

The combination of de Broglie's Harmony of the Phases and Mie's theory of gravity results in a Principle of Equivalence for Quantum Gravity

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RÉSUMÉ. Sous une transformation de Lorentz, la masse gravitationnelle de Mie de 1912 se comporte de la même manière que la fréquence du phénomène périodique interne de de Broglie. Il en va de même pour la masse d'inertie de Mie et la fréquence d'onde de de Broglie. Ceci permet d'interpréter l'Harmonie des Phases de de Broglie comme un "Principe d'Equivalence" pour une théorie de la Gravité Quantique. Ainsi, on peut donner au dualisme onde-particule une interprétation réaliste. L'interprétation "Mie-de Broglie" propose une correction du principe de variation de Hamilton dans le domaine quantique. L'équivalence des masses est interpretable comme "limite" classique de l'équivalence des phases.

ABSTRACT. Under a Lorentz-transformation, Mie's 1912 gravitational mass behaves identical as de Broglie's 1923 clock-like frequency. The same goes for Mie's inertial mass and de Broglie's wave-like frequency. This allows the interpretation of de Broglie's "Harmony of the Phases" as a "Principle of Equivalence" for Quantum Gravity. Thus, the particle-wave duality can be given a realist interpretation. The "Mie-de Broglie" interpretation suggests a correction of Hamilton's variational principle in the quantum domain. The equivalence of the masses can be seen as the classical "limit" of the quantum equivalence of the phases.

P.A.C.S.: 01.65.+g;03.65.Ta;03.75.-b;04.60.-m

1 Mie's foundations for a theory of matter.

In 1912-1913 Gustav Mie published his "Grundlagen einer Theorie der Materie" in a series of three papers in "Annalen der Physik" [1],[2],[3].

His concern was the failure of classical physics, mechanics and electrodynamics, in the sub-atomic domain. Mie tried to find a fundamental connection between the existence of quantized matter and the fact of gravity. He used the Hamiltonian method in his attempt to elucidate the existence of the electron, the quantum of action and the facts of gravity. He reduced the problem of finding a theory of matter to the problem of finding a universal function \mathcal{H} . This \mathcal{H} should be a function of the electromagnetic field, of the electric potentials and of the gravitational field. And this function, the universal Hamiltonian, should be invariant under a Lorentz transformation. The latter ensured the invariance of the formula Mie derived from this Hamiltonian. Mie assumed the existence of such a universal Hamiltonian and then searched its true expression.

Mie interpreted gravitation as a cohesive effect inherent in energy as such. In the theory of relativity the inertial energy-density \mathcal{E}_i entered as the last part of the stress-energy tensor, so he had to enter the complete tensor in the equations. But then other four-dimensional quantities should be incorporated as well and this led to such a level of mathematical complexity that Mie concluded it, in 1912, to be impossible to find a theory of gravity in this way. If, however, he used the Lorentz-invariant Hamiltonian energy-density \mathcal{H} instead of \mathcal{E}_i as having an inherent cohesive effect, then Mie considered it not too difficult to achieve the goal of finding a theory of gravity. Because \mathcal{H} was a Lorentz-scalar and because we have $dV = \frac{1}{\gamma}dV_0$, for a rest-system volume V_0 and a volume in a moving system \tilde{V} with

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (1)$$

Mie could combine this and conclude

$$\int_V \mathcal{H}dV = \frac{1}{\gamma}E_0 \quad (2)$$

and

$$\int_V \mathcal{E}_i dV = \gamma E_0 \quad (3)$$

for a moving particle ([3], p. 26). Mie interpreted \mathcal{E}_i as defining the inertial mass and \mathcal{H} as defining the gravitational mass ([3], p. 40):

$$m_i c^2 = \int_V \mathcal{E}_i dV = \gamma E_0 \quad (4)$$

$$m_g c^2 = \int_V \mathcal{H} dV = \frac{1}{\gamma} E_0 \quad (5)$$

with

$$m_i = \gamma m_0 \quad (6)$$

$$m_g = \frac{1}{\gamma} m_0. \quad (7)$$

Mie concluded that movements of matter influenced gravitational and inertial mass. Especially hidden movements of the elementary particles inside matter, heat, caused the inertial mass to increase and the gravitational mass to decrease ([3], p. 49). In his theory of gravity the Newtonian principle of equivalence, NEP or $m_{0i} = m_{0g}$, nowadays called the weak equivalence principle or WEP, only applied to a particle in its rest-system and became invalid in a moving frame. So the NEP was not Lorentz-invariant and could therefore not function as an axiom in his attempt to relativize gravity, dependent as it was on the relative motion of the observer. Mie didn't come up with an alternative principle of equivalence and he wasn't able to develop his theory any further. This motivated Einstein to ignore Mie's theory [12].

