[5] Taylor, G. I., *Proceedings of the Fifth International Congress for Applied Mechanics*, Cambridge, Mass., U.S.A. (1938), pp. 304–310, especially equation (45), p. 307.

[6] Synge, J. L., *Semicentennial Publications Amer. Math. Soc.*, Vol. II: Semicentennial Addresses, pp. 227–269 (1938); especially equation (11.23), p. 258.

[7] The conclusion that the plane Poiseuille flow may be unstable with respect to certain disturbances is in contradiction with that of Noether, F. (ZaMM, **6**, 232–243) (1926), and Pekeris, C. L. (*Jour. Aero. Sci.*, **5**, 237–240) (1938). The untenable points in their investigation are discussed in the present investigation.

[8] Squire, H. B., *Proc. Roy. Soc. London*, **A142**, 621–628 (1933).

[9] Rayleigh, Lord, *Scientific Papers*, Vol. 1, pp. 474–487.

[10] Tollmien, W., *Göttinger Nachrichten, Neue Folge*, **1**, 70–114 (1935).

[11] Rayleigh's treatment of this problem using broken linear velocity distributions has been denounced by Heisenberg, but his proof of the necessary condition is acceptable, except for the modification to be discussed below.

[12] The fact that an element with excessive vorticity is accelerated toward the region of higher vorticity was recognized and suggested to the author by Th. v. Kármán in connection with the theory of vorticity transfer in a fully developed turbulent flow.

[13] Kelvin, Lord, *Math. Phys. Papers*, Vol. 4, pp. 186–187.

FLAT SPACE-TIME AND GRAVITATION

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If one admits that physical events take place in a 4-dimensional space-time continuum (an idea abandoned in current quantum-mechanical theory) there are three interesting possibilities: classical space and time; flat or electromagnetic space-time; curved space-time. The appropriate corresponding mathematical languages are, respectively, those of 3-vectors, 4-vectors and 4-tensors.

In a certain sense the flat space-time, characteristic of the so-called special theory of relativity, is just as *absolute* as classical space and time, since the coördinates t, x, y, z require exactly 10 arbitrary constants for their complete specification in both cases. But, in the framework of flat space-time, the fundamental electromagnetic equations of Maxwell and Lorentz lose the artificiality which they possess in classical space and time.

The initial attempts to incorporate gravitational phenomena in flat space-time were not satisfactory. Einstein turned to the curved space-time suggested by his principle of equivalence, and so constructed his general theory of relativity. The initial predictions, based on this celebrated theory of gravitation, were brilliantly confirmed. However, the theory has not led to any further applications and, because of its complicated mathematical character, seems to be essentially *unworkable*. Thus curved space-time has come to be regarded by many as an auxiliary construct (Larmor) rather than as a physical reality.

In my opinion, the failure of the early attempts by Nordström and others to develop a theory of gravitation in flat space-time is to be attributed to the fact that a fundamental theoretic requirement was overlooked, namely, that the disturbance velocity in matter must be that of light.

With this requirement in mind, I have recently been led to a very simple theory of gravitation in flat space-time, concordant with all known gravitational phenomena and free of arbitrary constants. This theory was presented first in very brief form in 1942 at Tonanzintla, Mexico, and has been developed further in a Note in these PROCEEDINGS.[1] Furthermore, attention is to be directed to a Note by A. Barajas,[2] called forth by a review of H. Weyl, and to an article by A. Barajas, C. Graef, M. Sandoval Vallarta and myself, taking up the new theory from the physical point of view.[3] A very significant application of the theory to the two-body problem by Graef will be published shortly.[4]

Unfortunately, the foundation of the theory has not so far been adequately presented in its philosophic, postulational and mathematical aspects. My colleague, Professor Barajas, and I are planning to publish an extensive article remedying this deficiency.

The aim of the present Note is to present briefly these foundational considerations as I see them. It is especially necessary to do so in order to avoid further misunderstanding of the new theory. For example, Weyl says very recently,[5] referring to my theory: "Their [i.e., 'the field equations'] most general static centrally symmetric solution involves 3 arbitrary constants $a, b, l. \ldots$ From the present standpoint this is a serious disadvantage of B." His assertion is wrong since the general exact solution for the gravitational potentials h_{ij} is

$$h_{ij} = \delta_{ij}\frac{m}{r}$$

where r stands for radial distance, δ_{ij} is the familiar Kronecker δ, and the mass m is the single arbitrary constant which enters. This exact solution plays in my theory a rôle analogous to that of the Schwarzschild solution in the theory of Einstein. Weyl has overlooked the salient fact that the central body is composed of the basic "perfect fluid."

