Intersecting Brane Worlds and Their Effective Interactions

Dieter Lüst*)

Institut für Physik, Humboldt-Universität, Newtonstraße 15, D-12489 Berlin, Germany

In this review we describe the general geometrical framework of brane world constructions in orientifolds of type IIA string theory with D6-branes wrapping 3-cycles in a Calabi-Yau 3-fold. These branes generically intersect in points, and the patterns of intersections govern the chiral fermion spectra and issues of gauge and supersymmetry breaking in the low energy effective gauge theory on their world volume. We also specialize the discussion for the case of orbifold backgrounds with intersecting D6-branes. Then, in the second part of the paper, we discuss parts of the effective action of intersecting brane world models. Specifically we first compute from the Born-Infeld action of the wrapped D-branes the tree-level, D-term scalar potential, which is important for the stability of the considered backgrounds as well as for questions related to supersymmetry breaking. Second, we review the recent computation of one-loop gauge threshold corrections in intersecting brane world models, which are needed in order to give precise predictions for the values of the gauge couplings at low energies.

§1. Introduction

A central object of string phenomenology is to provide an existence proof for a string vacuum whose low energy approximation is reproducing the known physics of the Standard Model or of its supersymmetric and grand unified extensions. As a first approach one may concentrate on finding models with the correct light degrees of freedom, the right gauge group and chiral fermion spectra, leaving the details of their dynamics aside for the moment. Intersecting brane worlds[1)–23)] have proven to be a candidate framework of model building which offers excellent opportunity to meet this requirement. In these string compactifications, the standard model particles correspond to open string excitations which are located at the various intersections of the D-branes in the internal 6-dimensional space. At the moment, type IIA intersecting brane world models with D6-branes, which fill the 4-dimensional Minkowski space-time and are wrapped around internal 3-cycles, provide the most promising approach to come as close as possible to the standard model.**) Then the fermion spectrum is determined by the intersection numbers of certain 3-cycles in the internal space, as opposed for instance to the older approaches involving heterotic strings, where the number of generations was given by the Euler characteristic in the simplest case.

Going beyond these topological data in a second step, the computation of the effective interactions of the light (open) string modes is of vital importance in order to confront eventually the intersecting brane world models with experiment. In particular, the knowledge of the effective scalar potential is needed to discuss the

*) E-mail: luest@physik.hu-berlin.de

**) In a T-dual mirror description these models correspond to type IIB orientifolds including D9-branes with magnetic F-flux turned on.

question of stability of intersecting brane world models. At classical level parts of the effective scalar potential were computed in Refs. 8), 14); in this way one can determine the dynamics at least at the classical level. More recently also the one-loop gauge threshold corrections in supersymmetric intersecting brane world models were calculated.[19)] These are essential to get precise informations on the low-energy values of the standard model gauge couplings. Finally effective Yukawa couplings[12),18),21)] and quartic fermion interactions,[22),23)] relevant for flavor changing neutral currents, were also investigated.

In this work we will review the main aspects of the construction of intersecting brane world models as well as of the computation of the tree-level scalar potential and the one-loop threshold corrections. The constructive part will be fairly general, applicable for D6-branes wrapped around Calabi-Yau 3-cycles. Later, when we come to the computation of the one-loop gauge threshold corrections we restrict ourselves to orbifold and orientifold backgrounds.

Let us elaborate a little bit more on the general picture of intersecting brane world models. The internal Calabi-Yau space may develop one or more nodes which support 3-cycles wrapped by several D6-branes. On one of the nodes, the Standard Model (SM) fields are localized, while others may involve hidden sector (HS) gauge groups which couple only gravitationally to the visible sector (see Fig. 1).

This kind of scenario offers at least two possible ways to address the issue of space-time supersymmetry breaking in intersecting brane worlds. In the first class of models the Standard Model brane configuration is already non-supersymmetric from the beginning (this is true for many of the models considered so far, including the CY example in Ref. 14)). This means that supersymmetry is broken at the string scale M_{string}. In order to avoid the usual hierarchy problems M_{string} should be of the order of a few TeV, requiring that the volume of the internal CY space transverse to all Standard Model D-branes is large according to Refs. 24), 25). On the other hand it may happen that the Standard Model D-brane sector is itself supersymmetric ($\mathcal{N} = 1$ supersymmetric brane world models were constructed in Refs. 9), 10), 17)). Now

