Discrete Expressions for the Gravitational Constant Offer Improved Precision G=6.6740779428(56) 10−11 m3 kg−1 s−2

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Discrete Expressions for the Gravitational Constant Offer Improved Precision $G=6.6740779428(56)\ 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

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ABSTRACT

We present new expressions for resolving the gravitational constant calculated as a measure of the fine structure constant, electron mass, ground state orbital of an atom and the defined value for the speed of light. Using an approach to measure known as measurement quantization (MQ), a discrete approach to gravitational curvature is used to resolve a value of $G=6.6740779428(56)\ 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$. The solution extends the precision we are afforded today. Notably, MQ differs from modern interpretations of discrete frames of reference, identifying the internal frame of the universe as discrete while the system frame of the universe as non-discrete. Resolving the difference between these frames is what allows for this calculation.

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1. INTRODUCTION

The gravitational constant can be described as a function of either Newton's expression for force, $F=G(m_1m_2)/R^2$ or escape velocity $v_e=(2Gm_1m_2/R)^{1/2}$. That is, G is a function of mass and distance.

In this paper we present a new description of G that demonstrates quantum correspondence with these expressions. We do not present a solution to quantum gravity, which implies a description based on the principles of quantum mechanics. Instead, we present a discrete solution of motion ([1], Appx. C) in conjunction with an expanded understanding of frames of reference ([1], Appx. K). The physical significance of this interpretation is demonstrated and measured as described in the introductory paper entitled, *Measurement Quantization* [1] ([1], Table 2).

Importantly, where measure is discrete with respect to the internal frame of the universe, we recognize that length must be contracted, a property of all discrete frames where two or more dimensions are applicable ([1], Appx. D). The effect is greatest in magnitude for smaller distances and therein affects measures electromagnetic most significantly. To clarify how measures are resolved, we do not italicize terms resolved macroscopically (i.e., the measure of gravity G using a pendulum). All other measures are italicized (i.e., G, $\hbar$) but are most often resolved with an associated distance n_{Lr} of l_f we call the electromagnetic demarcation ([1], Appx. AB, AE, AN), approximately 84.60055 units of l_f.

With this, a value for G is resolved ([1], Appx. BM).

$$\mathrm{G} = \lim_{n_{Lr}\to\infty} \frac{Q_L n_{Lr} l_f c^3}{\theta_{si}} = \frac{l_f c^3}{2\theta_{si}} = 6.6740779428(56)\ 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}\,. \tag{1}$$

The term Q_L describes the fractional count n_{Lr} (i.e., $r=n_{Lr}l_f$) of a fundamental unit of measure l_f. The fundamental measures l_f, m_f, t_f and θ_{si} ([1], Appx. W) are more precise solutions to the Planck Units [2] that account for the length contraction effect – the *Informativity differential* ([1], Appx. D) – associated with discrete measure.

θ_{si} is the angle of polarization of the signal and idler of entangled X-rays at their degenerate frequency ([1], Appx. A) when aligned in a specific Bell state [3, 4]. $\theta_{si}=3.26239$ rad ± 2 μrad was first measured by Shwartz, et. al. [5, 6] and presented in a 2011 paper entitled, *Polarization Entangled Photons at X-ray Energies* [7]. The relation of angle and mass is further clarified by Po-Ning Chen, et. al. in their paper entitled, *Conserved quantities in general relativity – the view from null infinity* [8] whereby it is shown that each are conserved quantities supertranslation invariant. This corresponds to our own observations of equivalence at the lower Planck bound ([1], Appx. S).

Notably, this is one example of an approach to the measure of θ_{si}, constrained to six digits of precision. θ_{si} can also be resolved as a calculated function of the fine structure constant, mass of an electron, ground state orbital of an atom and the defined value for the speed of light ([1], Appx. AD). We reverse the calculation to resolve a value for G with thirteen digits of physical significance.