2 Louis de Broglie's Harmony of the Phases.

Ten years after the publication of Mie's papers, modern post-orbital or post-"Bohr-Sommerfeld" quantum mechanics began with de Broglie's hypothesis of the existence of matter waves connected to particles with inertial mass. De Broglie started with the assumption that every quantum of energy E should be connected to a frequency ν according to

$$E = h\nu \quad (8)$$

with h as Planck's constant [4],[5]. Because he assumed every quantum of energy to have an inertial mass m_o and an inertial energy $E_0 = m_0 c^2$ in its rest-system, he postulated

$$h\nu_0 = m_0 c^2. \quad (9)$$

De Broglie didn't restrict himself to one particular particle but considered a material moving object in general [4]. This object could be a photon (an atom of light), an electron, an atom or any other quantum of inertial energy. If this particle moved, the inertial energy and the associated frequency increased as

$$h\nu_i = E_i = \gamma E_0 = \gamma m_0 c^2 = \gamma h\nu_0 \quad (10)$$

so

$$\nu_i = \gamma\nu_0. \quad (11)$$

But the same particle should, according to de Broglie, be associable to an inner frequency which, for a moving particle, transformed time-like in the same manner as the atomic clocks with period τ_{atom} and frequency ν_{atom} do in Einstein's Special Theory of Relativity. We quote Arthur Miller from his 1981 study on Einstein's Special Theory of Relativity ([6], p. 211). In this quote, the rest frame is named k and the moving frame K .

In 1907 Einstein [.] defined a clock as any periodic process -for example, an atomic oscillator emitting a frequency ν_0 as measured in k . [.]..an observer in K measures the frequency:

$$\nu_{atom} = \frac{1}{\gamma}\nu_0. \quad (12)$$

[.]the clock at k 's origin registers a time observed from K of:

$$\tau_{atom} = \gamma\tau_0. \quad (13)$$

Einstein attributed a clock-like frequency to every atom. De Broglie generalized Einstein's view by postulating that every isolated particle with a rest-energy possessed a clock-like frequency. Thus, de Broglie gave every particle two, and not just one, frequencies, their inertial-energy frequency ν_i and their inner-clock frequency ν_c . The inner-clock frequency, of atoms and photons, was postulated by Einstein, the inertial-energy frequency was postulated by de Broglie. These frequencies were identical in a rest-system but fundamentally diverged in a moving frame according to

$$\nu_i = \gamma\nu_0 \quad (14)$$

$$\nu_c = \frac{1}{\gamma}\nu_0. \quad (15)$$

This constituted an apparent contradiction for de Broglie, but he could solve it by a theorem which he called "Harmony of the Phases". He assumed the inertial energy of the moving particle to behave as a wave-like phenomenon and postulated the phase of this wave-like phenomenon to be at all times equal to the phase of the inner clock-like phenomenon. Both inner-clock- and wave-phenomenon were associated to one and the

same particle, for example an electron, a photon or an atom. The inertial wave associated with a moving particle not only had a frequency ν_i but also a wave-length λ_i analogous to the fact that any inertial energy E_i of a moving particle had a momentum p_i associated to it. De Broglie used the four-vector notation to generalize the connection of a particles inertia to the associated wave-phenomenon ([5], Chap. II.5). This allowed him to incorporate the momentum \mathbf{p}_i and the wave-number \mathbf{k}_i :

$$P_\mu = (\mathbf{p}_i, \frac{i}{c}E_i) = h(\frac{1}{2\pi}\mathbf{k}_i, \frac{i}{c}\nu_i) = hO_\mu. \quad (16)$$

The phase φ_i of the wave-like inertial energy-momentum four-vector P_μ became

$$\varphi_i = 2\pi(\nu_i t - \frac{1}{2\pi}\mathbf{k}_i \cdot \mathbf{r}) = -2\pi O_\mu R^\mu \quad (17)$$

or, in energy-momentum expression

$$\varphi_i = -\frac{2\pi}{h}(E_i t - \mathbf{p}_i \cdot \mathbf{r}) = -\frac{1}{h}P_\mu R^\mu \quad (18)$$

which gave

$$\hbar\varphi_i = -P_\mu R^\mu. \quad (19)$$

De Broglie could show that his postulates ensured the law of the Harmony of the Phases, the inertial wave-like phase equaling the inner clock-like phase of the particle

$$\varphi_i = \varphi_c. \quad (20)$$

The proof of the principle of equivalence of the phases is based upon the Lorentz-transformation properties of four-vectors, especially the invariance of the inner product,

$$\varphi_i = -2\pi O_\mu R^\mu = -2\pi O_0 R^0 = 2\pi\nu_0 t_0, \quad (21)$$

and the transformation-properties of the inner clock-like frequency ν_c and the time-coordinate t

$$\varphi_c = 2\pi\nu_c t = \frac{1}{\gamma}2\pi\nu_0\gamma t_0 = 2\pi\nu_0 t_0. \quad (22)$$