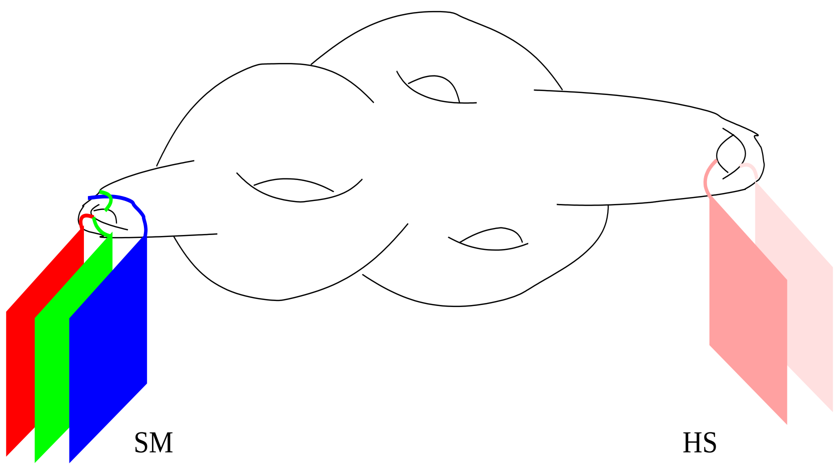

Fig. 1. Calabi-Yau space with intersecting standard model and hidden D6-branes.

suppose that the hidden sector preserves a different supersymmetry or is completely non-supersymmetric. Then the gravity mediated supersymmetry breaking appears very natural. In this case the following relation between the SUSY breaking scale in the Standard Model sector and the fundamental string scale is expected to hold:

$$M_{3/2} \simeq \frac{M_{\rm string}^2}{M_{\rm Planck}}\,. \tag{1·1}$$

With $M_{3/2}$ of order TeV one obtains an intermediate string scale, $M_{\rm string} \simeq 10^{11}$GeV, a scenario which was already discussed in Ref. 26). For D6-brane models the string scale, the string coupling constant $g_{\rm string}$, the typical length scale $R_{\|}$ of the internal D6-brane volume Vol(D6) $\sim R_{\|}^3$ and the scale $R_{\perp}$ of the transversal internal volume are related to the tree level gauge coupling $g_{\rm YM}$ and the effective Planck mass in the following way:

$$g_{\rm YM}^2 = g_{\rm string}(M_{\rm string}R_{\|})^{-3}\,, \qquad M_{\rm Planck} = \frac{M_{\rm string}^4}{g_{\rm string}}(R_{\|}R_{\perp})^{3/2}\,. \tag{1·2}$$

Assuming that $M_{\rm string} \simeq R_{\|}^{-1}$ this requires a moderately enlarged transversal space, namely $R_{\perp}^{-1} \simeq 10^9$GeV.

§2. Intersecting brane worlds on Calabi-Yau 3-folds

In the brane world scenarios we are going to consider here, there are D-branes filling out the entire four-dimensional space-time providing the degrees of freedom for an effective gauge theory. The overall transverse six-dimensional space is compact, such that the internal excitations decouple from the effective theory below the string scale. The global consistency conditions in string models with D-branes that fill out the non-compact space-time involve the cancellation of the RR charges. Furthermore, supersymmetry requires the cancellation of the brane tensions and the corresponding Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles as well. If the latter is neglected, one can achieve the RR charge cancellation within type II vacua by including anti-branes, but these vacua usually suffer from run-away instabilities, if not even tachyons. The only setting in which objects with negative tension arise naturally in string theory are orientifolds, where the orientifold O-planes can balance the charge and tension of the D-branes. Therefore, orientifolds provide the framework where supersymmetric brane worlds may be found within string theory.

2.1. *Definition*

According to the above reasoning we will consider orientifold compactifications on general Calabi-Yau spaces (see Refs. 14) - 16)), where the ten-dimensional space-time $\mathcal{X}$ is of the kind

$$\mathcal{X} = R^{3,1} \times \frac{\mathcal{M}^6}{\Omega\overline{\sigma}}. \tag{2·1}$$

Here $\mathcal{M}^6$ is a Calabi-Yau 3-fold with a symmetry under $\overline{\sigma}$, the complex conjugation

$$\overline{\sigma} : z_i \mapsto \bar{z}_i,\ i = 1, \cdots, 3, \tag{2·2}$$

in local coordinates $z_i = y_1^i + iy_2^i$. It is combined with the world sheet parity Ω to form the orientifold projection $\Omega\overline{\sigma}$. This operation is actually a symmetry of the type IIA string on $\mathcal{M}^6$. Orientifold O6-planes are defined as the fixed locus $\mathrm{Fix}(\overline{\sigma})$ of $\overline{\sigma}$, which is easily seen to be a supersymmetric 3-cycle in $\mathcal{M}^6$. It is special Lagrangian (sLag) and calibrated with respect to the real part of the holomorphic 3-form Ω_3. To see this define Ω_3 and the Kähler form J in local coordinates

$$\Omega_3 = dz_1 \wedge dz_2 \wedge dz_3, \quad J = i\sum_{i=1}^{3} dz_i \wedge d\bar{z}_i. \tag{2·3}$$

