}_{jk}$  are the structure constants. Now we consider the general coordinate transformation (4) and focus our attention on the very special transformation

$$\hat{\xi}^{\hat{\mu}}(x, y) = (0, \epsilon^i(x) K^{mi}(y)) \quad (29)$$

with  $\epsilon^i(x)$  arbitrary. Then from (4) we may compute the transformation rule for  $\hat{g}_{\mu n}(x, y)$  and hence from (27) that for  $A_\mu{}^i(x)$ . We find

$$\delta A_\mu{}^i(x) = \partial_\mu \epsilon^i(x) - f^i{}_{jk} A_\mu{}^j(x) \epsilon^k(x) \quad (30)$$

This is precisely the transformation law for a Yang-Mills field with gauge group  $G$ . Hence  $G$  is a subgroup of the  $D$ -dimensional general coordinate group. In summary, the basic idea is that what we perceive to be *internal symmetries* in four dimensions are really *spacetime symmetries* in the extra dimensions. Carrying this logic to its ultimate conclusion, one might be tempted to conclude that there is no such thing in nature as an internal symmetry, even apparent discrete internal symmetries like charge conjugation being just discrete spacetime transformations in the extra dimensions.

One can only speculate on how the course of twentieth century physics might have changed if, in groping towards non-abelian gauge fields in 1939 [38], Klein had applied his own ideas to a *sphere* instead of a circle.

## 6 Kaluza-Klein Supergravity

The history of Kaluza-Klein took a totally new turn with the advent of supergravity [48, 49] whose mathematical consistency requires  $D \leq 11$  dimensions [50]. Cremmer, Julia and Scherk [51] were able to construct the  $N = 1, D = 11$  supergravity Lagrangian, describing the interaction of the elfbein  $e_M^A$ , the gravitino  $\Psi_M$ , and the three-index gauge field  $A_{MNP}$ . The uniqueness of the field equations meant that not only did Kaluza-Klein enthusiasts have a guide to the dimensionality of spacetime but also a guide to the correct interactions with gravity and matter. In  $D \leq 11$ , supersymmetric theories are no longer unique but still very restrictive. It seemed that this



restriction on the dimension and the restrictions on the interactions, which supersymmetry provides, were essential to any successful Kaluza-Klein unification. Otherwise one is wandering in the wilderness: the problem of finding the Lagrangian of the world in  $D = 4$  is simply replaced by the problem of finding the the Lagrangian of the world in  $D > 4$  with the extra headache of which  $D$  to pick. Nothing has been gained by way of economy of thought. Weinberg [52] is fond of recalling the fable of the "stone soup" when discussing this problem. Just as the promise of delicious soup made from stones proved to mean stones plus meat and vegetables, so the Kaluza-Klein promise of a unified theory made only from gravity had proved to mean gravity plus a whole variety of matter fields with each author choosing his favorite ingredients. But by combining gravity and matter into one simple superfield to which no further supermatter may be added,  $D = 11$  realized Einstein's old dream of replacing the "base wood" of matter by the "pure marble" of geometry. Eleven-dimensional supergravity is "marble soup".

The early 1980s thus marked a major renaissance of extra dimensions. First Freund and Rubin [53] showed that the 3-form of  $D = 11$  supergravity provided a dynamical mechanism whereby 7 of the 11 dimensions compactify spontaneously. Then Witten [54] considered the search for a realistic Kaluza-Klein theory pointing out both the advantages of  $D = 11$  supergravity ( seven extra dimensions is the minimum to accommodate  $SU(3) \times SU(2) \times U(1)$ ) and its disadvantages ( quarks and leptons do not fit into the right represen-

tations, in particular the four dimensional theory cannot be *chiral*). Thirdly, Salam and Strathdee [8] laid the foundations for much of the subsequent research on harmonic expansions on coset spaces, essential for the understanding of the massive modes. All this sparked off the observation [55] that the seven extra dimensions of  $D = 11$  supergravity could, via the Freund-Rubin ansatz yield ground-state solutions of the form ( $D = 4$  Anti de Sitter space)  $\times S^7$  and that since  $S^7$  has isometry group  $SO(8)$ , this would give rise to a  $D = 4$  theory with  $SO(8)$  invariance. It was not difficult to prove that  $S^7$  also admits 8 *Killing spinors* and hence gives rise to  $N = 8$  supersymmetry in  $D = 4$ . It was therefore natural to conjecture [55, 56, 57] that the massless supermultiplet of spins  $(2, 3/2, 1, 1/2, 0^+, 0^-)$  in the  $SO(8)$  representations  $(1, 8, 28, 56, 35, 35)$  corresponded to the gauged  $N = 8$  theory of de Wit and Nicolai [58].