The relativistic expressions for the inertial phase of a moving particle allowed de Broglie to postulate a wave-length λ_i associated to the magnitude of the electrons inertial momentum \mathbf{p}_i

$$|\mathbf{p}_i| = \frac{h}{\lambda_i}. \quad (23)$$

This inertial momentum could be interpreted as generated by an inertial energy-flow $E_i \mathbf{v}_{group}$ with

$$\mathbf{p}_i = \frac{E_i}{c^2} \mathbf{v}_{group}. \quad (24)$$

The Harmony of the Phases resulted in a super-luminous wave-velocity v_{wave} connected to the particle-velocity $v_{particle}$ as

$$v_{wave} = \frac{c^2}{v_{particle}}, \quad (25)$$

but this was not in contradiction with the postulates of Einstein's Special Theory of Relativity because the wave couldn't carry energy and the group-velocity of the wave, v_{group} , equalled the velocity of the associated particle, $v_{particle}$. So the group velocity was connected to the moving inertial energy.

At first, these postulates were regarded as too fantastic to be true. But Einstein recognized it as important and reported it to the German physicist community. This allowed Schrödinger to use the ideas of de Broglie and in January 1926 he published his famous wave-equation based upon the postulates of de Broglie. The next year Davisson and Germer, working in the Bell lab in New York, obtained the first electron diffraction pattern by bombarding a crystal of nickel with a mono-velocity electron beam. The theoretical incorporation and experimental confirmation of the wave-aspect of particles with inertial mass prompted the general acceptance of this part of de Broglie's ideas. The interpretation of de Broglie's postulates soon became a central problem of the fast developing quantum theory. However, in the battle into which the interpretation-problem of quantum physics transformed, the idea of an inner, clock-like frequency associable to an electron as a particle disappeared from the scene. All attention got focussed on the nature of the matter waves connected to the inertial energy ([7], p. 27). Then in the final interpretation of the Copenhagen School, the moving electron completely evaporated in the wave and the inertial wave transformed into an abstract probability-wave disconnected from physical reality [8].

3 De Broglie's inner frequency and Mie's gravitational energy.

In the orthodox, Copenhagen School interpretation of quantum physics, the moving electron was only represented by its probabilistic wave aspect. When the moving "electron"-wave was stopped in an experiment

by placing a photographic plate in its path, the wave mysteriously "collapsed" or vanished and the particle miraculously reappeared as a dark spot on the photographic plate. The wave was no longer seen as an inertial wave, as in de Broglie's original papers, but as defining a probability density connected to the prediction of experimental outcomes, such as the chance of finding a dark spot on a particular area of the photographic plate. The particle-wave duality for moving particles, considered as a fundamental aspect of physical reality by de Broglie, was "resolved" by cancelling the particle-aspect and by interpreting the wave as an abstract mathematical entity used to predict the outcome of experimental setups. All references to an underlying physical reality in which particles and waves had a real existence, were carefully expelled from the theory [8]. This interpretation reflected the philosophical spirit dominating the scientific circles of the time, logical positivism in the line of Mach and the *Wiener Kreis*. Einstein and de Broglie opposed this particular philosophy, they inclined to common sense realism searching an explanation for the mysteries of nature in terms of models representing physical reality. Einstein and de Broglie wanted to retain the physical reality of both waves and particles, and they declared that

the formal concepts of the "orthodox" theory, while no doubt giving precise statistical representations, did not present a complete picture of physical reality [9].

They did not succeed in formulating a viable alternative for the interpretation of the Copenhagen School. However, if we connect Mie's theory to de Broglie's, an interesting realist interpretation arises, an interpretation that we can connect to a more modern approach based on relativistic tensor-dynamics.