From $\overline{\sigma}(\Omega_3) = \overline{\Omega}_3$ and $\overline{\sigma}(J) = -J$ it then follows that

$$\Im(\Omega_3)|_{\mathrm{Fix}(\overline{\sigma})} = 0, \quad J|_{\mathrm{Fix}(\overline{\sigma})} = 0. \tag{2·4}$$

It is also useful to define a rescaled 3-form

$$\widehat{\Omega}_3 = \frac{1}{\sqrt{\mathrm{Vol}(\mathcal{M}^6)}}\Omega_3\,. \tag{2·5}$$

This orientifold projection truncates the gravitational bulk theory of closed strings down to a theory with 16 supercharges in ten dimensions, leading to 4 supercharges and $\mathcal{N} = 1$ in four dimensions, after compactifying on the Calabi-Yau. In order to cancel the RR charge of the O6-planes it is required to introduce D6-branes into the theory as well, which will provide the gauge sector of the theory. If we label the individual stacks of D6_a-branes with multiplicities N_a by a label a, the gauge group of the effective theory will be given by

$$G = \prod_a U(N_a). \tag{2·6}$$

Here we exclude the possibility of branes which are invariant under the projection $\Omega\overline{\sigma}$. They would give rise to $SO(N_a)$ or $Sp(N_a)$ factors. It is no conceptual problem to include them as well, but they are of little phenomenological interest.

2.2. *RR charges and brane tension*

The charge cancellation conditions are often obtained by regarding divergences of one-loop open string amplitudes, but can also be determined from the consistency of the background in the supergravity equations of motion or Bianchi identities. The Chern-Simons action for Dp-branes and Op-planes are given by[27)–30)]

$$\mathcal{S}_{\mathrm{CS}}^{(\mathrm{D}p)} = \mu_p \int_{\mathrm{D}p} \mathrm{ch}(\mathcal{F}) \wedge \sqrt{\frac{\hat{\mathcal{A}}(\mathcal{R}_T)}{\hat{\mathcal{A}}(\mathcal{R}_N)}} \wedge \sum_q C_q,$$
$$\mathcal{S}_{\mathrm{CS}}^{(\mathrm{O}p)} = Q_p\mu_p \int_{\mathrm{O}p} \sqrt{\frac{\hat{\mathcal{L}}(\mathcal{R}_T/4)}{\hat{\mathcal{L}}(\mathcal{R}_N/4)}} \wedge \sum_q C_q. \tag{2·7}$$

The relative charge of the orientifold planes is given by $Q_p = -2^{p-4}$ and $\mathrm{ch}(\mathcal{F})$ denotes the Chern character, $\hat{\mathcal{A}}(\mathcal{R})$ the Dirac genus of the tangent or normal bundle, and the $\hat{\mathcal{L}}(\mathcal{R})$ the Hirzebruch polynomial. The physical gauge fields and curvatures are related to the skew-hermitian ones in (2·7) by rescaling with $-4i\pi^2\alpha'$. These expressions simplify drastically for sLag 3-cycles, where $\mathrm{ch}(\mathcal{F})|_{\mathrm{D}p} = \mathrm{rk}(\mathcal{F})$, the other characteristic classes become trivial and finally the only contribution in the CS-term (2·7) then comes from C_7.

In the following we denote the homology class of $\mathrm{Fix}(\overline{\sigma})$ by $\pi_{\mathrm{O6}} = [\mathrm{Fix}(\overline{\sigma})] \in H_3(\mathcal{M}^6)$ and the homology class of any given brane stack D6_a-brane by π_a. By our assumptions the π_a are never invariant under $\overline{\sigma}$ but mapped to image cycles π_a'. Therefore, a stack of D6-branes is wrapped on that cycle by symmetry, too. The RR charge cancellation can now easily be deduced by looking at the equation of motion of C_7

$$\frac{1}{\kappa^2}\, d \star dC_7 = \mu_6 \sum_a N_a\, \delta(\pi_a) + \mu_6 \sum_a N_a\, \delta(\pi_a') + \mu_6 Q_6\, \delta(\pi_{\mathrm{O6}}), \qquad (2\cdot 8)$$

where $\delta(\pi_a)$ denotes the Poincaré dual form of π_a, $\mu_p = 2\pi(4\pi^2\alpha')^{-(p+1)/2}$, and $2\kappa^2 = \mu_7^{-1}$. Upon integrating over $\mathcal{M}^6$ the RR-tadpole cancellation condition becomes a relation in homology

$$\sum_a N_a\,(\pi_a + \pi_a') - 4\pi_{\mathrm{O6}} = 0. \qquad (2\cdot 9)$$

In principle it involves as many linear relations as there are independent generators in $H_3(\mathcal{M}^6, R)$. But, of course, the action of $\overline{\sigma}$ on $\mathcal{M}^6$ also induces an action $[\overline{\sigma}]$ on the homology and cohomology. In particular, $[\overline{\sigma}]$ swaps $H^{2,1}$ and $H^{1,2}$, and the number of conditions is halved.

