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Generalization of the Hoyle-Narlikar Theory and Connection between Electromagnetism and Gravitation in the Generalized Theory

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Abstract—A generalization of the Hoyle-Narlikar theory is proposed, allowing for inclusion of electromagnetism into the scheme of this theory. This is carried out by modifying the equation for the scalar Green function. Electromagnetism is considered as direct particle interaction which is known to be equivalent to field electrodynamics from the viewpoint of experimentally observable results. The resulting generalized theory reduces to general relativity (GR) and to conventional electrodynamics in Minkowski space in the corresponding limiting cases. In the general case electromagnetism and gravitation turn out to be connected with each other in a nontrivial way which is of interest for further investigation.

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1. THE HOYLE-NARLIKAR THEORY OF GRAVITY: A BRIEF REVIEW

The Hoyle-Narlikar theory [1–3] is a modification of Einstein's classical theory. This theory has been constructed on the basis of Mach's principle treated by the authors in [1, 2] as the statement that the inertial properties of any physical body are determined by the whole set of all other bodies in the Universe. More specifically, the mass m_a of a particle a can in the general case depend on its coordinates and is determined by all other particles of the Universe through the relationship

$$m_a(X) = \sum_{b\neq a} m_a^{(b)}(X) = -\lambda \sum_{b\neq a} \int \tilde{G}(X,B)ds_b, \qquad (1)$$

where $m_a^{(b)}(X)$ is the contribution of particle b to the mass function of particle a at the point X; λ is a coupling constant, and $\tilde{G}(A,B)$ is a certain scalar Green function, symmetric with respect to its arguments. The integral in (1) is taken along the worldline of particle b. It is postulated that the Green function $\tilde{G}(X,A)$ and the metric $g_{\mu\nu}(X)$ for any points X and A obey the scalar wave equation

$$g^{\mu\nu}(X)\tilde{G}(X,A)_{;\,\mu_X\nu_X} + qR(X)\tilde{G}(X,A) = -\frac{1}{\sqrt{-g}}\delta^{(4)}_{(X,A)}, \qquad (2)$$

where q is a dimensionless constant, R is the scalar curvature, g is the determinant of the metric tensor at point X, and $\delta^{(4)}_{(X,A)}$ is the 4D delta function of the coordinate difference of the points X and A. The action functional of the Hoyle-Narlikar theory has a simple form coinciding with the free action in the classical theory of gravity:

$$S = -c\sum_a \int m_a ds_a = c\lambda \sum_a \sum_{b\neq a} \int\int \tilde{G}(A,B)ds_a ds_b. \qquad (3)$$

However, this expression is fundamentally different from the free action: the mass m_a of each particle is not fixed but is related to the distribution and characteristics of motion of all other particles of the Universe according to Eqs. (1) and (2). As a result, varying the action (3) with (2) with respect to the metric, $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$, the authors obtained an equation generalizing the Einstein equations (see the derivation in [1]),

$$\frac{2q}{\lambda}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)\sum_b \sum_{a<b} m^{(a)}m^{(b)} = -g_{\mu\rho}g_{\nu\sigma}T_m^{\rho\sigma} + \frac{2q}{\lambda}\sum_b\sum_{a<b}\Big[m^{(a)}(g_{\mu\nu}g^{\rho\sigma}m^{(b)}_{;\rho\sigma} - m^{(b)}_{;\mu\nu}) + m^{(b)}(g_{\mu\nu}g^{\rho\sigma}m^{(a)}_{;\rho\sigma} - m^{(a)}_{;\mu\nu})\Big] + \frac{1}{\lambda}\sum_b\sum_{a<b}[(1-2q)(m^{(a)}_{;\mu}m^{(b)}_{;\nu} + m^{(a)}_{;\nu}m^{(b)}_{;\mu})$$

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$$- (1 - 4q)g_{\mu\nu}m^{(a);\lambda}m^{(b)}_{;\lambda}], \tag{4}$$

where $T^{\rho\sigma}_m$ is the conventional stress-energy tensor of a set of point masses.

It should be noted that this theory is essentially macroscopic and does not describe a mass spectrum of particles. The mass determined by (1) depends on the coordinates of the point X but in no way depends on the particle type. A difference between the masses of particles of different types may be inserted into the theory artificially, for example, by introducing in (1) a number of coupling constants λ_i instead of the single constant λ. However, as pointed out by Hoyle and Narlikar (see, e.g., the review [3]), in this theory there is no necessity to introduce different coupling constants because this will not substantially alter the equations of the theory. Actually, the final equations contain the mass functions m_a, and their difference can be easily introduced by their redefinition.

