

# A flat space-time relativistic explanation for the perihelion advance of Mercury

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## Abstract

Starting with the flat space-time relativistic versions of Maxwell-Heaviside's toy model vector theory of gravity and introducing the gravitational analogues for the electromagnetic Lienard-Wiechert potentials together with the notion of a gravitational Thomas Precession; the observed anomalous perihelion advance of Mercury's orbit is here explained as a relativistic effect in flat (Minkowski) space-time, unlike Einstein's curved space-time relativistic explanation. In this new explanation for the old paradoxical observation of Mercury's perihelion shift, the predicted value of the effect happens to coincide with Einstein's predicted value in General relativity.

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# 1 Introduction

The explanation of the anomalous perihelion shift of Mercury's orbit represents one of the most famous classical tests of General Relativity (GR)- the relativistic gravitation theory in curved space-time. This is the only one involving relativistic effects on massive bodies - all other classical tests of GR being based on light propagation effects only. The perihelion advance of Mercury had been an unsolved problem in celestial mechanics for over half a century, since the announcement by Leverrier in 1859 [1] that, after the perturbing effects of the precession of the equinoxes on the astronomical co-ordinate system had been subtracted, there remained in the data an unexplained advance in the perihelion of Mercury. The modern value of this discrepancy is 43 arc-seconds per century[2]. A number of *ad hoc* proposals were made in an attempt to account for this excess, including , among others, to postulate [3, 4, 5] a magnetic-type component in gravity ( the gravitomagnetic field) for the gravitational influence of the Sun on the motion of planets ( the magnitude of this *ad hoc* component could be adjusted so as to account for the excess perihelion motion of Mercury), the existence of a new planet Vulcan near the Sun, a ring of planetoids, a solar quadrupole moment and a deviation from the inverse-square law of gravitation, but none was successful. The great interest in this paradoxical result vanished in 1915 when Einstein in his GR showed that the excess advance of Mercury's perihelion could be explained as a relativistic effect in curved space-time. In the GR the non-Newtonian "excess" advance of perihelion of Mercury's orbit is explained by using space curvature and Schwartzschild metric. Since the special relativistic approach to the problem of perihelion advance in the Kepler motion could not yield the observed effect (the existing approaches yield only one sixth of Einstein's predicted value in GR, see for example [6, 7] ), the observation of this effect played a role of one of the seemingly successful tests of the GR. In this work we, by the way reporting the establishment[8] of a compatibility of Newtonian gravity with special relativity in a non-general-relativistic way, offer a flat space-time relativistic explanation for the observed anomalous perihelion advance of Mercury from a new angle. In this new approach to the old and general relativistically solved problem we make use of the notions of gravitational Lienard-Wiechert potentials (GLWP) and the gravitational Thomas precession(GTP) which naturally follow from some extended special relativistic considerations. To this end we devote Sec.2 to the notion of gravitational Lienard-Wiechert potential (GLWP). In Sec.3 we show that the GLWP together with the GTP invoked in [9] is sufficient to explain the observed perihelion advance of Mercury within the framework of a flat space-time relativistic toy model vector theory of gravity named as "Maxwellian Gravity" by the authors of [8].In Sec.4 we have some concluding remarks of interest for relativistic gravity.