2.3. *Massless open string modes*

In this section we are going to present the most important input for constructing intersecting brane world models of particle physics, the formulae that determine the spectrum of the chiral fermions of the effective theory in terms of topological data of the brane configuration and the Calabi-Yau manifold. Roughly speaking, at any intersection point of two stacks of D6-branes a single chiral fermion is localized, transforming in the bifundamental representation of the two respective gauge groups. As was mentioned already, the search for a viable model close to the Standard Model particle content boils down to looking for Calabi-Yau spaces with an involution $\overline{\sigma}$ and an intersection form for its 3-cycles that allows to realize the desired particle spectrum at the intersections.

Catching up with the above discussion of the brane tension and the induced scalar potential, we can say a bit more: If we want to construct a supersymmetric intersecting brane world we need the desired intersection pattern to be realized within a set of sLag cycles all calibrated by the same 3-form, which makes the task a lot harder. In supersymmetric models, the total massless spectrum is easily found by adding superpartners to the fermions. One needs to take care that also non-chiral matter in sectors where intersection points of opposite orientation combine will stay massless. Upon breaking supersymmetry, it is to be expected that all fields except gauge bosons and chiral fermions will get masses through interactions.

To obtain the chiral spectrum of a given set of D6-branes wrapped on cycles π_a

with their images on π'_a and the O6-planes wrapped on π_{O6} a few considerations are necessary. The only novelty is that in addition to the standard operation by Ω a permutation of the branes and intersection points by $\overline{\sigma}$ occurs, formally encoded in acting by a permutation matrix on the Chan-Paton labels that determine the representation under the gauge group. First note that the net number of self-intersections of any stack vanish, due to the anti-symmetry of the intersection form (denoted by $\circ$). Whenever a brane intersects its own image, there are two cases to distinguish: The intersection can itself be invariant under $\overline{\sigma}$, such that the Chan-Paton labels are anti-symmetrized by Ω. Alternatively, it can also be mapped to a second intersection, such that no projection applies and the symmetric and antisymmetric parts are kept. Finally, if any two different stacks intersect, there are always bifundamental representations localized at the intersection. According to these rules, the spectrum of left-handed massless chiral fermions is shown in Table I.

Table I. Chiral fermion spectrum in $d = 4$.

Representation	Multiplicity
$[\boldsymbol{A}_a]_L$	$\frac{1}{2}\left(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a\right)$
$[\boldsymbol{S}_a]_L$	$\frac{1}{2}\left(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a\right)$
$[(\overline{\boldsymbol{N}}_{\boldsymbol{a}}, \boldsymbol{N}_{\boldsymbol{b}})]_L$	$\pi_a \circ \pi_b$
$[(\boldsymbol{N}_{\boldsymbol{a}}, \boldsymbol{N}_{\boldsymbol{b}})]_L$	$\pi'_a \circ \pi_b$

The above classification can be obtained directly from string amplitudes when a CFT description is available, e.g. in the orbifold limit, while at large volume one can apply the Atiyah-Singer index theorem to infer the zero-modes of the Dirac operator. This is actually a tautology, since the number of chiral modes is given as an integral over the point-like common world volume of any pair of D6-branes with a trivial integrand,

$$\int_{\mathrm{D6}_a \cap \mathrm{D6}_b} \mathrm{ch}(\mathcal{F}_a) \wedge \mathrm{ch}(\mathcal{F}_b^*) \wedge \hat{A}(\mathcal{R}) = \mathrm{rk}(\mathcal{F}_a)\mathrm{rk}(\mathcal{F}_b) \int_{\mathcal{M}^6} \delta(\pi_a) \wedge \delta(\pi_b), \qquad (2\cdot10)$$

which only counts the intersection numbers again. In the mirror symmetric type IIB picture chirality is in fact induced exclusively by the non-triviality of the gauge and spin connection.

Due to the topological nature of the chiral spectrum Table I should hold for every smooth Calabi-Yau manifolds and even the six-dimensional torus. Little can be said about the fate of the D-brane setting away from the limit of classical geometry, when venturing into the interior of the Kähler moduli space, where potentials may be generated. Therefore, the configuration will in general not be stable, but the important point is, whenever the setting is describable purely in terms of D6-branes on sLag 3-cycles Table I may apply.

To make a first check of the consistency of the spectrum, the non-abelian gauge anomaly $SU(N_a)^3$

$$A_{\text{non-abelian}} \sim \pi_a \circ \pi_a \qquad (2\cdot11)$$

vanishes due to the antisymmetry of the intersection form.

As discussed in Refs. 14), 16) the quintic Calabi-Yau provides a nice example of an intersecting brane world scenario with gauge group and (non-supersymmetric) chiral spectrum of the standard model. However in the following we want to give a few more details for intersecting brane world on orbifolds resp. orientifolds, which can be regarded as the limit in a Calabi-Yau moduli space where the twisted orbifold moduli are set to zero.