There is no mass term in the presented expression. This is not because mass is not a factor, but because the angle of polarization is taken such that the momentum of the beam is split and that corresponds to a count of half of the Planck Mass relative to a count of the Planck Length and Planck Time (i.e., $\theta_{si}=p=l_fm_f/2t_f$) ([1], Appx. S). To clarify, the value associated with a measure of momentum and angular measure overlap at the lower count bound.

We bring to the reader's attention that the resolved value for G also accounts for the effects of length contraction taken at the upper count limit, lim $n_{Lr\to\infty}$. This is not representative of all laboratory measurements and must be considered when resolving a calculation. With this in mind and such that the effects of length contraction for measurements greater than the ground state orbital of an atom affect measures at or greater than the 30th significant digit, using the upper count limit is physically sufficient for all macroscopic measurements.

This effect becomes a physically significant factor in experiments that use electromagnetic fields. One example would include an oil drop experiment, whereby G is a function of forces that balance gravitational and electromagnetic fields. Such that electromagnetic phenomena have associated demarcations of 84.60055 l_f – a distance strongly affected by the length contraction effects described by the *Informativity differential* – a more precise calculation is necessary to describe this measure.

2. METHODS

This paper uses classical expressions in a way that account for new properties of measure resolved using discrete expressions describing gravitational curvature, the speed of light, escape velocity and Heisenberg's uncertainty principle [9]. We identify this approach with the name Measurement Quantization. Notably, several examples in the publication record exist [10-15] whereby MQ is applied to problems in modern theory.

While not essential to understanding this presentation, this paper rests on physical evidence supporting the discreteness and countability of measure. Derivations predicting these properties and supporting measurement data are presented in the introductory paper entitled, *Measurement Quantization* [1]. All referenced appendices, along with a more extensive list of term and keyword definitions are included. A pre-print can also be found in Online Resources.

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Also available are two spreadsheets. Suppl. 1 contains calculations of all expressions in the introductory paper, which include calculations for expressions listed in the referenced appendixes. Suppl. 2 contains calculations of all expressions in this and 49 other papers that implement the MQ approach. Importantly, MQ is predicated on the physical support established in these resources.

3. RESULTS

1. Calculations

A calculation of the count Q_L of l_f is a distance sensitive property of the discrete internal frame of the universe. We call this effect the *Informativity differential* $Q_L n_{Lr}$ ([1], Appx. D). It is for this reason that we italicize G. When the measure of a phenomenon is taken at the upper count limit $\lim_{n_{Lr}\to\infty}$, terms are not italicized (i.e., G, ħ).

With this, we can then describe curvature as the loss of Q_L with each increment in elapsed time t_f per count n_{Lr} of l_f (i.e., Q_L/n_{Lr}) between an inertial frame and a center of mass. Thus, **side c** represents a non-dimensional description of gravitational curvature defined with respect to the non-discrete system frame of the universe.

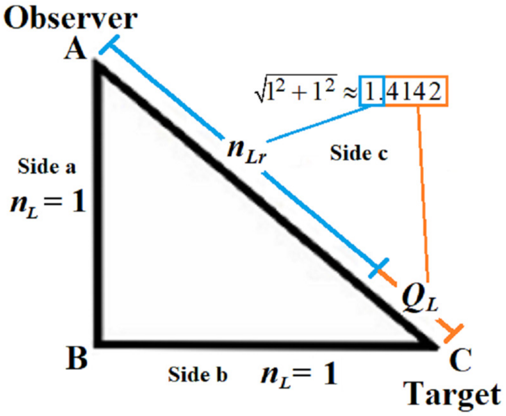

Fig 1. Count of l_f between an observer and an observable, adapted from [11].

To resolve its dimensional form (**side b**), we multiply by l_f and divide by t_f^2. Because the initial description is with respect to the discrete internal frame ([1], Appx. K) we must also apply a transform – the *metric differential* ([1], Appx. T) – thus, resolving the expression in terms of the internal Measurement Frame of the universe. We achieve this by multiplying by the rate of expansion c at the perimeter with respect to the perimeter. We then divide by the radial rate of expansion θ_{si} as defined with respect to the radius of the universe ([1], Appx. AZ). Combining terms, the transform is also described by $m_f/2$, or half a fundamental unit of mass, which is also known with regards to an analysis of the speed of light, escape velocity and the uncertainty principle. In other words, the *metric differential* associated with the phenomenon of gravitation is the reference count associated with fundamental mass (i.e., n_M=1/2) ([1], Appx. I).