## 2 The notion of GLWP

The notion of gravitational Lienard-Wiechert potential (GLWP) is the gravitational analogue of Lienard-Wiechert potential in classical electromagnetism. So to invoke the GLWP one has to establish a correspondence between the gravitation theory and electromagnetic theory. The close formal analogy between Newton's law of gravitation and Coulomb's law of electricity led many authors, in the past and also more recently, to investigate further similarities, such as the possibility that the motion of gravitational mass/charge could generate the analogs of a magnetic field. The magnetic field is produced by the motion of electric-charge, i.e. the electric current: the motion of gravitational mass/charge would produce what is called "gravitomagnetic" field. It is to be noted that such a correspondence had been proposed by J.C. Maxwell in 1865[10] and later further studied by Oliver Heaviside[11, 12] and O. Jefimenko[13]. In the linearized versions of Einstein's gravitational field equations, Maxwell-Lorentz-like equations for gravity have been obtained by several authors, see for examples[3, 14, 15, 16, 17, 18]. However from a closer look at these equations the reader can find that the linearized theory of GR is not perfectly isomorphic with electromagnetism which is commonly understood as a limitation of Linearized GR[17, 18, 19]. On the other hand there is no such limitation in the linear relativistic gravity developed in [8] under the name of "Maxwellian Gravity" (MG). In its formulation the relativistic nature of gravity and the source of gravity have been re-investigated by re-designing an often cited [20, 21] thought experiment [22] involving the motion of two point-like charged particles in Minkowski space-time. In the thought experiment we have considered a system of two point-like charged particles having such amount of rest masses and charges that the force of electrostatic repulsion balances that of gravitational attraction between the particles in an inertial frame of reference  $K'$  in which both the particles are at rest under equilibrium condition. Then we investigated in[8] the condition of equilibrium of the said particle system in  $K'$ -frame as well as in another inertial frame  $K$  moving with uniform velocity with respect to the  $K'$ -frame of reference. From the requirement of the frame-independence of the equilibrium conditions, we not only obtained a Lorentz-force law for gravitational interaction between the moving masses as expected [22] but also unexpectedly found the Lorentz-invariant rest masses of the interacting particles as representing their gravitational masses (or gravitational charges) in complete analogy with the Lorentz force law and the invariant electric charges of the classical electromagnetic theory. These findings are in conformity with Poincaré's[23] remark that *if equilibrium is to be a frame-independent condition, it is necessary for all forces of non-electromagnetic origin to have precisely the same transformation law as that of the Lorentz-force*. Having recognized these findings we have obtained four Faraday-Maxwell-type toy-model linear equations of gravity describing what we call "Maxwellian Gravity" following the known procedures of the electromagnetic theory. The equations

have a surprisingly rich and detailed correspondence with Faraday-Maxwell's field equations of the electro-magnetic theory. The field equations can be written in the following Faraday-Maxwellian-form:

$$\vec{\nabla} \cdot \vec{E}_g = -4\pi G\rho_0 = -\rho_0/\epsilon_{0g} \quad \text{where} \quad \epsilon_{0g} = 1/4\pi G \quad (1)$$

$$\vec{\nabla} \times \vec{B}_g = -\mu_{0g}\vec{j}_0 + (1/c^2)(\partial\vec{E}_g/\partial t), \quad \text{where} \quad \mu_{0g} = 4\pi G/c^2 \quad (2)$$

$$\vec{\nabla} \cdot \vec{B}_g = 0 \quad (3)$$

$$\vec{\nabla} \times \vec{E}_g = -\partial\vec{B}_g/\partial t \quad (4)$$

Where  $\rho_0$  = rest mass (or proper mass) density;  $\vec{j}_0$  = rest mass current density;  $G$  is Newton's universal gravitational constant;  $c$  is the speed of light in empty space; the gravito-electric and gravito-magnetic fields  $\vec{E}_g$  and  $\vec{B}_g$  respectively are defined by the gravitational Lorentz force on a test particle of rest mass  $m_0$  moving with uniform velocity  $\vec{u}$  as

$$\frac{d}{dt}[m_0\vec{u}/(1 - u^2/c^2)^{1/2}] = m_0[\vec{E}_g + \vec{u} \times \vec{B}_g] \quad (5)$$

where the symbols have their respective meanings in correspondence with the Lorentz force law in its relativistic form. It is to be noted that these set of gravitational Maxwell-Lorentz equations coincide with those speculated by Maxwell[10], Heaviside[11, 12] and discussed also by Peng[17] in the weak field and slow motion limit of Einstein's field equations. However the treatment made in [8] suggests the validity of these equations for relativistic speeds as well. In covariant formulation, introducing the space-time four vector  $x_\mu = (x, y, z, ict)$ , proper mass current density four vector  $j_\mu = (j_{0x}, j_{0y}, j_{0z}, ic\rho_0)$  and the second-rank antisymmetric gravitational field strength tensor

$$F_{\mu\nu} = \begin{pmatrix} 0 & B_{gz} & -B_{gy} & -iE_{gx}/c \\ -B_{gz} & 0 & B_{gx} & -iE_{gy}/c \\ B_{gy} & -B_{gx} & 0 & -E_{gz}/c \\ iE_{gx}/c & iE_{gy}/c & iE_{gz}/c & 0 \end{pmatrix} \quad (6)$$