Our association of Mie with de Broglie starts with the observation that de Broglie didn't connect a physical energy to the inner, clock-like frequency of the electron. He proved the Harmony of the Phases by using the wavelength and frequencies, not by means of the momentum and energies. We ask ourselves what kind of energy we should identify with the space left empty by de Broglie in the following sentence: the wave frequency belongs to the inertial energy of the particle as the inner clock-frequency belongs to the energy of the particle. The answer that will link his approach to Mie's is: the gravitational energy. If every quantum of energy has to be connected to a frequency, as de Broglie successfully postulated, then gravitational energy E_g has a gravitational

frequency ν_g with

$$E_g = h\nu_g. \quad (26)$$

If we connect the relativistic Hamiltonian part of Mie's theory of gravity to the basic postulates of the relativistic quantum-frequency theory of de Broglie, we get

$$\int_V \mathcal{H}dV = \frac{1}{\gamma}E_0 = E_{gravity} = h\nu_{gravity}. \quad (27)$$

This allows us to identify $\nu_{gravity}$ with ν_{clock} because

$$h\nu_c = \frac{1}{\gamma}h\nu_0 = \frac{1}{\gamma}E_0 = h\nu_g \quad (28)$$

so

$$\nu_{gravity} = \nu_{clock}. \quad (29)$$

The clock-like frequency belongs to an inner aspect of the particle, so we can associate gravity to the particle and inertia to the wave. Our interpretation results in a real particle-wave duality, because

$$\int_V \mathcal{H}dV = \frac{1}{\gamma}E_0 = E_{gravity} = h\nu_{gravity} = h\nu_{particle} \quad (30)$$

and

$$\int_V \mathcal{E}_i dV = \gamma E_0 = E_{inertial} = h\nu_{inertial} = h\nu_{wave}. \quad (31)$$

We will try to clarify this interpretation with the help of two strongly simplified figures A and B. The particle with its inner clock-like frequency is connected to the gravitational energy and gravitational mass. The inner clock-like frequency ν_c could also be called the particle-frequency ν_p or the frequency of the gravitational energy ν_g . In its rest-frame the gravitational mass is concentrated at the place of the particle, but in the moving frame the gravitational mass m_g decreases and becomes dislocated in the wave-packed. In its rest-frame, the space around the particle is interpreted as an inertial field with an inertial energy and a connected inertial frequency $\nu_i = \nu_0$. For the particle at rest, this inertial field is extended over the entire space so its density becomes infinitely small and can't be measured. There is no wave-length because the frequency and the connected energy are homogeneously spread out over all of space. But when the particle moves, the inertial field becomes inhomogeneous,

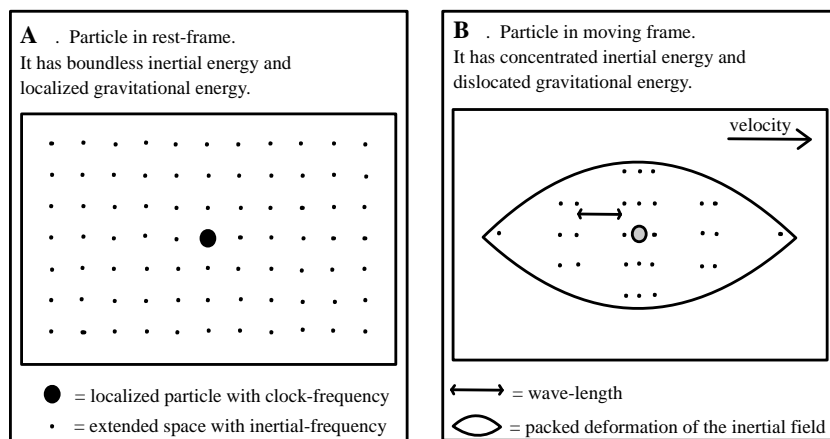


Figure 1: De Broglie's extended inertial frequency-field.

acquires a wavelength λ_i and the inertial mass m_i becomes concentrated in a small area "surrounding" the dislocated particle. The inertial frequency ν_i can now be called the wave-like frequency ν_w and makes a four-vector with the wave-number $\frac{1}{2\pi}\mathbf{k}_i$ or $\frac{1}{2\pi}\mathbf{k}_w$. This inertial wave in its four-vector representation was written by de Broglie as O_μ but is usually given in terms of the "angular velocity" ω_i and the wave-number k_i with $K_\mu = (\mathbf{k}_i, \frac{i}{c}\omega_i)$ and $K_\mu = 2\pi O_\mu$. De Broglie's association of a frequency-field to the inertial energy Lorentz-transforms into the four-vector relation for the inertial energy as a longitudinal wave $P_\mu = \hbar K_\mu$. This inertial wave can be connected to the gravitational energy, the latter being a fundamental property of the associated particle.