2.4. *Orbifold models*

First, we consider configurations of type II $D6$ branes wrapped on non-trivial three-cycles of a six-dimensional torus T^6. The torus is taken to be a direct product $T_6 = \prod_{j=1}^{3} T_2^j$ of three two-dimensional tori T_2^j with radii R_1^j, R_2^j and angles α^j w.r.t. to the compact dimensions with coordinates y_1^j and y_2^j. The Kähler and complex structure modulus of these tori are defined as usual:

$$U^j = \frac{R_2^j}{R_1^j} e^{i\alpha^j}, \quad T^j = b^j + iR_1^j R_2^j \sin\alpha^j \,, \tag{2·12}$$

with the torus B-field b^j. Furthermore, the three-cycle is assumed to be a factorizable into a direct product of three one-cycles, each of them wound around a torus T_2^j with the wrapping numbers (n^j, m^j) w.r.t. the fundamental 1-cycles of the torus. Hence the angle of the D6-brane with the y_1^j-axis is given by

$$\tan\phi^j = \frac{m^j R_2^j}{n^j R_1^j} \,. \tag{2·13}$$

Generally, two branes with wrapping numbers (n_a^j, m_a^j) and (n_b^j, m_b^j), are parallel in the subspace T_2^j, if their intersection number

$$I_{ab}^j = n_a^j m_b^j - n_b^j m_a^j \tag{2·14}$$

w.r.t. to this subspace vanishes, $I_{ab}^j = 0$. For later convenience let us also introduce:

$$\pi v^j := \text{arctanh}(\mathcal{F}^j) \,, \tag{2·15}$$

which implies $\phi^j = i\pi\, v^j$. Chiral fermions appear at (non-vanishing) intersections of two D6-branes.

In the T-dual picture, the D6-branes at angles ϕ^j are mapped to D9-branes with magnetic fluxes or background gauge fields F^j. Thereby the gauge field (magnetic flux) F^j on the brane is related to the angles (2·13) through:

$$iF^j = \frac{m^j}{n^j\ R_1^j R_2^j} \,. \tag{2·16}$$

Next let us consider the action of the orientifold and of the orbifold group, where the spatial orbifold group is by elements from Z_N (or $Z_N \times Z_M$). The latter are represented by the θ (and ω), describing discrete rotations on the compact coordinates

$y^i_{1,2}$. This action restricts the compactification lattice and fixes some of the internal parameter (2·12) to discrete values. The orientifold O6-planes describe the set of points which are invariant under the group actions $\Omega\overline{\sigma}$, $\Omega\overline{\sigma}\theta^k$, $\Omega\overline{\sigma}\omega^l$ and $\Omega\overline{\sigma}\theta^k\omega^l$. These planes are generated by rotations of the real y^j_1 axes by $\theta^{-k/2}\omega^{-l/2}$.

The condition for tadpole cancellations in IIA orientifold backgrounds in four space-time dimensions requires a system of D6 branes which has to respect the orbifold and orientifold projections. In particular, for consistency with the orbifold/orientifold group their orbifold/orientifold mirrors have to be introduced. Hence any stack a is organized in orbits, which represent an equivalence class $[a]$. For $N, M \neq 2$ the length of each orbit $[a]$ is at most $2NM$, but may be smaller, if e.g. stack a is located along an orientifold plane. Stacks within a conjugacy class $[a]$ have non-trivial intersections among each other and w.r.t. to stacks from a different class $[b]$ belonging to the gauge group G_b. Without going further into any details, it is appealing that one can check in several examples[14)] that the open string spectrum constructed following these rules in the toroidal ambient space precisely agrees with the geometrical spectrum discussed before, when considering the orbifold space as a limiting geometry of a Calabi-Yau manifold.

The requirement of R-tadpole cancellation leads to some constraints on the number and location of the D6 branes. Specifically, by requiring that all vacuum RR-tadpoles in the annulus, Klein-bottle and Möbius amplitudes vanish the homological tadpole conditions Eq. (2·9) take the following form:

$$\begin{aligned}\sum_{a=1}^{K} N_a\; n^1_a n^2_a n^3_a &= 16\,,\\ \sum_{a=1}^{K} N_a\; n^1_a m^2_a m^3_a &= -16\,,\\ \sum_{a=1}^{K} N_a\; m^1_a n^2_a m^3_a &= -16\,,\\ \sum_{a=1}^{K} N_a\; m^1_a m^2_a n^3_a &= -16\,.\end{aligned} \tag{2·17}$$

