On the New Theory of Gravitation of Hoyle and Narlikar

Toshiei KIMURA

Research Institute for Theoretical Physics
Hiroshima University, Takehara
Hiroshima-ken

January 18, 1965

Recently, Hoyle and Narlikar developed a new theory of gravitation.[1] They started from the consideration that the mass m_a is a direct particle field and arises from all other particles in the universe. As the action, they took the following equation

$$I=-\sum_a \frac{1}{2}\int m_a da=\nu \sum_{a<b}\sum \iint \overline{G}(a, b)\, da db\,, \tag{1}$$

where ν is a coupling constant and $\overline{G}(a, b)$ is the symmetric scalar Green function satisfying the equation

$$g^{ik}(x)\bar{G}(x,b)_{;ik}+\mu R\bar{G}(x,b)$$
$$=-(-\bar{g})^{-1/2}\delta^{(4)}(x,b), \qquad (2)$$

in which the symbol ";i" means the covariant differentiation with respect to x^i, R the scalar curvature and μ is a constant and is determined from the consistency of field equation (i.e. $\mu=1/6$). (It should be remarked that the signature of metric is $+---$.) They obtained the following main results: (i) The theory is equivalent to that of Einstein in the description of macroscopic phenomena. (ii) The gain over the Einstein theory is in that the sign of the constant of proportionality $-8\pi G$ which appears in the field equation $R^{ik}-(1/2)g^{ik}R=-8\pi GT^{ik}$ is both determinate and correct (attractive gravitational interaction). (iii) The magnitute of G follows from a determination of the mean density. (iv) Einstein's equation for an empty world is meaningless and the new theory demands that the least number of particles is two.

Though their approach is very interesting, it seems to us that their procedure may be not sufficient in that there is no guiding principle (such as the variational principle) to derive Eq. (2) and their theory is difficult to extend so that the particles described by the ordinary fields with spin are included. In this note, we shall then examine if a field theoretical version of the theory of Hoyle and Narlikar is possible.

In the Appendix of a previous paper,[2] we proposed a conformal invariant wave equation for the scalar field with the self-interaction term (corresponding to the mass term) in the expanding universe:

$$g^{ik}\phi_{;ik}+\mu R\phi-\lambda\phi\phi\cdot\phi=0\,, \qquad (3)$$

where $\mu=1/6$ and λ is a dimensionless constant, The Lagrangian density which derives Eq. (3) is

$$L=-\left(\frac{1}{2}\right)\sqrt{-g}\,\{g^{ik}\partial_i\phi\partial_k\phi$$
$$-\mu R\phi\phi+(\lambda/2)\phi\phi\cdot\phi\phi\}, \qquad (4)$$

μ being unspecified for the moment. The negative sign of $-(1/2)\sqrt{-g}$ corresponds to that of the right-hand side of Eq. (2). (The sign of the symmetrical Green function defined by Eq. (2) is reverse to that of the ordinary one.)

The variation with respect to g^{ik} yields

$$\mu\left[R_{ik}-\left(\frac{1}{2}\right)g_{ik}R\right]\phi\phi$$
$$=T_{ik}+\mu\{g_{ik}g^{pq}(\phi\phi)_{;pq}-(\phi\phi)_{;ik}\} \qquad (5)$$

with

$$T_{ik}=\partial_i\phi\partial_k\phi$$
$$-\left(\frac{1}{2}\right)g_{ik}\{\partial_l\phi\partial^l\phi+(\lambda/2)\phi\phi\cdot\phi\phi\}. \qquad (6)$$

Contracting Eq. (5) with respect to the indices i and k and using the wave equation (3) for ϕ, we have

$$\mu R(6\mu-1)\phi\phi=(6\mu-1)\,T, \qquad (7)$$

where $T=g^{ik}T_{ik}$. Thus, we arrive at the previous result $\mu=1/6$ in order that Eq. (7) reduces to an identity. It should be remarked that one cannot obtain this result if $\lambda\phi^4$ term in Eq. (4) is replaced by $\lambda\phi^n$ with $n\neq 4$ (for instance, $m^2\phi^2$).[3]