The field equations (1-4) can now be represented by the following two equations:

$$\sum_{\nu} \partial F_{\mu\nu} / \partial x_{\nu} = -\mu_{0g} \vec{j}_{\mu}, \quad \text{where} \quad \mu_{0g} = 4\pi G/c^2 \quad (7)$$

$$\partial F_{\mu\nu} / \partial x_{\lambda} + \partial F_{\nu\lambda} / \partial x_{\mu} + \partial F_{\lambda\mu} / \partial x_{\nu} = 0 \quad (8)$$

while the gravitational Lorentz force law (5) assumes the form :

$$c^2(d^2x_{\mu}/ds^2) = F_{\mu\nu}(dx^{\nu}/ds) \quad (9)$$

The absence of the rest mass of the test particle in its co-variant equation of motion (9) in the external gravitational field  $F_{\mu\nu}$  describes clearly the universal nature of gravitational interaction in conformity with Galileo's empirical law of universality of free fall (UFF) in a uniform gravitational field in the relativistic case as well. The law of UFF is one of the most fascinating character of gravity. For the Newtonian theory, the universality of free fall is of course a consequence of the equality of inertial mass and gravitational mass. Being fascinated by the mystery of the UFF observed in Galileo-Newtonian physics, Einstein in his development of the GR postulated that it holds generally, in particular also for large velocities (the relativistic case) and strong fields and for any form of mass-energy (i.e. he postulated the equality of inertial mass and gravitational mass for the UFF to hold generally). It is to be carefully noted that the general validity of the UFF is now a natural consequence of Maxwellian Gravity without the requirement of an Einsteinian postulate on the equality of inertial and gravitational masses. This is one of the interesting theoretical revelations of MG exacting the real UFF. The origins of the UFF may be traced to the Lorentz-invariant nature of the gravitational mass-charge (i.e. the rest mass or any form of Lorentz-invariant mass-energy) and the laws of physics as dictated by special relativity.

The fields  $\vec{E}_g$  and  $\vec{B}_g$  of MG are derivable from potential functions

$$\vec{B}_g = \nabla \times \vec{A}_g, \quad \vec{E}_g = -\nabla \cdot \Phi_g - \partial \vec{A}_g / \partial t \quad (10)$$

where  $\Phi_g$  and  $\vec{A}_g$  represents respectively the gravitational scalar and vector potential of MG. These potentials satisfy the inhomogeneous wave equations :

$$\nabla^2 \cdot \Phi_g - \frac{1}{c^2} \cdot \frac{\partial^2 \Phi_g}{\partial t^2} = 4\pi G \rho_0 = \rho_0 / \epsilon_{0g} \quad (11)$$

$$\nabla^2 \cdot \vec{A}_g - \frac{1}{c^2} \cdot \frac{\partial^2 \vec{A}_g}{\partial t^2} = \frac{4\pi G}{c^2} \vec{j}_0 = \mu_{0g} \vec{j}_0 \quad (12)$$

if the gravitational Lorenz[24] gauge condition

$$\vec{\nabla} \cdot \vec{A}_g + \frac{1}{c^2} \frac{\partial \Phi_g}{\partial t} = 0 \quad (13)$$

is imposed. These will determine the generation of gravitational waves by prescribed gravitational charge and current distributions. Particular solutions (in vacuum) are

$$\Phi_g(\vec{r}, t) = -G \int \frac{\rho_0(\vec{r}', t')}{|\vec{r} - \vec{r}'|} dv' \quad (14)$$

$$\vec{A}_g(\vec{r}, t) = -\frac{G}{c^2} \int \frac{\vec{j}_0(\vec{r}', t')}{|\vec{r} - \vec{r}'|} dv' \quad (15)$$

where  $t' = t - |\vec{r} - \vec{r}'|/c$  is the retarded time. These are called the retarded potentials. Thus we saw that retardation in gravity is possible in Minkowski space-time in the same procedure as we adopt in electrodynamics. This result seems to conflict with the view [25] that Newtonian gravity is entirely static, retardation is not possible until the correction due to deviations from Minkowski space is considered. Now in analogy with the electromagnetic case we have here the Gravitational Lienard-Wiechert potentials for a point particle of rest mass  $m_0$  moving with velocity  $\vec{v}$  as

