THE ORIGIN OF INERTIA

It's thought by some folks these days that the cause of inertial reaction forces isn't yet really understood, or that they have just succeeded in figuring out the explanation for these forces in terms of their new theory. These views are mistaken. The cause of inertial reaction forces has been understood to be the action of gravity for quite some time now. Back in 1953 Dennis Sciama showed that gravity could account for inertial reaction forces as long as the interaction of local stuff with the gravity field of distant matter was like the interaction of electric charges and currents with the electromagnetic field. It turns out, as a matter of fact, that this is true in general relativity theory, but it took a while to show this. (It was done by D.J. Raine back in the very early 1980s: *Reports of Progress in Physics*, **44**, 1151-1195 [1981].) The full-blown argument is rather formal and a bit daunting, but it's easy to see that gravity causes inertia in a simple little argument modeled on that presented by Sciama back in 1953.

You may remember from an undergraduate course in electricity and magnetism that the electric field of an electric charge can be represented by something called a "scalar potential" -- a "function" that assigns a single number to each point in space so that when the "gradient" of the function (the spatial rate of change of the function) is computed you get back the electric field strength (a vector quantity with magnitude and direction). Formally this looks like:

$$\boldsymbol{E} = -\nabla\phi \tag{1}$$

E is the electric field strength, ∇ the gradient "operator", and ϕ is the electric potential. This relationship, however, is only true for "static" electric fields: fields produced by electric charges that are all at rest and stay that way. When electric charges are in motion -- that is, when electric currents are present -- the electric field has to be modified to include a term that takes account of the motion of the charges. The electric field becomes:

$$\boldsymbol{E} = -\nabla\phi - (1/c)(\partial \boldsymbol{A}/dt) \tag{2}$$

c in the added term is the speed of light. $\partial \boldsymbol{A}/dt$ is the rate at which something called the "vector potential", ***A***, is changing. The vector potential is usually associated with the magnetic field (***B***) created by electric currents (electric charges in motion). Indeed, things are set up so that the "curl" of the vector potential (a measure of its "vorticity") is equal to the magnetic field strength. But it also contributes to the electric field, as above, when things are changing.

The way in which these "scalar" and "vector" potentials are computed is straight-forward, at least in principle. You just add up the contributions due to all the electric charges in the universe at any point in space and time you're interested in. The "sources" of the scalar electric potential are just the charges themselves; the sources of the vector potential are the currents produced by the motion of the electric charges. More distant electric charges and currents, of course, have a smaller effect than nearby sources, so the charges and currents have to be divided by the distance to them before they are added together. The vector potential, written out in terms of its sources, looks like:

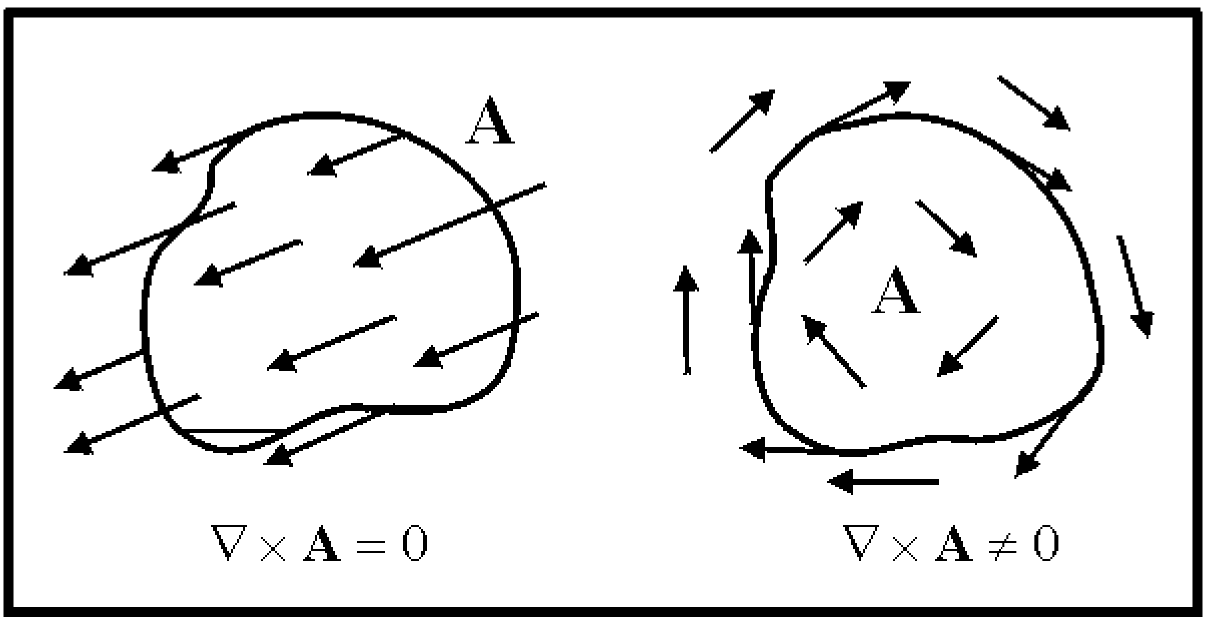
$$A = (1/c)\int(\rho\mathbf{v}/r)dV \tag{3}$$

