



Effective $F(T)$ gravity from the higher-dimensional Kaluza–Klein and Randall–Sundrum theories



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ABSTRACT

We explore the four-dimensional effective $F(T)$ gravity with T the torsion scalar in teleparallelism originating from higher-dimensional space–time theories, in particular the Kaluza–Klein (KK) and Randall–Sundrum (RS) theories. First, through the KK dimensional reduction from the five-dimensional space–time, we obtain the four-dimensional effective theory of $F(T)$ gravity with its coupling to a scalar field. Second, taking the RS type-II model in which there exist the five-dimensional Anti-de Sitter (AdS) space–time with four-dimensional Friedmann–Lemaître–Robertson–Walker (FLRW) brane, we find that there will appear the contribution of $F(T)$ gravity on the four-dimensional FLRW brane. It is demonstrated that inflation and the dark energy dominated stage can be realized in the KK and RS models, respectively, due to the effect of only the torsion in teleparallelism without that of the curvature.

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1. Introduction

The phenomenon of the accelerated expansion of the universe has been supported by various observations of Supernovae Ia [1], large scale structure [2] including the baryon acoustic oscillations [3], cosmic microwave background radiation [4], and weak lensing [5]. This is one of the most significant problems in modern cosmology. Provided that the universe is homogeneous, there exist two representative ways of accounting for the current cosmic acceleration: The first is to introduce “dark energy”, which has negative pressure, within general relativity (for recent reviews, see, e.g., [6]). The second is to modify the gravitational theory on large scale. As one of the latter approaches, “teleparallelism” [7] has recently been drawn much attention. The formulations are constructed with the Weitzenböck connection, and hence the action is described by the torsion scalar T , whereas in general relativity, the formulations are written with the Levi-Civita connection, and thus the action is represented by the scalar curvature R . It has been illustrated that in $F(T)$ gravity, inflation in the early universe [8] or the late-time cosmic acceleration [9–11] can be

realized. Also, it was verified that a non-minimal gravitational coupling of a scalar field in teleparallelism can explain the current cosmic acceleration [12]. Various theoretical issues of $F(T)$ gravity have extensively been discussed.

In this Letter, we examine the four-dimensional effective $F(T)$ gravity taking its origin from higher-dimensional space–time theories. As the first example, we consider the four-dimensional effective $F(T)$ gravity from the Kaluza–Klein (KK) theory [13–15]. By setting up the five-dimensional space–time, via the KK reduction to the four-dimensional space–time, we construct the four-dimensional effective theory, which is an $F(T)$ gravity model with the non-minimal coupling to a scalar field. Next, as the second example, we investigate the four-dimensional effective $F(T)$ gravity from the Randall–Sundrum (RS) [16,17] theory, which originates from a novel KK approach in the brane world description [18]. We take the RS type-II model where there are the five-dimensional Anti-de Sitter (AdS) space–time and the four-dimensional Friedmann–Lemaître–Robertson–Walker (FLRW) brane. In such a configuration, a contribution of $F(T)$ gravity on the FLRW brane will exist. It is shown that inflation or the dark energy dominated stage can be realized only by the effect of the torsion without that of the curvature. As a result, it can be interpreted that these models may be equivalent to the KK and RS models without gravitational effects of the curvature but just due to those of the torsion in teleparallelism. We use units of $k_B = c = \hbar = 1$

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and denote the gravitational constant $8\pi G$ by $\kappa^2 \equiv 8\pi/M_{\text{Pl}}^2$ with the Planck mass of $M_{\text{Pl}} = G^{-1/2} = 1.2 \times 10^{19}$ GeV.

The Letter is organized as follows. In Section 2, we introduce the formulations in teleparallelism and first explore the effective $F(T)$ gravity in the four-dimensional space-time coming from the five-dimensional Kaluza–Klein (KK) theory. Next, in Section 3, we examine the RS type-II model and show that the $F(T)$ gravity contribution will exist on the brane with the four-dimensional flat FLRW space-time. In Section 4, conclusions are finally given.

