Geon black holes and quantum field theory

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Abstract. Black hole spacetimes that are topological geons in the sense of Sorkin can be constructed by taking a quotient of a stationary black hole that has a bifurcate Killing horizon. We discuss the geometric properties of these geon black holes and the Hawking-Unruh effect on them. We in particular show how correlations in the Hawking-Unruh effect reveal to an exterior observer features of the geometry that are classically confined to the regions behind the horizons.

1. Introduction

A geon, short for "gravitational-electromagnetic entity", was introduced in 1955 by John Archibald Wheeler [1] as a configuration of the gravitational field, possibly coupled to other zero-mass fields such as massless neutrinos [2] or the electromagnetic field [3], such that a distant observer sees the curvature to be concentrated in a central region with persistent large scale features. The configuration was required to be asymptotically flat [4], allowing the mass to be defined by what are now known as Arnowitt-Deser-Misner methods. The examples discussed in [5] indicate that the configuration was also understood to have spatial topology $\mathbb{R}^3$, excluding black holes. These geons are however believed to be unstable, owing to the tendency of massless fields either to disperse to infinity or to collapse into a black hole [6].

In 1985, Sorkin [7] generalised Wheeler's geon into a *topological geon* by allowing the spatial topology to be nontrivial, as well as dropping the a priori requirement of persistent large scale features. The central regions may thus have complicated topology, as illustrated in Figure 1, and the time evolution of the initial data may lead into singularities and black holes [8, 9]. In particular, topological geons include quotients of stationary black hole spacetimes with a bifurcate Killing horizon, such that the two exteriors separated by the Killing horizon become identified [10, 11]. Such geon black holes are an intermediate case between conventional
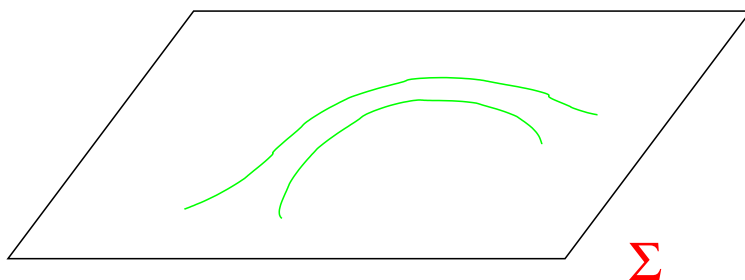
Figure 1. A spatial slice Σ of a topological geon. The geometry of Σ is asymptotically Euclidean, but the topology of Σ does not need to be $\mathbb{R}^3$, as illustrated by the handle drawn.

stationary black holes and dynamical black holes, in the sense that the nonstationary features are confined behind the horizons.

As the nontrivial spatial topology in a geon black hole is present since arbitrarily early times, one does not expect an astrophysical star collapse to result into a geon black hole. Instead, the interest of geon black holes is in their quantum mechanical properties. Because the Killing horizon now does not separate two causally disconnected exteriors, it is not possible to arrive at a thermal density matrix for a quantum field by the usual procedure of tracing over the second exterior. Nevertheless, as we shall discuss, a quantum field on a geon black hole does exhibit thermality in the standard Hawking temperature, although only for a restricted set of observations. We shall in particular see how the quantum correlations in the Hawking-Unruh effect reveal to an exterior observer features of the geometry that are confined to the regions behind the horizons.

We start by introducing in Section 2 the showcase example, the $\mathbb{RP}^3$ geon, formed as a quotient of Kruskal. Section 3 discusses a number of generalisations, including geons with angular momenta and gauge charges. Thermal effects in quantum field theory on geons are addressed in Section 4. Section 5 concludes by discussing the prospects of computing the entropy of a geon from a quantum theory of gravity.

2. Showcase example: $\mathbb{RP}^3$ geon

Before the Kruskal-Szekeres extension of the Schwarzschild solution was known, it had been observed that the time-symmetric initial data for exterior Schwarzschild can be extended into a time-symmetric wormhole initial data, connecting two asymptotically flat infinities, illustrated in Figure 2. Misner and Wheeler [5] noted that this wormhole admits an involutive freely-acting isometry that consists of the radial reflection about the wormhole throat composed with the antipodal map on the S^2 of spherical symmetry. The $\mathbb{Z}_2$ quotient of the wormhole by this isometry, illustrated in Figure 3, is hence a new time-symmetric spherically symmetric initial data for an Einstein spacetime.