Our interpretation puts the problem of the particle-wave duality in a new perspective. The problem of localizing the particle in the wave becomes a problem in the context of a theory of quantum gravity. In this interpretation the problem of the particle-localization is centered on the problem of the localization of the gravitational energy within the inertial-energy wave. The connection of the particle to the wave and the connection of the gravitational mass to the inertial mass are coinciding problems. The key to the enigma of the particle-wave duality lies in de Broglie's "Harmony of the Phases". In a Mie-de Broglie theory of

Quantum Gravity, the Newtonian principle of equivalence of the masses

$$m_g = m_i \quad (32)$$

is valid only in a rest-frame of a particle. It becomes invalid in a moving frame or for a moving observer. In a quantum context it has to be replaced by the principle of equivalence of the phases

$$\varphi_g = \varphi_i. \quad (33)$$

In this interpretation, the connection of the gravitational energy to the inertial energy according to de Broglie's "Harmony of the Phases" becomes the dynamical heart of quantum theory. The pilot wave interpretation of de Broglie can be re-evaluated in this new perspective. A particle moving, on a macroscopic scale, "uniformly" through space deforms the metric on a quantum local scale with its gravitational and inertial masses. In the process, the inertial energy flow $E_i \mathbf{v}_{group}$ becomes concentrated in a wave packet and the gravitational energy flow $E_g \mathbf{v}_{particle}$ becomes dislocated in this wave packet. From a macroscopical point of view, the two energies still coincide. On a quantum scale, the attempt of de Broglie to formulate a pilot-wave theory, in which a real particle is guided by a real wave on its real path in space ([10], p. 185-186), can, in the perspective of our present interpretation, only be accomplished in a fully developed theory of Quantum Gravity. Our pilot wave becomes the compressed inertial field or the quantum local deformed metric and the particle trajectory must be identified with the delocalized world tube of the gravitational energy within this deformation. This strongly and not coincidentally matches Vigier's description of his approach to unify general relativity and quantum mechanics ([11], p. 200).

4 The Harmony of the Phases, the Hamiltonian and relativistic dynamics.

Using the interpretation of the previous section it is quite easy to prove de Broglie's postulate of the Harmony of the Phases. If $\varphi_g = \varphi_i$ in the rest-frame and if they should remain equal when the frame is set in motion we must have

$$d\varphi_g = d\varphi_i. \quad (34)$$

For an infinitely small boost of the reference frame we can assume the energy-momentum to be unchanged and set

$$\hbar d\varphi_g = E_g dt = \frac{1}{\gamma} E_0 \gamma dt_0 = E_0 dt_0 \quad (35)$$

and

$$\hbar d\varphi_i = -P_\mu dR^\mu = E_i dt - \mathbf{p}_i d\mathbf{r} = E_0 dt_0 \quad (36)$$

So our interpretation makes the postulate of the Harmony of the Phases rather trivial.

We can modernize our interpretation further by showing that the Hamiltonian of Mie's theory is not that outdated as it seems. We can connect Mie's source of gravitational energy to the modern one, the trace of the inertial stress-energy tensor [12],[13]. A free moving particle with inertial mass m_i , momentum-density g_i and four-momentum density G_ν has an inertial stress-energy tensor

$$T_{\mu\nu} = V_\mu G_\nu \quad (37)$$

with, in a (+,+,+,-) metric,

$$\text{Trace}(T_{\mu\nu}) = V_\mu G^\mu = \mathbf{v} \cdot \mathbf{g}_i - \mathcal{E}_i. \quad (38)$$

For Mie's universal Hamiltonian we have

$$\int_V (\mathcal{H} - \mathcal{E}_i) dV = \left(\frac{1}{\gamma} - \gamma\right) E_0 \quad (39)$$

$$= \left(1 - \frac{v^2}{c^2} - 1\right) \gamma E_0 = -\frac{v^2}{c^2} E_0 = -\mathbf{v} \cdot \mathbf{p}_i \quad (40)$$

and so ([3], p. 52)

$$\int_V \mathcal{H} dV = \int_V \mathcal{E}_i dV + \int_V (\mathcal{H} - \mathcal{E}_i) dV \quad (41)$$

$$= E_i - \mathbf{v} \cdot \mathbf{p}_i = -V_\mu P^\mu, \quad (42)$$

which gives

$$\mathcal{H} = -V_\mu G^\mu = -\text{Trace}(T_{\mu\nu}) = \mathcal{E}_g. \quad (43)$$

[In Mie's treatment, the identification of \mathcal{H} with $-V_\mu G^\mu$ depends on a specific state of internal oscillating motion of the system in consideration (see [3], p. 52 for further details).]

