Is there a quantum equivalence principle?

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The understanding that has been gained of accelerated vacua sheds new light on the classic issue of the self-force suffered by a uniformly accelerated charge. Moreover, the fact that, as a result of quantum theory, the radiative decay of an excited atom can be viewed equivalently as the result of the vacuum fluctuations of the electromagnetic field or as the result of the radiative self-force of the electron is shown to be nontrivially linked with the equivalence principle.

INTRODUCTION

The aim of this paper is to point out that the understanding that has been gained in recent years of accelerated vacuum states, motivated in large part by a desire to understand the phenomenon of black-hole radiance, sheds new light on the classic issue of the radiation reaction suffered by a uniformly accelerated charge and also on related questions regarding the radiation emitted by a charge that is either at rest in or freely falling through a static gravitational field. We begin with the case of uniform acceleration. The feature of this motion that has attracted so much attention over the years is the seemingly paradoxical relation between the radiation rate and the radiation-reaction force.

In order to avoid boundary effects that would otherwise obscure the issue, we shall consider only motions of the charge such that the agency producing the acceleration has finite duration. A realization of such a motion is: the charge initially moves freely, at time $t=0$ it enters a region where there is a uniform electric field, it remains in the field until $t=t_1$, at which time it leaves the region. Owing to the phenomenon of preacceleration the acceleration of the charge is nonzero before $t=0$ and nonuniform before $t=t_1$. However, if the time t_1 is long compared with the characteristic time

$$\tau=\frac{2}{3}\frac{e^2}{mc^3}$$

(the time that light takes to traverse the classical electron radius), then the motion of the charge may be considered as being effectively inertial for $t<0$, uniformly accelerated for $0<t<t_1$, and inertial again for $t>t_1$ though, as we shall see presently, the initial and final periods of nonuniform acceleration play a crucial role in the discussion.

UNIFORM ACCELERATION: THE NATURE OF THE PROBLEM

Following the detailed work of Bradbury[1] (see also the recent article by Boulware[2]) we note that the following facts are pertinent:

(i) By direct computation from the Lienard-Wiechart potentials it is straightforward to show that at each instant of retarded time the charge radiates energy at a rate P given by the Larmor formula

$$P=\frac{2}{3}\frac{e^2a^2}{c^3}$$

with a the magnitude of the proper acceleration of the charge. In particular it radiates at a uniform rate whenever a is constant.

(ii) The classical radiation-reaction force is given, in the instantaneous rest frame of the charge, by

$$F_{\text{rad}}=\frac{2}{3}\frac{e^2}{c^3}\frac{d^2}{dt^2}v\ ,$$

which depends on the second derivative of the charge's velocity and hence vanishes when a is constant.

(iii) The magnetic field of the charge is zero everywhere in the accelerated frame of the charge while it undergoes uniform acceleration. More precisely, if the charge follows the worldline $\xi = a^{-1}$ in the Rindler coordinate frame of the Appendix then the magnetic components of the electromagnetic field tensor $F_{\xi y}$, $F_{\xi z}$, and F_{yz} are zero on the Rindler manifold.

The apparent paradox is the seeming contradiction between (i) and (ii) whereby during the period of uniform acceleration there is a uniform rate of radiation of energy yet no radiation-reaction force. That there is in fact no blatant violation of the principle of the conservation of energy follows from the following fact.

(iv) The radiation-reaction force acts, during the initial and final periods of *non*uniform acceleration, in just such a way as to ensure that the total work done by the agency accelerating the charge is equal to the sum of the change in the charge's kinetic energy and the total amount of energy radiated to infinity.

This last statement amounts to the assertion that the time integral of the rate at which work is done against the radiation-reaction force is equal to the total amount of energy radiated and is assured, mathematically, by an integration by parts. The integrated term vanishes provided that the motion is inertial at sufficiently early and sufficiently late times.