$$\Phi_g = - \frac{G m_0}{r (1 - \frac{v^2 \sin^2 \theta}{c^2})^{1/2}} \quad (16)$$

$$\vec{A}_g = - \frac{G m_0 \vec{v}}{c^2 r (1 - \frac{v^2 \sin^2 \theta}{c^2})^{1/2}} \quad (17)$$

where  $r$  is the magnitude of the instantaneous position vector  $\vec{r}$  of the field point from the position of the particle and  $\theta$  is the angle between  $\vec{r}$  and  $\vec{v}$  at the instant of time. Thus for a planet with rest mass  $m_0$  having relative velocity  $\vec{v}$  with respect to the the Sun (with rest mass  $M_\odot$ ) the instantaneous gravitational Lienard-Wiechert (LW) potential energy is given by

$$U_{gLW} = - \frac{G M_\odot m_0}{r (1 - \frac{v^2 \sin^2 \theta}{c^2})^{1/2}} \quad (18)$$

Considering the angular momentum of the planet as  $L = m_0 r v \sin \theta$  Eq.(18) can be re-written as

$$U_{gLW} = - \frac{G M_\odot m_0}{r (1 - \frac{L^2}{m_0^2 c^2 r^2})^{1/2}} \quad (19)$$

which in the first approximation can be reduced to

$$U_{gLW} = - \frac{G M_\odot m_0}{r} - \frac{G M_\odot L^2}{2 m_0 c^2 r^3} = - \frac{k}{r} - \frac{h_0}{r^3} \quad (20)$$

where

$$k = G M_\odot m_0 \quad \text{and} \quad h_0 = \frac{G M_\odot L^2}{2 m_0 c^2} \quad (21)$$

Thus we saw that the consideration of the GLWP introduced a  $1/r^3$  potential into the Kepler problem. Its effect on the planetary motion will be considered in the following section together with the GTP effect.

### 3 The GTP, GLWP and The perihelion advance

The Thomas precession[26, 27, 7] is purely kinematical in origin [27]. If a component of acceleration ( $\vec{a}$ ) exists perpendicular to the velocity  $\vec{v}$ , for whatever reason, then there is a Thomas Precession, independent of other effects [27]. When the acceleration is caused by a gravitational force field, the corresponding Thomas Precession is reasonably referred to as the Gravitational Thomas Precession (GTP). Given the physics involved in the Thomas Precession, the possibility of the existence of the GTP in planetary motion can not be ruled out in principle. The Thomas Precession frequency  $\vec{\omega}_T$  in the non-relativistic limit (i.e., when  $v \ll c$ ) is given by [7, 27]

$$\vec{\omega}_T = \frac{1}{2c^2}(\vec{a} \times \vec{v}), \quad (22)$$

where the symbols have their usual meanings. For a planet (say Mercury) moving around the Sun, the acceleration  $\vec{a}$  is predominately caused by the Newtonian gravitational field of the Sun, viz.,

$$\vec{a} = -\frac{GM_\odot}{r^3} \vec{r}, \quad (23)$$

where the symbols have their usual meanings. Thus, from Eqs.(22) and (23) we get the GTP frequency of the planet in question as

$$\vec{\omega}_{gT} = -\frac{GM_\odot}{2c^2 r^3} (\vec{r} \times \vec{v}), \quad (24)$$

where  $\vec{v}$  is the velocity of the planet. In terms of the angular momentum of the planet  $\vec{L} = m_0(\vec{r} \times \vec{v})$ , Eq.(24) can be re-written as

$$\vec{\omega}_{gT} = -\frac{GM_\odot}{2m_0 c^2 r^3} \vec{L}, \quad (25)$$

If, as Thomas first pointed out, that coordinate system rotates, then the total time rate of change of the angular momentum  $\vec{J}^\dagger$  or more generally, any vector  $\vec{A}$  is given by the well known result [7, 27],

$$\left(\frac{d\vec{A}}{dt}\right)_{\text{nonrot}} = \left(\frac{d\vec{A}}{dt}\right)_{\text{rest frame}} + \vec{\omega}_T \times \vec{A} \quad (26)$$

