

On the Nature of Quantum Geometroynamics

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Classical gravitation, electromagnetism, charge, and mass are described in a preceding article in terms of curved empty space and nothing more. In advance of the detailed quantization of this pure Einstein-Maxwell geometrodynamics, an attempt is made here (1) to bring to light some of the most important properties to be expected for quantized geometrodynamics and (2) to assess whether this theory, without addition of any inventive elements, can contribute anything to the understanding of the elementary particle problem. Gravitational field fluctuations are concluded to have qualitatively new consequences at distances of the order of $(\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}$ cm. They lead one to expect the virtual creation and annihilation throughout all space of pairs with electric charges of the order $\sim(\hbar c)^{1/2}$ and energies of the order $(\hbar c^5/G)^{1/2} = (2.18 \times 10^{-5} \text{ g})c^2 = 2.4 \times 10^{22} mc^2$.

The problem is discussed, to what extent these charges can be identified with the unrenormalized or "undressed" charges of electron theory. Decisive for the future usefulness of quantum geometrodynamics is the question whether spin shows itself as an inevitable geometrical concomitant of quantization, or whether it and other ideas have to superposed on this purely geometrical description of nature.

Classical gravitation, electromagnetism, charge, and mass can be described in terms of the Rainich specialization of the Riemannian geometry of a curved empty space, and nothing more, according to an accompanying article (1). In this description we add nothing to the generally accepted equations of Maxwell and Einstein. We only recall that those equations, as first shown by Rainich, can be put into an "already unified form" where nothing but geometrical quantities make an appearance. We also abandon the tacit assumption made hitherto that space is simply connected, an assumption for which the Einstein-Maxwell equations themselves give no foundation. Then *classical* charge appears as the flux of lines of force trapped in a multiply connected metric. In this way we arrive at a description of classical physics which is extraordinarily far reaching. It is nevertheless purely geometrical and based throughout on the most firmly established principles of electromagnetism and general relativity. No changes are made in those principles nor are any free inventive elements added.

The charge and mass which appear in this analysis obey the inequality

$$m \geq G^{-1/2} q = (3.9 \times 10^3 \text{ g/esu})q, \quad (1)$$

are unquantized, and have no direct relation whatsoever to the quantized charge and mass of elementary particles.

What then is the relation between geometrodynamics and the world of particle physics?

To attempt to answer this question it would be conceivable to add to pure geometrodynamics electron fields, meson fields, neutrino fields, and other kinds of fields. One would then run into all the familiar difficulties of field theory and would be forced in addition to bring in coupling constants and characteristic mass values as primitive unexplained elements in physical theory. In this way one would have lost the features that distinguish geometrodynamics as a description of nature at the *classical* level—features that may be stated briefly as follows: (1) Space time is not an *arena* for physics, it is *all of classical* physics. (2) There are no “constants of nature” to be explained—neither c nor G . The velocity of light is only a factor of conversion between two historical units of distance, the light sec and the cm, just as 5280 is the factor of conversion between miles and feet. Similarly, the inertial properties of mass are expressed by a purely geometrical quantity, the Schwarzschild radius, which may be measured either in cm or an older unit, endowed like the mile with a name of its own, the gram. From the standpoint of general relativity the ratio between the two units of length, $G/c^2 = 0.74 \times 10^{-28}$ cm/g, is as accidental and historical in its origin as the number 5280. Lengths alone enter classical geometrodynamics. (3) There are no “coupling constants” and there are no independently existing fields to be coupled with each other. The electromagnetic field is not a new object; it is a construct from first derivatives of the Ricci curvature.

Shall it be claimed that these distinctive features of geometrodynamics ought to be abandoned? Is there anyone who knows enough about physics to say that the pattern established by Maxwell and Einstein is the wrong pattern for the description of nature? How can one demand that quantized fields and coupling constants must be added when no one has traced out the consequence of quantizing pure Einstein-Maxwell geometrodynamics as it stands? Let these questions motivate a close look at quantum geometrodynamics!