In addition the twisted tadpole conditions on the Chan paton factors of the open strings ends have to hold:

$$\mathrm{Tr}\gamma^a_{\theta^{N/2}} = 0 \quad , \quad \mathrm{Tr}\gamma^a_{\omega^{M/2}} = 0 \quad , \quad \mathrm{Tr}\gamma^a_{\theta^{M/2}\omega^{M/2}} = 0\ . \tag{2·18}$$

Further restrictions arise in the case of space-time supersymmetry, where NS vacuum tadpole cancellation follows from R vacuum tadpole cancellation (see also the chapter on the scalar potential). All the 3-cycles on the torus including the orientifold 3-cycle, constructed in the way described above, are supersymmetric, i.e. sLag's, for all values of n^i_a and m^i_a. For supersymmetric models one therefore has to check whether they conserve the same supersymmetries as the orientifold plane. Specifically one obtains the following conditions on the angles v^j_a of a stack of supersymmetric D6$_a$ branes:

(i) $\mathcal{N}=4$ sectors: $v_a^1=v_a^2=v_a^3=0$,

(ii) $\mathcal{N}=2$ sectors: $v_a^i=0$ w.r.t. the i-th plane and $v_a^j\pm v_a^l=0$,

(iii) $\mathcal{N}=1$ sectors: $v_a^1\pm v_a^2\pm v_a^3=0$.

Specific examples of orbifold intersecting brane world models with $\mathcal{N}=1$ supersymmetry in $D=4$ have been introduced in Refs. 9), 10), 17).

§3. The effective action of intersecting brane world models

3.1. *The scalar potential*

Non-supersymmetric brane configurations are in general unstable. On the one hand, depending on the intersection angles there can be tachyons localized at the intersection points. Phenomenologically it was suggested that these tachyons might be interpreted as Standard Model Higgs fields, where in particular in Ref. 12) it was demonstrated that the gauge symmetry breaking is consistent with this point of view. In general we expect that tachyons can well be avoided in a large subset of the parameter space, as was due in the simpler setting of toroidal compactifications. On the other hand, even if tachyons are absent one generally faces uncanceled NSNS tadpoles, which might destabilize the configuration.[8),14)] In Ref. 8) it was shown that for appropriate choices of the D-branes the complex structure moduli can be stabilized by the induced tree level potential. The stabilization of the dilaton remains a major challenge as in all non-supersymmetric string models.

For supersymmetric intersecting brane worlds we can expect much better stability properties. First tachyons are absent in these models due to the Bose-Fermi degeneracy. However, since for orientifolds on Calabi-Yau spaces the configuration only preserves $\mathcal{N}=1$ supersymmetry, in general non-trivial F-term and D-term potentials can be generated.

There are strong restrictions known for the contributions that can give rise to corrections to the effective $\mathcal{N}=1$ superpotential of a type II compactification on a Calabi-Yau 3-fold with D6-branes and O6-planes on supersymmetric 3-cycles. The standard arguments about the non-renormalization of the superpotential by string loops and world sheet α' corrections apply. The only effects then left are non-perturbative world sheet corrections, open and closed world-sheet instantons. In general, these are related to non-trivial CP^1 and RP^2 with boundary on the O6-plane in the Calabi-Yau manifold for the closed strings and discs with boundary on the D6-branes for open strings. In fact, only the latter contribute to the superpotential. The typical form for the superpotential thus generated is known, but explicit calculations are only available for non-compact models. Usually, they make use of open string mirror symmetry arguments. In many cases, there is an indication that the non-perturbative contributions to the superpotentials tend to destabilize the vacuum, and it would be a tempting task to determine a class of stable $\mathcal{N}=1$ supersymmetric intersecting brane models.

The tension of the D6-branes and O6-planes in addition introduces a vacuum energy which is described in terms of D-terms in the language of $\mathcal{N}=1$ supersymmetric field theory. These depend only on the complex structure moduli and do not affect the Kähler parameter of the background. The most general form for such a potential is given by

$$\mathcal{V}_{\text{D-term}} = \sum_a \frac{1}{2g_a^2}\left(\sum_i q_a^i |\phi_i|^2 + \xi_a\right)^2, \tag{3·1}$$

with g_a the gauge coupling of a $U(1)_a$, ξ_a the FI parameter, and the scalar fields ϕ_i are the superpartners of some bifundamental fermions at the intersections. They become massive or tachyonic for non-vanishing ξ_a, depending on their charges q_a^i. Due to the positive definiteness of the D-term, $\mathcal{N}=1$ supersymmetry will only be unbroken in the vacuum, if the potential vanishes.