The proposed theory of gravitation in flat space-time may be characterized as follows in its fundamental features:

1. In the 4-dimensional framework of flat space-time, matter is regarded as the occupant of certain tubular regions made up of the worldlines of

identifiable points. Point-particles are abandoned once and for all, except as a limiting possibility.

Thus there is a duality of matter and space-time in my theory, whereas the monistic concept of space-time conditioned by matter prevails in Einstein's theory. Whichever point of view is destined to figure in ultimate physical theory, it seems the part of obvious common sense to explore both possibilities fully.

The simplest available form of matter in flat space-time is that characterized by a certain stream 4-vector v^i of space type, i.e., with

$$\rho^2 = (v^1)^2 - (v^2)^2 - (v^3)^2 - (v^4)^2 > 0;$$

here ρ is the scalar length of the 4-vector v^i. Or, alternatively, we may think of matter of this kind as characterized by a density ρ and a velocity 4-vector u^i, where $v^i = \rho u^i$ and

$$(u^1)^2 - (u^2)^2 - (u^3)^2 - (u^4)^2 = 1.$$

The principle of local causation is taken to hold for an isolated portion of this kind of matter, as embodied in the differential equations:

$$\frac{\partial v^i}{\partial t} = F^i\left(v^1, \ldots v^4, \frac{\partial v^1}{\partial x}, \ldots \frac{\partial v^4}{\partial z}\right)$$

where $t = x^1, x = x^2, y = x^3, z = x^4$, and where F^i are taken to be rational and integral in the partial derivatives involved. These four differential equations assert that the time rates of change of the components of the stream vector are functions of these components and their space rates of change, being rational and integral in the latter.

The requirements of the underlying language of 4-vectors, based on the Lorentz group, then indicate as the *only* available possibility, when referred to instantaneous rest coördinates ($v^1 = \rho, v^2 = v^3 = v^4 = 0$):

$$\frac{\partial v^1}{\partial t} = F(\rho)\left(\frac{\partial v^2}{\partial x} + \frac{\partial v^3}{\partial y} + \frac{\partial v^4}{\partial z}\right),$$

$$\frac{\partial v^i}{\partial t} = G(\rho)\frac{\partial v^1}{\partial x^i} \qquad (i = 2, 3, 4).$$

These equations involve a pair of arbitrary functions $F(\rho)$, $G(\rho)$.

By appropriate normalization of the scalar ρ (that is by multiplicative modification of the 4-vector v^i) the preceding equations may be expressed in the normal form

$$\frac{\partial T^{i\alpha}}{\partial x^\alpha} = 0 \qquad (T^{ij} = v^i v^j - p(\rho)g^{ij})$$

where $g^{11} = -g^{22} = -g^{33} = -g^{44} = 1, g^{ij} = 0$ for $i \neq j$. Here there appears only one arbitrary function $p(\rho)$. This normalized density ρ is determined up to a unit of magnitude, while the scalar $p(\rho)$ is determined up to an additive constant.

2. It is granted that free equilibrium is possible for a certain equilibrium density, $\rho_0 > 0$, i.e., that a possible state is $v^i = v_0^i$ provided only that we have

$$(v_0^1)^2 - (v_0^2)^2 - (v_0^3)^2 - (v_0^4)^2 = \rho_0^2.$$

Because of the thoroughgoing analogy of the equations obtained with those of a homogeneous adiabatic fluid in classical physics, it is natural to assume that along a free boundary we have always $\rho = \rho_0$, and furthermore that at collision ρ takes on the same value on both sides of the common boundary until separation occurs for $\rho = \rho_0$ subsequently.

At this stage the behavior of a collection of freely moving portions of this "fluid" has been completely specified, whether or not collisions occur. The equations involved present a more familiar aspect if the velocities u^i are introduced, with

$$T^{ij} = \rho u^i u^j - p(\rho) g^{ij}.$$

This type of equations has always been taken to be appropriate for a general homogeneous adiabatic fluid in flat space-time. The symmetric tensor T^{ij} has been called the energy tensor; ρ, the density; and $p(\rho)$, the pressure.