There then followed a deluge of activity in both  $D = 11$  and  $D < 11$  Kaluza-Klein supergravity (and in Kaluza-Klein cosmology and quantum effects in Kaluza-Klein theories) by Abbott, Alvarez, Appelquist, Aurelia, Awada, Bais, Barkanroth, Barr, Bars, Bele'n Gavela, Berkov, Biran, Candelas, Castellani, Ceresole, Chapline, Cho, Chodos, Coquereax, D'Auria, De Alwis, Dereli, Detweiler, deWit, Duff, Ellis, Emel'yanov, Englert, Fre, Freedman, Freund, Fujii, Gell-Mann, Giani, Gibbons, Gunaydin, Gurse, Hurni, Jadcyk, Inami, Ito, Kato, Kogan, Koh, Kolb, Lukierski, Maeda, Manton, MacDowell, McKenzie, Mecklenburg, Minnaert, Moor-

house, Morel, Nicolai, Nikitin, Nilsson, Nixon, Ohta, Orzalesi, Page, Panahimoghaddam, Pilch, Pollard, Pope, Randjbar Daemi, Romans, Rooman, Rozenenthal, Rubakov, Salam, Sahdev, Schaposnikov, Sezgin, Shafi, Slansky, Sorokin, Spindel, Strathdee, Sudbery, Tanii, Tkach, Toms, Townsend, Tucker, Tze, van Baal, van Nieuwenhuizen, Volkov, Voronov, Wang, N. Warner, R. Warner, Weinberg, West, Wetterich, Wu, Yamagishi, Yasuda, Zwiebach, to name but some.

## 7 Superstrings

Thus up until the summer of 1984 various proposals were put forward combining supersymmetry and the Kaluza-Klein idea but none with complete success. Those based on conventional field theory suffered from various problems, not least of which was the traditional objection to a non-renormalizable theory of gravity. Those based on superstrings seemed better from this point of view, and also from the point of view of chirality, but had problems of their own. The realistic-looking strings appeared to suffer from inconsistencies (anomalies akin to the triangle anomalies of the standard model) while the anomaly-free strings did not appear realistic. In particular, they seemed to live in ten spacetime dimensions rather than undergoing a spontaneous compactification to four spacetime dimensions as demanded by the Kaluza-Klein idea. This was the sorry state of affairs until the September 1984 superstring revolution:

1) Green and Schwarz [59] discovered that the gravitational and Yang-Mills anomalies of the ten-dimensional superstrings all cancel provided the gauge group is either  $SO(32)$  or  $E_8 \times E_8$ ;

2) Gross, Harvey, Martinec and Rohm [60] discovered the heterotic (hybrid) string with the above gauge groups;

3) Candelas, Horowitz, Strominger and Witten [61] discovered that the  $E_8 \times E_8$  heterotic string admits spontaneous compactification to four dimensions on a six-dimensional Calabi-Yau manifold. The resulting four-dimensional theory resembles a GUT theory based on the group  $E_6$ . In particular, there are chiral families of quarks and leptons.

This is probably the place to admit the supreme irony of the Kaluza-Klein unification story. It is that the heterotic superstring [60], which is currently the favorite way to unify gravity with the other forces, while making use of (and indeed demanding) extra spacetime dimensions a la Kaluza-Klein, nevertheless eschews the route of getting the Yang-Mills gauge group from the general coordinate group. Instead, the Yang-Mills fields are already present in the  $D = 10$  dimensional formulation! Moreover, the favorite way subsequently to compactify the theory to  $D = 4$  invokes a Calabi-Yau manifold [61]; an  $M_6$  with no isometries at all! Does this mean that the idea of non-abelian gauge fields from extra dimensions is dead? Actually, no. The heterotic string construction involves taking the right-moving modes on the 2-dimensional worldsheet to correspond to a 10-dimensional superstring and

