The Isotropy of the Universe+

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C. W. Misner

Dicke's conception[1)] of primordial blackbody radiation not only allowed for the interpretation[2)] and publication of data confirming[3)] its existence, and led to further observations,[4-7)] but with these observations there has come new life and order into theories[8-15)] of the early epochs of our universe ten billion years in the past. The most accurate observations[16)] of this radiation concern its isotropy and show that the radiation coming from various different directions all has the same temperature to within 1% . As the most accurate observational datum in cosmology this measurement surely deserves a better explanation than is provided by the postulate that the universe from the beginning was remarkably symmetric. We shall attempt to provide some explanation here, in a preliminary way, subject to severe limitations on the complexity of the conditions we are capable of describing in the early epochs.

The traditional problem of relativistic cosmology has been to collect and correlate sufficient observational data to be able to distinguish among the small number of homogeneous and isotropic cosmological solutions of Einstein's equations. Although this problem is not yet solved we propose that a beginning be made on a wider problem which might show relativistic cosmology to be a theory with some predictive as well as descriptive value. We propose that rather few assumptions, some supported by observational data, might be sufficient to provide a theoretical explanation of whatever degree of homogeniety and isotropy of the universe the observations may eventually reveal. To illustrate this line of research we make what we hope will turn out to be unnecessarily strong assumptions about the universe, and derive from them the result that the blackbody radiation now should be isotropic to better than 1% . Our main assumptions are 1) Einstein's equations, 2) primordial thermal neutrinos as well as photons, 3) a $p = \frac{1}{3}\epsilon$ perfect fluid description of matter above $\sim 10^{11}$ °K , the thermal π meson threshold, 4) a weak assumption about the initial anisotropy, and 5) a universe with the same homogeniety as the closed Friedmann model. Of these assumptions we would hope that (5) could be drastically weakened by techniques as yet unknown, while (3) is primarily a convenience except that $p = \epsilon$ would require (4) to be strengthened somewhat. Assumption (4) is stated below, and the force of our argument evidently depends critically on the "weakness" of this assumption.

The metric for these cosmological models depends on a length $R(t)$ giving the volume of the universe, and a trace-less symmetric matrix β describing its anisotropy (ratio of different principle diameters). The Einstein equations show $R(t)$ governed by a Friedmann-like equation which now contains "anisotropy energy" terms added to the usual matter and radiation energy density terms, while β is found to perform oscillations in a time-dependent potential well. Above $\sim 10^{11}$ °K our perfect fluid assumption leads to no direct coupling between the radiation and the anisotropy coordinate β , and the β potential well is weak except at extreme values of β . But for these large amplitude β oscillations we find an adiabatic invariant which shows that the anisotropy energy decreases faster (by a factor R^2) than the matter-radiation energy. For more modest anisotropy amplitudes (diameter ratios $< 10^5 : 1$) we show only that this amplitude does not increase.

As these models have an infinite expansion from an initial singularity at $R(o) = 0$, we think there is some sense in a claim that "most" of these homogeneous cosmologies will show the anisotropy energy to have been reduced below its range of rapid decreases before any given epoch such as 10^{11} °K , so that we need only admit "modest" $10^5 : 1$ anisotropy at that epoch. This is then our assumption (4) above.

We overlook the viscous damping[17] of anisotropy by the long neutrino mean free paths during the transition period from 10^{11} to 10^{10} °K , except to assume that it will result in the anisotropy of the neutrino momentum distribution being in step with the anisotropy of the geometry so both could vanish simultaneously.

During the expansion phase below 10^{10} °K when the neutrinos are collisionless, there can be large anisotropic stresses in $T^{\alpha\beta}$, and the β oscillations are strongly coupled to the neutrino radiation and are sufficiently rapid that once again an adiabatic invariant controls their amplitude. We then find a decrease of anisotropy energy faster (by at least a factor R) than the decrease of radiation energy, and that a diameter ratio of $10^5 : 1$ at 10^{10} °K is reduced to $1.01 : 1$ already at 10^4 °K . This is adequate to explain the $< 1\%$ anisotropy now when $T = 3$°K.

