Let θ be the angle between $\mathbf{v}$ and $\mathcal{P}$ [Figure 1(a)]. By trichotomy exactly one of the following conditions is true:

- $\theta > \alpha$: $\mathcal{P} \cap \mathcal{C}$ is $\mathbf{V}$, and $\ell \cap \mathcal{C}$ is empty.
- $\theta = \alpha$: $\mathcal{P} \cap \mathcal{C}$ is the tangent line of $\mathcal{P}$ and $\mathcal{C}$, and $\ell \cap \mathcal{C}$ consists of at most one point.
- $\theta < \alpha$: $\mathcal{P} \cap \mathcal{C}$ consists of two lines, and $\ell \cap \mathcal{C}$ consists of at most two points.

However, using θ directly is not as efficient as using $\cos\theta$, since the latter can be obtained easily as follows. Let $\phi = \theta + 90°$ be the angle between $\mathbf{n}$ and $\mathbf{v}$ [Figure 1(b)]. Therefore, $\cos\phi = \mathbf{n} \cdot \mathbf{v}$ and

$$\cos\theta = \cos(\phi - 90°) = \sin\phi = (1 - \cos^2\phi)^{1/2} = (1 - (\mathbf{n} \cdot \mathbf{v})^2)^{1/2}.$$

Since the cosine function is monotonically decreasing between $0°$ and $90°$, $\cos(x) > \cos(y)$ if and only if $x < y$ for $0° \le x, y \le 90°$. Therefore, with $\cos\alpha$ and $\cos\theta$, tests $\theta > \alpha$, $\theta = \alpha$, and $\theta < \alpha$ can be replaced by $\cos\theta < \cos\alpha$, $\cos\theta = \cos\alpha$, and $\cos\theta > \cos\alpha$, respectively.

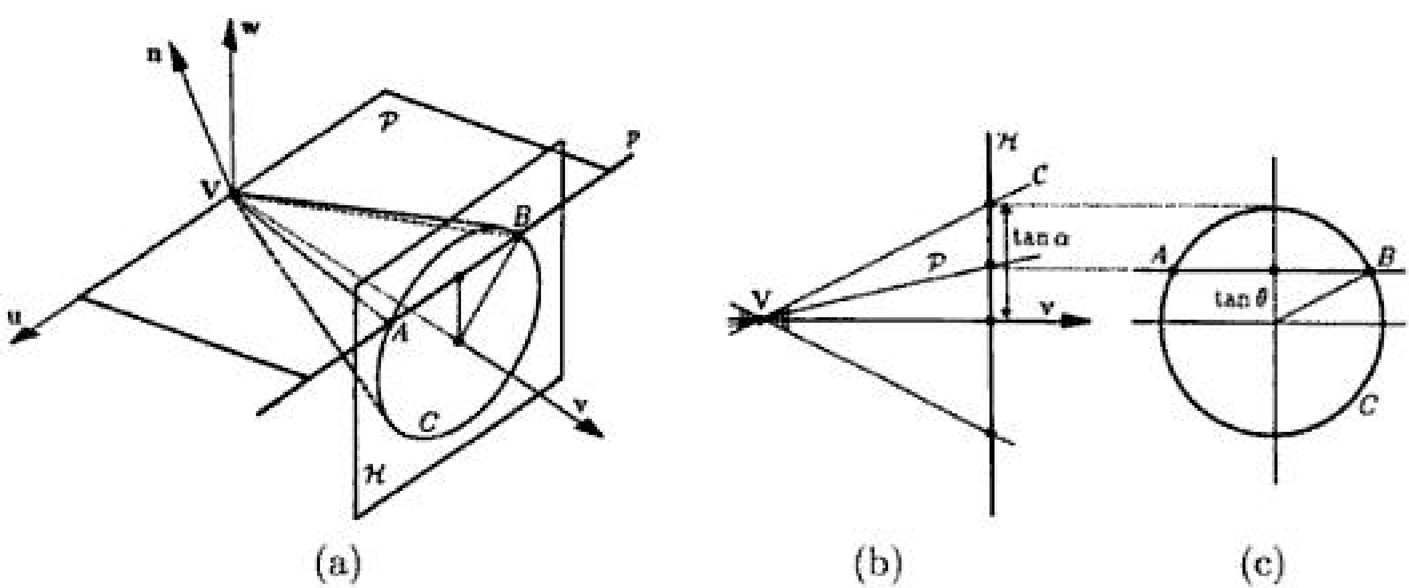

Figure 2. The *u*-*v*-*w* coordinate system and related information.