2. From the Kaluza–Klein (KK) theory

In teleparallelism, orthonormal tetrad components $e_A(x^\mu)$ with $A = 0, 1, 2, 3$ are adopted. Here, an index A is for the tangent space at each point x^μ of the manifold. With orthonormal tetrad components, the metric is expressed as $g_{\mu\nu} = \eta_{AB} e_A^\mu e_B^\nu$ with μ and ν , where $\mu, \nu = 0, 1, 2, 3$, coordinate indices on the manifold, and accordingly e_A^μ is equivalent to the tangent vector of the manifold. This is called the vierbein. The relation $e_\mu^A e_A^\nu = \delta_\mu^\nu$ defines the inverse of the vierbein. The torsion and contorsion tensors are defined as $T^\rho{}_{\mu\nu} \equiv \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu} = e_A^\rho (\partial_\mu e_\nu^A - \partial_\nu e_\mu^A)$ with $\Gamma^\rho{}_{\nu\mu} \equiv e_A^\rho \partial_\mu e_\nu^A$, being the Weitzenböck connection without curvature, and $K^{\mu\nu}{}_\rho \equiv -(1/2)(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu})$ the contorsion tensor, respectively. The torsion scalar is constructed as $T \equiv S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu} = (1/4)T^{\rho\mu\nu} T_{\rho\mu\nu} + (1/2)T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}{}^\rho T^{\nu\mu}{}_\nu$, where $S_\rho{}^{\mu\nu} \equiv (1/2)(K^{\mu\nu}{}_\rho + \delta_\rho^\mu T^{\alpha\nu}{}_\alpha - \delta_\rho^\nu T^{\alpha\mu}{}_\alpha)$ is the superpotential. Consequently, the teleparallel Lagrangian density is described by the torsion scalar T , although the Einstein–Hilbert action is represented by the scalar curvature R in general relativity. The modified teleparallel action describing $F(T)$ gravity [10] with matter is

$$S = \int d^4x |e| \left(\frac{F(T)}{2\kappa^2} + \mathcal{L}_M \right), \quad (1)$$

where $|e| = \det(e_\mu^A) = \sqrt{-g}$ with g the determinant of the metric $g_{\mu\nu}$ and $\mathcal{L}_M$ is the matter Lagrangian. In what follows, we concentrate on the part of gravitation of the action.

First, we explore the four-dimensional effective $F(T)$ gravity from the KK theory. We suppose that the procedure of the KK reduction [13–15] can be applied to the modified teleparallel gravity in the same manner as in general relativity. The action of $F(T)$ gravity in the five-dimensional space-time is expressed as [19]

$${}^{(5)}S = \int d^5x |{}^{(5)}e| \left| \frac{F({}^{(5)}T)}{2\kappa_5^2} \right|, \quad (2)$$

$${}^{(5)}T \equiv \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{cba} - T_{ab}{}^a T^{cb}{}_c, \quad (3)$$

where ${}^{(5)}e = \sqrt{{}^{(5)}g}$ with ${}^{(5)}g$ the determinant of the metric ${}^{(5)}g_{\mu\nu}$ in the five-dimensional space-time, $\kappa_5^2 \equiv 8\pi G_5 = ({}^{(5)}M_{\text{Pl}})^{-3}$ with G_5 the gravitational constant and $M_{\text{Pl}}^{(5)}$ the Planck mass in the five-dimensional space-time. Here, the superscript or subscript of (5) or 5 mean the quantities in the five-dimensional space-time. In addition, ${}^{(5)}T$ is the torsion scalar in the five-dimensional space-time, where the Latin indices $a, b, \dots$ run over 0, 1, 2, 3, 5 and “5” denotes the component of the fifth coordinate. The form in Eq. (3) is equivalent to that in the four-dimensional space-time shown above [19]. We now consider the following original KK compactification scenario in case of the five-dimensional space-time. One of the dimensions of space is compactified to a small circle and the four-dimensional space-time is extended infinitely. The radius of the fifth dimension is taken to be of order of the Planck length in order for the KK effects not to be seen. Thus, the size of the circle is so small that phenomena in sufficiently low energies cannot be