We then set the expression equal to Newton's expression for curvature G/r^2 and solve for G. In practice, the effects described by the *Informativity differential* for any distance greater than the ground state orbital of an atom are less than the thirtieth digit of physical significance. We should also note that we commonly use the fundamental terms for length, mass, and time, but their values are resolved after we have established their physical significance using the expression for gravitational curvature derived here. Their values are not needed for this derivation and are intended only as conceptual representations whose physical significance is resolved separately. For reference, the terms Q_L and n_{Lr}, the latter defined as equal to n_{Lb}, are described by,

$$\overline{\mathrm{AC}} = \left(1+n_{Lb}^2\right)^{1/2}, \tag{2}$$

$$Q_L = \left(1+n_{Lb}^2\right)^{1/2} - n_{Lb}. \tag{3}$$

We can then describe curvature as Q_L/n_{Lr} and scale to SI units whereby,

$$\frac{Q_L l_f}{n_{Lr} t_f^2}, \tag{4}$$

$$\frac{Q_L l_f}{n_{Lr} t_f^2}\frac{c}{\theta_{si}} = \frac{Q_L c^2}{n_{Lr} t_f \theta_{si}} = \frac{Q_L l_f c^2}{n_{Lr} l_f t_f \theta_{si}} = \frac{Q_L c^3}{r\theta_{si}}, \tag{5}$$

$$\frac{Q_L c^3}{r\theta_{si}} = \frac{G}{r^2}. \tag{6}$$

Before we resolve a value for G, we also need a more precise solution to the value of θ_{si}. To this end, we utilize the expression ([1], Appx. AC),

$$n_{Lr}\theta_{si} = 276, \tag{7}$$

which describes the count of θ_{si} at the *charge coupling demarcation* ([1], Appx. AE) with respect to the non-discrete frame of the universe. Along with the discrete solution to gravity, we then reduce the MQ expression for the inverse fine structure constant ([1], Appx. AA),

$$\alpha_c^{-1} = 2Q_L n_{Lr}\left(84\theta_{si} - \mathrm{RND}\left(42\theta_{si}\right)\right), \tag{8}$$

to resolve that ([1], Appx. AD),

$$2\alpha_c = \frac{1}{n_{Lr}\theta_{si}} \frac{\theta_{si}}{\left(\theta_{si}^2 + \left(n_{Lr}\theta_{si}\right)^2\right)^{1/2} - n_{Lr}\theta_{si}} \frac{\theta_{si}}{\left(84\theta_{si} - \mathrm{RND}\left(42\theta_{si}\right)\right)}. \tag{9}$$

And such that $n_{Lr}\theta_{si}$=276, we solve for,

$$\theta_{si} = 3.26239030392(48). \tag{10}$$

Notably, the units depend on the frame of reference and the context of the experiment.

Next, we resolve more precise values for each of the fundamental measures ([1], Appx. BM) and with those we resolve a 13-digit value for the gravitational constant. Granted, in that the speed of light is defined, there is some disconnect with the physical significance of this result, perhaps constraining its relevance to nine digits.

Returning to the right-angle triangle and such that $\overline{\mathrm{AC}}$ is the non-discrete count n_L of l_f between an observer and an observable, we solve for Q_L and then for G. Also, for this calculation, we consider the length contraction effects described by the *Informativity differential* at a distance of one meter.