the left-moving modes to correspond to a 26-dimensional bosonic string. The  $D = 10$  formulation is obtained by compactifying the extra 16 dimensions on a specially chosen  $T^{16}$  corresponding to an even, self-dual lorentzian lattice. This leads uniquely to the anomaly-free [59], dimension 496, gauge groups  $E_8 \times E_8$  or  $SO(32)$ . The Cartan subalgebra  $U(1)^{16}$  admits a Kaluza-Klein interpretation, but the remaining 480 gauge bosons arise as solitons from the very different Frenkel-Kac mechanism. However, from the point of view of conformal field theory, a string moving on such a  $k$ -torus is indistinguishable from a string moving on a simply laced group manifold  $G$  with  $k = \text{rank} G$ . In fact, therefore, it is possible to understand *all* of the 496 gauge bosons as Kaluza-Klein gauge bosons having arisen from compactification on the group manifold [62, 63]. (One gets Yang-Mills fields of  $G$  rather than  $G \times G$  because the construction applies only to the left-movers and not the right-movers.) It has to be admitted, however, that such a traditional Kaluza-Klein interpretation, though valid, has not lead to any new insights into string theory.

Nevertheless, many other discoveries of the Kaluza-Klein supergravity era do continue to influence current thinking in string theory. These include: the connection between holonomy, Killing spinors and unbroken supersymmetry in four dimensions; Calabi-Yau manifolds (first proposed as a way of going from 10 to 6 on K3); orbifolds; fermion condensates and the cosmological constant; the higher dimensional interpretation of the Higgs mechanism as a distortion of the extra dimensional geometry; the importance of topology,

index theorems and zero modes in determining the massless spectrum. All this work is summarized in the Physics Report by Duff, Nilsson and Pope [64], where the corresponding references may be found.

Historical reviews of string theory frequently dwell on the early days of Regge theory, the Veneziano model and the dual resonance model of hadrons and then jump to 1984 as though nothing much happened in between. In my opinion, however, the renaissance of string theory in 1984 owed more to Kaluza-Klein supergravity than it did to the dual resonance model.

In addition to Calabi-Yau manifolds many sophisticated ways of arriving at a consistent four-dimensional heterotic string theory have been studied in the last ten years, including: self-dual Lorentzian lattices, symmetric and asymmetric orbifolds, fermionic formulations, etc. These were reviewed in [65] and I will not repeat the story here. Suffice it to say that despite a good deal of technical progress, we are still no closer to resolving what is perhaps the most important issue in string theory, namely the vacuum degeneracy problem. Finding the right compactification has become synonymous with finding the right  $c = 9$  superconformal field theory and there are literally billions (maybe an infinite number?) of candidates. For the time being therefore, the phrase *superstring inspired phenomenology* can only mean sifting through these billions of heterotic models in the hope of finding one that is realistic. The trouble with this needle-in-a-haystack approach is that even if we found a model with good phenomenology, we would be left wondering

in what sense this could be described as a *prediction* of superstrings.

The general consensus now is that this problem will never be resolved provided we remain within the confines of a weak coupling perturbation expansion. I would therefore like to finish with some very recent developments in string theory which address this strong coupling problem, and I am delighted to say that they rely heavily on the Kaluza-Klein idea.

## 8 Kaluza-Klein states as extreme black holes

The idea that elementary particles might behave like black holes is not a new one [66, 67, 68]. Intuitively, one might expect that a pointlike object whose mass exceeds the Planck mass, and whose Compton wavelength is therefore less than its Schwarzschild radius, would exhibit an event horizon. In the absence of a consistent quantum theory of gravity, however, such notions would always remain rather vague. Superstring theory, on the other hand, not only predicts such massive states but may provide us with a consistent framework in which to discuss them. In this section we shall summarize the results of [30] and confirm the claim [20] that certain massive excitations of four-dimensional superstrings are indeed black holes. Our results thus complement those of [69, 70, 71] where it is suggested that all black holes are single string states. Of course, non-extreme black holes would be unstable due to the Hawking effect. To describe stable elementary particles, therefore, we must focus on extreme black holes whose masses saturate a Bogomol'nyi

bound.