The most novel element in our model is the neutrino stress tensor. The homogeniety of the universe provides enough first integrals of the geodesic equation to give a relatively simple description of the momentum distribution and resultant stresses in homogeneous collisionless radiation. For a homogeneous metric we take

$$ds^2 = - dt^2 + R^2(t)(e^{2\beta})_{ab}\sigma^a \sigma^b \tag{1}$$

where $\beta_{ab}(t)$ is a trace-less symmetric matrix and σ^a are three well known differential 1-forms on S^3 satisfying $d\sigma^1 = \sigma^2 \wedge \sigma^3$ and its cyclic permutations. The σ^a are invariant under transformations generated by three vector fields with "angular momentum" commutation relations which become Killing vectors of ds^2. For collisionless radiation which is homogeneous in space and which has a standard Planck thermal spectrum when $\beta_{ab} = 0$ we then find

$$T^{oo} = a_r R^{-4} \left[1 + V_\nu (\beta) \right] \tag{2}$$

Here the first factor is the standard aT^4 of isotropic radiation, and the second term gives the internal energy that would be supplied by adiabatically increasing β_{ab} from zero while holding R constant. The detailed computations give

$$V_\nu (\beta) = \left\langle \left[n_a(e^{-2\beta})_{ab} n_b \right]^{\frac{1}{2}} \right\rangle_\Omega - 1 \tag{3}$$

where $\langle \ \rangle_\Omega$ indicates the standard Euclidean angle average over unit vectors n_a satisfying $n_a n_a = 1$. Evidently $V_\nu (0) = 0$, and one finds that this is the unique stationary point of $V_\nu(\beta)$ and that V_ν is positive definite. For small β one has

$$V_\nu = \frac{4}{15} \beta_{ab}\beta_{ab} + O(\beta^3) \tag{4}$$

while for large β one has

$$V_{\nu} > \frac{1}{3} \exp \left[\left(\frac{1}{6} \beta_{ab}\beta_{ab} \right)^{\frac{1}{2}} \right] - 1 \quad . \tag{5}$$

It is not necessary to carry out a similarly detailed study of the T_{ij} because one can establish the formula

$$(-g)^{\frac{1}{2}} T_{ij} = 2 \frac{\partial}{\partial g^{ij}} \left[(-g)^{\frac{1}{2}} T^{oo} \right] \quad . \tag{6}$$

In the preceeding discussion $T^{\alpha\beta}$ referred only to collisionless radiation. In the cosmological models we wish to discuss, a more general form is assumed where

$$T^{oo} = a_{\gamma} R^{-4} + a_{\nu} R^{-4}(1 + V_{\nu}) + mR^{-3} \tag{7}$$

The first term here represents collision dominated radiation, the second represents collisionless radiation, and the third is pressure-free matter (dust). Each of a_{γ}, a_{ν} and m is constant.

In writing the Einstein equations for the metric (1) we will assume that β is diagonal. [Among the solutions with two diagonal elements equal are the Taub[18,19] universe with $T^{\alpha\beta} = 0$ and the Behr[20] numerical solutions with $a_{\gamma} = 0 = a_{\nu}$.] The Einstein equations fall into four groups: i) the equation $0 = T_{oi}$ which our $T^{\alpha\beta}$ satisfies, ii) a T^{oo} equation giving $\dot{R}$, iii) equations for $\ddot{\beta}_{ab}$ involving $T_{ab} - \frac{1}{3} g_{ab} T^{k}{}_{k}$ and iv) a $T^{k}{}_{k}$ equation related to ii) and iii) by Bianchi identities. This system becomes manageable when we find that the $\ddot{\beta}_{ab}$ equations can be restated by varying β_{ab} in the action integral $I = \int L dt$ where

$$8\pi L = \frac{1}{2} R^3 \dot{\beta}_{ab}\dot{\beta}_{ab} - RV_g(\beta) - 8\pi a_\nu R^{-1} V_\nu (\beta) \qquad (8)$$