Ultimately we can relate the Harmony of the Phases to a very general tensor equation. If we define the action tensor as $S_{\mu\nu} = P_\mu R_\nu$ and use a four-volume $d\tau = dx dy dz dicit$, then we have the invariant relation

$$T_{\mu\nu} = \frac{\partial ic S_{\mu\nu}}{\partial \tau}. \quad (44)$$

We can write this as a differential equation

$$T_{\mu\nu}d\tau = dicS_{\mu\nu} \quad (45)$$

and with the approximation

$$dP_\mu R_\nu \approx P_\mu dR_\nu \quad (46)$$

we get an equation that could well be the Harmony of the Phases in differential form and in its tensor generalization,

$$T_{\mu\nu}d\tau = icP_\mu dR_\nu. \quad (47)$$

If we concentrate on the trace of both sides we have

$$V_\mu G^\mu d\tau = icP_\mu dR^\mu. \quad (48)$$

With $\hbar d\varphi_i = -P_\mu dR^\mu$ and $d\varphi_i = d\varphi_g$ we must have

$$-V_\mu G^\mu d\tau = ic\hbar d\varphi_g, \quad (49)$$

an equation which we interpret as the Harmony of the Phases in its differential tensor trace expression. We can write it as an integral equation

$$\hbar\varphi_g = -\frac{1}{ic} \int_\tau V_\mu G^\mu d\tau = - \int_R P_\mu dR^\mu = \hbar\varphi_i. \quad (50)$$

The left side parts of this expression incorporate the particle aspect and the right side parts the wave aspect. The right side wave aspect of equation (50) can be connected to Sommerfeld's quantum rule for the phase integral or scalar action S , (see [14], p. 102):

$$-S = - \int_R P_\mu dR^\mu = n_k \hbar, \quad (51)$$

with n_k as the quantum number connected to the k -th degree of freedom. According to Sommerfeld, the occurrence in nature of a minimum variation in the action (δS), connected to the jump of a quantum system from one phase-state to the next, was the fundamental reason for the appearance of Planck's constant \hbar . According to Sommerfeld ([14], p. 97), the minimum variation of the phase integral, or scalar action S , was

$$-\delta S = -\delta \int_R P_\mu dR^\mu = \hbar. \quad (52)$$

The left side particle aspect of equation (50) can be rewritten as

$$\int_{\tau} \frac{\mathcal{H}}{ic} d\tau = \hbar\varphi_g. \quad (53)$$

This can be connected to Mie's application of Hamilton's principle and to de Broglie's identification of Fermat's principle with the principle of least action of Maupertius. Mie formulated his relativistic version of Hamilton's variational principle ($\delta S = 0$) as ([1], p. 527)

$$\delta \int_{\tau} \frac{\mathcal{H}}{ic} d\tau = 0 \quad (54)$$

This relativistic version of Hamilton's variational principle was called *Mie's axiom of the world function* (\mathcal{H}) by Hilbert and was transformed by the same Hilbert into a general covariant variational principle. Hilbert's version was assimilated by the theorists of general relativity [15]. But in our Mie-de Broglie theory of Quantum Gravity, we have to correct Mie's use of Hamilton's variational principle. A minimum variation of the action integral can't be zero any more, because we have

$$\delta \int_{\tau} \frac{\mathcal{H}}{ic} d\tau = \delta\hbar\varphi_g \quad (55)$$

and the quantum minimum of variation is one unit of action \hbar , when $\delta\varphi_g = 1$. This means that the Mie-de Broglie version of Hamilton's variational principle in the quantum domain should be

$$\delta \int_{\tau} \frac{\mathcal{H}}{ic} d\tau = \hbar. \quad (56)$$

In the classical limit, on a scale where $\hbar \approx 0$, we get the usual non-quantum version of Hamilton's principle. This explains the failure of Mie to repair the breakdown of classical physics, where the minimum variation of the action-integral is assumed to be zero, in the quantum domain, where the minimum variation equals Planck's constant \hbar . So the Harmony of the Phases, or the principle of equivalence of the phases, as expressed in equation (50), leads to a very fundamental correction of Mie's use of Hamilton's variational principle in the quantum domain ($\delta S = \hbar$ instead of $\delta S = 0$).