The calculation begins with fundamental length l_f and time t_f. Notably, both are calculated as a function of G and $\hbar$ and for this reason we do not use their CODATA values. Rather, we resolve each as a function of the defined value for the speed of light, the more precise measure of θ_{si} as a function of the fine structure constant ([1], Appx. AD), electron mass m_e and the ground state orbital of an atom a_0 [16].

$$l_f = \frac{m_e a_0 \alpha_p}{m_f} = 1.6161999120(35)\ 10^{-35}\ \mathrm{m}. \tag{11}$$

$$t_f = \frac{l_f}{c} = 5.3910626131(72)\ 10^{-44}\ \mathrm{s}. \tag{12}$$

Collectively, all measures give us results with thirteen digits of physical significance ([1], Appx. BM). This also resolves a count of n_{Lr}=6.1873533871(26) 10^{34} associated with one meter.

$$Q_L = \overline{\mathrm{AC}} - \overline{\mathrm{BC}} = \sqrt{1^2 + \overline{\mathrm{BC}}^2} - \overline{\mathrm{BC}}, \tag{13}$$

$$Q_L = \sqrt{1 + (6.1873533871(26)\ 10^{34})^2} - 6.1873533871(26)\ 10^{34}, \tag{14}$$

$$Q_L = 8.0809995601(73)\ 10^{-36}. \tag{15}$$

$$G = r\frac{Q_L c^3}{\theta_{si}} = 1\frac{8.0809995601(73)\ 10^{-36}\ \ 299792458^3}{3.26239030392(48)}, \tag{16}$$

$$G = 6.6740779428(56)\ 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}\,. \tag{17}$$

For a solution appropriate to cosmological scales, knowing that the *Informativity differential* approaches a value of 1/2 at the upper count limit, we make the substitution to replace $Q_L n_{Lr}$.

$$\mathrm{G} = n_{Lr} l_f \frac{Q_L c^3}{\theta_{si}} = Q_L n_{Lr} \frac{l_f c^3}{\theta_{si}} = \frac{1}{2}\frac{l_f c^3}{\theta_{si}}, \tag{18}$$

$$\mathrm{G} = 1\frac{1.6161999120(35)\ 10^{-35}\ \ 299792458^3}{2\ \ 3.26239030392(48)}. \tag{19}$$

$$\mathrm{G} = 6.6740779428(56)\ 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}\,. \tag{20}$$

While this can make the presentation simplistic, it should be noted that there is an upper count bound to l_f just as the uncertainty principle describes a lower count bound to a measure of the fundamental measures ([1], Appx. B). The upper bound does not approach infinity. Thus, if a macroscopic value for G is desired, a more precise calculation must be performed.

2. Experimental Support

To gain a physical understanding of the magnitude of the *Informativity differential*, we compare the MQ and Newton expressions at various distances. To do so we must begin with some anchor value for G. We use the 2014 published value of 6.67408(31) 10^{-11} m^3 kg^{-1} s^{-2} [17] as this is most comparable to our calculations as a function of the fine structure constant (Table 1).

Table 1. Difference between $Q_L c^3/r\theta_{si}$ and G/r^2 with respect to $\lim_{n_{Lr}\to\infty} f(\mathrm{G})$, reprinted from [1].

	84.6005 l_f	500 l_f	10,000 l_f	a_0 [a]	1 m	13.794 Gly
Difference	3.49 10^{-5} %	1.00 10^{-6} %	2.5 10^{-9} %	0.0 10^{-30} [b]	0.0 10^{-30} [b]	0.0 10^{-30} [b]

[a] Describes the distance associated with the ground state orbital of an atom.

[b] Exceeds the physical significance of the source measures.

The effect decreases with increasing distance, most significant when mixing measures of electromagnetic and macroscopic phenomena. By example mixing is most problematic when resolving the value of G using Planck's unit expressions ([1], Appx. N) and has been a significant hindrance to understanding the physical significance of Planck's expressions.

Also note differences at measurement distances greater than the ground state orbital of an atom. This difference is physically insignificant as it extends beyond the precision of G. Importantly, we are reminded that measure less than a reference is physically insignificant, yet the notion of gravitational curvature is a function of Q_L, counts of l_f many orders in magnitude smaller than the reference. This indicates that gravity is not a physically significant phenomenon with respect to the internal frame of the universe, but instead a geometric phenomenon, an emergent property of the frame difference with respect to elapsed time.

Notably, we have intentionally selected the first comparison distance at 84.6005 l_f. This is the corresponding Measurement Frame count of l_f associated with electromagnetic demarcations ([1], Appx. AB, AE, AN), rounded down.