The passage from classical theory to quantum theory is direct, according to Feynman. The expression

$$\langle C_2\sigma_2 | C_1\sigma_1 \rangle = \sum_H S \exp (iI_H/\hbar) \quad (2)$$

gives the key quantity needed to evaluate all physically meaningful magnitudes: the probability amplitude to transit from a configuration C_1 on the space like surface σ_1 to C_2 on σ_2 . Here H is any history of the system between σ_1 and σ_2 that has as boundary values the configuration C_1 and C_2 . The quantity I_H is the classical action associated with that history. The symbol S denotes a summation over all histories, classically allowed or not, with equal weighting for each, and with such a normalization that the propagator (2) is unitary.

To say that all of quantum geometrodynamics is contained in principle in the prescription (2), is not the same as translating this prescription into practice! On this substantial task Misner has made a decisive beginning (2).¹ Before a proper treatment is completed one can make order of magnitude estimates along lines familiar from quantum electrodynamics: The phase in the Feynman-Huygens exponent can be written qualitatively in the form

$$I_{\hbar}/\hbar \sim \int [(c^3/8\pi\hbar G)(\partial g/\partial x)^2 + (1/8\pi\hbar c)(\partial A/\partial x)^2](-g)^{1/2} d^4x. \quad (3)$$

Consider the change in this integral due to alterations Δg in a typical component, $g_{\mu\nu}$, of the metric, and ΔA in a typical component, A_μ , of the electromagnetic potential, over a space-cotime region with dimensions of the order $L \times L \times L \times L$:

$$\Delta(\text{phase}) = \Delta I/\hbar \sim (c^3/\hbar G)L^2(\Delta g)^2 + (1/\hbar c)L^2(\Delta A)^2. \quad (4)$$

We conclude that field variations over such regions contribute to the sum over histories without significant destructive interference ($\Delta(\text{phase}) \sim 1$ radian) only when they are of the following order of magnitude or less (Table I). One arrives

TABLE I

ORDER OF MAGNITUDE OF FIELD FLUCTUATIONS. HERE L^* IS AN ABBREVIATION FOR THE QUANTITY $(\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}$ cm.

	Electromagnetic	Gravitational
Potential	$\Delta A \sim (\hbar c)^{1/2}/L$	$\Delta g \sim L^*/L$
Field	$\Delta F \sim (\hbar c)^{1/2}/L^2$	$\Delta(\text{acceleration}) \sim c^2 L^*/L^2$
Space curvature	...	$\Delta R \sim L^*/L^3$

at the same estimates by considering a quantum of energy confined to a region of the order L . The energy is of the order $\hbar c/L$, the energy density of the order $\hbar c/L^4$, and thus the fields of the order $(\hbar c)^{1/2}/L^2$, with or without appropriate dimensional factors for transformation into familiar units.

Both the fluctuations in the metric and the static alterations in the metric are fantastically small at atomic distances and even at the characteristic localization distance, $L_m = \hbar/mc$, from an electron:

$$\begin{aligned} \Delta g_{\text{static}} &\sim 2Gm/c^2 L_m \sim (L^*/L_m)^2 \sim 10^{-44}, \\ \Delta g_{\text{fluct}} &\sim L^*/L \sim 10^{-22}. \end{aligned} \quad (5)$$

¹ See Everett (3) for the method of application of the quantum principle to a system like that of geometrodynamics, which is self contained, not subject to external observation.

Moreover, the static field does not increase by orders of magnitude as one goes to orders of magnitude smaller distances, due to the lack of localizability of the energy and mass of an electron. In contrast, the calculated fluctuations in the metric are unaffected by proximity to a particle and, like electromagnetic field fluctuations, have only to do with the scale of the region of observation. On an atomic scale the metric appears flat, as does the ocean to an aviator far above. The closer the approach, the greater the degree of irregularity. Finally, at distances of the order L^* , the fluctuations in the typical metric component, $g_{\mu\nu}$, become of the same order as the $g_{\mu\mu}$ themselves. Then the character of the space undergoes an essential change, as indicated schematically by Fig. 1. Multiple connectedness develops, as it does on the surface of an ocean where waves are breaking. Of course it is not necessary to propose a method to observe these fluctuations in order to note how directly and inescapably they follow from the quantum theory of the metric. One does not have to use the word "observation" at all. One can say simply: (1) the propagator is expressed as a sum over his-