Here we return to the humble torus and consider the four-dimensional heterotic string obtained by toroidal compactification. At a generic point in the moduli space of vacuum configurations the unbroken gauge symmetry is  $U(1)^{28}$  and the low energy effective field theory is described by the  $N = 4$  supergravity coupled to the 22 abelian vector multiplets of section (4). We shall consider the Schwarz-Sen [26, 27]  $O(6, 22; Z)$  invariant spectrum of elementary electrically charged massive  $N_R = 1/2, N_L = 1$  states of this four-dimensional heterotic string, and show that the spin zero states correspond to extreme limits of the Kaluza-Klein black hole solutions of section (3) which preserve  $1/2$  of the spacetime supersymmetries. By supersymmetry, the black hole interpretation then applies to all members of the  $N = 4$  supermultiplet [72, 73], which has  $s_{max} = 1$ . Here  $N_L$  and  $N_R$  refer to the number of left and right oscillators respectively. The  $N = 4$  supersymmetry algebra possesses two central charges  $Z_1$  and  $Z_2$ . The  $N_R = 1/2$  states correspond to that subset of the full spectrum that belong to the 16 complex dimensional ( $s_{max} \geq 1$ ) representation of the  $N = 4$  supersymmetry algebra, are annihilated by half of the supersymmetry generators and saturate the strong Bogomol'nyi bound  $m = |Z_1| = |Z_2|$ . As discussed in [35, 36, 26, 27], the reasons for focussing on this  $N=4$  theory, aside from its simplicity, is that one expects that the allowed spectrum of electric and magnetic charges is not renormalized by quantum corrections, and that the allowed mass spectrum



of particles saturating the Bogomol'nyi bound is not renormalized either.

Following [22, 23, 24], Schwarz and Sen have also conjectured [26, 27] on the basis of string/fivebrane duality [74, 75] that, when the solitonic excitations are included, the full string spectrum is invariant not only under the target space  $O(6, 22; Z)$  ( $T$ -duality) but also under the strong/weak coupling  $SL(2, Z)$  ( $S$ -duality). The importance of  $S$ -duality in the context of black holes in string theory has also been stressed in [25]. Schwarz and Sen have constructed a manifestly  $S$  and  $T$  duality invariant mass spectrum.  $T$ -duality transforms electrically charged winding states into electrically charged Kaluza-Klein states, but  $S$ -duality transforms elementary electrically charged string states into solitonic monopole and dyon states. We shall show that these states are also described by the extreme magnetically charged black hole solutions discussed in [20]. Indeed, although the results of this section may be understood without resorting to string/fivebrane duality, it nevertheless provided the motivation. After compactification from  $D = 10$  dimensions to  $D = 4$ , the solitonic fivebrane solution of  $D = 10$  supergravity [76] appears as a magnetic monopole [77, 78] or a string [29] according as it wraps around 5 or 4 of the compactified directions <sup>5</sup>. Regarding this dual string as fundamental in its own right interchanges the roles of  $T$ -duality and  $S$ -duality. The solitonic monopole states obtained in this way thus play

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<sup>5</sup>It could in principle also appear as a membrane by wrapping around 3 of the compactified directions, but the  $N = 4$  supergravity theory (24) obtained by naive dimensional reduction does not admit the membrane solution [29].

the same role for the dual string as the elementary electric winding states play for the fundamental string. The Kaluza-Klein states are common to both. Since these solitons are extreme black holes [20], however, it follows by  $S$ -duality that the elementary Kaluza-Klein states should be black holes too! By  $T$ -duality, the same holds true of the elementary winding states. Rather than invoke  $S$ -duality, however, we shall proceed directly to establish that the elementary states described above are in one-to-one correspondence with the extreme electric black holes <sup>6</sup>. Now this leaves open the possibility that they have the same masses and quantum numbers but different interactions. Although we regard this possibility as unlikely given the restrictions of  $N = 4$  supersymmetry, the indirect argument may be more compelling in this respect (even though it suffers from the drawback that  $S$ -duality has not yet been rigorously established). Of course, elementary states are supposed to be singular and solitonic states non-singular. How then can we interchange their roles? As we saw in section (3), the way the theory accommodates this requirement is that when expressed in terms of the fundamental metric  $e^{\sqrt{3}\phi}g_{\mu\nu}$  that couples to the worldline of the superparticle the elementary solutions are singular and the solitonic solutions are non-singular, but when expressed in terms of the dual metric  $e^{-\sqrt{3}\phi}g_{\mu\nu}$ , it is the other way around [19, 20].

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<sup>6</sup>The idea that there might be a dual theory which interchanges Kaluza-Klein states and Kaluza-Klein monopoles was previously discussed in the context of  $N = 8$  supergravity by Gibbons and Perry [79].