The positive definite function $V_g(\beta)$ is defined by

$$V_g(\beta) = \frac{1}{4} \mathrm{tr}(e^{4\beta} - 2e^{-2\beta} + 1) \qquad (9)$$

where tr forms the trace of the indicated matrix (which we continue to assume is diagonal). Because the Lagrangian (8) contains explicit time dependence through $R(t)$. the corresponding Hamiltonian

$$8\pi h = \tfrac{1}{2} R^3 \dot{\beta}_{ab}\dot{\beta}_{ab} + RV_g(\beta) + 8\pi a_\nu R^{-1} V_\nu (\beta) \qquad (10)$$

is not conserved. Its time derivative is, however, easily computed from $dh/dt = - \partial L/ \partial t$ and this equation plays an important role in our analysis. Evidently dh/dt contains a factor $\dot{R}$, so that when this factor is cancelled from the time derivative of the T^{oo} equation

$$\dot{R}^2 - \frac{8\pi}{3} \left[(a_\gamma + a_\nu)R^{-2} + mR^{-1} + hR^{-1} \right] = - \frac{1}{4} \qquad (11)$$

one obtains the equation (iv) for $\ddot{R}$.

The quantity hR^{-3} which enters Eq. (11) on exactly the same footing as the "isotropic" contributions to the energy density T^{oo} , namely $(a_\gamma + a_\nu)R^{-4}$ and mR^{-3} , we will call the "density of anisotropy energy" or simply the anisotropy energy.

To apply the preceeding equations to models of the universe we make estimates of the constants involved as follows ($G = c = 1$):

$$\begin{aligned} a_\gamma &= 5 \times 10^{14} \text{ ly} \\ a_\nu &= \frac{7}{4} a_\gamma \\ m &= 10^{55} \text{ gm} = 10^9 \text{ ly} \end{aligned} \qquad (12)$$

and have

$$RT_{10} = 3 \text{ ly} \tag{13}$$

where $T_{10} = T/10^{10}\ {}^{o}K$. These values are appropriate from $10^3\ {}^{o}K$ to $10^{10}\ {}^{o}K$ and correspond to $T_{now} = 3\ {}^{o}K$, $(\dot{R}R^{-1})_{now} = (10^{10}\text{ly})^{-1}$, $(mR^{-3})_{now} \approx 10^{-29}\ \text{gm/cm}^3$.

At temperatures above $10^{11}\ {}^{o}K$ where $a_{\nu} = 0$ (neutrinos collision dominated) we compute $dh/dt = -\ \partial L/\partial t$ from Eq. (8) to find

$$8\pi\dot{h} = -\ 3(\dot{R}R^{-1})(\tfrac{1}{2}R^3\ \dot{\beta}^2{}_{ab}) + (\dot{R}R^{-1})RV_g \quad . \tag{14}$$

The equivalent equation

$$8\pi d(hR^{-1})/dt = -\ 2\dot{R}R\dot{\beta}_{ab}^{\ 2} \tag{15}$$

shows hR^{-1} to be non-increasing during any expansion. Now at a turning point β_{max} of a β oscillation where $\dot{\beta}_{ab} = 0$, $V_g(\beta_{max}) = 8\pi hR^{-1}$, so we have shown that the amplitude of a β oscillation can never increase. For large amplitude β oscillations we do better, for then the β oscillations are limited by sharp collisions with the exponential walls of the potential V_g and on a time average over the β oscillations one has

$$\langle 8\pi h \rangle \gtrsim \langle \tfrac{1}{2}R^3\dot{\beta}^2{}_{ab} \rangle \gg \langle RV_g \rangle \tag{16}$$

Thus Eq.(14) yields an adiabatic invariant

$$h_o = hR^3 \simeq \text{const} \tag{17}$$

when one takes this average. Whenever this adiabatic approximation is valid then, the anisotropy energy $hR^{-3} \simeq h_o R^{-6}$ decreases much faster than the fluid energy $a_\gamma R^{-4}$. This rapid decrease stops when $\omega_\beta < \omega_R$ (which is the opposite of the adiabatic assumption) where $\omega_R = \dot{R}R^{-1}$ gives the expansion time scale and ω_β gives the time scale for collisions of the β_{ab} coordinates with the walls of the V_g potential. Thus

$$\omega_\beta = V_g^{-1} \dot{V}_g \approx 4\dot{\beta} \approx e^{2\beta} R^{-1} \tag{18}$$