5 On the relativity of the principle of equivalence.

We thus "derived" the postulate of the Harmony of the Phases from the tensor-equation (44)

$$T_{\mu\nu} = \frac{\partial icS_{\mu\nu}}{\partial \tau}. \quad (57)$$

We will now investigate under which circumstances we may apply the equivalence of the masses and/or the equivalence of the phases. Let's assume an observer in a reference frame K to be capable of measuring the values for R_μ and P_ν within a certain degree of accuracy. Then he is able to calculate the associated action tensor $S_{\mu\nu}$ and the stress-energy tensor $T_{\mu\nu}$ unambiguously. This allows him to define the inertial mass and the gravitational mass as in Mie's theory as

$$m_i c^2 = \int_V \mathcal{E}_i dV = - \int_V T_{44} dV \quad (58)$$

$$m_g c^2 = \int_V \mathcal{H} dV = - \int_V T_{\mu\nu}^{trace} dV. \quad (59)$$

This observer is able to check the conditions under which the inertial mass equals the gravitational mass (the weak principle of equivalence). He will set

$$\int_V T_{\mu\nu}^{trace} dV = \int_V T_{44} dV, \quad (60)$$

which gives

$$\int_V (T_{\mu\nu}^{trace} - T_{44}) dV = 0 \quad (61)$$

and

$$T_{\mu\nu}^{trace} - T_{44} = \mathbf{v} \cdot \mathbf{g} - \mathcal{E}_i + \mathcal{E}_i = \mathbf{v} \cdot \mathbf{g} = 0, \quad (62)$$

with finally

$$\mathbf{v} \cdot \mathbf{g} = 0 \quad (63)$$

as the condition for which $m_i = m_g$. Because $\mathbf{v} \cdot \mathbf{g}$ equals the pressure p , this implies a pressureless situation as a necessary condition for the equivalence of gravitational and inertial masses. The observer in K will conclude that the equivalence of the masses is not a Lorentz-invariant condition and cannot be transformed into a fundamental axiom or law of nature. The same observer can define the inertial phase and gravitational phase as

$$\hbar\varphi_i = - \int_R P_\mu dR^\mu \quad (64)$$

and

$$\hbar\varphi_g = -\frac{1}{ic} \int_{\tau} T_{\mu\nu}^{trace} d\tau. \quad (65)$$

The observer is able to check the conditions under which the inertial phase equals the gravitational phase. He will find equation (57) as the necessary condition. He will conclude that the equivalence of the phases is a Lorentz-invariant condition and that this equivalence can be seen as a fundamental law of nature. If this observer 1 wants to communicate his findings to a second observer in reference frame K' , he must first instruct observer 2 to measure R'_μ and P'_ν within a certain degree of accuracy in K' . Then observer 2 must follow an identical procedure as observer 1, but with the values of his own K' in order to check the conditions for $m'_i = m'_g$ and $\varphi'_i = \varphi'_g$. He will find the conditions to be

$$\mathbf{v}' \cdot \mathbf{g}' = 0 \quad (66)$$

for the first and

$$T'_{\mu\nu} = \frac{\partial icS'_{\mu\nu}}{\partial \tau'} \quad (67)$$

for the second. Both observers will conclude that there are only particular reference frames in which the inertial mass equals the gravitational mass but that the inertial phase equals the gravitational phase in every reference frame. The particular set are the reference frames in which the pressure vanishes. The whole procedure is in accordance with the principles of relativity and not a single absolute value or reference frame is introduced.

We can connect this issue to a discussion between Schrödinger and Einstein regarding the stress-energy tensor for the universe as a whole. In 1918 Schrödinger argued that a stress-energy tensor with a trace $T = -p - p - p + (\rho - p) = 0$ was possible as a solution for the Einstein Equations, with negative pressure $-p$ and density ρ [16]. Einstein answered to have considered such a possibility but that he rejected it because this negative pressure couldn't vanish in free space, which would implicate a negative mass distribution throughout interstellar space, a concept that meant the negation of free space [17]. In his General Theory of Relativity, Einstein mainly considered situations in which the total pressure p vanished and the only non-zero component of the stress-energy tensor was T_{44} . This matched Laue's condition for completely static systems, by which Laue meant all systems for which equation (61) holds ([12], p. 58). So Laue's and Einstein's completely static systems are

exactly those for which Mie's definitions give $m_i = m_g$. We quote John Norton from his 1992 study of the emergence of Einstein's gravitational theory ([12], p. 58):

Notice that Einstein can only say he does justice to the equality of inertial and gravitational mass "up to a certain degree", since this result is known to hold only for completely static systems and then only in their rest frame.

