Therefore, the intersection lines of $\mathcal{P}$ and $\mathcal{C}$ are simply $\ell_1(\mathbf{V}, \delta_1)$ and $\ell_2(\mathbf{V}, \delta_2)$. Without loss of generality, assume $||\delta_1|| = ||\delta_2|| = 1$. Note that if $\mathcal{P}$ is tangent to $\mathcal{C}$, $\alpha = \theta$, and $\ell_1 = \ell_2$.

Finally, computing $\ell_1 \cap \ell$ and $\ell_2 \cap \ell$ yields the desired result. Determining the intersection point of two coplanar lines is not difficult. If δ_1 and $\mathbf{d}$ have the same or opposite direction (i.e., $\mathbf{d} \times \delta_1 = \mathbf{0}$, or equivalently $||\mathbf{d} \cdot \delta_1|| = 1$), ℓ_1 and ℓ are parallel to each other and there is no intersection point. Otherwise, there exist r and s such that $\mathbf{D} + r\mathbf{d} = \mathbf{V} + s\delta_1$. Since $\mathbf{g} \times \mathbf{g} = \mathbf{0}$ holds for any nonzero vector $\mathbf{g}$, computing the cross product with δ_1, the preceding formula gives

$$r\mathbf{d} \times \delta_1 = (\mathbf{V} - \mathbf{D}) \times \delta_1.$$

Computing the inner product with $\mathbf{d} \times \delta_1$ yields

$$r = \frac{[(\mathbf{V} - \mathbf{D}) \times \delta_1] \cdot (\mathbf{d} \times \delta_1)}{||\mathbf{d} \times \delta_1||^2}.$$

Thus, $\ell_1 \cap \ell$ is computed. Replacing δ_1 with δ_2 yields $\ell_2 \cap \ell$.

In practice, the computation for r could be simpler. Let $\pi_i(\mathbf{x})$ be the ith component of vector $\mathbf{x}$. Then

$$r = \frac{\pi_i((\mathbf{V} - \mathbf{D}) \times \delta_1)}{\pi_i(\mathbf{d} \times \delta_1)},$$

where $\pi_i(\mathbf{d} \times \delta_1)$ is a nonzero component of vector $\mathbf{d} \times \delta_1$.

Remark. Since a cylinder is a cone with its vertex at infinity, the algorithm presented here provides another way of computing the intersection of a line and a cylinder. In this case, $\mathcal{P}$ is the plane that is parallel to the cylinder axis and contains the given line, and $\mathcal{P} \cap \mathcal{C}$ degenerates to a pair of parallel lines. Consequently, the computation is reduced to computing the intersection points of this pair of lines with the given one.

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◇ Bibliography ◇

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