Although there is in reality no difficulty posed by overall conservation of energy, the fact that the force of radiative reaction vanishes during the period of uniform acceleration seems counterintuitive, especially in light of the observation first made by Callen and Welton[3] in their celebrated paper on the fluctuation-dissipation theorem of the intimate relation between the force of radiation reaction and the zero-point fluctuations of the electromagnetic field. Indeed, for an oscillating dipole, they show that these quantities are related by precisely such a fluctuation-dissipation theorem. This relation was further elucidated by Senitzky[4] and by Milonni *et al.*[5] who showed that the decay of an excited atom can be viewed equivalently as arising from the perturbing effect of the vacuum electric field fluctuations or from the radiative reaction due to the electrons self-field or indeed any linear combination of these processes. The net effect cannot, of course, be altered, a definite answer being obtained for any observable quantity. However the blame for a decay, say, may be apportioned at will between vacuum fluctuation and radiative reaction, the relative proportion being determined by the ordering chosen for *commuting* operators.[6] This somewhat remarkable state of affairs indicates that neither vacuum fluctuation nor radiation reaction furnishes a complete or fully consistent statement of the quantum-mechanical reality each being an oversimplification motivated by a desire to attach a physical picture to equations that arise at an intermediate stage of calculation. Thus, for example, if one thinks solely in terms of vacuum fluctuation it is difficult to understand why an atom in its ground state should never be promoted to a higher state as the result of electric field fluctuations. If on the other hand one thinks solely in terms of radiation reaction, then one is confronted with the familiar difficulty of understanding the stability of the ground state. Despite these shortcomings the fluctuation and radiation-reaction pictures furnish considerable intuitive understanding of a number of processes, a good example of this is provided by Welton's computation of the Lamb shift[7] viewed as the shift in the energy level of the electron as the result of its interaction with the vacuum fluctuation of the electric field.

It is instructive to examine the absence of radiative reaction on a uniformly accelerated charge in the light of this duality between the fluctuation and radiative-reaction pictures. We know from (ii) above that the self-force vanishes by virtue of the vanishing of the second derivative of the velocity. What we seek here is an understanding of this fact in terms of fluctuations. This is provided by the observation first made by Unruh[8] that to a uniformly accelerated observer whose acceleration is α the Minkowski vacuum takes on the appearance of a thermal mixture of temperature $\alpha/2\pi$. We might say that the charge perceives the vacuum fluctuations as comoving and comprising a thermal bath. Thus if the charge is constrained to move with constant acceleration *there can be no net transfer of energy or momentum* between the charge and the vacuum as seen in the accelerated frame. This explains (ii) as well as drawing a close parallel with (iii) which is the statement that the field of the charge is nonradiative as seen in the accelerated frame.

It is also worth noting that the Unruh heat bath observed in the constantly accelerating frame is subject to the Gaussian fluctuations of energy density which a conventional heat bath would possess. If the charge were acted on by a constant force, the pressure fluctuations associated with these energy fluctuations would confer on the charge an irregular motion. This motion would represent a nonconstant acceleration and so would also lead to a systematic radiation damping force acting on the charge. We would expect this combination of forces to lead to a situation the importance of which was so often emphasized by Einstein, namely, that in which an irregular activating force and a systematic damping one lead to a steady state in which the system's momentum distribution is that given by Maxwell for thermal equilibrium. In the present case this would mean that a charge subject to a constant external force would come to have the momentum distribution appropriate to the Unruh temperature of the ambient quantum vacuum. If, on the other hand, the charge were constrained to have exactly constant acceleration, then the external force would have to fluctuate to compensate for the pressure fluctuations in the Unruh heat bath.

A converse argument can also be made. By demanding that the spectrum of vacuum fluctuations be such that there be no net transfer of energy or momentum between the field and the charge it may be inferred that the Minkowski vacuum appears to a uniformly accelerated observer to comprise a thermal bath.

We turn now to a consideration of the radiation-reaction force experienced by a freely falling charge.