The new time-symmetric initial data has topology $\mathbb{RP}^3 \setminus \{\text{point}\}$, the omitted point being at the asymptotically flat infinity [12]. The time evolution is a topological geon spacetime, called the $\mathbb{RP}^3$ geon [10], and it can be obtained as a $\mathbb{Z}_2$ quotient of Kruskal as shown in Figures 4 and 5. A general discussion of quotients of Kruskal can be found in [13].

The $\mathbb{RP}^3$ geon is a black and white hole spacetime, it is spherically symmetric, and it is time and space orientable. It inherits from Kruskal a black hole singularity and a white hole singularity, but the quotient has introduced no new singularities. The distinctive feature is that is has only one exterior region, isometric to exterior Schwarzschild.

As is well known, Kruskal admits a one-parameter group of isometries that coincide with Schwarzschild time translations in the two exteriors. These isometries do however not induce globally-defined isometries on the $\mathbb{RP}^3$ geon, because the quotienting map changes the sign of the corresponding Killing vector, as shown in Figures 6 and 7. As a consequence the geon exterior has a distinguished surface of constant Schwarzschild time, shown in Figure 5, even though one needs to probe the geon geometry beyond the exterior to identify this surface. The existence of this distinguished spacelike surface will turn out significant with the Hawking-Unruh effect in Section 4.

3. Other geons
3.1. Static
The quotienting of Kruskal into the $\mathbb{RP}^3$ geon generalises immediately to the higher-dimensional spherically symmetric generalisation of Schwarzschild [14]. As the antipodal map on a sphere preserves the sphere's orientation in odd dimensions but reverses the sphere's orientation in even dimensions, the geon is space orientable precisely when the spacetime dimension is even.
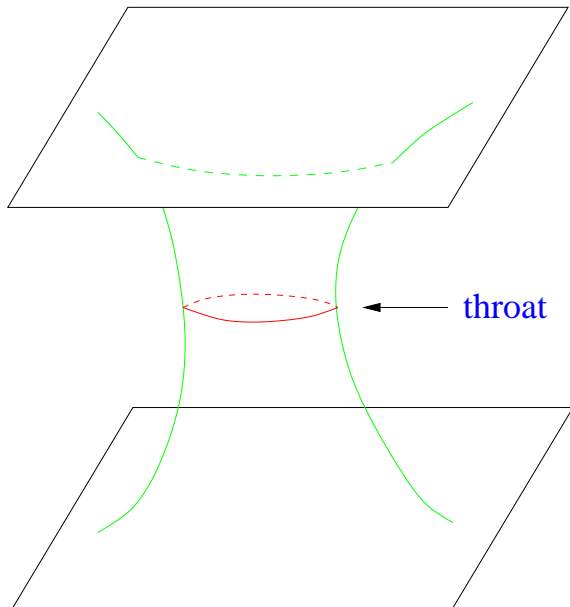
Figure 2. Time-symmetric wormhole initial data for Kruskal manifold, with the radial dimension drawn vertical and the two-spheres of spherical symmetry drawn as horizontal circles. The wormhole connects two asymptotically flat infinities. It admits a freely-acting involutive isometry that consists of the radial reflection about the wormhole throat composed with the antipodal map on the S^2.
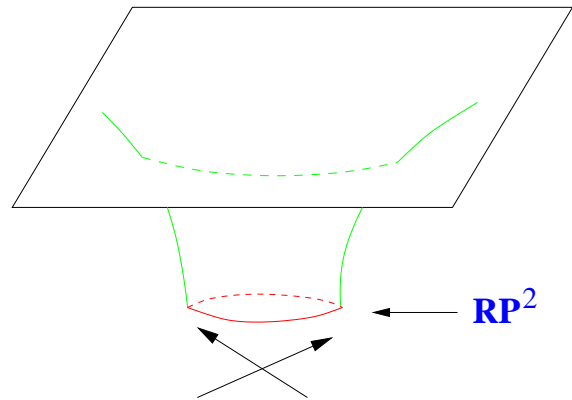