detected [13–15]. Provided that the metric in the five-dimensional space-time is described as the following diagonal form

$${}^{(5)}g_{ab} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -\phi^2 \end{pmatrix}, \quad (4)$$

with $\phi \equiv \varphi/\varphi_*$ a homogeneous scalar field depending only on time, where ϕ is a dimensionless quantity, φ is a homogeneous scalar field having a mass dimension and φ_* is a fiducial value of φ . We represent $\phi^2 = \mathcal{R}^2 \theta^2$, where $\mathcal{R}$ is the radius of the compactified space, and the orthonormal tetrad components in the one-dimensional compactified space is written by the dimensionless coordinates θ such as an angle. We also find $\sqrt{{}^{(5)}g} = \sqrt{-g} \mathcal{R} \sqrt{\hat{g}}$. Here, $\hat{g}$ is the determinant of the metric corresponding to the pure geometrical part represented by θ and relevant to the compactified space volume $V_{\text{com}} = \int \hat{g} d\theta$ [15]. In this case, we take $e_a^A = \text{diag}(1, 1, 1, 1, \phi)$ and the $\eta_{ab} = \text{diag}(1, -1, -1, -1, -1)$. For the action in the five-dimensional space-time in Eq. (2) with Eq. (3), by adopting the above expressions of e_a^A and η_{ab} to analyze ${}^{(5)}S$ and ${}^{(5)}T$, the effective action in the four-dimensional space-time through the KK compactification mechanism explained above can be described as

$$S_{\text{KK}}^{\text{eff}} = \int d^4x |e| \left| \frac{1}{2\kappa^2} \phi F(T + \phi^{-2} \partial_\mu \phi \partial^\mu \phi) \right|. \quad (5)$$

The appearance of ϕ on the right-hand side in Eq. (5) in front of the function F comes from the relation $|{}^{(5)}e| = \phi |e|$ due to the KK dimensional reduction. Furthermore, the form of ${}^{(5)}T$ is the same as that of T , and the part of the 0, ..., 3 of ${}^{(5)}g_{ab}$ in Eq. (4) is $g_{\mu\nu}$, i.e., the metric in the four-dimensional space-time. Hence, the form of the torsion scalar through the KK dimensional reduction to the four-dimensional space-time, which is the argument of the function F on the right-hand side in Eq. (5), would consist of T and the other part in terms of ϕ , which is related to the size of the compactified space. We note that the form of the function F itself would not be changed by the KK dimensional reduction. Our KK reduced action in Eq. (5) is compatible with the results in Ref. [20]. Also, we mention that the investigations in the case with the non-diagonal form of the metric in the five-dimensional space-time have also been executed in Ref. [21]. Here, as a simplest example, we consider the case of teleparallelism, i.e., $F(T) = T - 2\Lambda_4$ in Eq. (1) with $\Lambda_4 (> 0)$ a positive cosmological constant in the four-dimensional space-time. In this case, the action in Eq. (5) is similar to the one describing the Brans–Dicke theory with the cosmological constant. If we define a scalar field σ as $\phi \equiv \xi \sigma^2$ with $\xi = 1/4$, we can rewrite the action in Eq. (5) into the one where the kinetic term of σ becomes canonical as [15] $S_{\text{KK}}^{\text{eff}}|_{F(T)=T-2\Lambda_4} = \int d^4x |e| (1/\kappa^2) [(1/8)\sigma^2 T + (1/2)\partial_\mu \sigma \partial^\mu \sigma - \Lambda_4]$. The metric of the flat FLRW universe is written as $ds^2 = dt^2 - a^2(t) \sum_{i=1,2,3} (dx^i)^2$ with a the scale factor and $H \equiv \dot{a}/a$ the Hubble parameter, where the dot denotes the time derivative of $\partial/\partial t$. For this space-time, we have $g_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2)$ and $e_\mu^A = \text{diag}(1, a, a, a)$. These expressions lead to the relation $T = -6H^2$. In this background, the gravitational field equations read $(1/2)\dot{\sigma}^2 - (3/4)H^2\sigma^2 + \Lambda_4 = 0$ and $\dot{\sigma}^2 + H\sigma\dot{\sigma} + (1/2)\dot{H}\sigma^2 = 0$ [12]. Furthermore, the equation of motion of σ becomes $\ddot{\sigma} + 3H\dot{\sigma} + (3/2)H^2\sigma = 0$. In deriving these equations, we have used the relation $T = -6H^2$. By combining the above gravitational field equations, we have $(3/2)H^2\sigma^2 - 2\Lambda_4 + H\sigma\dot{\sigma} + (1/2)\dot{H}\sigma^2 = 0$. Hence, we can obtain a solution for this equation as $H = H_{\text{inf}} = \text{constant} (> 0)$, which corresponds to the Hubble parameter at the inflationary stage, and $\sigma = b_1(t/t_1) + b_2$, where b_1 is a constant and $b_2 (> 0)$ a positive one, and t_1 denotes a time. In the limit $t \rightarrow 0$, we can acquire an approximate expression as $H_{\text{inf}} \approx$