FALLING CHARGES

For the case of charge moving inertially at nonrelativistic velocity through a static gravitational field it has been shown that "the field in the immediate vicinity of the particle tends to fall freely with the particle, and although it suffers a local tidal distortion characteristic of an explicit occurrence of the Riemann tensor the net retarding force due to this distortion is zero when integrated over solid angle. The deviation of the particle motion from geodetic when $F^{\text{in}}_{\mu\nu}=0$ is caused not by the local field of the particle but by a field which originates well outside the classical radius ... Physically the nonlocal term arises from a back-scatter process in which the Coulomb field of the particle, as it sweeps over the 'bumps' in space-time, receives 'jolts' which are propagated back to the particle."[9]

This effect is analogous to the nonzero self-force which acts on a charge even when it is held at rest, for example, in the gravitational field of a stationary black hole. The distortion of the Coulomb field of the charge due to the Riemann tensor of the background gravitational field leads to the existence of such a self-force, which we could call a polarization force. Of course, for a general gravitational field which lacks a timelike Killing vector there is no invariant definition of "at rest," and the self-force, which will continue to depend on the charge's motion, cannot be invariantly decomposed into a radiation and a polarization force.

Returning to the radiation problem, we now seek to understand from the fluctuation point of view the absence of radiation emanating from the region near a charge moving inertially at nonrelativistic velocity through a static gravitational field, and for simplicity we shall confine the discussion to spherically symmetric gravitational fields. Naturally the classical analysis makes no reference to the state of motion of the electrodynamic vacuum. We know now that there are three natural vacua that are associated with the spacetime geometry.[10] These are the following: (i) the Boulware vacuum, which is the natural vacuum state for the spacetime geometry of an extended massive body such as a neutron star, and may be thought of as a vacuum state which has come to equilibrium loaded under the action of the gravitational field in a manner not wholly dissimilar from an equilibrium atmosphere; (ii) the Hartle-Hawking vacuum which corresponds to the natural vacuum state of a black hole enclosed by a (sufficiently small) box; this state corresponds to a black hole in equilibrium with a bath of blackbody radiation; and (iii) the Unruh vacuum which is the natural vacuum state to assign to a black hole that results from the collapse of an extended object.

The question that comes naturally to mind is whether the classical calculation applies with equal validity to a charge falling freely through each of these three vacua. To put the matter graphically we might care to think of the Hartle-Hawking vacuum as a state in which the vacuum fluctuations move inertially, so that it is intuitively plausible that an inertially moving charge should find itself in harmony with the vacuum field fluctuations and hence that there should be little systematic retarding effect on the charge due to the fluctuations. However in this picture it is not quite so evident that a charge falling freely through the Boulware vacuum, the fluctuations of which may be thought of as comprising a static ether, should not be subject to a systematic force due to its interaction with these fluctuations. Furthermore, returning to the case of the Hartle-Hawking vacuum, a fact that now requires explanation is that a charge held fixed in the gravitational field of a black hole (and hence accelerated) should not be able to extract energy from the freely falling vacuum fluctuations and hence radiate in contravention of the classical result. That this does not occur is due to the fact that in the Hartle-Hawking vacuum the fluctuations are distributed with a thermal spectrum so that there can be no systematic exchange of energy between the charge and the field. The former case of a charge falling freely through the Boulware vacuum requires a more detailed analysis, which we present in the Appendix; the result of which is that for the case of a charge moving inertially in Minkowski space-time through an *accelerated* vacuum the spectrum of field fluctuations perceived by the charge *is the same as if the vacuum were unaccelerated,* i.e., the spectrum of field fluctuations is the same as that of the Minkowski vacuum. From this we may understand why a charge that falls freely in a gravitational field is not subject to a local reactive force due to its interaction with the field fluctuations, but only the reactive force referred to above, which arises in virtue of the field lines feeling out the "bumps" in spacetime and which originates well outside the classical radius. We may understand in the same way the fact that the radiation emitted by a freely falling charge does not originate at the charge.