$\int(\ \ )dV$, the volume integral of ( ), just means sum up the contributions, at some location, of the stuff inside the parentheses at all points in space. $\rho\mathbf{v}$ is the electric charge density (ρ) times its velocity (**v**), that is, the electric current density. And *r* is the distance of the current being considered to the point where the potential is being computed. An important point should be noted here. The time taken for the influence of the current at some point to get to the place where the potential is being calculated must be kept track of. In general, the influences of charges and currents are not felt instantly over large distances. The scalar potential, by the way, is:

$$\phi = \int (\rho / r) dV \tag{4}$$

It's noteworthy that if the density of the charge is uniform throughout space, then the most distant charge dominates the potential. Although its influence decreases with r, its amount goes up as $\mathbf{r}^2$. This is unimportant in electrodynamics, because the mean charge density in space is essentially zero. That's not true in gravity.

Now if gravity behaves like electromagnetism, we can simply hijack the mathematical formalism of electromagnetism in order to calculate gravitational effects. This isn't always true. But for the simple case we're going to consider it is. We look at a "test particle", a massive object that's so small that it doesn't disturb any of the stuff it finds itself in. It's located in a universe much like ours, but to keep things simple we assume that everything in the universe, other than the test particle, is smeared out smoothly throughout space. We now let our test particle move along a straight line in this stuff and ask: What is the force of gravity on the test particle due to the rest of the stuff? In general, both "gravito-electric" and "gravito-magnetic" fields, the gravitational counterparts of electric and magnetic fields, may act. In this case, however, the gravito-magnetic field itself doesn't act, so we can ignore it. The reason why is that the rest of the stuff out there doesn't "circulate", so the "curl" of the vector potential vanishes. (Taking the curl of a vector field consists of adding up its contributions keeping track of the way they point along a closed path in space. If the field has no circulation, the non-zero contributions from one part of the path will be canceled by contributions from some other part of the path. This is illustrated in the accompanying Figure.)



Since the gravito-magnetic field is equal to the curl of the vector potential, the gravito-magnetic field disappears too. Another way of looking at this is to note that each particle of the stuff moving past the test particle produces a gravito-magnetic field at the test particle. But when we add up all of the gravito-magnetic fields at the test particle the sum is zero because the field due to one part of the current is canceled out by the field due to another part of the current owing to its symmetry.

Gravito-magnetic forces *per se* may not act in this case, but we can't get rid of the effects of the vector potential. The gravito-electric field is given by Equation (2) above, and it has the term involving the rate of change of the vector potential in it. So we have to figure out what the vector potential is at the test particle. That's Equation (3) with the electric charge density ρ replaced by the gravitational "charge" density $G\rho$, where ρ is now the mass density and *G* is Newton's constant of universal gravitation. The summation of the effects of the matter currents everywhere in space is done using the standard technique of integration. Sciama noted that before the integral is computed, the velocity can be removed from the calculation of the integral. His justification for doing this was that it appears that the universe is moving rigidly from the point of view of the test particle. As a result, in the frame where the test particle is at rest we can assign the velocity - **v** to every part of the universe at any instant. Since **v** in this frame of reference is independent of location (and thus independent of *r*) it can be treated as a constant and removed from the integration.