$(2/b_2)\sqrt{\Lambda_4/3}$ and $\sigma \approx b_2$. Furthermore, with the equation of motion of σ , for $t \rightarrow 0$, we find $b_1 \approx -(1/2)b_2 H_{\text{inf}} t_1 \approx -\sqrt{\Lambda_4/3} t_1$. As a result, when $t \rightarrow 0$, an exponential inflation with the scale factor $a \approx \bar{a} \exp(H_{\text{inf}} t)$, where $\bar{a} (> 0)$ is a positive constant, can be realized approximately. It is significant to emphasize that the contribution of the effect of the KK compactification, namely, the role of extra dimensions, is to lead to the scalar field ϕ in the gravitational field equations.

3. From the Randall–Sundrum (RS) theory

Next, we explore the four-dimensional effective $F(T)$ gravity from the RS theory with the procedure in Ref. [15]. In the RS type-I model [16], there are a positive tension brane at $y = 0$ and a negative one at $y = s \rightarrow \infty$, where y is the fifth direction. Suppose that the metric describing the five-dimensional space-time is given by $ds^2 = e^{-2|y|/l} g_{\mu\nu}(x) dx^\mu dx^\nu + dy^2$ with $l = \sqrt{-6/\Lambda_5}$, where $e^{-2|y|/l}$ is the warp factor and $\Lambda_5 (< 0)$ is the negative cosmological constant in the bulk. It is known that for the RS type-I model, the effective gravity theory in four dimensions is the Brans–Dicke (BD) theory with the BD parameter $\omega_{\text{BD}} = (3/2)(e^{\pm s/l} - 1)$, where the sign $(\pm)$ corresponds to that of the brane tension [22].