Figure 3. Time-symmetric initial data for the $\mathbb{RP}^3$ geon, obtained as the $\mathbb{Z}_2$ quotient of the wormhole by the freely-acting involutive isometry. The topology of the wormhole is $S^2 \times \mathbb{R} \simeq S^3 \setminus \{2 \text{ points}\}$ and that of the quotient is $\left(S^3 \setminus \{2 \text{ points}\}\right)/\mathbb{Z}_2 \simeq \mathbb{RP}^3 \setminus \{\text{point}\}$, each of the omitted points being at an asymptotically flat infinity.
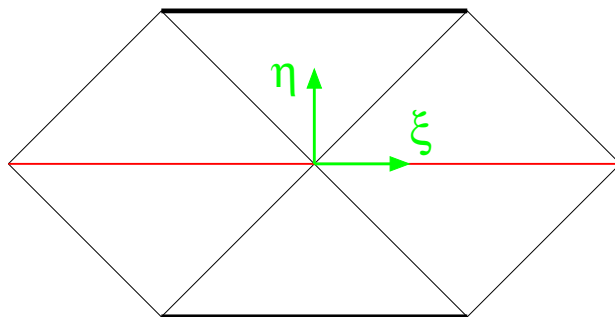

Figure 4. Conformal diagram of Kruskal manifold, with the S^2 orbits of spherical symmetry suppressed and the Kruskal time and space coordinates (η, ξ) shown. The wormhole of Figure 2 is the horizontal line $\eta = 0$. The freely-acting involutive isometry that preserves time orientation and restricts to that of the wormhole is $J_K : (\eta, \xi, \theta, \varphi) \mapsto (\eta, -\xi, P(\theta, \varphi))$, where P is the S^2 antipodal map.
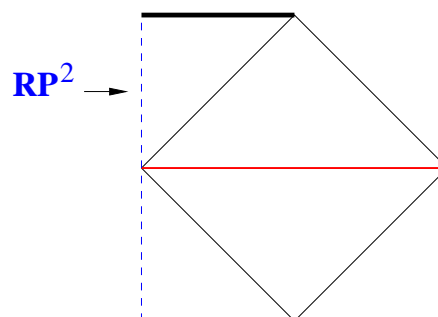
Figure 5. Conformal diagram of the $\mathbb{RP}^3$ geon, obtained as the $\mathbb{Z}_2$ quotient of Kruskal by the freely-acting involutive isometry J_K. The suppressed orbits of spherical symmetry are S^2 except on the dashed line where they are $\mathbb{RP}^2$. The initial data of Figure 3 is shown as the horizontal line.
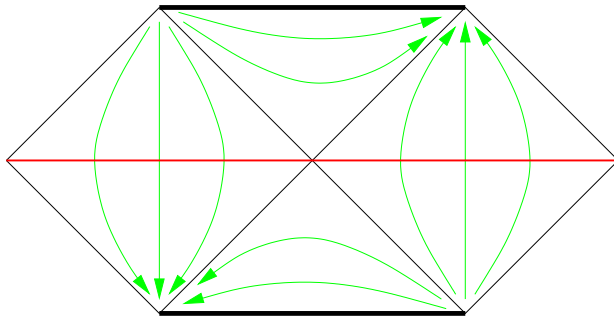
Figure 6. Orbits of Killing time translations on Kruskal. In the exteriors these isometries are translations in Schwarzschild time.
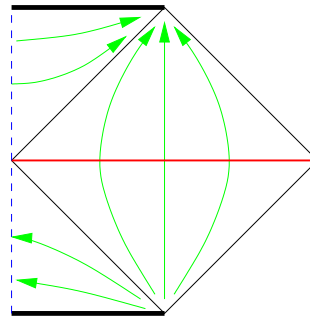
Figure 7. Orbits of the local $\mathbb{RP}^3$ geon isometry inherited from the Killing time translations on Kruskal. The isometry is not globally defined because the corresponding Killing vector has a sign ambiguity in the black and white hole interiors.