Also notable, difference measures at 500 l_f and 10,000 l_f respectively resolve values 1.00000 10^{-6} % and 2.50000000 10^{-9} %. For now, we should treat the coincidence of these values with whole unit values as such, but the coincidence is with significant magnitude. The latter demonstrates a coincidence of eight digits. Combined with the prior six digit result, there may be an important geometry that favors the rate of *universal expansion* we observe.

Finally, we bring to the reader's attention that the most direct confirmation of the MQ approach is physically demonstrated in the existing measurement record, the measure of the effects of length contraction with respect to the measure of *G* (Table 2).

Table 2. Matching CODATA values for *ħ*, *G*, a function of Planck Units with MQ Calculated Values. Reprinted from [1].

Year	$\hbar$ (10^{-34} J s)	l_p (10^{-35} m)	t_p (10^{-44} s)	m_p (10^{-8} kg)	G (10^{-11} m^3 kg^{-1} s^{-2})
2010 [a]	1.054571726(47)	1.616199(97)	5.39106(32)	2.17651(13)	6.67384(80)
2014 [a]	1.054571800(13)	1.616229(38)	5.39116(13)	2.176470(51)	6.67408(31)
2018 [a]	1.054571817	**1.616255(18)**	**5.391247(60)**	**2.176434(24)**	**6.67430(15)**
f(ħ,*G*) [b]	1.0545349844(45)	1.6161716871(50)	5.3909684650(90)	2.1764705491(26)	6.6738448362(53)
f(ħ,G) [b]	1.0545349844(45)	1.6161999120(35)	5.3910626131(72)	2.1764325398(24)	6.6740779428(56)
f(*ħ*,G) [b]	1.0545718176(46)	1.6162281374(12)	5.3911567628(97)	2.1764705491(26)	6.6740779428(56)
f(*ħ*,*G*) [b]	1.0545718176(46)	1.6161999120(35)	5.3910626131(72)	2.1765085590(91)	6.6738448362(53)

[a] CODATA publications for 2010, 2014 and 2018 [16-18].

[b] Using precise *G* and *ħ*, we account for the *Informativity differential* at the *blackbody demarcation* n_{Lr}= 84.6005496647(07) (i.e., italicized), the upper count limit (i.e., $\lim_{n_{Lr}\to\infty}$) (not italicized), or a mix of the two.

If one uses a macroscopic approach to the measure of G, for instance a pendulum, they will resolve a value consistent with the upper count limit, as demonstrated by the CODATA collaboration in their 2010 publication [16]. Conversely, if they resolve an electromagnetic measure of *G*, for instance via an oil drop experiment, they will resolve a value consistent with the electromagnetic demarcation ([1], Appx. AB, AE, AN), as demonstrated by the CODATA collaboration in their 2014 publication [17]. All these results are more fully described in the reference paper, *Measurement Quantization* [1], in the text following Table 2 of that paper.

4. DISCUSSION

A MQ description of *G* stands in sharp contrast to existing approaches. That is, we approach a description of gravitation as a difference between the internal discrete frame and the non-discrete frame of the universe. Along with an understanding of the *fundamental expression* ([1], Appx. V), its scalability ([1], Appx. AY) and physical significance ([1], Appx. I, J), we construct a description of gravitational curvature. The result demonstrates accuracy throughout the measurement domain. Importantly, it also considers the effects of length contraction associated with the discrete internal frame.

We add that this presentation carries with it an often overlooked observation, that the fundamental measures are physically significant references in nature. Given that they are irreducible ([1], Appx. I, J), this means that the phenomenon of length has no component features, such as curvature. A closer look at the immeasurable count Q_L of l_f shows that the notion of curvature is a loss of measurement counts that arise from the discreteness of measure relative to the internal frame of the universe. Space itself is not curved, at least not in a physically significant way.