We now turn to the electric and magnetic charge spectrum. Schwarz and Sen [26, 27] present an  $O(6, 22; Z)$  and  $SL(2, Z)$  invariant expression for the mass of particles saturating the strong Bogomol'nyi bound  $m = |Z_1| = |Z_2|$ :

$$m^2 = \frac{1}{16}(\alpha^a \ \beta^a)\mathcal{M}^0(M^0 + L)_{ab} \begin{pmatrix} \alpha^b \\ \beta^b \end{pmatrix} \quad (31)$$

where a superscript 0 denotes the constant asymptotic values of the fields. Here  $\alpha^a$  and  $\beta^a$  ( $a = 1, \dots, 28$ ) each belong to an even self-dual Lorentzian lattice  $\Lambda$  with metric given by  $L$  and are related to the electric and magnetic charge vectors  $(Q^a, P^a)$  by  $(Q^a, P^a) = (M_{ab}^0(\alpha^b + \lambda_1^0 \beta^b)/\lambda_2^0, L_{ab}\beta^b)$ . The fundamental charge  $Q$  is normalized so that  $Q^2 = e^2/2\pi$  and we have set  $\kappa^2 = 16\pi$ . As discussed in [26, 27] only a subset of the conjectured spectrum corresponds to elementary string states. First of all these states will be only electrically charged, i.e.  $\beta = 0$ , but there will be restrictions on  $\alpha$  too. After performing an  $O(6, 22)$  rotation  $\Omega$  of the background  $M^0$  transforming it into the 28-dimensional identity matrix  $I$  and an accompanied change of basis to  $\hat{\alpha} = L\Omega L\alpha$  the mass formula (31) can be rewritten [26, 27] as

$$m^2 = \frac{1}{16\lambda_2^0}\hat{\alpha}^a(I + L)_{ab}\hat{\alpha}^b = \frac{1}{8\lambda_2^0}(\hat{\alpha}_R)^2 \quad (32)$$

with  $\hat{\alpha}_R = \frac{1}{2}(I + L)\hat{\alpha}$  and  $\hat{\alpha}_L = \frac{1}{2}(I - L)\hat{\alpha}$ . In the string language  $\hat{\alpha}_{R(L)}$  are the right(left)-moving internal momenta. The mass of a generic string state in the Neveu-Schwarz sector (which is degenerate with the Ramond sector) is given by

$$m^2 = \frac{1}{8\lambda_2^0} \{(\hat{\alpha}_R)^2 + 2N_R - 1\} = \frac{1}{8\lambda_2^0} \{(\hat{\alpha}_L)^2 + 2N_L - 2\}. \quad (33)$$

A comparison of (32) and (33) shows that the string states satisfying the Bogomol'nyi bound all have  $N_R = 1/2$ . One then finds

$$N_L - 1 = \frac{1}{2} \left( (\hat{\alpha}_L)^2 - (\hat{\alpha}_R)^2 \right) = \frac{1}{2} \alpha^T L \alpha, \quad (34)$$

leading to  $\alpha^T L \alpha \geq -2$ .

We shall now show that the extreme Kaluza-Klein black holes are string states with  $\alpha^T L \alpha$  null ( $N_L = 1$ ). To identify them as states in the spectrum we have to find the corresponding charge vector  $\alpha$  and to verify that the masses calculated by the formulas (19) and (31) are identical. The action (24) can be consistently truncated by keeping the metric  $g_{\mu\nu}$ , just one field strength ( $F^1$  say), and one scalar field  $\phi$  via the ansatz  $\Phi = \phi/\sqrt{3}$  and  $M_{11} = e^{2\phi/\sqrt{3}} = M_{77}^{-1}$ . All other diagonal components of  $M$  are set equal to unity and all non-diagonal components to zero. Now (24) reduces to (8). (This yields the electric and magnetic Kaluza-Klein (or " $F$ ") monopoles. This is not quite the truncation chosen in [20], where just  $F^7$  was retained and  $M_{11} = e^{-2\phi/\sqrt{3}} = M_{77}^{-1}$ . This yields the electric and magnetic winding (or " $H$ ") monopoles. However, the two are related by  $T$ -duality). We shall restrict ourselves to the purely electrically charged solution with charge  $Q = 1$ , since this one is expected to correspond to an elementary string excitation. The charge vector  $\alpha$  for this solution is obviously given by  $\alpha^a = \delta^{a,1}$  with  $\alpha^T L \alpha = 0$ . Applying (31) for the mass of the state we find  $m^2 = 1/16 = Q^2/16$ , which coincides with (19) in the extreme limit. This agreement confirms the claim that this extreme Kaluza-Klein black hole is a state in the Schwarz-

Sen spectrum and preserves 2 supersymmetries. From this solution we can generate the whole set of supersymmetric black hole solutions with  $\alpha^T L \alpha = 0$  in the following way: first we perform an  $O(6, 22; R)$  transformation to obtain a vector proportional to the desired lattice vector. This clearly leaves the mass invariant, but the new charge vector  $\alpha'$  will in general not be located on the lattice. To find a state in the allowed charge spectrum we have to rescale  $\alpha'$  by a constant  $k$  so that  $\alpha'' = k\alpha'$  is a lattice vector. Clearly the masses calculated by (19) and (31) still agree (this is obvious by reversing the steps of rotation and rescaling), leading to the conclusion that all states obtained in this way preserve 1/2 of the supersymmetries. Therefore all states in the spectrum belonging to  $s_{max} = 1$  supermultiplets for which  $N_R = 1/2, N_L = 1$  are extreme Kaluza-Klein black holes.