When **v** is removed from the integration in Equation (3), the remaining expression to be computed turns out to be the computation that gives the scalar potential. If we ignore factors of the order of one (like π), then this integration yields *GM*/*R*. *M* and *R* are the mass and radius of the observable universe, and accordingly *GM*/*R* is the scalar gravitational potential due to all of the matter in the universe, ϕ. Consequently, in these circumstances we have:

$$A = (1/c)\phi\mathbf{v} \tag{5}$$

up to constant factors of the order of one anyway. Now we examine Equation (2), taking it to be the gravito-electric force, to find out what force our test particle experiences. We note immediately that since ϕ is the same everywhere (the test particle, by assumption, is too small to louse this up), $\nabla\phi$ vanishes. Since $\nabla\phi$ is what we normally think of as the origin of gravitational forces, we see that our normal intuition about gravity contributes nothing to inertia.

This means that if gravity is to account for inertia, it must be the vector potential part of the gravito-electric force that does the trick. To see if this will work all we have to do is put ***A*** from Equation (5) into Equation (2) (and set $\nabla\phi$ equal to zero):

$$\boldsymbol{E} = -(1/c^2)\,\partial(\phi\mathbf{v}/dt) \tag{6}$$

Since ϕ, being a constant, doesn't depend on time we can write:

$$\boldsymbol{E} = -(\phi/c^2)\,\partial\mathbf{v}/dt \tag{7}$$

Now when the test particle is moving with constant velocity *E* vanishes because $\partial\mathbf{v}/dt$ is zero -- just as we expect should be true. But if an external force makes the test particle accelerate, then $\partial\mathbf{v}/dt$ isn't zero and the distant matter in the universe produces a gravito-electric force on the particle that opposes the accelerating force. If ϕ/c^2 (together with the other constant factors of order unity that we've ignored) are just equal to one, then the gravito-electric field strength ***E*** is precisely the right strength to account for inertial reaction forces. If you go read Sciama's 1953 paper, you'll find that this also works for rotation and "centrifugal" forces.

You may be wondering: Well, all of this is fine, but maybe we don't have to take the effects of gravito-magnetism and the vector potential seriously. Perhaps they're so minute they can be ignored. Turns out that that's not true. As Ken Nordtvedt pointed out in 1988 [*International Journal of Theoretical Physics*, **27**, 1395-1404], gravito-magnetic effects must be taken into account in even the simplest planetary orbit calculations. Only a moment's reflection is needed to see that this must be right. From Newtonian mechanics we know that the gravitational force acting on a planet must act along the instantaneous line of centers of the planet and the Sun if an elliptical orbit is to be recovered. (That is, the force exerted by the Sun must propagate to the planet instantaneously. This fact is the reason why Newtonian gravitation is called an "action-at-a-distance" force. Newton privately thought this preposterous; but he never found a way around it.) If relativity is right, then the gravito-electric field (*i.e.*, Newtonian gravity) must propagate at the speed of light, and the corresponding gravitational force on the planet wouldn't point along the instantaneous line of centers. So, if the gravito-magnetic contribution to the total force weren't included, the force of the Sun on the Earth, for example, would point in the wrong direction and its orbit wouldn't be elliptical. (Nordtvedt arrives at this conclusion by a variant of this argument. He shows that the motion of a test particle around the Sun is elliptical for an observer at rest with respect to the Sun. In this frame of reference the field is stationary and everywhere points toward the Sun at all times, so the force is always along the instantaneous line of centers. If the observer moves with respect to the Sun [for example, with the planet], however, and doesn't take into account the gravito-magnetic vector potential, the predicted orbit "blows up". [This example is a neat illustration of the fact that "coordinate transformations" in general relativity theory are equivalent to "gauge" transformations in electrodynamics. The observer at rest with respect to the Sun is effectively in the Coulomb gauge, and the one moving with the planet in the Lorentz gauge.]) Despite the widespread belief that they're inconsequential, gravito-magnetic effects are real, and often large. [Nordtvedt's formal argument in his *IJTP* article makes it very easy to show that general relativity theory leads to precisely the same result as Sciama's little calculation. I show this in, "Nordtvedt's Remarks on Gravitomagnetism", in case you're interested. The formalism involved is no more daunting than that we've already used.]

When Raine showed that Sciama's argument was true for all realistic universes in general relativity theory, the gravitational origin of inertial forces (that is, Mach's principle) ceased to be an area of active work for more than a decade. Some subtleties attendant to Mach's principle, however, weren't fully appreciated and worked out in the 1970s. They began to attract attention in the early 1990s. Some of them are related to the business of transient mass fluctuations. So I'll tell you a bit about them. Be prepared. They're pretty weird.