3. Such a fluid has the property that a certain divergence vanishes:

$$\frac{\partial}{\partial x^\alpha}(e^{-\int dp/\rho}\, v^\alpha) = 0$$

This ensures that the 3-dimensional integral

$$\int \rho e^{-\int dp/\rho} dv$$

over the rest-volume is invariable.[6]

Keeping in mind the hydrodynamic analogy, it appears to be absolutely essential to suppose that this divergence vanishes under all conditions. Otherwise the fluid might undergo a full cyclic return to a set of initial velocities without a cyclic return of densities, such as is always observed to occur with ordinary matter. This requirement means that we have always

$$v_\beta \frac{\partial T^{\beta\alpha}}{\partial x^\alpha} \equiv 0.$$

4. There is, however, a fundamental theoretic difficulty in the theoretic employment of the general adiabatic fluid. In fact if two portions of the fluid collide with oppositely directed velocities nearly equal to that of light

(1)[7] the equations of motion will break down if the disturbance velocity of the fluid is less than that of light. Since it is physically inadmissible that this velocity v (relative to rest coördinates) exceed that of light, we are led to require that this velocity, namely,

$$v = \sqrt{dp/(d\rho - dp)},$$

equals that of light, so that at all densities $dp/d\rho = {}^1/_2$.

Thus we obtain by integration the only physically allowable constitutive equation $p = \rho/2$. The corresponding special form of the general adiabatic fluid is called the *perfect fluid.*

As far as this determination of T^{ij} is concerned, we might have written more generally $p = {}^1/_2\rho + c$, and so have obtained an additional term of the form $-cg^{ij}$ in T^{ij}. This modification would not affect the equations of motion, however. Reason will be given later on for the special choice made of $c = 0$ in fixing completely the energy tensor T^{ij}.

For the perfect fluid the invariable integral reduces to the simple form

$$\int \sqrt{\rho}dv$$

where dv is the 3-dimensional rest volume.

The perfect fluid may be looked upon as the counterpart in flat space-time of the homogeneous adiabatic incompressible fluid in classical space and time, which has infinite disturbance velocity. Physically speaking, the perfect fluid is very nearly incompressible and thus possesses very nearly invariable mass $\int \rho dv$.

If electricity of density σ be attached to the perfect fluid, the ratio $\sigma/\sqrt{\rho}$, called the substance coefficient, remains forever constant along any worldline.

In what follows the perfect fluid is regarded as the single primordial form of matter.

5. Suppose now that the perfect fluid, with energy tensor

$$T^{ij} = v^iv^j - {}^1/_2(v_\alpha v^\alpha)g^{ij} \tag{1}$$

is not free but is subject to body forces. Formally, the suggestion is obvious that we *define* a force vector f^i by means of

$$\frac{\partial T^{i\alpha}}{\partial x^\alpha} = f^i \tag{2}$$

where because of the postulated invariance of the integral

$$\int \sqrt{\rho}dv \tag{3}$$

we have

$$f_\alpha v^\alpha \equiv 0 \tag{4}$$

Thus the force vector is required to be identically orthogonal to the stream (or velocity) vector. It may be recalled that the acceleration vector a^i along any worldline has the same property.

In view of the identity (4) it is clear that f^i cannot be independent of the vector v^i. For the case of electrically charged matter f^i is known to be linear homogeneous in the 4-vector v^i and identically orthogonal to it. For the gravitational forces it is the simplest possible hypothesis to assume that in a purely gravitational field f^i is homogeneous and quadratic in v^i and proportional to ρ. We write therefore in that case

$$f^i = \rho\varphi^i_{\alpha\beta}v^\alpha v^\beta \tag{5}$$

where the components of the tensor $\varphi^i_{jk} = \varphi^i_{kj}$ are functions of t, x, y, z defined throughout space-time.

Without going into any detail it is to be stressed that this is the most natural assumption which can be made about the manner in which f^i depends on the stream vector v^i. This is especially the case in view of the reversibility of gravitational phenomena in time.

6. At this stage of our genetic account of the theory under consideration, in full accordance with the tradition of the past, it will be supposed to begin with that non-gravitational forces, such as electrical forces, need not be considered in gravitational problems. This hypothesis is justified by the more complete form of the theory given in the Note already alluded to.[1]

For the case of a rest system the x, y, z components of the gravitational forces take the form

$$f_x = \rho\varphi^2_{11}, \qquad f_y = \rho\varphi^3_{11}, \qquad f_z = \rho\varphi^4_{11} \tag{5'}$$

with $f_t = 0$ of course.