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<sup>†</sup>For Thomas  $\vec{J} = \vec{S}$ , the spin angular momentum; but here we consider a more general term  $\vec{J} = \vec{L} + \vec{S}$ ,  $\vec{J}$  representing the total (orbital + spin) angular momentum of the particle under consideration.

where  $\vec{\omega}_T$  is the angular velocity of rotation found by Thomas. When applied to the total angular momentum  $\vec{J}$ , Eq.(26) gives an equation of motion:

$$\left(\frac{d\vec{J}}{dt}\right)_{\text{non-rot}} = \left(\frac{d\vec{J}}{dt}\right)_{\text{rest frame}} + \vec{\omega}_T \times \vec{J} \quad (27)$$

The corresponding energy of interaction is

$$U = U_0 + \vec{J} \cdot \vec{\omega}_T = U_0 + \vec{L} \cdot \vec{\omega}_T + \vec{S} \cdot \vec{\omega}_T \quad (28)$$

where  $U_0$  is the energy corresponding to the coupling of  $\vec{J}$  to the external fields - say the Coulomb field in atomic case, nuclear field in nuclear case and the Newtonian gravitational field in the planetary case. The origin of the Thomas precessional frequency  $\vec{\omega}_T$  is the acceleration experienced by the particle as it moves under the action of external forces[27]. Since the nature of the external forces is not specified, the result obtained in Eq.(28) is valid for all type of force fields which cause accelerations of whatever nature. When applied to the gravitodynamic problems in solar system where the acceleration of a planet with respect to the Sun is predominately caused by a force arising out of the Newtonian scalar potential, Eq.(28) takes the form

$$U_g = U_{0g} + \vec{J} \cdot \vec{\omega}_T = U_{0g} + \vec{L} \cdot \vec{\omega}_{gT} + \vec{S} \cdot \vec{\omega}_{gT} \quad (29)$$

where  $U_{0g}$  is the Newtonian potential energy of the planet under consideration and  $\vec{\omega}_{gT}$  is given by Eq.(25). We then have

$$U_g = -\frac{k}{r} - \frac{h_1}{r^3} - \frac{h_2}{r^3} \quad (30)$$

where  $k = GM_\odot m_0$  and

$$h_1 = \frac{GM_\odot L^2}{2m_0 c^2} \quad (31)$$

$$h_2 = \frac{GM_\odot}{2m_0 c^2} (\vec{L} \cdot \vec{S}). \quad (32)$$

Thus we see the gravitational Thomas precession in the non-relativistic limit introduced two potentials of the form  $1/r^3$  into the classical Kepler problem. If in place of the  $U_{0g}$  in Eq.(29) we take the gravitational Lienard-Wiechert (LW) potential energy  $U_{gLW}$  given by Eq.(20), we would then have the following equation in place of Eq.(30) :

$$U_g = -\frac{k}{r} - \frac{h_0}{r^3} - \frac{h_1}{r^3} - \frac{h_2}{r^3} = \frac{k}{r} - \frac{h_{MG}}{r^3} \quad (33)$$

where

$$h_{MG} = h_0 + h_1 + h_2 = \frac{GM_{\odot}L^2}{m_0 c^2} \left[ 1 + \frac{(\vec{L} \cdot \vec{S})}{2L^2} \right]. \quad (34)$$

What effect will result from the introduction of the potential(s) of the form  $1/r^3$  into the Kepler problem? It is shown in [7] that if a potential with  $1/r^3$  form is added to a central force perturbation of the bound Kepler problem, the orbit in the bound problem is an ellipse in a rotating coordinate system. In effect the ellipse rotates, and the periapsis appears to precess. If the perturbation Hamiltonian is

$$\Delta H = -\frac{h}{r^3}, \quad (h = \text{some constant}) \quad (35)$$

then it predicts [7] a precession of the perihelion of a planet arising out of the perturbation Hamiltonian (of the form as in Eq.(35)) at an average rate of

$$\dot{\omega} = \frac{6\pi k m_0^2 h}{\tau L^4} \quad (36)$$

where  $k = GM_{\odot}m_0$  and  $\tau$  is the classical period of revolution of the planet around the sun. It is worth-noting from [7, 28] that the so-called Schwarzschild spherically symmetric solution of the Einstein field equations corresponds to an additional Hamiltonian in the Kepler problem of the form of Eq.(35) with