Since the definiteness of μ plays, in the theory of Hoyle and Narlikar, an important role in the derivation of the attractive gravitational interaction, we shall show that ϕ^4 term in Eq. (4) is essential in our approach to give both the mass of scalar field and the correct gravitational interaction. Now, we replace the term $\phi\phi$ which appears in the non-linear in Eqs. (3) and (4) with its vacuum expectation ualue, i.e.

$$g^{ik}\phi_{;ik}+\mu R\phi-\lambda\langle\phi\phi\rangle_0\phi=0, \qquad (3')$$

$$\left[R_{ik}-\left(\frac{1}{2}\right)g_{ik}R\right]\langle\phi\phi\rangle_0=6T_{ik}$$
$$+\{g_{ik}g^{pq}(\phi\phi)_{;pq}-(\phi\phi)_{;ik}\}, \qquad (5')$$

where

$$\langle\phi\phi\rangle_0 = \lim_{x'\to x}\langle T(\phi(x), \phi(x'))\rangle_0$$
$$= -\frac{1}{2}\Delta_F(x)_{x\to 0} \approx -K^2/8\pi^2 \quad (8)$$

in the approximation of the Minkowski space, $\Delta_F(x)$ being Feynman's propagation function and K a cutoff momentum for $\Delta_F(o)$. If we take

$$K^2 = 6\pi/G, \quad (9)$$

Eq. (5′) reduces to Einstein's gravitational equation, for the second term on the right-hand side of Eq. (5′) does not contribute if we take the smooth fluid approximation or a suitable expectation value. Since T_{ik} is positive (the sign of the present T_{ik} defined by Eq. (6) is reverse to that of the one derived from Eq. (4) by the ordinary method), the reduced equation (5′) gives the correct attractive gravitational interaction. On the other hand, Eq. (3′) gives, in the Minkowski space, the ordinary equation of scalar field with mass $\lambda K^2/8\pi^2$.

Thus, it is considerd that the same result as that of Hoyle and Narlikar is achieved by starting from the Lagrangian density which gives our non-linear equation for the scalar field. There is, however, a difficult point concerning the concept state vectors. Our vacuum state which leads to Eq. (8) means that no creation of quanta of negative energy is possible. Though this condition formally ensures that the vacuum is a state of minimum energy, the definition of the vacuum which forbids creation process is unfamiliar.[4] On the other hand, if we start from the Lagrangian whose sign is reverse to that of Eq. (4), the repulsive gravitational interaction is obtained unless the quantization is carried out in Hilbert space with indefinite metric.[5] We may conclude that in order to treat the theory of Hoyle and Narlikar in terms of field theory one must use properly c- and q-number theories by using expectation values suitably.

1) F. Hoyle and J. V. Narlikar, Proc. Roy. Soc. **A282** (1964), 191.
2) T. Kimura, Prog. Theor. Phys. **33** (1965), 510.
3) The term $\lambda m^2(-g)^{1/4}\phi^2$ has the desired character but is not a scalar density.
4) Formally the conventional interpretation is possible provided that the indefinite metric is introduced in Hilbert space. (See, for instance, S. N. Gupta, Canad. J. Phys. **35** (1957), 961). In this case, the expecttion value of T_{ik} is also positive.
5) If we take $\langle\phi\phi\rangle_0 = (1/2)\lim_{t'\to t}\langle\{\phi(\boldsymbol{x}, t), \phi(\boldsymbol{x}, t')\}\rangle_0 = (1/2)\lim_{t'\to t}\Delta^{(1)}(\boldsymbol{x}, t; \boldsymbol{x}, t')$ in place of Eq. (8), $\langle\phi\phi\rangle_0$ becomes negative and an attractive interaction is obtained. However, such a definition is not satisfactory.