In the corresponding Newtonian situation the force components are:

$$f_x = \rho\frac{\partial g}{\partial x}, \qquad f_y = \rho\frac{\partial g}{\partial y}, \qquad f_z = \rho\frac{\partial g}{\partial z},$$

where g is the Newtonian potential defined by Poisson's equation

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2} = -4\pi\rho, \text{ or 0 in empty space,}$$

together with the requirement that g is finite (or vanishes) at infinity.

By formal analogy one is led to require that φ^i_{jk} are linear homogeneous in the first derivatives of the gravitational potential defined by means of an appropriate Poisson equation.

A very simple possibility would be to set up a scalar gravitational potential h defined by

$$\frac{\partial^2 h}{\partial t^2} - \frac{\partial^2 h}{\partial x^2} - \frac{\partial^2 h}{\partial y^2} - \frac{\partial^2 h}{\partial z^2} = 8\pi T, \text{ or 0 in empty space,}$$

where we have written

$$T = g_{\alpha\beta}T^{\alpha\beta} = \rho/2$$

for the contracted energy tensor. However, with this form of f^i it is not possible to fulfil the vital condition of orthogonality, embodied in (4).

The alternative, equally simple, hypothesis from the formal point of view is to take as the analog of the Poisson equation

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right] h_{ij} = 8\pi T_{ij}, \text{ or } 0 \text{ in empty space,} \quad (6)$$

with the additional requirement that h_{ij} are finite (or vanish) at infinity. The gravitational potential h_{ij} thus introduced is a symmetric tensor of the second order. No essential limitation is introduced by use of the special numerical factor 8π on the right.

It is now readily seen that the same condition of orthogonality (4) leads to the following uniquely determined form for the gravitational force vector f^i:

$$f_i = \left(\frac{\partial h_{i\alpha}}{\partial x^\beta} - \frac{\partial h_{\alpha\beta}}{\partial x^i}\right)v^\alpha v^\beta \quad (7)$$

The theory has now been completely formulated for the limited form in which only gravitational forces enter. It may be extended so as to include electrical and atomic forces in a natural and interesting manner (see reference 1 and Section 11 below). In the limited but important form now under consideration, with use of the usual tensorial subscript and superscript notation, the theory is embodied in the pair of equations

$$\begin{aligned} \frac{\partial T^{i\alpha}}{\partial x^\alpha} &= g^{i\gamma}\left(\frac{\partial h_{\gamma\alpha}}{\partial x^\beta} - \frac{\partial h_{\alpha\beta}}{\partial x^\gamma}\right)v^\alpha v^\beta, \\ g^{\alpha\beta}\frac{\partial^2 h_{ij}}{\partial x^\alpha \partial x^\beta} &= 8\pi T_{ij}, \text{ or } 0 \text{ in empty space,} \end{aligned} \quad (8)$$

where T^{ij} is given by equation (1).

At this stage it is clear why it was natural to take the undetermined constant c in T^{ij} to be 0. Otherwise we should have had

$$T^{ij} = v^i v^j - {}^1/_2(v_\alpha v^\alpha)g^{ij} - cg^{ij} = \rho(u^i u^j - {}^1/_2 g^{ij}) - cg^{ij}$$

and we should obtain as the Poisson equation for the limiting case $\rho = 0$, inside of the fluid,

$$g^{\alpha\beta}\frac{\partial^2 h_{ij}}{\partial x^\alpha \partial x^\beta} = -8\pi c g_{ij},$$

which seems inappropriate unless c is 0.

On the basis of the theory thus obtained it is found that isolated bodies have a static, centrally symmetric distribution, and that a collection of spherical bodies move with very high approximation according to the Newtonian law of gravitation, relative to a convenient rest system.

7. Let us now turn to the case of a static, centrally symmetric distribution of the perfect fluid. It is not difficult to determine the radial distribution of the density ρ, but for our present purposes it suffices to observe that we have precisely (r, radial distance),

$$T^{ij} = {}^1/_2\rho(r)\delta_{ij}.$$

Our extended Poisson equation takes the form

$$\frac{\partial^2 h_{ij}}{\partial x^2} + \frac{\partial^2 h_{ij}}{\partial y^2} + \frac{\partial^2 h_{ij}}{\partial z^2} = -4\pi\rho(r)\delta_{ij}, \text{ or } 0 \text{ in empty space.}$$

Here, of course, the boundary of the (spherical) distribution occurs for some value r_0 of r where $\rho(r_0) = \rho_0$. Thus we find as the *exact* solution for the centrally symmetric case

$$h_{ij} = \frac{m}{r}\delta_{ij}$$

in empty space ($r > r_0$), involving, as the single arbitrary constant, the mass m of the fluid.