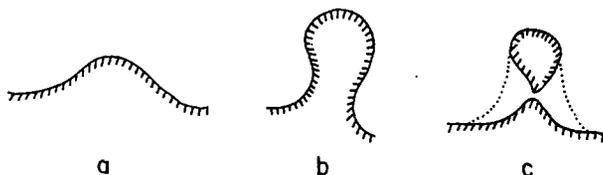


FIG. 1. Effect of a local fluctuation in the metric in introducing a multiple connectedness. In (c) the upper region of space is not disconnected from the lower one, as might appear from a single cross section of the space; hence the dotted lines to symbolize the connection between the two regions that would appear if the cut were made in another place.

tories (2) a history is a sequence of configurations, and (3) the type of configuration symbolized by Fig. 1 contributes importantly to the sum over histories. The word "fluctuation" is then only a shorthand language to speak about the configurations that contribute most to the sum over histories.

Along with the fluctuations in the metric there occur fluctuations in the electromagnetic field. In consequence the typical multiply connected space, such as that in Fig. 1(c), has a net flux of electric lines of force passing through the "wormhole". These lines are trapped by the topology of the space. These lines give the appearance of a positive charge at one end of the wormhole and a negative charge at the other.

For an estimate of the typical charge associated with a wormhole, denote the size scale of the wormhole by L . Then the area over which the flux passes is of the order L^2 . The fluctuation field is of the order $(\hbar c)^{1/2}/L^2$. Consequently the integrated flux and the charge are of the order

$$q_{\text{fluct}} \sim (\hbar c)^{1/2} \sim 12e. \tag{6}$$

independent of the size of the wormhole.

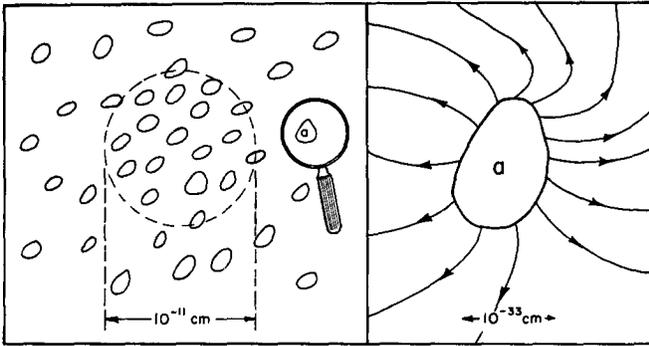


FIG. 2. Slice at a constant time through a field history of the kind that will contribute heavily to the sum over histories in the Feynman propagator. The small circles indicate wormhole mouths associated with the multiple connectedness of the space. The typical wormhole is connected with a fluctuation like that illustrated in Fig. 1, and has dimensions of the order $(\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}$ cm, far smaller than any dimension with any direct relevance to the elementary particle problem, $\hbar/mc = 3.9 \times 10^{-11}$ cm. Associated with a typical wormhole such as *a* is not only a great curvature of space, but also a fluctuation electromagnetic field, as indicated by the lines of force in the enlarged view of *a* at the right.

The existence of such a pair of charges is forced on one by the most elementary considerations of quantum theory and Maxwell-Einstein geometrodynamics. However, such a charge has no direct relation whatsoever to the charge of an elementary particle. (1) The typical charge is one order of magnitude greater than the elementary quantum of charge. (2) This charge is not quantized. On the contrary, configurations have to be taken into account in the sum over histories for which this charge has all values. However, contributions to this sum cancel out by destructive interference when the charge exceeds in order of magnitude the q_{fluct} of Eq. (6). (3) The mass of the electromagnetic field associated with one such wormhole of typical dimension $L \sim L^*$ is of the order

$$c^{-2}E \sim c^{-2}(\Delta F)^2 L^3 \sim \hbar/cL \sim (\hbar c/G)^{1/2} = 2.2 \times 10^{-5} \text{ g}, \tag{7}$$

completely incompatible with the masses of elementary particles. (4) Most important of all, these fluctuation charges are not a property of particles, they are a property of all space.²

Figure 2 gives a symbolic representation of the typical configuration that contributes heavily to the sum over histories. In this configuration, wormhole mouths occur everywhere, with typical spacings and typical dimensions of the order of the characteristic length L^* . The enlargement of one wormhole at the

² The structure discussed here and previously (4) is to be contrasted with the "Swiss-cheese structure" described by Belinfante (5), which is due solely to preexisting "real masses" such as are not envisaged here.

right shows lines of force issuing from it, such as likewise emerge from or converge onto every other typical wormhole mouth. This situation is appropriately described in the following words: (1) Quantization of the physics of Maxwell and Einstein forces on space a foam-like structure. Space has not only a macrocurvature on the scale of the universe, but also a microcurvature on the scale L^* . (2) In the vacuum, virtual pairs of charges are being continually created and annihilated. (3) With these pairs are associated charges and especially electromagnetic masses far larger than anything familiar from the elementary particle problem.