Finally, we note that an MQ description of G can easily be extended with this approach.

$$G = \frac{Q_L r c^3}{\theta_{si}} = \frac{Q_L n_{Lr} l_f c^3}{\theta_{si}} = \frac{c^3 l_f}{2\theta_{si}} = \frac{c^3 t_f}{m_f} = \frac{l_f}{t_f}\frac{l_f}{t_f}\frac{l_f}{t_f}\frac{t_f}{m_f}. \tag{21}$$

G is rarely expressed this way. Most prefer to reduce it to G=$c^2 l_f/m_f$. This practice is physically counterintuitive. In its expanded form, G describes the relation between length and time with respect to each of the three spatial dimensions. This phenomenon is then coupled with the relation of mass to time.

In that MQ is a nomenclature of counts, we identify n_L/n_T as the *length frequency* and n_M/n_T as the *mass frequency*, more specifically addressing the count relation, not the measures. We often refer to the measure relations (i.e., l_f/t_f) by the same name, as the counts are bound to the reference measures by definition. The gravitational constant then becomes an automatic and entirely predictable relation that does not need to be derived. Specifically, gravitational curvature is *length frequency*, cubed – once for each of the three spatial dimensions – divided by the *mass frequency*. A significant advantage of the MQ approach is that descriptions of phenomena are coupling relations of the fundamental measures or upper and lower count bounds of those measures.

We also call to the reader's attention that the usual Planck solution to G as a function of Planck units (i.e., G=$(l_p/t_p)^3$ (t_p/m_p) is not a physically correlated result. Recall, Planck's unit expressions are an arrangement of the terms c, G and $\hbar$ such that each arrangement exposes one dimension. Their physical significance cannot be assessed. In contrast, the MQ approach is derived from the underlying geometry. The derivation carries with it a physical instantiation which establishes physical significance of the derived result.

5. APPENDICES

APPENDIX A: TERMS

Complete set of term and keyword definitions in the paper entitled, *Measurement Quantization* [1].

- θ_{si} can be measured as the polarization angle of quantum entangled X-rays at the degenerate frequency of a maximal Bell state. As an angle θ_{si}=3.26239 rad ± 2 µrad [5-7]; as a momentum θ_{si}=3.26239030392(48) kg m s^{-1}; and with respect to the Target Frame, θ_{si} has no units. Notably, the relation of angle and momentum are clarified as conserved quantities supertranslation invariant, by Po-Ning Chen, et. al. ([8], Sec. 4.4.2) consistent with our own solution of equivalence at the lower count bound.
- c is the speed of light which may also be written as $c=n_L l_f/n_T t_f$ =299,792,458 m/s [18] such that $n_L=n_T$=1 is physically significant.
- G is the gravitational constant, 6.6740779428(56) 10^{-11} m^3 kg^{-1} s^{-2} such that its value considers the effects of length contraction associated with discrete measure at the upper count limit. Italicized G identifies a measure not at the limit (e.g., G=6.6738448362(53) 10^{-11} m^3 kg^{-1} s^{-2} at the *blackbody demarcation*).
- l_f, m_f and t_f are the fundamental measures, more precise expressions for Planck's units – length, mass, and time – that consider the effects of length contraction associated with discrete measure.
- l_p, m_p and t_p are Planck's units for length, mass, and time.
- n_L, n_M and n_T are physically significant discrete counts of l_f, m_f and t_f respectively.
- n_L describes the count of l_f representative of a change in position of an observable measured with respect to the observer's frame of reference.