Multiplying (2) by λ and integrating along the world line of particle a, we obtain an equation for the contribution of particle a to the mass function at point X (we omit the subscript at the mass function because it does not depend on the particle type; the index X is also omitted everywhere where it does not cause an ambiguity; we follow [1–3]):

$$g^{\mu\nu}m^{(a)}_{;\mu\nu} + qRm^{(a)} = \lambda \int \frac{\delta^{(4)}_{(X,A)}}{\sqrt{-g}} ds_a. \tag{5}$$

The set of equations[1] (4), (5) for determining the metric $g_{\mu\nu}(X)$ and the contributions to the mass function $m^{(a)}(X)$ has qualitatively different properties in the cases $q = 1/6$ and $q \neq 1/6$. As was noted by the authors of [1–3], in the case $q = 1/6$ the theory is conformally invariant, and Eqs. (4), (5) become dependent. In this case the set of equations is underdeterminate: if the set of functions $[g_{\mu\nu}(X); m^{(a)}(X)]$ is one of its solutions, then any other set of functions $[\zeta^2 g_{\mu\nu}(X); \zeta^{-1}m^{(a)}(X)]$, distinct from the first one by the scalar function $\zeta(X)$, is also a solution. An interpretation of this fact is that *all solutions that differ from each other by the choice of the function* $\zeta(X)$*, are physically equivalent* [2]. Constraints to be imposed on the function $\zeta(X)$ in this case are discussed in the review [3] and references therein.

In [2] the authors concluded that a correct theory of gravity corresponds to the case $q = 1/6$. A ground for this choice is discussed in [3]. In this case, if one of the solutions $[g_{\mu\nu}(X);\ m^{(a)}(X)]$, is known, one can specify the function $\zeta(X)$ in such a way that in another solution $[\zeta^2 g_{\mu\nu}(X); \zeta^{-1}m^{(a)}(X)]$ the mass of any particle k is coordinate-independent:[2]

$$\zeta^{-1}(X)m_k(X) = \zeta^{-1}(X)\sum_{a\neq k} m^{(a)}(X) = m_0 = \text{const}. \tag{6}$$

The condition (6) singles out, from all possible solutions of (4), a special class corresponding to the class of reference frames which has been named conformal [2]. In this case, the terms in the r.h.s. of [4], containing sums of derivatives of $m^{(a)}$, turn to zero, and (4) turns into the Einstein equations: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{\lambda}{2qM}T_{\mu\nu}$, where it is denoted $M = \sum\sum_{a<b} m^{(a)}m^{(b)}$. A fundamental difference between this theory and GR is the fact that the gravitational constant is here no more a free parameter whose value is found from the experiment, but is connected with matter distribution and motion in the whole Universe. An expression for it[3] follows from comparison of the equation obtained with the standard form of the Einstein equations.

It is of interest to note that there exists a relationship between Mach's principle in the Hoyle-Narlikar theory with the cosmological coincidence. This relationship has been discussed, in particular, by Vladimirov and Romashka [4, 5]. In [5] it was established that the coincidence of the observed radius of the Universe with its gravitational radius is a consequence of Mach's principle. It has also been noted that two other cosmological coincidences considered in [5] follow from each other in the framework of the Hoyle-Narlikar theory.

2. CLASSICAL ELECTRODYNAMICS IN TERMS OF DIRECT PARTICLE INTERACTION

Classical electrodynamics can be formulated in terms of direct particle interaction. Such a formulation was considered in papers by Schwarzschild, Tetrode and Fokker and was later developed by Wheeler and Feynman [6, 7]. It is completely equivalent to Maxwell's electrodynamics in its experimentally observed effects, but this theory is lacking the notion of a field as an independent physical category.

[1] In the Hoyle-Narlikar theory, as in GR, it is implied that to find specific solutions, Eqs. (4), (5) should be supplemented with appropriate boundary and coordinate conditions.

[2] It is assumed that the Universe consists of a sufficiently large number of particles, and they are distributed in such a way that the quantity under consideration is the same for all particles with high precision. Only in this case it is possible to simplify Eq. (4) as is described below.