A wider class of generalisations occurs when the sphere is replaced by a (not necessarily symmetric) Einstein space. In $D \geq 4$ spacetime dimensions, the vacuum solution can be written in Schwarzschild-type coordinates as [15]

$$ds^2 = -\Delta dt^2 + \frac{dr^2}{\Delta} + r^2 d\bar{\Omega}_{D-2}^2 \quad , \quad \Delta = \tilde{\lambda} - \frac{\mu}{r^{D-3}} - \tilde{\Lambda} r^2, \tag{3.1}$$

where $d\bar{\Omega}_{D-2}^2$ is the metric on the $(D-2)$-dimensional Einstein space $\mathcal{M}_{D-2}$, $\tilde{\lambda}$ is proportional to the Ricci scalar of $\mathcal{M}_{D-2}$, $\tilde{\Lambda}$ is proportional to the cosmological constant and μ is the mass parameter. Suppose that Δ has a simple zero at $r = r_+ > 0$ and $\Delta > 0$ for $r > r_+$. The solution has then a Kruskal-type extension across a bifurcate Killing horizon at $r = r_+$, separating two causally disconnected exteriors. If now $\mathcal{M}_{D-2}$ admits a freely-acting involutive isometry, a geon quotient can be formed just as in Kruskal [11]. For a negative cosmological constant these geons are asymptotically locally anti-de Sitter.

For $D = 3$ and a negative cosmological constant, the above $\mathbb{Z}_2$ quotient generalises immediately to the nonrotating Bañados-Teitelboim-Zanelli (BTZ) hole [16]. The BTZ hole admits however also geon versions that arise as other quotients of anti-de Sitter space, even with angular momentum [17, 18, 19].

If a spacelike asymptopia at $r \to \infty$ is not assumed, further quotient possibilities arise, for example by quotienting de Sitter space or Schwarzschild-de Sitter space [20, 21, 22]. We shall not consider these quotients here.

3.2. Spin

Including spin is delicate. To see the problem, consider the Kerr black hole, shown in Figure 8. The two exteriors are isometric by an isometry that preserves the global time orientation, but this isometry inverts the direction of the rotational angle and has hence fixed points in the black hole region and in the white hole region. Kerr does therefore not admit a geon quotient. A way to think about this is that if the angular momentum at one infinity is J, the angular momentum at the opposing infinity is $-J$.

The generalisation of Kerr(-AdS) to $D \geq 5$ dimensions [23, 24, 25, 26] has $[(D - 1)/2]$ independent angular momenta. Each of these angular momenta has a different sign at the opposing infinities. This suggests that if the angular momenta are arranged into multiplets, such that within each multiplet the magnitude is constant but the signs alternate, there might
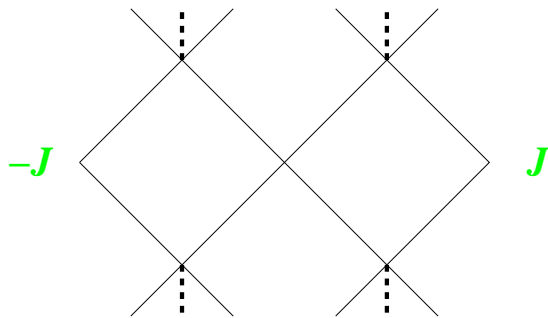
Figure 8. Conformal diagram of Kerr. The angular momenta at the opposing infinities are equal in absolute value but opposite in sign.
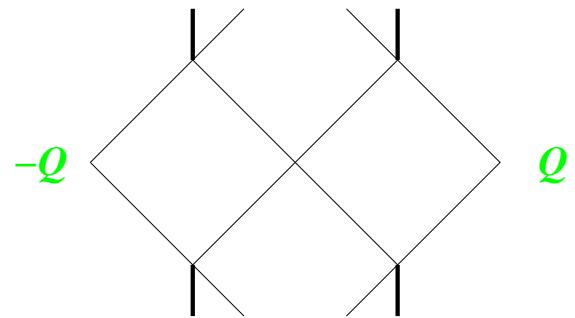
Figure 9. Conformal diagram of Reissner-Nordström. The electric charges at the opposing infinities are equal in absolute value but opposite in sign.

exist a geon quotient by a map that suitably permutes the rotation planes within each multiplet. This turns out to work when D is odd and not equal to 7, but not for other values of D at least in a way that would be continuously deformable to the nonrotating case [11].

For a negative cosmological constant it is possible to define angular momenta also on a torus, rather than on a sphere. The geon quotient must then again involve a permutation of angular momenta with equal magnitude and opposite signs [11].

Finally, the five-dimensional Emparan-Reall black ring [27] has a geon quotient [28], despite having just one nonvanishing angular momentum.