It should also be clear that the purely magnetic extreme black hole solutions [20] obtained from the above by the replacements  $\phi \rightarrow -\phi, F \rightarrow e^{-\sqrt{3}\phi} F$  will also belong to the Schwarz-Sen spectrum of solitonic states. Starting from either the purely electric or purely magnetic solutions, dyonic states in the spectrum which involve non-vanishing axion field  $\Psi$  can then be obtained by  $SL(2, Z)$  transformations. Specifically, a black hole with charge vector  $(\alpha, 0)$  will be mapped into ones with charges  $(a\alpha, c\alpha)$  with the integers  $a$  and  $c$  relatively prime [26, 27].

We have limited ourselves to  $N_R = 1/2, N_L = 1$  supermultiplets with  $s_{min} = 0$ . Having established that the  $s = 0$  member of the multiplet is

an extreme black hole, one may then use the fermionic zero modes to perform supersymmetry transformations to generate the whole supermultiplet of black holes [72, 73]. Of course there are  $N_R = 1/2$  multiplets with  $s_{min} > 0$  coming from oscillators with higher spin and our arguments have nothing to say about whether these are also extreme black holes.

As discussed in [30] the  $N_R = 1/2, N_L > 1$  states (and their dual counterparts) are also extreme electric (magnetic) black holes. However, whereas the Kaluza-Klein black holes have a scalar-Maxwell coupling  $e^{-a\phi} F^{\mu\nu} F_{\mu\nu}$  with  $a = \sqrt{3}$ , the  $N_L > 1$  states have  $a = 1$  and correspond to the supersymmetric dilaton black hole [16]. It is well-known that the non-supersymmetric  $a = 1$  case provides a solution of the heterotic string [80, 81] but it was only recently recognized that the supersymmetric  $a = 1$  case and also the  $a = \sqrt{3}$  case are also solutions [20, 30]. The electric solutions are in fact *exact* [82] with no  $\alpha'$  corrections for both  $a = \sqrt{3}$  and  $a = 1$ . There are also  $a = 0$  Reissner-Nordstrom black holes [30], but these do not belong to the  $N_- = 1/2$  sector of string states. None of the spinning  $N_R = 1/2$  states is described by extreme *rotating* black hole metrics because they obey the same Bogomol'nyi bound as the  $s_{min} = 0$  states, whereas the mass formula for an extreme *rotating* black hole depends on the angular momentum  $J$ . Rather it is the fermion fields which carry the spin. (For the  $a = 0$  black hole, they yield a gyromagnetic ratio  $g = 2$  [73]; the  $a = \sqrt{3}$  and  $a = 1$  superpartner  $g$ -factors are unknown to us.) It may be that there are states

in the string spectrum described by the extreme rotating black hole metrics but if so they will belong to the  $N_R \neq 1/2$  sector<sup>7</sup>. Since, whether rotating or not, the black hole solutions are still independent of the azimuthal angle and independent of time, the supergravity theory is effectively *two-dimensional* and therefore possibly integrable. This suggests that the spectrum should be invariant under the larger duality  $O(8, 24; Z)$  [22, 23], which combines  $S$  and  $T$ . The corresponding Kac-Moody extension would then play the role of the spectrum generating symmetry [86].

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<sup>7</sup>The gyromagnetic and gyroelectric ratios of the states in the heterotic string spectrum would then have to agree with those of charged rotating black hole solutions of the heterotic string. This is indeed the case: the  $N_L = 1$  states [11] and the extreme rotating  $a = \sqrt{3}$  black holes [83] both have  $g = 1$  whereas the  $N_L > 1$  states [71] and the extreme rotating  $a = 1$  [84] (and  $a = 0$  [85]) black holes both have  $g = 2$ . In fact, it was the observation that the Regge formula  $J \sim m^2$  also describes the mass/angular momentum relation of an extreme rotating black hole which first led Salam [67] to imagine that elementary particles might behave like black holes!

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