[3] This expression is a constant only in a certain approximation (the continuum approximation [1], as well as regions far enough from point masses). Equation (4) near a point mass requires a more detailed consideration which has been performed, e.g., in [2].

The expressions corresponding to field potentials and strengths can be introduced as auxiliary mathematical objects, and they obey the *identities* whose form coincides with that of Maxwell's equations. The equation of motion of a charged particle is obtained in this theory from a variational principle and a postulate that determines the form of the action of a system of particles, known as the Fokker principle. In Minkowski space-time, the Fokker action has the form [6]

$$S = -c\sum_a m_a \int ds_a - \sum_a \sum_{b<a} \frac{e_a e_b}{c} \int\int u_a^\mu u_{b\mu} \delta(s^2(a,b)) ds_a ds_b, \quad (7)$$

where $u_a^\mu = e_a dx_a^\mu/ds_a$, and the delta function of the squared interval can be presented in the form

$$\delta(s^2(a,b)) = \frac{1}{2r_{ab}}[\delta(ct_{ab} - r_{ab}) + \delta(ct_{ab} + r_{ab})].$$

It is then clear that in (7) *the retarded and advanced interactions* are represented symmetrically, whereas in field electrodynamics only retarded interactions have a physical meaning. A solution of this problem was suggested by Wheeler and Feynman [7]. The problem of excluding the advanced interaction was solved on the basis of the general idea of Mach's principle, i.e., taking into account the contribution to an interaction between any two charges from all other charges in the Universe, a kind of the Universe's response to the interaction. The authors showed that the influence of the surrounding world *eliminates the advanced interaction whereas the retarded one is doubled*. Another result of principle importance that follows from the Universe's response is that in this theory *there automatically emerges a radiation friction force, which turns out to result from the action of all particles of the surrounding Universe upon the radiating particle* [7].

A generalization of Fokker's principle to arbitrary Riemannian spaces of GR was obtained by Hoyle and Narlikar in [8]. In this case the action has the form

$$S = -c\sum_a m_a \int ds_a - \sum_a \sum_{b<a} \frac{4\pi e_a e_b}{c} \int\int \bar{G}_{\mu_A \nu_B} dx_a^\mu dx_b^\nu, \quad (8)$$

where $\bar{G}_{\mu_A\nu_B}$ is the vector Green function, such that $\bar{G}_{\mu_A\nu_B} = \bar{G}_{\mu_B\nu_A}$, satisfying the condition

$$g^{\alpha\beta}(X)\bar{G}_{\mu_X\nu_A;\,\alpha_X\beta_X} + R_{\mu\cdot}^{\cdot\sigma}(X)\bar{G}_{\sigma_X\nu_A} = -\frac{1}{\sqrt{-g}}\delta^{(4)}_{(X,A)}\bar{g}_{\mu_X\nu_A}. \quad (9)$$

Here $R_{\mu\cdot}^{\cdot\sigma}$ is the Ricci tensor while $\bar{g}_{\mu_X\nu_A}$ is the parallel transport matrix [8, 9]. The expressions corresponding to the electromagnetic field 4-potential and tensor created by particle a at point X have the form [8]

$$A_\mu^{(a)}(X) = 4\pi e_a \int \bar{G}_{\mu_X\nu_A} dx_a^\nu,$$
$$F_{\mu\nu}^{(a)}(X) = A_{\nu;\mu}^{(a)}(X) - A_{\mu;\nu}^{(a)}(X). \quad (10)$$

For these quantities, the usual superposition principle holds. But they are half-retarded and half-advanced: they contain both retarded ("ret") and advanced ("adv") parts because the Green function itself contains both these parts:

$$A_\mu = \frac{1}{2}(A_\mu^{\text{ret}} + A_\mu^{\text{adv}}),$$
$$F_{\mu\nu} = \frac{1}{2}(F_{\mu\nu}^{\text{ret}} + F_{\mu\nu}^{\text{adv}}),$$
$$\bar{G}_{\mu_X\nu_A} = \frac{1}{2}(\bar{G}^{\text{ret}}_{\mu_X\nu_A} + \bar{G}^{\text{adv}}_{\mu_X\nu_A}). \quad (11)$$

However, it has been shown [8] that, taking into account the Universe's response to the interaction of any two particles, the advanced interaction is eliminated while the retarded one is doubled, and, moreover, there emerges a radiation friction force in the equation of motion of a charged particle. Thus all results of Wheeler and Feynman's paper [7] have been extended to curved space-times of GR (under certain assumptions on the properties of the Universe as a whole). This has shown the equivalence of field electrodynamics and the theory of direct particle interaction in a sufficiently general case.