The energy density of the vacuum as just described appears at first sight to be completely unreasonable. Multiplying the mass-energy per typical wormhole, \hbar/cL^* , by a number of virtual pairs per unit volume of the order $1/L^{*3}$, one comes to a mass density so great,

$$\rho \sim \hbar/cL^{*4} = c^5/\hbar G^2 = (2.2 \times 10^{-5} \text{ g})/(1.6 \times 10^{-33} \text{ cm})^3 = 5 \times 10^{93} \text{ g/cm}^3, \quad (8)$$

that even a Compton wavelength includes more than the total estimated mass of the universe:

$$\begin{aligned} \rho \cdot L_m^3 &= 5.0 \times 10^{93} (3.87 \times 10^{-11})^3 = 2.9 \times 10^{62} \text{ g;} \\ M_{\text{universe}} &\sim c^2 R_{\text{univ}}/G \sim (1.35 \times 10^{28} \text{ g/cm}) \\ &(0.94 \times 10^{18} \text{ cm/yr}) 5 \times 10^9 \text{ yr} \sim 6 \times 10^{55} \text{ g.} \end{aligned} \quad (9)$$

This difficulty is not a new one. It is the problem of the zero point energy of the electromagnetic field. Let one sum the energy of the typical field oscillator, $\frac{1}{2}\hbar\omega$, over all frequencies down to a wavelength, $c/\omega \sim L^*$, where the microcurvature of space completely alters the character of the field oscillators. Then one obtains the mass-energy of (8). It is customary to try, not to solve this problem, but to side step it. One subtracts off the zero point energy. This procedure is legitimate in special relativity. However, in general relativity there is no disposable additive constant of energy and this procedure is not justified. Moreover, the subtracted term is not a constant, but depends upon the curvature of space.

We have neglected up to now any gravitational contribution to the density of energy and mass. However, the electromagnetic fields in the typical wormhole fluctuation are very intense. They have gravitational interactions that cannot be neglected. The typical electromagnetic mass energy for one wormhole is

$$m_1 \sim (\hbar c/G)^{1/2}, \quad (10)$$

and the typical separation of two such field disturbances is of the order $L^* \sim (\hbar G/c^3)^{1/2}$. From these estimates it follows that the gravitational energy of interaction of two nearby wormholes is of the order

$$E_{\text{grav}} \sim -Gm_1^2/L^*. \quad (11)$$

There is a resultant decrease in the overall mass of the pair of neighboring wormholes,

$$m_{\text{grav}} = E_{\text{grav}}/c^2 \sim -(\hbar c/G)^{1/2}, \quad (12)$$

which is of the same order as the positive electromagnetic masses of those two concentrations of energy. In other words, circumstances are favorable for the local *compensation* of electromagnetic energy by gravitational energy. Moreover, to the extent that this compensation holds locally, nearby wormholes exert no gravitational attraction on remote concentrations of mass-energy. The possibility is open to have a vacuum state with zero net energy density. In any case we conclude that *it is essential to allow for fluctuations in the metric and gravitational interactions in any proper treatment of the compensation problem*—the problem of compensation of “infinite” energies that is so central to the physics of fields and particles.³

The theory of general relativity tells one that it is not possible to give a well-defined meaning to *the energy of the electromagnetic field in a curved space* (6). Only the sum of the energy of the electromagnetic field and the gravitational energy has a well-defined meaning in geometrodynamics, and then only when the space is asymptotically flat, or possibly also when it is closed (7). This circumstance strengthens the conclusion that gravitation cannot be overlooked in any satisfactory account of the zero energy density of the vacuum.

We do not pretend to have shown that quantum geometrodynamics solves the compensation problem within its sphere of applicability. That point can only be tested by a detailed investigation, for which the tools are still only in course of manufacture (8).¹

If this energy compensation shall be established, however, then we will have a divergence-free quantum theory of the continuum such as has never before been in our grasp.