$$h = h_E = \frac{GM_{\odot}L^2}{m_0 c^2} \quad (37)$$

so that Eq.(36) becomes

$$\dot{\omega}_E = \frac{6\pi k^2}{\tau L^2 c^2} = \frac{6\pi GM_{\odot}}{\tau c^2 a (1 - e^2)} \quad (38)$$

where we have used the relation  $L^2 = GM_{\odot} m_0^2 a (1 - e^2)$ . Eq.(38) represents Einstein's expression for the anomalous perihelion advance of a planet's orbit. Likewise the contributions to the perihelion advance arising out of the GLWP and GTP in the framework of Maxwellian Gravity can be estimated by taking the  $h$  in Eq.(36) as

$$h = h_{MG} = \frac{GM_{\odot}L^2}{m_0 c^2} \left[ 1 + \frac{(\vec{L} \cdot \vec{S})}{2L^2} \right] = h_E \left[ 1 + \frac{(\vec{L} \cdot \vec{S})}{2L^2} \right]. \quad (39)$$

Then Maxwellian Gravity can predict the relativistic perihelion advance of a planet at

$$\dot{\omega}_{MG} = \dot{\omega}_E \left[ 1 + \frac{(\vec{L} \cdot \vec{S})}{2L^2} \right]. \quad (40)$$

For Mercury, the value of  $\dot{\omega}_E = 42 \cdot 98$  arc-seconds/century - a well known data [2, 7, 29, 30]. Hence the relativistic perihelion advance of Mercury's orbit in the flat space-time Maxwellian Gravity could be predicted at

$$\dot{\omega}_{MG} = 42 \cdot 98 \left[ 1 + \frac{(\vec{L} \cdot \vec{S})}{2L^2} \right] \text{arc-seconds/century} \quad (41)$$

The additional  $L - S$  term viz.,  $\frac{(\vec{L} \cdot \vec{S})}{2L^2}$ , that appears in Eq.(41) has a numerical value of the order of  $10^{-10}$  when the physical and orbital parameters of Mercury are used and is therefore utterly negligible. So by neglecting this  $L - S$  term we get

$$\dot{\omega}_{MG} = \dot{\omega}_E = 42 \cdot 98 \text{ arc-seconds/century for Mercury.} \quad (42)$$

## 4 Concluding Remarks

In this work we saw the possibility of explaining the observed anomalous advance of the perihelion of Mercury's orbit in flat space-time relativistic gravity. This new approach to the old gravitodynamic problem of perihelion advance may serve as a test of the validity of special relativity in the domain of gravitation. Again this also implies a test of the physics of Thomas Precession in gravitational phenomena. It is to be noted that forces and accelerations (of whatever origin) are well within the scope of special relativity (SR) because the SR in its entirety no where forbids one to study the force of gravity within its versatile scope. In this connection we would like to quote an important observation made by Denisov and Logunov[31]:

*“ ...,it must be noted that the literature not infrequently contains statements claiming that the special theory of relativity deals with the description of phenomena in the inertial reference frames, while the description of phenomena in non-inertial reference frames is the prerogative of the GTR.*

*These statements are wrong. .... . Because of this, it is quite conceivable to describe the physical phenomena either by the special theory of relativity or within non-inertial reference frames. This point was transparently clear to Fock [13]. ”*

By the way we remark that we are not proposing a new theory of gravity as MG. In our work on MG we only investigated some unexplored aspects of relativistic gravity in flat space-time and elevated the status of Maxwell-Heaviside's gravity to that of a test theory for testing the foundations of both special and general relativity. It is to be carefully noted that MG is now a natural outcome of some well established principles,theories and

methods of study in physics. Therefore the predictions of the MG may not be totally false. The theory might be working somewhere in some domain of physics yet unexplored. So we have to explore the situations where and when the MG was/is/might be operating in the evolution of the physical world. The authors make an appeal to the redears not to consider the MG as an alternative theory of gravity to the GR, because MG has to be made compatible with many other experimental data or observational results for its elevation to that status. So we now prefer MG to be treated as a toy model vector theory of gravity in flat space-time.

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