Further on, there emerged the question of how the direct particle interaction affects the properties of space-time. According to GR, non-geometric matter affects the space-time through the stress-energy tensor. Therefore there emerged the question of introducing the stress-energy tensor in the formulation of electrodynamics under consideration. This question was considered by Wheeler and Feynman [6] for the case of Minkowski space, and two forms of the stress-energy tensor were suggested, which lead to identical results in the case of pure electrodynamics. Later Narlikar [10] solved this problem for an arbitrary space-time of GR. The sought-for tensor is determined uniquely and has the form

$$T^{\mu\nu} = \frac{1}{8\pi}\sum_b \sum_{a<b} \left(\frac{1}{2}g^{\mu\nu}F^{\alpha\beta(a)\text{ret}}F^{(b)\text{adv}}_{\alpha\beta} - F^{\mu\cdot(a)\text{ret}}_{\cdot\sigma}F^{\nu\sigma(b)\text{adv}} - F^{\mu\cdot(b)\text{adv}}_{\cdot\sigma}F^{\nu\sigma(a)\text{ret}}\right). \quad (12)$$

3. GENERALIZATION OF THE HOYLE-NARLIKAR THEORY: INCLUSION OF THE ELECTROMAGNETIC INTERACTION

The purpose of the present paper is to include electromagnetism into the scheme of the Hoyle-Narlikar theory. In the original paper [1], a theory of gravity was obtained with the exceedingly simple action (3), and a hypothesis was expressed that gravitation and electromagnetism can be jointly described by a theory whose action also has a simple form like (3). In that paper, on page 193, Hoyle and Narlikar wrote: *"Having at hand this result, the purely gravitational theory on the basis of a single expression with a double integral, it would be natural to try to perform a further simplification of the total action*[4] *by combining its terms into a unified expression. However, we will not consider this problem in the present paper. We shall restrict ourselves to the remark that we see in that a correct step on the way to construction of a unified theory of gravitation and electricity."*

In the present paper we suggest a method of realizing this idea: we build a theory that turns in the corresponding limiting cases either to a pure gravitational theory or to electrodynamics. The action of this theory has, as before, the form (3), but the equation for the Green function is modified in such a way that the theory includes both gravity and electromagnetism. This modification rests on an analogy between the scalar curvature R and the squared electromagnetic field tensor $F_{\mu\nu}F^{\mu\nu}$. The scalar $F_{\mu\nu}F^{\mu\nu}$ can be introduced into Eq. (2) along with R, i.e., R can be replaced with an expression like $R + kF_{\mu\nu}F^{\mu\nu}$, where k is a certain coefficient. However, if we take for $F_{\mu\nu}$ the electromagnetic field tensor with independent degrees of freedom, corresponding to the field version of electrodynamics, we will not obtain the desired unification. In this case, as is easily verified, we would obtain a theory with a free electromagnetic field in curved space-time, i.e., with a field without sources, which does not interact with particles. To describe such interaction, it is necessary to introduce an additional term into the action. But it turns out that it is possible to avoid this and to preserve the original structure of the Hoyle-Narlikar theory by including electrodynamics *in its formulation as a direct particle interaction*. To do so, let us compare the tensor (12) and the conventional stress-energy tensor of field electrodynamics. The latter contains the scalar $F_{\mu\nu}F^{\mu\nu}$. Comparing it with the tensor (12), it is easy to see that an analogue of this scalar in the direct particle interaction theory is the expression $\sum_a \sum_{b\neq a} F^{\mu\nu(a)\text{ret}} F^{(b)\text{adv}}_{\mu\nu}$, where $F^{\mu\nu(a)\text{ret}}$ and $F^{(b)\text{adv}}_{\mu\nu}$ are no longer fields with independent degrees of freedom but are completely determined by charged particles according to (10) and (11). It is therefore possible to assume that we can include electromagnetism into the Hoyle-Narlikar theory by replacing (2) with the following equation (all quantities in parentheses are calculated at point X):

$$
\begin{aligned}
&g^{\mu\nu}\tilde{G}(X,A)_{;\,\mu_X\nu_X} \\
&+ q\left(R + k\sum_a\sum_{b\neq a} F^{\mu\nu(a)\text{ret}} F^{(b)\text{adv}}_{\mu\nu}\right)\tilde{G}(X,A) \\
&= -\frac{1}{\sqrt{-g}}\delta^{(4)}_{(X,A)}. \qquad (13)
\end{aligned}
$$