3.3. U(1) charge

Including the electromagnetic field is also delicate. To see the problem, consider electrically-charged Reissner-Nordström, shown in Figure 9. Given the global time orientation, Gauss's law says that if the charge at one infinity equals Q, the charge at the opposing infinity equals $-Q$. While there is an involutive isometry by which to take a geon quotient of the spacetime manifold, this isometry reverses the sign the field strength tensor, and a quotient is thus not possible with the usual understanding of the electromagnetic field as a U(1) gauge field [5]. The situation is the same for the electrically-charged generalisations of (3.1) [15].

There is however a way around this minus sign problem. Maxwell's theory has a discrete involutive symmetry that reverses the sign of the field strength tensor as well as the signs of all charges: charge conjugation. The usual formulation of electromagnetism as a U(1) gauge theory treats charge conjugation as a global symmetry rather than as a gauge symmetry. However, it is possible to promote charge conjugation into a gauge symmetry: the gauge group is then no longer $U(1) \simeq SO(2)$ but $O(2) \simeq \mathbb{Z}_2 \ltimes U(1)$, where in the semidirect product presentation the nontrivial element of $\mathbb{Z}_2$ acts on U(1) by complex conjugation [29]. The disconnected component of the enlarged gauge group then reverses the sign of the field strength tensor. A geon quotient is now possible provided the quotienting map includes a gauge transformation in the disconnected component of the enlarged gauge group [11]. The geon's charge is well-defined only up to the overall sign, but detecting the sign ambiguity would require observations behind the horizons.

Within the magnetically-charged generalisations of (3.1) [15], the options depend on the $(D-2)$-dimensional Einstein space $\mathcal{M}_{D-2}$ [11]. In some cases (including magnetic Reissner-Nordström in even dimensions) the situation is just as with electric charge. In other cases (including magnetic Reissner-Nordström in odd dimensions) the field strength tensor is invariant under the relevant involutive isometry and the usual U(1) formulation of Maxwell's theory suffices. Further options arise if the spacetime has both electric and magnetic charge [11].

3.4. SU(n) charge
Black holes with a non-Abelian gauge field also admit a geon quotient. For generic static, spherically symmetric SU(n) black holes with $n > 2$ [30, 31, 32] this requires enlarging the gauge group to $\mathbb{Z}_2 \ltimes \mathrm{SU}(n)$, where the nontrivial element of $\mathbb{Z}_2$ acts on SU(n) by complex conjugation and the quotienting map includes a a gauge transformation in the disconnected component [33, 34]. By contrast, static SU(2) black holes with spherical [35, 36, 37, 38, 39, 40, 41] or axial [42, 43] symmetry admit a geon quotient without enlarging the gauge group [11, 33, 34].

4. Quantum field theory
The geon black holes that we have discussed are genuine black and white holes, and the exterior region is isometric to that in an ordinary stationary black hole with a bifurcate Killing horizon. Does the geon also have a Hawking temperature, and if so, in what sense?

4.1. Real scalar field on the $\mathbb{RP}^3$ geon
As the prototype, consider a real scalar field on the $\mathbb{RP}^3$ geon.

Recall first the usual setting for a quantised scalar field on Kruskal. There is a distinguished vacuum state $|0_\mathrm{K}\rangle$, the Hartle-Hawking-Israel (HHI) vacuum [44, 45], which is the unique regular state that is invariant under all the continuous isometries [46]. $|0_\mathrm{K}\rangle$ does *not* coincide with the Boulware vacuum state $|0_\mathrm{K,B}\rangle$ [47], which static observers in the exteriors see as a no-particle state. Instead, we have the expansion [44, 45]

$$|0_\mathrm{K}\rangle = \sum_{ij\ldots} f_{ij\ldots} \underbrace{a^\dagger_{R,i} a^\dagger_{L,i}}_{\text{corr}} \underbrace{a^\dagger_{R,j} a^\dagger_{L,j}}_{\text{corr}} \cdots |0_\mathrm{K,B}\rangle, \tag{4.1}$$

where $a^\dagger_{R,i}$ and $a^\dagger_{L,i}$ are creation operators of particles seen by the static observers in respectively the right exterior R and the left exterior L, and i stands for the quantum numbers labelling these particles. The creation operators occur in correlated pairs, with one particle in each exterior, as shown in Figure 10. From the properties of the expansion coefficients $f_{ij\ldots}$ it follows that measurements in (say) R see the pure state $|0_\mathrm{K}\rangle$ as a Planckian density matrix, in a temperature that at asymptotically large radii equals the Hawking temperature

$$T_H = \frac{1}{8\pi M}, \tag{4.2}$$

where M is the mass, and at finite radii is corrected by the gravitational redshift factor.