- n_{Lr} describes the count of l_f representative of the position of an observable with respect to the frame of a center of mass.
- n_{Lc} describes the count of l_f representative of a change in position of light measured with respect to the observer's frame of reference.
- n_θ is the Target Frame count of θ_{si} resolved as a function of the Measurement Frame demarcation adjusted for the *metric differential* (i.e., RND(84.60055/2)=42).
- RND(n) is a function; round to the nearest whole-unit value.
- Q_L is the fractional portion of a count of l_f when engaging in a more precise calculation.
- $Q_L n_{Lr}$, also known as the *Informativity differential*, describes the contraction of length associated with discrete measure.
- $\hbar$ is the reduced Planck constant, 1.054571817 10^{-34} m^2 kg s^{-1}. When accounting for the *Informativity differential* at the upper count bound, this term is not italicized (i.e., ħ=1.0545349844(45) 10^{-34} m^2 kg s^{-1}).
- m_e is the mass of an electron.
- a_0 is the ground state orbital of an atom.
- α_f^{-1}, α_p^{-1}, and α_c^{-1} are the fundamental, Planck-like, and classical forms of the inverse fine structure constant. $\alpha_f^{-1}=42\theta_{si}$ is defined with respect to the Target Frame; the remaining two against the Measurement Frame such that α_c^{-1} differs from α_p^{-1}, accounting for the length contraction associated with discrete measure.
- γ is a collection of terms that describe the geometry between the Target and Measurement frames.

APPENDIX B: MEASUREMENT QUANTIZATION APPENDICES [1]

A. Angular Description of θ_{si}
B. Heisenberg's Uncertainty Principle
C. Discrete Gravity
D. Length Contraction (*Informativity differential*)
E. Classical Solution to G
F. Discrete Solution to G
G. Escape Velocity as a Function of Classical G
H. Escape Velocity as a Function of Discrete G
I. Solving the Minimum Measure Counts
J. Discreteness of Measure
K. Measurement Frameworks
L. Precise Description of $\hbar$
M. Precise Description of G
N. Solutions to G using the CODATA
O. Bounds to Measure
P. Electromagnetic Description of θ_{si}
Q. Momentum Description of θ_{si}
R. Angular Interpretation of $\hbar$
S. Correlation Angular Measure and Momentum at the Bound
T. Metric Differential
U. The Metric Offset
V. Fundamental Expression
W. Fundamental Measures
X. Correlating the Planck Units with the Fundamental Measures
Y. Correlating the Newton and MQ expressions for Gravitational Curvature
Z. Energy Relation of Baryonic and Electromagnetic Phenomena
AA. Fine Structure Constant
AB. Blackbody Demarcation
AC. Quantization Ratios
AD. θ_{si} as Measured with the Fine Structure Constant
AE. Charge Coupling Demarcation
AF. Upper Count Bound to Fundamental Mass
AG. Observable Mass Bound
AH. Effective Mass of a Galaxy
AI. Effective Velocity of a Star
AJ. Correlating Star Velocity to Kinetic Energy
AK. Galactic Rotation
AL. Quantization Crossover
AM. Elementary Charge
AN. Solutions to Elementary Charge Using the Gamma Approach
AO. Elementary Charge Demarcation
AP. Electric Constant
AQ. Magnetic Constant
AR. Coulomb's Constant
AS. Gamma – Target to Measurement Frame Transform
AT. Unification
AU. Contraction and Dilation with Inertial Mass
AV. Contraction and Dilation with Gravitational Mass

AW. Equivalence between Inertial and Gravitational Mass
AX. Unity Expression
AY. Physical Significance of the Fundamental Expression for All Applications Throughout the Measurement Domain
AZ. Correlating Diameter, Age of Universe to Fundamental Mass
BA. Hubble’s Constant (for the Expansion of Space)
BB. Physically Significant Domains of the Observable Universe
BC. Alternative Domain Relations
BD. Curvature of Universe: Flat
BE. Mass Accretion in the Universe
BF. Quantum Referencing
BG. Quantum Epoch
BH. Quantity, Age, Density, Temperature of the CMB
BI. Age of Universe as a Function of CMB Temperature
BJ. Mass of the Universe as a Function of CMB Temperature
BK. Time Dilation/Length Contraction Between Epochs
BL. Ground State Orbital of an Atom
BM. Extending the Physical Constants
BN. How to Derive the Planck Expressions
BO. Physical Support for Measurement Quantization
BP. Symbol Definitions
BQ. Term Definitions
BR. Derivations of Basic Physical Principles in MQ

STATEMENTS AND DECLARATIONS

The author has no relevant financial or non-financial interests to disclose.
The author has no competing interests to declare that are relevant to the content of this article.

DATA AVAILABILITY

The manuscript has data included as electronic supplementary material.

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