On the other hand, in the RS type-II model [17], there is only one brane with the positive tension floating in the AdS bulk space and hence the negative-tension brane does not exist. This configuration can be realized by the RS type-I model [16] with two branes in the limit $\tilde{s} \rightarrow \infty$. We start with the equation in the five-dimensional space-time with the brane whose tension is a positive constant. We consider that the vacuum solution in the five-dimensional space-time is AdS one, and that the brane configuration is consistent with the equation in the five-dimensional space-time. This implies that the brane configuration with a positive constant tension connecting two vacuum solutions in the five-dimensional space-time, namely, the condition of the configuration is nothing but the equation for the brane. In Ref. [23], using the analysis in Ref. [24], the RS type-II model in teleparallelism has been considered. The procedure is as follows. (i) The corresponding Gauss–Codazzi equations in teleparallelism, namely, the induced equations on the brane, are examined by using the projection vierbein of the five-dimensional space-time quantities into the four-dimensional space-time brane. (ii) The Israel's junction conditions to connect the left-side and right-side bulk spaces sandwiching the brane are investigated. The first junction condition is that the vierbeins induced on the brane from the left side and right side of the brane should be the same with each other. Moreover, the second junction condition is that the difference of the superpotential between the left side and right side of the brane comes from the energy–momentum tensor of matter, which is confined in the brane. (iii) Provided that there exists Z_2 symmetry, i.e., $y \leftrightarrow -y$, in the five-dimensional space-time, the quantities on the left and right sides of the brane are explored. The difference between the scalar curvature and the torsion scalar is a total derivative of the torsion tensor [19,25]. This may affect the boundary. It has been shown that in comparison with the gravitational field equations in general relativity [24,26], the induced gravitational field equations on the brane have new terms, which comes from the additional degrees of freedom in teleparallelism. These extra terms correspond to the projection on the brane of the vector portion of the torsion tensor in the bulk.

Through the procedure explained above, we find that for $F(T)$ gravity, in the flat FLRW background the Friedmann equation on the brane is given by

$$H^2 \frac{dF(T)}{dT} = -\frac{1}{12} \left[F(T) - 4\Lambda - 2\kappa^2 \rho_M - \left(\frac{\kappa_5^2}{2} \right)^2 \mathcal{Q} \rho_M^2 \right], \quad (6)$$

with $\mathcal{Q} \equiv (11 - 60w_M + 93w_M^2)/4$. We note that $\mathcal{Q}$ includes the contributions from teleparallelism, which do not exist in general relativity [23]. Here, $w_M \equiv P_M/\rho_M$ with the energy density ρ_M and pressure P_M of matter, assumed to be a perfect fluid, is the equation of state parameter for matter confined to the brane, the effective cosmological constant in the brane is $\Lambda \equiv \Lambda_5 + (\kappa_5^2/2)^2 \lambda^2$ with $\lambda (> 0)$ the tension of the brane and $G = [1/(3\pi)](\kappa_5^2/2)^2 \lambda$. Clearly, the significant contributions from the fifth dimension to the Friedmann equation on the brane are the second term and the fourth term proportional to ρ_M^2 on the right-hand side in Eq. (6). Furthermore, the function of $F(T)$ induced on the brane would be considered to be the same as that in the five-dimensional space-time. In the dark energy dominated stage, the energy density of non-relativistic matters with $w_M = 0$, i.e., cold dark matter and baryon, is so much smaller than that of the cosmological constant that the third and fourth terms on the right-hand side can be neglected. For teleparallelism with the cosmological constant in the five-dimensional space-time, $F(T) = T - 2\Lambda_5$ in Eq. (6), we obtain an approximate de Sitter solution on the brane $H = H_{\text{DE}} = \sqrt{\Lambda_5 + \kappa_5^4 \lambda^2/6} = \text{constant}$ and $a(t) = a_{\text{DE}} \exp(H_{\text{DE}} t)$ with $a_{\text{DE}} (> 0)$ a constant, where we have used $T = -6H^2$. Therefore, for the late time cosmic acceleration can be realized. We mention that for $F(T) = T$, $\Lambda = 0$ and $\mathcal{Q} = 8/3$ realizing if $w_M = -5.5 \times 10^{-3}$, we find $H^2 = (\kappa^2/3)\rho_M[1 + \rho_M/(2\lambda)]$, which is equivalent to the Friedmann equation in the brane world scenario [27]. Moreover, for a power-law model such as $F(T) = T^2/\bar{M}^2 + \alpha\Lambda_5$ in Eq. (6), where $\bar{M}$ is a mass scale and α is a constant, we find a similar approximate de Sitter solution $H = H_{\text{DE}} = [(M^2/108)\mathcal{J}]^{1/4} = \text{constant}$ with $\mathcal{J} \equiv (\alpha - 4)\Lambda_5 - 4(\kappa_5^2/2)^2 \lambda^2$, where $\mathcal{J} (\geq 0)$ has to be larger than or equal to zero, so that this can lead to a constraint on α as $\alpha \geq 4 + (\kappa_5^2 \lambda^2)/\Lambda_5$. Here, we have used an approximation that on the right-hand side of Eq. (6), the first and second terms, which corresponds to the components of dark energy, are much larger than the third and fourth terms proportional to ρ_M and ρ_M^2 , respectively. This approximation can be appropriate when the universe is considered to be the dark energy (sufficiently) dominated stage and thus the energy density of non-relativistic matters ρ_M can be negligible in comparison with the dark energy density. In addition, we note that in deriving the above solution, we have used the relation $T = -6H^2$, and that as a result, both the left-hand side of Eq. (6) and the first term of the right-hand side are proportional to H^4 . It is emphasized that the generic formulation for the gravitational field equation on the brane in teleparallel gravity has been derived in Ref. [23]. On the other hand, the new ingredients obtained in this Letter would be considered to describe the Friedmann equation (6) in the flat FLRW space-time and to acquire the solutions to realize the current accelerated expansion of the universe for two concrete $F(T)$ models. It should also be cautioned that the condition on $F(T)$ gravity for the AdS configuration in the bulk to be realized has to be shown in future work. In addition, four-dimensional bouncing $F(T)$ cosmologies [28] unifying inflation with the late-time cosmic acceleration due to dark energy have been discussed. Such cosmologies may be reconstructed also in $F(T)$ gravity from the RS brane world scenario.