In all cases, therefore, the classical results regarding the radiation emitted by and the radiative-reaction force on an electron undergoing the various states of motion that we have discussed can be understood in terms of the spectrum of field fluctuations perceived by the charge. This is a remarkable fact since *a priori* the classical results might have been incorrect either because the quantum-mechanical equations might differ from the classical ones by terms of the order of Planck's constant or because the classical results for a given motion might apply for one vacuum state but not for another.

An understanding of the fact that neither possibility is realized can be gained under the assumption that the Heisenberg operator equations of motion take the same form as the classical equations (this point is not entirely trivial since the classical equations are not derived directly from a Hamiltonian but rather by a process of successive approximation involving, among other operations, the renormalization of the charge's mass). This is in fact known to be the case at least for a charge bound to an atom that is at rest in flat spacetime.[6] It then follows in virtue of the duality between the radiation-reaction and vacuum-fluctuation pictures referred to earlier that the spectrum of the field fluctuations for a given motion and vacuum must be such as to accord with the classical result. In particular since the Heisenberg operator equations of motion make no reference to the state of the field, the classical results must apply with equal validity to each of the distinct vacua.

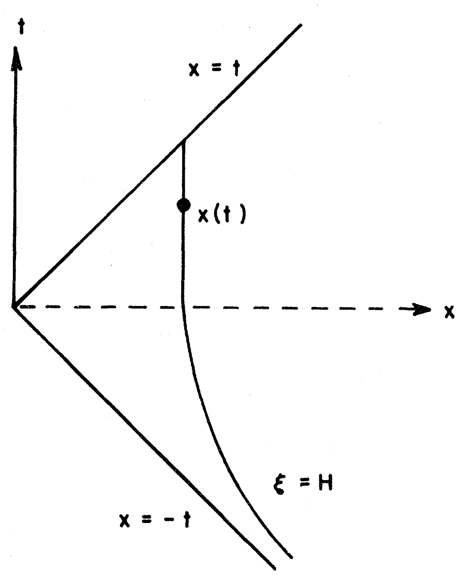

FIG. 1. The world line of an observer who is uniformly accelerated with acceleration H^{-1} before $t=0$ and who moves inertially thereafter.

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APPENDIX: CALCULATION OF THE RESPONSE OF AN UNRUH BOX THAT MOVES INERTIALLY THROUGH AN ACCELERATED VACUUM

We study in this appendix the extent to which an observer who moves inertially through an accelerated vacuum state is, by virtue of his interaction with the vacuum fluctuations, able to detect his motion relative to the ether. In order to concentrate on effects which might be directly attributed to the motion of the vacuum relative to the observer rather than effects which might be attributed to the effects of spacetime curvature, we shall perform the calculation for an observer moving inertially through the Fulling vacuum, to be denoted by $|F\rangle$, which can be thought of as the natural vacuum state above a uniformly accelerated plane mirror or equivalently as a state representing a static vacuum in the gravitational field of an infinite "flat earth."[10]

A necessary complication is that an observer moving inertially can only remain in the Rindler wedge for a finite time although the observer's world line can be chosen so as to render this time arbitrarily long (in terms of our flat-earth picture a freely falling observer released from rest will strike the earth after the elapse of a finite time). We specify the initial conditions by supposing that before $t=0$ the observer, who is equipped with an Unruh box, is subject to uniform acceleration and in fact that he follows the world line $\xi=H$ of a Rindler coordinate system; after $t=0$ we shall suppose that he moves

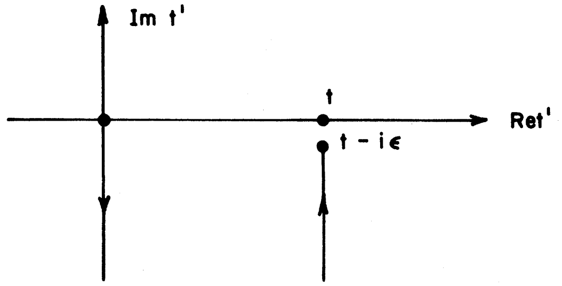

FIG. 2. The contour appropriate to the evaluation of $\Pi(\omega,t)$ for the case $\omega>0$.