In this way we obtain the gravitational potentials around a static, centrally symmetric body like the Sun. These potentials are not observably affected by random atomic motion.

8. Now consider a comparatively small approximate sphere of the perfect fluid, forming in the limit a kind of ideal particle of mass 0. First, we have essentially $\rho = \rho_0$ throughout, so that we may write

$$\frac{\partial T^{i\alpha}}{\partial x^\alpha} = \rho_0\frac{\partial u^i u^\alpha}{\partial x^\alpha} = \rho_0 u^\alpha\frac{\partial u^i}{\partial x^\alpha}$$

with high approximation, since (4) yields with similar approximation

$$\frac{\partial u^\alpha}{\partial x^\alpha} = 0.$$

But we have

$$u^\alpha\frac{\partial u^i}{\partial x^\alpha} = \frac{du^i}{ds} = \frac{d^2x^i}{ds^2} \qquad (ds^2 = dt^2 - dx^2 - dy^2 - dz^2)$$

along the worldline of the ideal particle, so that we may write

$$\frac{\partial T^{2\alpha}}{\partial x^\alpha} = \rho_0 \frac{d^2x}{ds^2}, \quad \frac{\partial T^{3\alpha}}{\partial x^\alpha} = \rho_0 \frac{d^2y}{ds^2}, \quad \frac{\partial T^{4\alpha}}{\partial x^\alpha} = \rho_0 \frac{d^2z}{ds^2}.$$

Thus there are obtained the differential equations of motion for a comparatively small body which moves in the field of a larger central body. These lead to essentially the same result for the perihelial advance of a planet and for the bending of a light photon in the field of the Sun as does the theory of Einstein.

Furthermore, if we assume that the Planck formula

$$E = h\nu$$

determines the frequency of radiation ν, the result of Einstein for the "red shift" in light reaching the Earth from the Sun is obtained. Inasmuch as the precise mechanism of radiation is unknown, it seems more correct to employ this basic formula, than to give an explanation in which the light-carrying photon plays no rôle, such as is afforded by the Einstein theory.

9. A real test of the availability of the new theory in other directions is afforded by the problem of two or more bodies.

As a first approximation to this problem, it is natural to consider the limiting case of n "ideal particles" of masses $m_1, m_2 \ldots m_n$, respectively, obtained by taking the equilibrium density ρ_0 to be very large. It is clear that in the neighborhood of each particle P_l of mass m_l, the corresponding gravitational potentials should have a principal part

$$\frac{m_l}{r_l}\delta_{ij}$$

in instantaneous rest coördinates.

Graef has already shown (see references 3 and 4) that the calculations involved can be effectively carried out in the case of two bodies of masses m_1 and m_2. Presumably his method can be extended to the case of more than two bodies. Furthermore it should be possible to investigate in a similar spirit all of the fine-structure corrections to the Newtonian theory which lie within the limits of observation.

10. In order to generalize the theory so as to admit cosmological terms one has only to write the Poisson equation in the form

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right]h_{ij} = 8\pi T_{ij} + Kg_{ij}$$

where K is the small cosmological constant. This really means that we allow a form of energy tensor T^{ij} containing the term $-cg^{ij}$ previously indicated, with $c = -K/8\pi$ small but not 0.

The conditions at ∞ have then clearly to be lightened to the form that

h_{ij} becomes regularly infinite to the second order at infinity, in which case it is concluded that for a uniquely determined space-time origin we have

$$h_{ij} = h^*_{ij} + \frac{K}{8}(t^2 - x^2 - y^2 - z^2)g_{ij}$$

where h^*_{ij} satisfy the previously indicated form of the Poisson equation and boundary conditions. In this case there is obtained an expanding universe, about the space-time origin $t = x = y = z = 0$ in the flat space-time under consideration.