where we have used the large β asymptotic form $V_g \sim \frac{1}{4} e^{4\beta}$ and have taken a typical $\dot{\beta}$ value from $\dot{\beta}^2 R^3 \approx RV_g$. Rather than analyse the complicated transition region $10^{10}\ {}^oK$ to $10^{12}\ {}^oK$ where neutrinos are becoming collisionless, we simply accept the limit $\omega_\beta < \omega_R$ from $a_\nu = 0$ theory even at $10^{10}\ {}^oK$ which should be a considerable overestimate. It reads

$$|\beta| < \frac{1}{4} \ln \left(\frac{8\pi}{3} a_\gamma R^{-2} \right) \simeq 8.4 + \tfrac{1}{2} \ln T_{10} \quad . \tag{19}$$

In the next expansion phase, below $10^{10}\ {}^oK$, where $a_\nu \neq 0$, we neglect RV_g compared to $a_\nu R^{-1} V_\nu$ since their ratio $a_\nu^{-1} R^2 = 2 \times 10^{-14} \times T_{10}^{-2}$ is small above $10^4\ {}^oK$. Averaging the $\dot{h}$ equation again leads to adiabatic invariants showing that $(V_\nu)_{max} \sim hR$ decreases as R^2 when β oscillations are large enough to see hard (exponential) walls for the V_ν potential, or only as R^1 for small β where V_ν is quadratic. But these small oscillations are not safely adiabatic at first glance since their frequency ω_β satisfies

$$\frac{\omega_R^2}{\omega_\beta^2} = \frac{5}{8}\,\frac{a_\nu + a_\gamma}{a_\nu} = \frac{55}{56} \approx 1 \qquad (20)$$

Under these small oscillation conditions however, h is negligible in Eq.(11); and above 10^5 °K so is m, giving $R^4 t^{-2} = (32\pi/3)(11/4)a_\gamma$. Then each of the two β oscillations modes satisfies

$$\ddot{\beta} + \frac{3}{2t}\dot{\beta} + \frac{14}{55t^2}\beta = 0 \qquad (21)$$

which has the solution

$$\beta = (t/t_o)^{-\frac{1}{4}} \sin\left[\left(\frac{14}{55} - \frac{1}{16} \right)^{\frac{1}{2}} \ln (t/to) \right] \qquad (22)$$

showing that $\beta^2 \propto t^{-\frac{1}{2}} \propto R^{-1}$ as was predicted by the adiabatic law, which we therefore accept. Now start from $V_\nu \approx \frac{1}{2}e^{8.4} = 10^{3.4}$ corresponding to $\beta = 8.4$ at $T_{10} = 1$, reduce this by the R^2 law to $V_\nu = 1$ at $T_{10} = 10^{-1.7}$, and then by the R^1 law to $V_\nu = 5 \times 10^{-5}$ at $T_{10} = 10^{-6}$. With $V_\nu \sim \frac{2}{5}\beta^2$ this then gives $|\beta| < .01$ at 10^4 °K as desired. In addition to neglecting viscous reductions in hR^{-3} above 10^{10} °K , we have also not bothered to argue that h (not β_{max}) is conserved as a_ν changes, so our present limit on β could be reduced by morethan an order of magnitude in a lengthier treatment of the problem.

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The Isotropy of the Universe

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The cosmic blackbody ($3^{\circ}K$) radiation which is assumed to originate in the early epochs of a "hot big bang" cosmology has been observed to have remarkably little anisotropy (< 1%). We seek to "explain" this observation, which is the most accurate cosmological datum known, within relativistic cosmology theory. A new Lagrangian technique for studying homogeneous cosmological models allows us to discuss, analytically, solutions of greater complexity than were previously manageable. We find that the neutrinos which should also be thermally produced early in the big bang have important gravitational effects once they become collisionless, due to the large anisotropic stresses they can develop in an anisotropically expanding universe. In conjunction with the overall expansion these neutrino restoring forces can reduce the anisotropy (expansion ratio in different directions) from $10^5:1$ just before neutrino collisions cease to less than 1.01:1 before photon scattering stops, and there are additional mechanisms which should make the reduction in anisotropy even more dramatic.

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