Is this continuum picture plainly incompatible with the world of elementary particle physics? Let us begin with electrons. In electron theory one distinguishes between the mass and charge of the “undressed electron” and the mass and charge of the experimental electron. The factors of conversion from one quantity to the other are divergent, but only logarithmically divergent. Because of this divergence it is sometimes claimed that only the renormalized theory has any physical significance. However, if the continuum picture makes sense, then it is reasonable to believe that the factors of conversion are not infinite, but only very large. When the logarithm of the ratio of two wave numbers appears as the argument of the logarithm, we shall assume that the upper limit is not infinity, but L^{*-1} , because at such wave numbers the micromultiple connectedness of space makes an essential change in the conventional analysis. Such a logarithm appears

³ As especially emphasized by Niels Bohr in lectures and discussions.

in the leading term of the standard expression for the electromagnetic self energy or self mass, δm , of the electron (8):

$$\delta m/m \sim (3/2\pi) (e^2/\hbar c) \ln (k_{\max}/k_m). \tag{13}$$

We will expect that k_{\max} should be set equal to a quantity of the order L^{*-1} , and that we should add to (13) an expression of a different mathematical form to represent the contribution to the energy due to wave numbers from k_{\max} to ∞ . In view of the orders of magnitude,

$$k_{\max}/k_m \sim 10^{33}/10^{11} \sim e^{50}, \tag{14}$$

we cannot say that Eq. (13) is incompatible with the long-advocated view of Lorentz, that all of the energy of the electron is of electromagnetic origin.

We are therefore led to consider the view that the electron is nothing but a collective state of excitation of the foam-like medium symbolized by Fig. 2. This collective disturbance is suggested in Fig. 2 by the slightly closer spacing of the wormholes within the dashed circle. The fractional increase in the concentration of electromagnetic mass-energy within the electron,

$$m/L_m^3 \sim m^4 c^3/\hbar^3 = 1.57 \times 10^4 \text{ g/cm}^3, \tag{15}$$

is fantastically small compared to the concentration of electromagnetic energy already present in the vacuum, $5 \times 10^{98} \text{ g/cm}^3$ [Eq. (8)]. In other words the electron is not a natural starting point for the description of nature, according to the present reinterpretation of the views of Lorentz. Instead it is a first order correction to vacuum physics. That vacuum, that zero order state of affairs, with its enormous concentrations of electromagnetic energy and multiply-connected topologies, has to be described properly before one has the starting point for a proper perturbation theoretic development. On this view it is a marvellous achievement of subtraction physics that it can deal with such a wide range of questions as it does without having at its disposal a convenient theory of the charge and mass of the electron. The accuracy of its predictions, as well as the divergence of its foundations, are attributed on the present view to the smallness of the characteristic lengths, $L \sim L^*$, of vacuum physics, compared to the lengths, $L \sim L_m$, of electron physics.

The charge of the experimental electron is small compared with the “undressed charge”, according to well-known considerations from the quantum theory of the electron. Despite appearances, this result does not contradict the atomicity of electric charge even when a unique value is assigned to the “undressed charge”. The theory assigns to the experimental charge something of the character of an expectation value that neither needs to be nor is an integral multiple of the undressed charge. Consequently there is no obvious con-

TABLE II

DISCREPANCY BETWEEN OBSERVATION AND ELEMENTARY EXPECTATIONS DERIVED FROM THE MOST OBVIOUS UNITS OF LENGTH AND ENERGY (1) FOR ELECTRON PHYSICS (2) FOR SUPERCONDUCTIVITY

	Observation <i>versus</i> First expectation	
	Electron physics	Superconductivity
Distance discrepancy	10^{-11} to 10^{-13} cm <i>vs</i> 10^{-33} cm	10^{-3} cm <i>vs</i> 10^{-8} cm
Discrepancy in mass-energy	10^{-27} g <i>vs</i> 10^{-5} g	10^{-3} to 10^{-4} ev <i>vs</i> 10 ev

tradition between a quantized charge e for quantized collective disturbances and an unquantized charge of the order $q_{\text{fluct}} \sim (\hbar c)^{1/2}$ for the undressed charges.