Let us show that the theory with the action (3) and the new Green function satisfying (13) does really contain electrodynamics and gravitation theory as limiting cases. To begin with, let us vary (3) with (13) with respect to the metric. This procedure is similar to the one presented in the original paper [1], therefore we only dwell upon some points. A new feature is variation in the metric applied to the expression $\sum_a \sum_{b\neq a} F^{\mu\nu(a)\text{ret}} F^{(b)\text{adv}}_{\mu\nu}$. In a general form, this variation can be written as follows:

$$
\begin{aligned}
&\delta\left[\sum_a\sum_{b\neq a} F^{\mu\nu(a)\text{ret}} F^{(b)\text{adv}}_{\mu\nu}\right] \\
&= \sum_a\sum_{b\neq a}\left(F^{\mu\nu(a)\text{ret}}\delta F^{(b)\text{adv}}_{\mu\nu} + F^{\mu\nu(b)\text{adv}}\delta F^{(a)\text{ret}}_{\mu\nu}\right) \\
&+ \sum_a\sum_{b\neq a}\Big(F^{\sigma\cdot(a)\text{ret}}_{\cdot\,\mu} F^{(b)\text{adv}}_{\sigma\nu} \\
&+ F^{\sigma\cdot(b)\text{adv}}_{\cdot\,\mu} F^{(a)\text{ret}}_{\sigma\nu}\Big)\delta g^{\mu\nu}. \qquad (14)
\end{aligned}
$$

With the l.h.s. of (14) one can perform all calculations similar to those done with the quantity δR in [1]. Furthermore, in the emerging expression with an integral over space, one can pass over from $\delta F^{(a)\text{ret, adv}}_{\mu\nu}$ to variations of the 4-potentials $\delta A^{(a)\text{ret, adv}}_{\mu}$. To do so, one can use the second relation (10), interchange the variations and make use of the Gauss theorem to transfer the derivatives from the variations $\delta A^{(a)\text{ret, adv}}_{\mu}$ to the remaining part of the integrand. After that it is necessary to obtain explicit expressions for variations of the 4-potentials. To this end, one can use the results of [10], where variations of the vector Green function $\delta\bar{G}_{\mu_A\nu_B}$ are calculated. Taking into account the details of this work, it is also not difficult to write down the expressions for $\delta\bar{G}^{\text{ret}}_{\mu_A\nu_B}$ and $\delta\bar{G}^{\text{adv}}_{\mu_A\nu_B}$ taken

[4] The authors mean the summed action including several terms describing gravitation and electromagnetism, the expression (1) in [1].

separately. With these expressions at hand, one can calculate variations of the 4-potentials in agreement with the first relation (10). Following [10], we denote for convenience

$$Q_{\mu_X\nu_X\sigma_A} = \bar{G}_{\nu_X\sigma_A;\mu_X} - \bar{G}_{\mu_X\sigma_A;\nu_X}. \tag{15}$$

We also introduce the additional Green function $G(A,B)$ [10], satisfying an equation of the type (13), but without the term with parentheses. Using the relations where integration is performed along the world line of particle a,

$$F^{(a)}_{\mu\nu}(X) = 4\pi e_a \int Q_{\mu_X\nu_X\lambda_A} dx^\lambda_a,$$
$$\int G^{\rm ret}(X,A)_{,\,\mu_A} dx^\mu_a = \int G^{\rm adv}(X,A)_{,\,\mu_A} dx^\mu_a = 0, \tag{16}$$

it is easy to calculate a variation of the 4-potential created by charge a at point X:

$$\delta A^{(a)\rm ret}_\alpha(X) = \int d^4y\sqrt{-g} \times \Big[-\frac{1}{2}g_{\mu\nu}g^{\rho\sigma}G^{\rm ret}(Y,X)_{,\,\alpha_X\sigma_Y}A^{(a)\rm adv}_\rho(Y) + G^{\rm ret}(Y,X)_{,\,\alpha_X\nu_Y}A^{(a)\rm adv}_\mu(Y) - g^{\rho\sigma}F^{(a)\rm ret}_{\rho\nu}(Y)Q^{\rm adv}_{\mu_Y\sigma_Y\alpha_X} + \frac{1}{4}g_{\mu\nu}g^{\lambda\eta}g^{\rho\sigma}F^{(a)\rm ret}_{\rho\eta}(Y)Q^{\rm adv}_{\lambda_Y\sigma_Y\alpha_X}\Big]\delta g^{\mu\nu}. \tag{17}$$