11. In the general case there is an electromagnetic 4-potential φ_i satisfying Maxwell's equations, and an atomic potential ψ constant along every worldline. This leads to the complete theory with (covariant) force components:

$$f_i = \rho\frac{\partial\psi}{\partial x^i} + \sigma\left(\frac{\partial\varphi_i}{\partial x^\alpha} - \frac{\partial\varphi_\alpha}{\partial x^i}\right)u^\alpha + \rho\left(\frac{\partial h_{i\alpha}}{\partial x^\beta} - \frac{\partial h_{\alpha\beta}}{\partial x^i}\right)u^\alpha u^\beta \qquad (9)$$

where the terms corresponding to atomic, electric and gravitational forces are homogeneous of degrees 0, 1 and 2, respectively, in the velocity vector, and are linear homogeneous in the first partial derivatives of the corresponding potentials ψ, φ^i and h^{ij}, respectively.

It will require further mathematical investigation in order to determine the serviceability of this conjectural theory in the domain of atomic physics. I have previously indicated how an equation much like the Schrödinger wave equation may be obtained on the basis of the atomic potential ψ.[8] However, my attempt was based on a background of curved space-time, and I had not then realized that the potentials must all be of zero dimensions, so that I used $\partial\psi/\partial x^i$ in place of $\rho\partial\psi/\partial x^i$.

12. Professors Barajas, Graef, Sandoval Vallarta and I are examining further these and other physical problems in the light of the new theory. Meanwhile, it seems clear that the theory promises well and deserves careful study because of its striking simplicity, completeness and mathematical consistency. Furthermore, as will be seen from what precedes, the theory is independent of all ideas of curved space-time and of the corresponding Einstein theory.

No doubt, in view of the substantial successes of the Einstein theory, it is worth while to attempt to reflect that theory on to flat space-time, and so to obtain a degenerate theory, which in a certain sense is only the shadow of a shadow. However, the objections made by Barajas[2] to the form of degenerate theory considered by Weyl[5] seem to be substantially valid.[9]

As far as I can see, the Einstein principle of equivalence ("that inertia and gravitation are one," Weyl, loc. cit.) is at bottom a mathematical principle signifying only that certain equations in the Einstein theory are linear homogeneous in the density of matter—a fact just as true of the theory of gravitation here proposed as of the general theory of relativity. To me there is only the following mathematical fact in the comparison of the basic points of view of the Einstein theory and my own: in his theory there is no underlying framework of independent variables t, x, y, z valid throughout space-time, such as are present in the theory based on flat space-time. The real question is whether or not the theory based on such special coördinates is simpler and more useful for the description and prediction of the physical facts. This is not a question to be decided by *a priori* considerations. What is required is rather a study of the new theory and its physical applications.

[1] Birkhoff, G. D., "Matter, Electricity and Gravitation in Flat Space-Time," these PROCEEDINGS, **29**, 231 (1943). My lecture at Tonanzintla is about to appear under the title "El Concepto de Tiempo y la Gravitacíon," *Boletín de la Sociedad Matemática Méxicana*, **1**, No. 4 (1944).

[2] Barajas, A., "Birkhoff's Theory of Gravitation and Einstein's for Weak Fields," these PROCEEDINGS, **30**, 54 (1944).

[3] Barajas, A., Birkhoff, G. D., Graef, C., and Sandoval Vallarta, M., "On Birkhoff's New Theory of Gravitation," *Physical Review*, **66**, 138 (1944).

[4] In vol. 1, No. 5 (1944) of the *Boletín de la Sociedad Matemática Méxicana*.

[5] Weyl, H., "Comparison of a Degenerate Form of Einstein's with Birkhoff's Theory of Gravitation," these PROCEEDINGS, **30**, 205 (1944).

[6] See for instance, Birkhoff, G. D., *Relativity and Modern Physics*, Chaps. VII and XI, Cambridge, 1923, 1927.

[7] The velocity of light c is 1 since the lightsecond is regarded as the unit of distance.

[8] Cf. two notes in these PROCEEDINGS, **13**, 160, 165 (1927).

[9] In his Note Barajas is more than fair to the "degenerate theory", which, strictly speaking, is no more usable than is the early Nordström theory. See M. Wyman, *Math. Rev.* **5**, 218 (1944). Since *all* of the relativistic theories of gravitation take the classical Newtonian theory as prototype, the formal resemblance between them is inevitably considerable. This fact is stressed, for example, in my article, *Newtonian and Other Forms of Gravitational Theory*, Scientific Monthly, **58**, 49 and 136 (1944). It is to be looked upon as the source of the formal resemblance between Einstein's general theory of relativity, based on curved space-time, and my own theory, based on flat space-time, which Weyl refers to.