We conclude that Mie's definitions of m_i and m_g are in accordance with the (weak) principle of equivalence as used by Einstein. The cosmological successes of General Relativity were applications restricted to completely static systems for which equation (61) holds.

John Norton has identified two other formulations of the principle of equivalence, the first in Einstein's original writing and the second in the work of those who based themselves on Einstein's General Theory of Relativity. The first was expressed by Einstein in 1918 and states that inertia and gravity are identical in essence (*wesensgleich*) ([18], p. 233). This realist version of the principle of equivalence seems to be an absolute statement on the nature of gravity and inertia, independent of any reference frame, and cannot be sustained in our present interpretation. In the context of a Mie-de Broglie theory of Quantum Gravity, inertia and gravity seem to be "*wesensungleich*" (with double quotes, one for the language and one to indicate the ambiguity of the use of "*wesen*" in our case) because gravity seems to be a particle aspect of elementary particles and inertia a wave aspect. Gravity and inertia exist together as a fundamental and real particle-wave duality. This duality is not an absolute statement but it is inferred or induced from many experiments. The Harmony of the Phases makes this duality compatible with the principle of relativity.

The second formulation was the infinitesimal formulation of the principle of equivalence, attributed by Norton to Pauli and which is now common in the context of the modern treatment of General Relativity. It assumes the equality of inertial and gravitational mass to hold only locally, in infinitesimal regions of space-time. This version of the principle, also called the strong principle of equivalence, was never accepted by Einstein ([18], p. 238). In Pauli's version the infinitely small world region $\Delta \tau$ is so small that the space-time variation of gravity is supposed to be negligible in it ([18], p. 235). The local version has been

criticized not to be in accordance with the appearance of tidal effects that do not vanish inside the local cabin, however small it is made [18]. In our context, this version can be accepted in empty space where all pressure vanishes and matter doesn't move, so applied to completely static systems for which $\mathbf{v} \cdot \mathbf{g} = 0$. However, if this principle is used inside matter and in situations with non-zero pressure, the infinitesimal principle can't be in accordance with the basic empirical principles of Quantum Mechanics. The strong principle just implies $\mathbf{v} \cdot \mathbf{g} = 0$ on a infinitesimal scale. This holds in free space, but on a quantum scale in matter we have Heisenbergs uncertainty relations $\Delta \mathbf{p} \cdot \Delta \mathbf{r} \geq \hbar$ and so $\mathbf{v} \cdot \mathbf{g} \Delta \tau \geq ic\hbar$ or

$$\mathbf{v} \cdot \mathbf{g} = p_{quantum} \geq ic \frac{\hbar}{\Delta \tau}. \quad (68)$$

On the infinitesimal scale of $\Delta \tau$ in matter, there always is a quantum pressure because there always exists a non zero action four-density. This implicates that wherever Heisenbergs uncertainty relations practically limit the attainable accuracy of measurements, the infinitesimal principle of equivalence of inertial mass and gravitational mass becomes invalid and only the equivalence of the phases may be used.

So in our interpretation, both the strong and the weak principle of equivalence of the masses can be seen as the classical "limit" of the principle of equivalence of the phases, when the approximation $\hbar \approx 0$ is valid *and* when inertial wave-like clocks behave identical as gravitational inner-particle clocks, that is when $\gamma \approx 1$.

6 Conclusion

We belief that our interpretation based on the connection of Mie's theory of matter and de Broglie's Harmony of the Phases contains a new perspective on the old problem of the correct formulation and use of the principle of equivalence. We do not consider a scalar theory of gravity, as Mie's approach was, as definitive. But if it is possible to integrate Quantum Physics to a certain extend with a scalar theory of gravity, like Mie's, then such a Scalar Quantum Gravity, however primitive, must contain vital clues for the future development of a fully covariant tensor-dynamical Quantum Gravity. The scalar theory of quantum gravity can be developed further by connecting de Broglie's Theory of the Double Solution, specially his attempt to connect the inner clock-like frequency of particles to an inner warmth Q ([7], p. 40), to Mie's connection of thermodynamics and gravity([3], p. 47-50), but for the time being we

consider that as a subject for future study. Last but not least we conclude that if the connection made between Mie and de Broglie proves to be physically valid, then a theory of Quantum Gravity based on the orthodox interpretation of the Copenhagen School will be utterly impossible. One cannot deny the existence of the particle in the wave *and* connect something in that wave to the gravitational energy! And the wave as defining only probabilities blocks the view on its inertial properties needed in a theory of Quantum Gravity. It seems to be an either/or situation, in which the interpretation of the Copenhagen School dominates past and present and Quantum Gravity has the future.

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