The disc level tension can be determined by integrating the Dirac-Born-Infeld effective action. It is proportional to the volume of the D-branes and the O-plane, so that the disc level scalar potential reads

$$\mathcal{V} = T_6 \frac{e^{-\phi_4}}{\sqrt{\text{Vol}(\mathcal{M}^6)}}\left(\sum_a N_a \left(\text{Vol}(\text{D6}_a) + \text{Vol}(\text{D6}'_a)\right) - 4\text{Vol}(\text{O6})\right). \tag{3·2}$$

The potential is easily seen to be positive semidefinite and its vanishing imposes conditions on some of the moduli, freezing them to fixed values. Whenever the potential is non-vanishing, supersymmetry is broken and a classical vacuum energy generated by the net brane tension. It is easily demonstrated that the vanishing of $\mathcal{V}$ requires all the cycles wrapped by the D6-branes to be calibrated with respect to the same 3-form as are the O6-planes. In a first step, just to conserve supersymmetry on their individual world volume theory, the cycles have to be calibrated at all, which leads to

$$\mathcal{V} = T_6\, e^{-\phi_4}\left(\sum_a N_a \left|\int_{\pi_a}\widehat{\Omega}_3\right| + \sum_a N_a \left|\int_{\pi'_a}\widehat{\Omega}_3\right| - 4\left|\int_{\pi_{\text{O6}}}\widehat{\Omega}_3\right|\right). \tag{3·3}$$

Since $\widehat{\Omega}_3$ is closed, the integrals only depend on the homology class of the world volumes of the branes and planes and thus the tensions also become topological. If we further demand that any single D6_a-brane conserves the same supersymmetries as the orientifold plane the cycles must all be calibrated with respect to $\Re(\widehat{\Omega}_3)$. We can then write

$$\mathcal{V} = T_6\, e^{-\phi_4}\int_{\sum_a N_a(\pi_a+\pi'_a)-4\pi_{\text{O6}}} \Re(\widehat{\Omega}_3). \tag{3·4}$$

In this case, the RR charge and NSNS tension cancellation is equivalent, as expected in the supersymmetric situation.

To apply Eq. (3·1) we have to use the properly normalized gauge coupling

$$\frac{1}{g^2_{U(1)_a}} = \frac{N_a}{g_a^2} = \frac{N_a\, M_s^3}{(2\pi)^4}\, e^{-\phi_4}\left|\int_{\pi_a}\widehat{\Omega}_3\right|. \tag{3·5}$$

Hence, the FI-parameter ξ_a can be identified as

$$\xi_a^2 = \frac{M_s^4}{2\pi^2} \frac{\left|\int_{\pi_a} \widehat{\Omega}_3\right| - \int_{\pi_a} \Re(\widehat{\Omega}_3)}{\left|\int_{\pi_a} \widehat{\Omega}_3\right|}, \tag{3·6}$$

which vanishes precisely if the overall tension of the branes and planes cancels out, i.e. if all are calibrated with respect to the same 3-form. Since the FI-term is not a holomorphic quantity one expects higher loop corrections to the classical potential Eq. (3·2).

3.2. *Gauge threshold corrections*

Now we turn to the question of computing one-loop gauge threshold corrections in intersecting brane world models, which is also very important from the phenomenological point of view. Unlike what happens, e.g., in perturbative heterotic string vacua, the tree-level gauge couplings for the various gauge groups, arising from different stacks of branes, are not the same at the string scale. The tree-level gauge couplings follow from dimensional reducing the Born-Infeld action of a D6-brane on a 3-cycle of the internal space and are essentially determined by the volume of the 3-cycle (see Eq. (3·5)). E.g., for a six-torus $T_6 = \prod_{j=1}^{3} T_2^j$ and a special 3-cycle embedded with the wrapping numbers (n_a^j, m_a^j) w.r.t. to the two-tori T_2^j the tree-level gauge couplings are given by

$$g_{a,\text{ tree}}^{-2} = M_{\text{string}}^3 e^{-\phi_4} \prod_{j=1}^{3} \sqrt{(n_a^j)^2 (R_1^j)^2 + (m_a^j)^2 (R_2^j)^2 + 2 n_a^j m_a^j R_1^j R_2^j \cos\alpha_a^j}\ , \tag{3·7}$$

with the 4-dimensional string coupling constant $e^{-\phi_4}$. Hence a priori there is no unification of gauge couplings at the string scale (at string tree-level). One-loop gauge threshold corrections Δ_a (to the gauge group G_a), which take into account Kaluza-Klein and winding states from the internal dimensions and the heavy string modes, may change this picture.[31)] For certain regions in moduli space these corrections may become huge and thus have a substantial impact on the unification scale.