To summarise, observers in an exterior region of Kruskal see the HHI vacuum $|0_\mathrm{K}\rangle$ as a thermal equilibrium state in the Hawking temperature (4.2).

Consider now a scalar field on the $\mathbb{RP}^3$ geon. By the quotient from Kruskal, the HHI vacuum $|0_\mathrm{K}\rangle$ induces on the geon a vacuum state $|0_\mathrm{G}\rangle$, which we call the geon vacuum. Again, $|0_\mathrm{G}\rangle$ does not coincide with the geon's Boulware state $|0_\mathrm{G,B}\rangle$, which static observers in the exterior see as a no-particle state. Instead, $|0_\mathrm{G}\rangle$ can be expanded as in (4.1), with creation operators of exterior particles acting on $|0_\mathrm{G,B}\rangle$, and these creation operators come again in correlated pairs, as illustrated in Figure 11. For generic observations in the exterior region, the geon vacuum state does hence not appear as a mixed state, let alone a thermal one.

However, the correlated particle pairs in the geon exterior have a special structure: if one particle is localised at asymptotically late times, then the correlated particle is localised at asymptotically early times, and vice versa. Here, "early" and "late" are defined with respect to the distinguished constant Schwarzschild time hypersurface shown in Figure 5. This means that $|0_\mathrm{G}\rangle$ appears as a mixed state when probed by observers at asymptotically late (or early) times, and this mixed state is again Planckian in the Hawking temperature (4.2) [48].
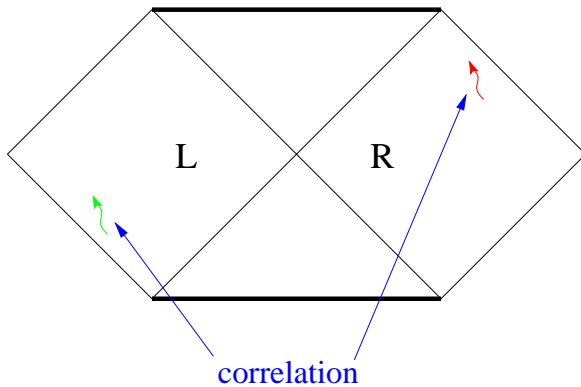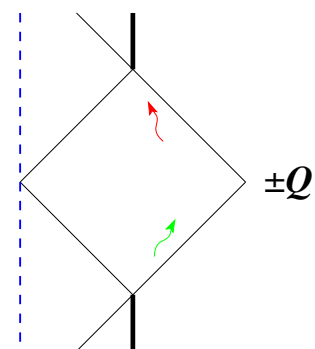
Figure 10. A correlated pair of exterior particles in the expansion (4.1) of the Kruskal HHI vacuum $|0_K\rangle$ as excitations on the Boulware vacuum. The two particles are in the opposite exteriors. From the invariance of $|0_K\rangle$ under Killing time translations it follows that if the particle in R is localised in the far future, the correlated particle in L is localised in the far past, and vice versa, regardless the precise sense of the localisation.
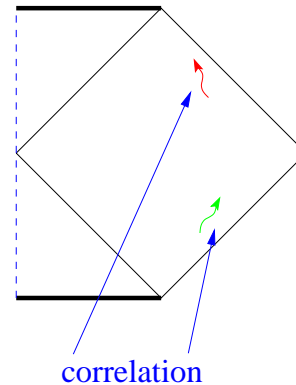
Figure 11. A correlated pair of exterior particles in the expansion of the $\mathbb{RP}^3$ geon HHI vacuum $|0_G\rangle$ as excitations on the Boulware vacuum. Both particles are in the one and only exterior. If one particle is localised in the far future, the correlated particle is localised in the far past, and vice versa.
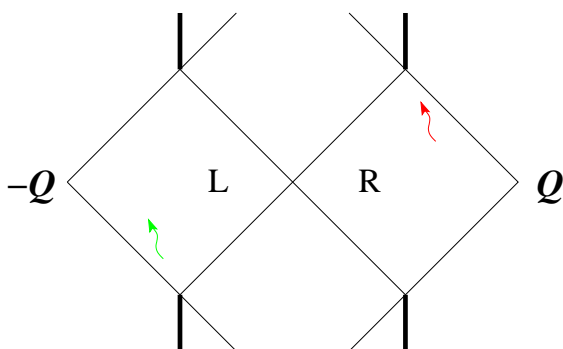
Figure 12. A correlated pair of exterior particles in the expansion of the electrically-charged Reissner-Nordström HHI vacuum as excitations on the Boulware vacuum. The two particles have opposite charge.