Here the metric tensor and its determinant are calculated at point Y, in whose coordinates the integration is carried out. The variation of the advanced potential $A^{(a)\rm adv}_\alpha(A)$ has a form similar to (17), but all retarded quantities are replaced there by advanced ones and vice versa. Collecting all that and completing the derivation similar to that of [1], we obtain an equation generalizing (4). We present it without the expressions with sums of derivatives of $m^{(a)}$ standing on the r.h.s. of (4) and denoting for brevity $\sum\sum_{k>i} m^{(i)}(X)m^{(k)}(X) = M$:

$$\Big(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\Big)M = 16\pi k M T^{(e)}_{\mu\nu} - \frac{\lambda}{2q}T^{(m)}_{\mu\nu} - 2k\sum_a\sum_{b\neq a}\Big(MF^{\alpha\beta(a)\rm adv}\Big)_{;\,\beta}\int d^4y\sqrt{-g} \times\Big[-\frac{1}{2}g_{\mu\nu}g^{\rho\sigma}G^{\rm ret}(Y,X)_{,\,\alpha_X\sigma_Y}A^{(b)\rm adv}_\rho(Y) + G^{\rm ret}(Y,X)_{,\,\alpha_X\nu_Y}A^{(b)\rm adv}_\mu(Y) - g^{\rho\sigma}F^{(b)\rm ret}_{\rho\nu}(Y)Q^{\rm adv}_{\mu_Y\sigma_Y\alpha_X} + \frac{1}{4}g_{\mu\nu}g^{\lambda\eta}g^{\rho\sigma}F^{(b)\rm ret}_{\rho\eta}(Y)Q^{\rm adv}_{\lambda_Y\sigma_Y\alpha_X}\Big] - 2k\,\{\text{ret}\leftrightarrow\text{adv}\}. \tag{18}$$

In this equation $T^{(e)}_{\mu\nu}$ is the tensor (12), $T^{(m)}_{\mu\nu}$ is the stress-energy tensor of the system of point masses; all quantities outside the integral over space are taken at point X, while in the integrand the metric tensor and its determinant are taken at point Y over which the integration is carried out. The last term, denoted as $-2k\{\text{ret}\leftrightarrow\text{adv}\}$, is similar to the previous expression with an interchange of advanced and retarded quantities. The terms containing $F^{\alpha\beta(a)\rm ret}_{;\,\beta}$ and $F^{\alpha\beta(a)\rm adv}_{;\,\beta}$ can be transformed taking into account that $(F^{\alpha\beta(a)\rm ret} - F^{\alpha\beta(a)\rm adv})_{;\,\beta} \equiv 0$ in the whole space. In fact, $F^{\alpha\beta(a)\rm ret} - F^{\alpha\beta(a)\rm adv} \equiv 2F^{\alpha\beta(a)\rm rad}$ is the quantity characterizing the radiation friction force applied to charge a, as was shown in [11] and extended to Riemannian spaces in [12]. It is finite and has zero divergence at all points, including the world line of particle A. With all this, and taking into account the results of [8], one can write

$$F^{\alpha\beta(a)\rm ret}_{;\,\beta}(X) = F^{\alpha\beta(a)\rm adv}_{;\,\beta}(X) = -\frac{4\pi}{c}j^{\alpha(a)}(X) = -4\pi e_a\int \bar{g}^{\alpha_X\cdot}_{\cdot\lambda_A}\frac{\delta^{(4)}_{(X,A)}}{\sqrt{-\det(\bar{g}_{\mu_X\nu_A})}}dx^\lambda_a, \tag{19}$$

where $j^{\alpha(a)}(X)$ is the charge density, and $\bar{g}_{\mu_X\nu_A}$ is the parallel transport matrix. However, Eq. (18) still remains to be integro-differential, which is a substantial distinction from GR. We will not study it in a general form in this paper, but remark that it is of interest to study it in various approximations. It is evident that in the case of electrically neutral matter our resulting equation passes over to that of the Hoyle-Narlikar theory, which in turn passes over to the Einstein equations in the case $M = \text{const}$.