inertially in such a way that his velocity suffers no discontinuity at $t=0$. This motion is depicted in Fig. 1. Since the observer will leave the Rindler wedge after a time H he can only make reliable inferences about components of the vacuum fluctuations of frequency ω for frequencies such that $\omega H \gtrsim 1$. We shall therefore suppose H to be very large. At time t the acceleration of the vacuum with respect to the observer has magnitude $(H^2-t^2)^{-1/2}$. Since we are primarily interested in effects that might be ascribed to the relative motion of the vacuum with respect to the observer and less so with transient effects that might be ascribed to the discontinuity in the observer's acceleration at $t=0$ we shall ultimately take the limit $(H,t)\to\infty$ in such a way that the acceleration $(H^2-t^2)^{-1/2}$ remains constant.

We shall assume that the Unruh box with which the observer is equipped interacts with the vacuum fluctuations of a scalar field $\hat{\phi}$ via a monopole charge $\hat{\mu}(t)$ such that the interaction Lagrangian is

$$\hat{\mathscr{L}}_{\rm int}=\hat{\mu}(t)\hat{\phi}(t)$$

evaluated at the instantaneous position of the box, and that at $t=0$ the box is in an energy eigenstate $|m\rangle$.

Standard analysis reveals that the probability of finding the box to be in an energy eigenstate $|n\rangle$ at time t is

$$P_{mn}(t)=|\langle n|\hat{\mu}(0)|m\rangle|^2\int_0^t dt_1\int_0^t dt_2 e^{-i\omega(t_1-t_2)}\langle F|\hat{\phi}(t_1)\hat{\phi}(t_2)|F\rangle ,$$

where

$$\omega=E_n-E_m$$

and $\hat{\phi}(t)$ means $\hat{\phi}(x)$ evaluated at

$$x(t)=(t,H,y,z) .$$

Differentiating $P_{mn}(t)$ we obtain an expression for the transition rate in the form

$$\frac{d}{dt}P_{mn}(t)=|\langle n|\hat{\mu}(0)|m\rangle|^2\Pi(\omega,t)$$

with

$$\Pi(\omega,t)=2\,\mathrm{Re}\int_0^t dt' e^{+i\omega(t-t')}\langle F|\hat{\phi}(t)\hat{\phi}(t')|F\rangle .$$

If we now substitute an explicit expression for the Wightman function occurring on the right-hand side of this relation and take note of the fact that, in the sense of distributions,

$$\langle F|\hat{\phi}(t_1)\hat{\phi}(t_2)|F\rangle\sim-\frac{1}{4\pi^2(t_1-t_2-i\epsilon)^2}$$

as $t_1\to t_2$, where ϵ is an infinitesimal, then we find

$$\Pi(\omega,t)=-\frac{1}{2\pi^2}\,\mathrm{Re}\int_0^{t-i\epsilon}\frac{dt' e^{+i\omega(t-t')}}{(t^2-t'^2)}\left\{\frac{1}{\ln[(H+t)/(H+t')]}+\frac{1}{\ln[(H-t)/(H-t')]}\right\} .$$

It remains now only to evaluate this integral. For $\omega>0$ we deform the contour of integration into the lower-half t' plane until we arrive at the contours of Fig. 2.

It is easy to see that the contribution to $\Pi(\omega,t)$ of the contour that extends from the origin to $-i\infty$ approaches

$$-\frac{\sin\omega t}{\omega t^2}\left\{\frac{1}{\ln[H/(H+t)]}+\frac{1}{\ln[H/(H-t)]}\right\}$$

in the asymptotic limit. The evaluation of the contribution to Π of the other contour, which extends from $t-i\infty$ to $t-i\epsilon$, requires a certain amount of care. Setting $t'=t-iy$ we see that this contribution is the real part of

$$I=\frac{1}{2\pi^2}\int_\epsilon^\infty \frac{dy\,e^{-\omega y}}{y(2t-iy)}\left\{\frac{1}{\ln[1-iy/(H+t)]}+\frac{1}{\ln[1+iy/(H-t)]}\right\}.$$