In the type IIA picture with intersecting D6-branes these threshold correction Δ_a will depend on the homology classes on the 3-cycles (open string parameters) and also on the closed string geometrical moduli. In toroidal models these corrections will be given in terms of the wrapping numbers n_a^j, m_a^j and the radii R_i^j of the torus. All D6-branes have in common their four-dimensional (non-compact) world volume. Hence their gauge fields are located on parallel four-dimensional subspaces, which may be separated (in the cases $I_{ab}^j \neq 0$ and $I_{aa'}^j \neq 0$) in the transverse internal dimensions. One-loop corrections to the gauge couplings are realized through exchanges of open strings in that transverse space. The open string charges q_a, q_b at their ends couple to the external gauge fields sitting on the branes. Only annulus and Möbius diagrams contribute, as torus and Klein bottle diagrams refer to closed string states. In Ref. 19) the one-loop corrections to the gauge couplings were computed by the background field method: one turns on a (space-time) magnetic field, e.g., $F_{23} = BQ_a$ in the X^1-direction and determines the dependence of the open string partition function on that field. Here, Q_a is an appropriately normalized generator of the gauge group G_a under consideration. This leads to the so-called gauged open string partition function. The second order of an expansion w.r.t. to B of the gauged partition function gives the relevant piece for the one-loop gauge couplings.

In the following we will omit all details of the calculations, referring the reader to the paper;[19] instead we give only the end results for the one loop threshold corrections:

(i) $\mathcal{N} = 4$ sectors: $\varDelta_a = 0$,

(ii) $\mathcal{N} = 2$ sectors:

If the stacks a and b preserve $\mathcal{N} = 2$ supersymmetry, i.e., they are parallel within some torus T_2^i, we obtain for the gauge group G_a:

$$\varDelta_{ab}^{\mathcal{N}=2} \sim b_{ab}^{\mathcal{N}=2} \ \ln(T_2^i V_a^i |\eta(T^i)|^4) + \text{const} \ , \tag{3·8}$$

with the wrapped brane volume

$$V_a^i = \frac{1}{U_2^i} |n_a^i + U^i m_a^i|^2 \,, \tag{3·9}$$

and the Kähler modulus T^i defined in Eq. (2·12).

(iii) $\mathcal{N} = 1$ sectors:

In the case, that the branes from a and b preserve $\mathcal{N} = 1$ supersymmetry, the one-loop correction to the gauge coupling of G_a takes the form:

$$\varDelta_{ab}^{\mathcal{N}=1} = -b_{ab}^{\mathcal{N}=1} \ \ln \frac{\varGamma(1-\frac{1}{\pi}\phi_{ba}^1)\ \varGamma(1-\frac{1}{\pi}\phi_{ba}^2)\ \varGamma(1+\frac{1}{\pi}\phi_{ba}^1+\frac{1}{\pi}\phi_{ba}^2)}{\varGamma(1+\frac{1}{\pi}\phi_{ba}^1)\ \varGamma(1+\frac{1}{\pi}\phi_{ba}^2)\ \varGamma(1-\frac{1}{\pi}\phi_{ba}^1-\frac{1}{\pi}\phi_{ba}^2)} \ . \tag{3·10}$$

This expression depends on the closed string moduli of the underlying toroidal geometry, since the the difference of the angles ϕ_a^j and $\phi_{a'}^j$ are related to the radii through:

$$\coth(\pi v_{aa'}^j) = i\cot(\phi_{a'}^j - \phi_a^j) = i \ \frac{n_a^j n_{a'}^j \frac{R_1^j}{R_2^j} + m_a^j m_{a'}^j \frac{R_2^j}{R_1^j}}{n_a^j m_{a'}^j - n_{a'}^j m_a^j} \ . \tag{3·11}$$

Note that this type of moduli dependence of the $\mathcal{N} = 1$ threshold functions in intersecting brane world models is completely new, as in heterotic string compactifications the $\mathcal{N} = 1$ thresholds are moduli independent constants.[32]

In addition one should emphasize that in supersymmetric brane world models there are no UV divergences in the one-loop $\mathcal{N} = 2, 1$ thresholds. In fact one can show that cancellation of the vacuum RR tadpoles (see Eq. (2·17)) implies that in supersymmetric models also all RR and NS tadpoles are absent in the one-loop 2-points functions for the gauge couplings, i.e. in the gauged open string partition functions. This proves the finiteness of the supersymmetric one-loop gauge thresholds in the considered class of orbifold models.

§4. Outlook — M-theory on G_2 manifolds

The supersymmetric intersecting brane models considered here map in the strong coupling limit to compactifications of M-theory on certain singular G_2 manifolds. Thus our $\mathcal{N} = 1$ gauge threshold function is possibly related to the recently calculated Ray–Singer torsion of singular G_2 manifolds.[33)] Besides it may give a hint on the form of non-perturbative corrections to the gauge couplings on the heterotic side. The one-loop gauge thresholds for $\mathcal{N}$ =1 sectors describe 1/4 BPS saturated couplings w.r.t. to the 16 supersymmetries of two intersecting branes. Hence the $\mathcal{N} = 1$ one-loop threshold represents a counting function for 1/4 BPS states in close analogy to Ref. 34). Hence on the M-theory side the threshold function maps to a topological quantity, most likely to the elliptic genus of the singular G_2 compactification manifold.

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