From the requirement that the coefficients of $T^{(e)}_{\mu\nu}$ and $T^{(m)}_{\mu\nu}$ should be equal, we obtain an expression for the factor k: $k = -\lambda/(32\pi qM) = -G/(2c^4)$.

The equation of motion of a test particle i can also be obtained from the action (3) with (13) by varying the particle world line under the condition of a fixed metric. Then the expression $\sum_a\sum_{b\neq a}F^{\mu\nu(a)\rm ret}\times F^{(b)\rm adv}_{\mu\nu}$ at each point X should be considered as a functional of the world line of the selected particle, and a variation of this functional should be found. In the approximation in which Eq. (6) is valid, this equation has the form

$$m_i c\frac{d^2x^\mu_i}{ds^2_i} = -m_i c\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha_i}{ds_i}\frac{dx^\beta_i}{ds_i}$$

$$+ \frac{e_i}{c}\frac{dx_i^\lambda}{ds_i}\sum_{k\neq i}\left(\frac{1}{2}F^{\mu\ \cdot(k)\text{ret}}_{\cdot\ \lambda} + \frac{1}{2}F^{\mu\ \cdot(k)\text{adv}}_{\cdot\ \lambda}\right). \quad (20)$$

Using the results of [8], Eq. (20) transforms to the conventional equation of motion of a charged particle, where in the r.h.s. there is a pure retarded field, $\sum_{b\neq a} F^{\mu\ \cdot(b)\text{ret}}_{\cdot\ \lambda}$ and the radiation friction force (for which an expression in the case of an arbitrary metric has been obtained in [12]). However, a new aspect is that the metric is determined in a more complicated manner, using Eq. (18). Thus we have demonstrated that the theory obtained passes over to a pure gravitational theory in the case where all particles have a zero electric charge, and to electrodynamics in flat space-time in the case of the corresponding matter distribution. In the general case, the presence of new terms in Eq. (18) indicates the possibility of some new influence of charged particles on the space-time metric, which is of interest for a further study.

REFERENCES

1. F. Hoyle and J. V. Narlikar, *A new theory of gravitation*, Proc. Roy. Soc. A **282**, 191 (1964).
2. F. Hoyle and J. V. Narlikar *A conformal theory of gravitation*, Proc. Roy. Soc. A **294**, 138 (1966).
3. J. V. Narlikar, *Action at a distance and cosmology: a historical perspective*, Annu. Rev. Astron. Astrophys. **41**, 169 (2003).
4. Yu. S. Vladimirov and M. Yu. Romashka, *Mach's principle in the Hoyle-Narlikar theory and in the unary relational approach. Part I*, Vestnik RUDN, ser. Mathematics, Informatics, Physics, No. 1, 121 (2011).
5. Yu. S. Vladimirov and M. Yu. Romashka, *Mach's principle in the Hoyle-Narlikar theory and in the unary relational approach. Part II*, Vestnik RUDN, ser. Mathematics, Informatics, Physics, No. 3, 119 (2011).
6. J. A. Wheeler and R. P. Feynman, *Classical electrodynamics in terms of direct interparticle action*, Rev. Mod. Phys. **24**, 425 (1949).
7. J. A. Wheeler and R. P. Feynman *Interaction with the absorber as the mechanism of radiation*, Rev. Mod. Phys. **17**, 157 (1945).
8. F. Hoyle and J. V. Narlikar, *Time-symmetric electrodynamics and the arrow of time in cosmology*, Proc. Roy. Soc. A **277**, 1 (1963).
9. J. L. Synge, *Relativity: the General Theory* (Amsterdam: North-Holland, 1960).
10. J. V. Narlikar *Stress-energy tensors in the theories of direct interparticle action*, J. Phys. A **7**, 1274 (1974).
11. P. A. M. Dirac, *Classical theory of radiating electrons*, Proc. Roy. Soc. A **167**, 148 (1938).
12. B. S. DeWitt and R. W. Brehme, *Radiation damping in a gravitational field*, Ann. Phys. **9**, 220 (1960).