It is convenient to separate I into the sum of three terms,

$$I=J+K+L$$

with

$$J=\frac{1}{2\pi^2}\int_\epsilon^\infty \frac{dy\,e^{-\omega y}}{y(2t-iy)}\frac{1}{\ln[1-iy/(H+t)]},$$

$$K=\frac{1}{2\pi^2}\int_\epsilon^\infty \frac{dy\,e^{-\omega y}}{y(2t-iy)}\left\{\frac{1}{\ln[1+iy/(H-t)]}-\frac{(H-t)}{iy}-\frac{1}{2}\right\},$$

$$L=\frac{1}{2\pi^2}\int_\epsilon^\infty \frac{dy\,e^{-\omega y}}{y(2t-iy)}\left\{\frac{(H-t)}{iy}+\frac{1}{2}\right\}.$$

We shall consider these integrals in turn. It is easy to see that in the limit $t\to\infty$

$$J=\frac{i(H+t)}{4\pi^2 t}\int_\epsilon^\infty \frac{dy\,e^{-\omega y}}{y^2}-\frac{H}{8\pi^2t^2}\int_\epsilon^\infty \frac{dy\,e^{-\omega y}}{y}+O\left[\frac{1}{\omega t^2}\right],$$

the last term indicating a quantity independent of ϵ.

The quantity inside the braces in K has been chosen such that it is $O(y)$ as $y\to 0$. Thus it is redundant to retain the infinitesimal ϵ in this term and we may therefore take the lower limit of integration to be zero. Setting also $y=(H-t)u$ and taking the asymptotic limit we find

$$K\sim\frac{1}{4\pi^2 t}\int_0^\infty \frac{du}{u}e^{-\omega(H-t)u}\left[\frac{1}{\ln(1+iu)}-\frac{1}{iu}-\frac{1}{2}\right]$$

$$\sim-\frac{1}{8\pi^2 t}\int_{u_0}^\infty \frac{du}{u}e^{-\omega(H-t)u}$$

$$\sim\frac{1}{8\pi^2 t}\ln[\omega(H-t)].$$

The final integral is straightforward. We find

$$L=-\frac{i(H-t)}{4\pi^2 t}\int_\epsilon^\infty \frac{dy}{y^2}e^{-\omega y}+\frac{H}{8\pi^2t^2}\int_\epsilon^\infty \frac{dy}{y}e^{-\omega y}+O\left[\frac{1}{\omega t^2}\right].$$

Combining the expressions for J, K, and L we find that

$$\mathrm{Re}I\sim\frac{1}{8\pi^2 t}\ln[\omega(H-t)].$$

Thus we have shown that for ω positive and $t\to\infty$ $\Pi(\omega,t)$ differs from zero only by transient terms.

The analysis for ω negative is entirely similar apart from the fact that the contour is deformed into the upper half plane (see Fig. 3). The portions of the contour that are parallel to the imaginary axis represent transient terms just as in the previous case, while the semicircular portion of the contour yields (in the limit $\epsilon\to 0$) the contribution $\omega/2\pi$.

Thus finally we have, in the asymptotic limit,

$$\Pi(\omega,t)\to\frac{\omega}{2\pi}\theta(-\omega),$$

where θ denotes the step function. This is precisely the result that would obtain for an Unruh box moving inertially through the Minkowski vacuum.

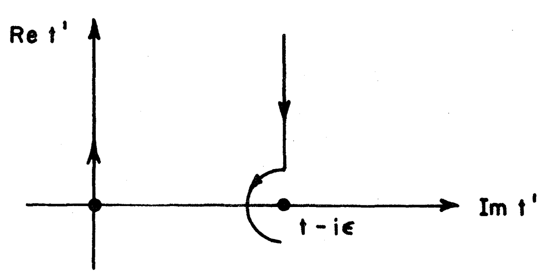

FIG. 3. The contour appropriate to the evaluation of $\Pi(\omega,t)$ for the case $\omega<0$.

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