THE SUBTLETIES

Problems arise when we ask *how*, in detail, inertial reaction forces are produced by the distant matter in the cosmos. The foregoing argument may leave you with the impression that the distant matter in the universe generates a vector potential field throughout space that acts on bodies immediately when external forces cause them to accelerate. This notion is reinforced by the Image of the rigid relative motion of the universe invoked by Sciama to justify removing **v** from the calculation of the integral in computing the vector potential. All of this, however, is a bit misleading. The principle of relativity tells us that real physical influences that are involved in accelerations and the transfer of energy must propagate with finite speed -- namely, at or less than the speed of light. If all of the contributions to $\boldsymbol{A}$ that are responsible for any inertial reaction force an object experiences propagate at the speed of light, it would seem that the currents that generate $\boldsymbol{A}$ for any particular acceleration episode must have happened in the very distant past. We are then left with the question: How did the stuff out there in the distant past know that we would try to accelerate, say, our car at any specific instant and, in the distant past, move in just the right way to launch the right $\boldsymbol{A}$ field in our direction?

You may be inclined to think that this is some sort of trick question intended to set you up somehow. It's actually a very serious, rather profound problem. Inertial reaction forces are instantaneous; there's no doubt whatsoever about that. When you push on something, it pushes back on you immediately. If they're caused chiefly by the most distant matter in the universe, how can that be?

Only three answers to this question seem to be available:

1. Relativity notwithstanding, the force really is propagated instantaneously. The occurrence of so-called "non-local" interactions in quantum phenomena (reported even in the popular press of late) might make such a scheme seem plausible.

2. Some sort of a local field, maybe not our $\boldsymbol{A}$ field, is really the cause of inertia.

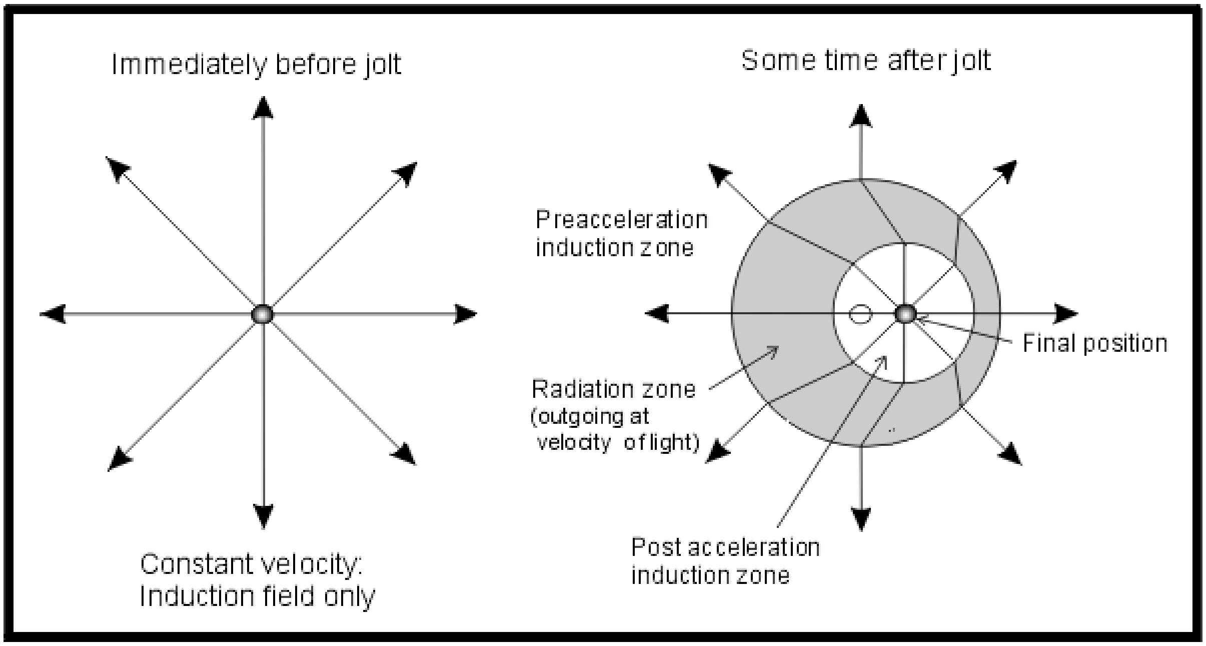
3. When you push on an object a gravitational disturbance goes propagating off into either the past or the future. Out there in the past or future the disturbance makes the distant matter in the universe wiggle. The wiggling stuff out there makes up the currents that cause disturbances to propagate from the past or the future back to the object. They all arrive from the past or future just in time to produce the inertial reaction force you feel.