Figure 13. A correlated pair of exterior particles in the expansion of the electrically-charged Reissner-Nordström geon HHI vacuum as excitations on the Boulware vacuum. The two particles have the *same* charge.

To summarise, observers in the geon exterior see the geon vacuum $|0_\mathrm{G}\rangle$ as thermal in the Hawking temperature (4.2) at asymptotically late (or early) times. At intermediate times the non-thermal correlations in $|0_\mathrm{G}\rangle$ can be observed by local measurements.

4.2. Generalisations

When the scalar field is replaced by a fermion field, a new issue arises in the choice of the spin structure [49]. As Kruskal has only one spin structure, there is only one HHI vacuum on Kruskal. The $\mathbb{RP}^3$ geon has however two spin structures, and each of them inherits a HHI vacuum from that on Kruskal. Observations made at asymptotically late (or early) exterior times see each of these vacua as a thermal state in the usual Hawking temperature, and are in particular unable to distinguish the two vacua. The vacua, and their spin structures, can however be distinguished by observations of the non-thermal correlations at intermediate times.

When the geon has a gauge field, a new issue arises from its coupling to the quantised matter field. For concreteness, consider a complex scalar field on electrically-charged Reissner-Nordström [50, 51]. The HHI vacua on the two-exterior hole and on the geon again contain exterior particles in correlated pairs as shown in Figures 12 and 13, and the sense of thermality is as in the uncoupled case but with an appropriate chemical potential term [52]. However, while on the two-exterior hole the correlated pairs consist of particles with opposite charge, on the geon the pairs consist of particles with the *same* charge. This is a direct consequence of the charge conjugation that appears in the quotienting map. A geon exterior observer can hence infer from the non-thermal correlations at intermediate times that charges on the spacetime are not globally defined, even though this phenomenon only arises from the geometry of the geon's gauge bundle behind the horizons.

When the geon has anti-de Sitter asymptotics, the geometry induces a state for a quantum field that lives on the geon's conformal boundary, via gauge-gravity correspondence [53, 54]. The boundary state exhibits thermality for appropriately restricted observations, and the non-thermal correlations in this state bear an imprint of the geometry behind the geon's horizons [16, 55, 56, 57].

5. Concluding remarks: geon entropy?

We have seen that a geon black hole, formed as a $\mathbb{Z}_2$ quotient of a stationary black hole with a bifurcate Killing horizon, has a Hartle-Hawking-Israel vacuum that appears thermal in the usual Hawking temperature at asymptotically late (and early) times. Does the geon have also an entropy?

For a late time exterior observer, laws of classical black hole mechanics do not distinguish between a geon, a conventional eternal black hole with a bifurcate Killing horizon, or indeed a black hole formed in a star collapse. Combining the laws of classical black hole mechanics with the late time Hawking temperature, the late time observer will hence conclude that the geon has the usual Bekenstein-Hawking entropy.

Could the geon entropy be obtained directly from a quantum theory of gravity? If the theory is formulated in a way that allows focussing on the quantum spacetime in the distant future, state-counting computations for the geon should proceed essentially as for the hole with two exteriors. This is in particular the case in loop quantum gravity, where the counting of black hole microstates is expressly set up on the future horizon [58, 59, 60, 61]. By contrast, it is not clear whether Euclidean path-integral methods allow a localisation at late times. While the geon has a regular Euclidean-signature section, a naïve application of Euclidean path-integral methods to this section yields for the entropy a result that is only half of the late time Bekenstein-Hawking entropy [48, 62].

It would be of interest to develop a geon entropy computation within the gauge-gravity correspondence. Case studies [16, 55, 56, 57] suggest that an appropriate formalism for this

could be late time entanglement entropy in the boundary field theory. The challenge would be to give a concrete implementation in which the gauge theory side of the correspondence is well understood.

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