4. Conclusions

We have studied the four-dimensional effective $F(T)$ gravity coming from the higher-dimensional KK and RS space-time theories. With the KK reduction from the five-dimensional space-time to the four dimensions, we have built the four-dimensional effective theory of $F(T)$ gravity coupling to a scalar field. Moreover, for the RS type-II model consisting of the five-dimensional AdS space-time and the four-dimensional FLRW brane, we have also

shown that the contribution of $F(T)$ gravity appears on the four-dimensional FLRW brane. Furthermore, it has been verified that inflation or the late time cosmic accelerated expansion can occur only through the effect of the torsion without that of the curvature. Thus, these models can be regarded as the KK and RS models constructed by not the curvature effect but only the torsion one in teleparallelism.

What it has been executed in this Letter is to explicitly demonstrate that in the four-dimensional effective $F(T)$ gravity theories obtained by the KK reduction from the five-dimensional space-time and those on the four-dimensional FLRW brane in the RS type-II model, inflation in the early universe and the accelerated expansion in the late time universe can be realized, respectively, owing to the effect of the torsion of the space-time and not the curvature effect. Indeed, this is the first work on the concrete cosmological solutions to describe the cosmic accelerated expansion of the KK and RS models in $F(T)$ gravity. These results may imply that phenomenological $F(T)$ gravity models in the four-dimensional space-time can be derived from more fundamental theories. Since $F(T)$ gravity models can lead to the current accelerated expansion of the universe, namely, a resolution of the dark energy problem, this study may present us a clue to explore the origin of extensions of gravity from general relativity including $F(T)$ gravity.

Finally, it should be remarked that the observational constraints on the derivative of $F(T)$ with respect to T until the fifth order have been presented in Ref. [29] with cosmographic parameters acquired from the observational data of Supernovae Ia and the baryon acoustic oscillations. In this Letter, as concrete examples of $F(T)$ gravity models, we have considered $F(T) = T$ plus an effective cosmological constant and $F(T) = T^2/\bar{M}^2$ plus a constant term corresponding to an effective cosmological constant. These two models can be consistent with the results obtained in Ref. [29].

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