In the transcription of the idea of Lorentz that we are trying to assess, the electron mass energy is viewed as the lowest characteristic energy for a stable collective disturbance in the Einstein-Maxwell field. Why should this mass energy be so small compared to the only characteristic mass energy that has so far appeared in quantum geometrodynamics, $m_1 = (\hbar c/G)^{1/2} = 2.18 \times 10^{-5}$ g? In Table II we put this question into juxtaposition with another puzzle, the origin of the phenomenon of superconductivity. The evidence indicates that superconductivity is a collective phenomenon associated with weak residual interactions (9). Obviously there is no reason to expect any direct analogy between collective disturbances in the Einstein-Maxwell field and those that take place in a crystal lattice. Let it nevertheless be insisted that one shall leave no stone unturned in searching for special phenomena associated with the propagation of energy through space. Then one is led to consider the relation between effective frequency and effective wave number (Fig. 3), a relation that would be linear in flat space. However, disturbances of very short wavelength, $\sim 10^{-33}$ cm, will feel out the microcurvature of space and will not propagate normally. The same will be true of disturbances of very long wavelength, comparable to the radius of the universe. Consequently a disturbance that is originally localized will be dispersed. However, if the dispersion curve possesses a point of inflection, as indicated in Fig. 3, then disturbances made out of wavelengths near the point of inflection will keep together for a long time in the linear approximation, and could even be imagined to remain completely stable when one goes beyond the elementary superposition approximation. It is easy to write down a mathematical expression for a dispersion curve which will put the point of inflection at wave numbers of the order of 10^{11} to 10^{13} cm^{-1} , such as are characteristic of electron physics. There is no point in investigating such ideas now. We are trying here to find out, not whether pure quantum geometrodynamics can account for elementary particle physics, but whether there is some way to prove that it *cannot*. We see

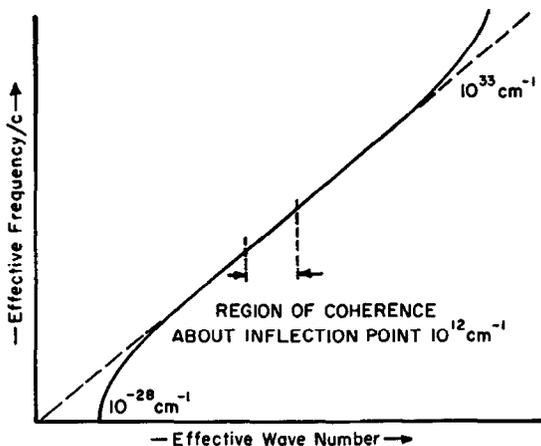


FIG. 3. Highly schematic representation of the nature of propagation of disturbances in a space which possesses both a macrocurvature on the scale of the universe and a microcurvature on the scale of the elementary length $(\hbar G/c^3)^{1/2}$ that is characteristic of gravitational fluctuations.

no simple way short of detailed analysis, to disprove the possibility that the masses of electrons and other elementary particles correspond to characteristic states of excitation of collective disturbances in the metric, as symbolized by Fig. 2.

What about nuclear forces? Will two localized collective disturbances interact via pure electromagnetic and gravitational field fluctuations with enough strength to account for the binding of two nucleons? In this connection one recalls that the observed strength of this interaction is only one or two orders of magnitude greater than ordinary electrostatic interactions. One also recalls that chemical forces between atoms were long thought to be different in origin from electrical forces until the connection was recognized, thanks not least to G. N. Lewis and P. Debye. It appears most difficult to rule out in advance a purely geometrodynamical account of nuclear forces.

Having touched on the orders of magnitude of particle masses, and of nuclear forces, we come to the final and crucial point: spin. How can a classical theory endowed with fields of integral spin possibly give on quantization a spin $\frac{1}{2}$ such as is required to account for the properties of the neutrino, the electron, and other particles? From the beginning Pauli referred to spin as a "nonclassical two valuedness". Is there anything about the process of formulating Feynman's sum over histories, anything about two choices for the orientation of each elementary space-time volume, or any other feature, that forces the introduction of any such nonclassical two-valuedness? Unless there is, *pure quantum geometrodynamics must be judged deficient as a basis for elementary particle physics*. There-

fore the question of the origin of spin is decisive for the assessment of quantum geometrodynamics.

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