Given these choices, you may be inclined to think that number 2 must be the right answer. Although number 2 sounds pretty good, it turns out to be the least likely explanation of inertia. I'll explain why after we look into the other explanations a bit. To explore them we'll need to know about something called "gauge invariance". If you've had a course in electromagnetism, you'll probably recall that the equations for the electric and magnetic fields and the scalar and vector potentials, by themselves, aren't enough to let you calculate much of anything. The problem is that the field equations are so general that they aren't completely defined. In addition to the field equations you have to specify a choice of "gauge" (within certain broad constraints) if you want to actually do any calculations. [You'll find all this explained in any good text on electromagnetism.] As long as the gauge is selfconsistent, you can choose any gauge you like. In practice, two gauges are commonly used. One is called the "Lorentz" gauge [after H.A. Lorentz who created much of this theory around the turn of the century]. In this gauge both of the potentials and both of the fields explicitly propagate at the speed of light. The other gauge is called the "Coulomb" or "radiation" gauge [after C.A. Coulomb because in this gauge the scalar potential propagates instantaneously, as does the force between electric charges at rest according to "Coulomb's law"].

You might think that we can solve our problem of instantaneous inertial reaction forces by simply choosing a Coulomb type gauge so that the gravito-electric field propagates instantaneously. The problem with this is that while the scalar potential propagates instantaneously in this gauge, the vector potential still propagates at the speed of light. And the part of the gravito-electric field that produces inertial reaction forces is the part that depends on the vector potential. So this doesn't work when you get down to the nitty-gritty. It might seem to you, as a result, that we can kiss off simple instantaneous action explanations of inertia. Almost, but not quite. It has been forcefully argued in the past few years [notably by I. Ciufolinni and J.A. Wheeler in their recent book, *Gravitation and Inertia* (Princeton, 1995)] that inertia arises in a similar, but more subtle way.

Roughly, the modern instantaneous action argument goes as follows. In general relativity theory matter "there" tells space "here" how to curve, and space "here" tells matter "here" how to move. (Matter "here" also tells space "there" how to curve.) Thus, in order to talk about any situation in dynamics we must specify the distribution and motion of matter throughout space. (Strictly speaking, we must provide "initial data" on some suitably chosen "three dimensional spacelike hypersurface".) The usual field equations for gravity (Einstein's equations) are not enough, by themselves, to do this it turns out. Because of the finite propagation velocity built into them, we might specify some distribution of matter that subsequently leads to idiotic results. To make sure this doesn't happen, our distribution of matter has to satisfy some additional equations called "constraint" equations. The neat thing about these constraint equations is that, unlike the field equations, they're instantaneous. (Technically, they're "elliptic" rather than "hyperbolic" differential equations.) It's then claimed that inertia is conveyed by the constraint equations -- instantaneously. The use of constraint equations to communicate real physical influences instantaneously is justified by appeal to the instantaneous propagation of stationary electric fields in the Coulomb gauge.

Appealing as constraint conveyance of inertia may at first appear, it is arguably unsatisfying. It seems a rather artificial (if very clever) solution to a very serious problem. This is especially true when one considers that inertial reaction forces are accounted for by the field equations themselves. The problem is that no plausible choice of gauge will make them instantaneous. This shouldn't be very surprising, for they have all of the earmarks of a "radiative" interaction: they depend on accelerations (not velocities) and they have the characteristic $1/r$ distance dependence of radiation. (The difference between radiative and "static" or "inductive" field phenomena is neatly seen in the Figure below where the electric field around a charged particle is shown in "lines of force representation" immediately before and some time after its been subjected to some accelerations to move it from its first location. The "kink" in the induction field produced by the accelerations of the charge is the radiation that propagates outward at the speed of light. Note that if the charge in this Figure is moving with some constant velocity, nothing changes. The induction field moves rigidly with the charge. [At relativistic velocities length contraction effects make the field appear distorted.] Only changes in the velocity of the charge cause propagating kinks in the field.) If we reject constraint conveyance of real physical influences as implausible, we are left with left with gravitational disturbances propagating back and forth in time to account for the seemingly instantaneous communication of inertial reaction forces. This may not seem like much of an improvement, but it is.



It has been known for ages that the "wave" equations that describe the propagation of radiation have two equally valid types of solutions: ones that propagate forward in time, and ones that propagate backward in time. (Technospeak: the equations have the symmetry of "time reversal invariance".) The reason for this peculiar state of affairs is pretty simple. Imagine that you are something that's disturbed (moved) by the passage of some type of wave. When a wave goes past, you move up and down. Can you tell from your motion whether the wave that makes you move comes from the left or the right? No. Similarly, you can't tell from your motion whether the wave is coming from the past or the future either. Waves that move backward in time are called "advanced" waves because their "effects" in the past occur in advance of their "causes" in the future.

You may think the whole idea of advanced waves coming from the future pretty preposterous, but they solve some rather nasty problems. P.A.M. Dirac used them in an epochal study of the nature of electrons in the 1930s, and R.P. Feynman and J.A. Wheeler elaborated their "absorber" theory of electrodynamics in the 1940s on the basis of them. It turns out that electromagnetic radiation reaction (the reaction force on a source produced when radiation is launched) is neatly accounted for in terms of a combination of "retarded" waves (normal waves propagating forward in time) and advanced waves. [Radiation reaction, intimately connected to transient mass fluctuations, is addressed in another document.] Precisely the same thing evidently happens with inertial reaction forces. The act of pushing on something causes a disturbance in the gravitational field to go propagating off into the future. It makes stuff (the "absorber") out there wiggle. When the stuff wiggles it sends disturbances backward (and forward) in time. All the backward traveling disturbances converge on what we're pushing and generate the inertial reaction force we feel. No physical law is violated in any of this. And nothing moves faster than the speed of light. It only seems so because of the advanced waves traveling at the speed of light in the backward time direction.

If you're new to all this, you may suspect me of making all or most of it up, or at least embellishing more prosaic ideas of others. Actually, I'm not. You'll find this discussed in John Gribbin's book, *Schrödinger's Kittens*, and many of the popularization's by Paul Davies (for example, *Other Worlds* and *About Time*). John Cramer has done occasional columns on these and related issues for *Analog* (all but the most recent of which are now available on his website). The second edition of Hawking's book, *The Illustrated a Brief History of Time*, and Kip Thorne's, *Black Holes and Timewarps*, are also quite good. Should you have a more technical bent, you might want to look at Paul Davies, *The Physics of Time Asymmetry* (U.C. Press, 1973), or *Physical Origins of Time Asymmetry* (ed. Halliwell, Perez-Mercader, and Zurek, Cambridge U.P., 1994), or *Time's Arrows Today* (ed. S. Savitt, Cambridge, 1995).

Bibliography notwithstanding, all of this weirdness may convince you that the ambient, pre-existing field explanation (that is, explanation number 2) must be right. The problem with this is that in order to avoid all of the really strange stuff we've just considered, we have to assume that the "field" that produces inertial reaction forces has a real, independent physical existence apart from the sources that create it. Better yet, we'd like to have a field that doesn't have any sources like the distant matter in the cosmos at all. Then we wouldn't have to worry about pesky influences propagating around. A scheme of this sort has recently been proposed by Haisch, Rueda, and Puthoff (HRP) [*Physical Review A*, **49**, 678-694 (1994)]. They invoke local "quantum vacuum fluctuations" (instead of distant matter) as the source of their field: fluctuations of the electromagnetic field as it happens.

The energy-time Uncertainty relation of quantum mechanics can be interpreted as allowing fleeting violations of the conservation of energy. In this view "photons" (quanta) of the electromagnetic field and other stuff can pop up into existence very briefly. Any specific particles created in this way can only exist for a minute fraction of a second. But since this process is going on everywhere all the time, it turns out that the vacuum should have an utterly idiotic amount of energy in it if quantum vacuum fluctuations really exist. Need I say that this has encouraged some far out speculation on how this putative local energy might be exploited.

Now you might think that if the vacuum is a thick soup of photons (and other stuff), things moving around in it should detect its presence. Neatly, however, it turns out that if the photons in the vacuum have the right "spectral energy density", anything moving in a straight line at constant speed -- that is, moving inertially -- is unaffected by the photons. (The spectral energy density needed to pull this off is that the energy density of photons increases with the cube of their frequency.) Any other spectrum doesn't work. If electric charges are accelerated with respect to the soup of photons, however, they experience a force that's directly proportional to the magnitude of the acceleration. Here, it seems, we have a candidate explanation for inertial reaction forces that avoids all that peculiar stuff about gauges.

Alas, vacuum fluctuations are not a plausible explanation for inertia. Several compelling arguments reveal the electromagnetic vacuum fluctuation explanation of inertia to be a delusion. The most obvious is that if the vacuum really did have some idiotically stupendous energy density, since energy is a source of the gravitational field, the universe would be curled up into a miniscule little ball according to general relativity theory. The proponents of vacuum fluctuations as the origin of inertia have concocted some unconvincing arguments to try to deflect this objection.

Quite apart from the ridiculousness of a highly energetic vacuum, there are other compelling reasons to believe that inertia is not an electromagnetic interaction with a "zero point" field. For example, if inertial reaction forces are electromagnetic forces that accelerating electric charges experience as they interact with a zero point field, then the inertial reaction forces protons, neutrons, and other elementary particles experience should depend on some function of how much charge they contain. Since the reaction force points in the same direction for both positive and negative electric charge, it follows that the force must depend on the square of the electric charge.

Consider now the neutron and proton. The neutron consists of an "up" and two "down" quarks. The proton consists of two "up" and one "down" quarks. Down quarks have minus 1/3 of an electron charge while up quarks have plus 2/3 of an electron charge. (These are the smallest units that electric charge comes in.) It follows immediately that the sum of the absolute values of the quark charges in a neutron is 4/3 electron charges and the corresponding value for the proton is 5/3. The ratio of the proton to neutron charge is thus 5/4. The ratio of the inertial reaction forces they should experience for equal accelerations through the zero point field is the square of this ratio if the HRP theory is right. It follows that the inertial mass of the proton should be 1.56 times as large as that of the neutron. In fact, the neutron is very slightly more massive than the proton.

If you are determined to believe that inertia is the consequence of electromagnetic zero point vacuum fluctuations, you might try to construct some convoluted *ad hoc* scheme to get the total charge of the constituents of protons and neutrons to come out the same. But the simple message of the neutron and proton seems to be that such machinations hardy constitute the "natural" explanation of inertia advocates of zero point fields have claimed. Indeed, the message HRP's scheme seems quite clearly to be that vacuum fluctuations of any field (other than quantum gravity perhaps) will almost certainly succumb to the same difficulty. Schemes of this sort evidently are wrong, destined to fail.

HRP's work contains another, less obvious message. It is that in order to make a real local field explanation of inertia work we have to use vacuum fluctuations, or their equivalent, since the usual field approach has the weird gauge properties that we've already looked into. This has strange consequences because of something called the "fluctuation-dissipation theorem". Roughly, what this theorem says is that fluctuations and dissipative forces are opposite sides of the same coin. For example, frictional forces are dissipative since they act to slow down the motion of normal objects. They are always accompanied by microscopic thermal fluctuations. In the case of electromagnetic quantum vacuum, Peter Milonni [in *The Quantum Vacuum* (Academic, 1993)] has shown that this means that zero point fluctuation processes can equally well be regarded as due to radiation reaction (or some combination of the two). Since radiation reaction can be viewed as a Wheeler-Feynman "absorber" interaction with the distant matter in the universe in the far future, it seems that vacuum fluctuation schemes are really no different from the simple field approach. Indeed, the absorber interpretation is to be preferred because it doesn't load up the vacuum with a lot of energy that quite obviously isn't there.

Trying to ascribe inertia to some origin other than gravity, we see, gets us into rather deep water. We are left with the fact that the least implausible explanation of the origin of inertia is gravitational disturbances that propagate to and from the distant future out there. Support for this view of reality can be found in Wheeler and Feynman's absorber theory that accounts for electromagnetic radiation reaction forces in essentially the same way. All this suggests that radiation reaction is likely to be an important aspect of gravity and inertia, and that it is worth exploring radiation reaction a bit.

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