A geometric theory of charge and mass

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A geometric model of a charge is constructed by defining several geometries on the same spacetime manifold. A Riemannian geometry describes the vacuum. On the same spacetime, two Weyl geometries are constructed for the charge description. The geometries are constrained by a variational principle. Energy conservation requires the equality of active and passive mass. Chargeless particles have essentially no mass. The treatment of radiation relies on the approximate nature of the wave equation. Variable mass terms in the wave equation cause the $2S$–$2P$ levels in hydrogen to separate by 30 000 Mhz. This unobserved transition together with the lack of spin sets a limit to the correspondence of the model to real electrons.

I. INTRODUCTION

The general theory of relativity provided support for the viewpoint that the course of events which physics describes can most simply be expressed as the result of geometric constraints on a spacetime manifold. This report extends the geometry and the constraints so that the geometry itself provides the physical description of charges.

Basic to the model is the observation that several different geometries can be constructed on the same manifold. On spacetime a Riemannian geometry is defined to describe the classical gravitational vacuum. Several Weyl[1] geometries can then be defined to describe the charges. We use two Weyl geometries per charge. The metrics of the Weyl geometries are conformal to the metric of the Riemannian geometry. The conformal factor is essentially the density of the charge. Since charges in matter are localized, nonzero lengths in the Weyl geometries occur locally. Thus the Weyl geometries are trivial except in the neighborhood of the charge. The only geometry to have long range effects is the vacuum Riemannian geometry.

The number of Weyl geometries per charge was determined by the effects in our theory caused by gauge dependence. The notion of gauge and of gauge invariance was introduced by Weyl and is implicit in his geometry. Our theory relies heavily on the gauge dependence of the field equations. This dependence requires a unique gauge to be determined by the physical interpretation. Gauge variables are present in both the form of the electromagnetic current and the form for the mass. The current is linear in the gauge terms and the mass is quadratic. The physical interpretation requires the gauge terms in the mass formula to remain, but the terms in the current must vanish. To do this, we use two Weyl geometries to describe the charge, their gauge terms being additive inverses. This device gets rid of the unwanted terms in the current since they are linear, and cancel, yet keeps the required quadratic terms in the mass formula. Therefore, to have the proper physical interpretation, two Weyl geometries are required to describe a single charge.

The conformal scalar curvatures of the Weyl geometries must be modified to use in the variational principle. The field equations consist of a Klein–Gordon type equation for the charge motion and the source equations for gravity and electromagnetism. Conservation of energy and charge follow from identities. All the equations are covariant, but none are gauge invariant.

Inherent in the construction of a Weyl geometry is an electromagnetic vector potential. This is assumed to contribute additively to the total potential of the vacuum. To avoid self-energy problems, the potentials of the charge geometry is assumed to be due to other sources. Thus radiation is carried away by a different potential. These assumptions are not time reversal invariant. Furthermore, radiation and conservation of energy together require a change in the state of the charge, since energy radiated must be lost by the charge. By assumption, only the external vector potential can change the state of the charge. Therefore, to treat radiation, we must assume the wave equation is not exact, relying instead on the source equation and conservation of energy equation to describe the radiation.

The usual concept of mass includes two separate notions: mass as the source of gravity and mass as inertia, known as active and passive mass, respectively.[2] Passive mass enters in the wave equation; active mass occurs in the energy equation of gravity. Each type has a rest mass which is a constant in the theory. Their equality arises as follows. The energy an atomic electron absorbs from the external field changes its active mass. The amount of energy lost in radiation can be found from the conservation of energy equation and the electromagnetic source equation. These two energies must be equal if an atom that absorbs radiation and subsequently emmits radiation is not a source or sink of energy. The formula for the radiated energy contains the ratio of active rest mass to passive. This ratio must be one if the atom is not a source or sink of energy.

This is a theory of electromagnetic charges, i.e., electrons. But the theory is spinless, so the charges do not reproduce the behavior of electrons. The question of many charge statistics is tied to spin, so we treat only the single charge. The fine structure of spectra is also linked to spin so the details of spectroscopy cannot be reproduced. Further evidence of this failure is the prediction of a 30 000 Mhz shift in the $2S$–$2P$ levels of hydrogen. This shift is due to variable mass terms in the wave equation.

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The difference with previous geometric models of charge are apparent. We avoid singularities by spreading out the charge as in wave mechanics. Geometric models of point charges have used unusual topologies to represent the inherent singularities of point sources, for example, the multiconnected topology due to Wheeler.[3] Breaking gauge invariance is basic to our approach. Weyl[4] required this invariance in his theory of electromagnetism. Flint[5] used the conformal factor in a Weyl geometry as the square of the wavefunction, but failed to employ the concept of many geometries on the same surface.

Weyl's geometry has appeared to conflict with fundamental atomic phenomena. Given an atomic clock and the speed of light, a well-defined unit of length is determined. Weyl's geometry rests on a concept of indeterminate length. The way to avoid this conflict is to introduce several geometries: In one atomic lengths are fundamental; in the others Weyl's geometry holds. In our theory, the Riemannian geometry of the vacuum measures atomic lengths, and the Weyl geometries are localized to atomic dimensions, as discussed above. Dirac[6] has used two metrics: one measuring length with the atomic standard, one to which Weyl's theory applies. He uses the Weyl geometry to describe effects of the large numbers hypothesis.

Since Weyl's geometry may be unfamiliar, we provide a quick derivation of the results needed here. Following this is the variational principle and the discussion of the field equations.

II. GEOMETRY

Riemannian geometry in the limit of zero curvature reduces to Euclidean geometry. In particular, vectors of equal components have equal lengths. Weyl's geometry retains curvature in the limit of a Euclidean metric. The lengths of two vectors located at points on the manifold separated by coordinate differences, dx^σ, differ according to the formula

$$dl = l(a \cdot dx), \tag{1}$$

where a^σ is a vector and l is the Riemannian length of the vectors. Thus, even when the Riemannian curvature is zero, the affine connections cannot be null. The vector a^σ was interpreted by Weyl as the electromagnetic vector potential.

To derive the affine connections, recall that, in Riemannian geometry with metric $g_{\alpha\beta}$, the equation $d(l^2)=0$ suffices. For a vector with components V^σ, this means

$$(\partial_\sigma g_{\alpha\beta}) V^\alpha V^\beta dx^\sigma + g_{\alpha\beta}[\partial_\sigma (V^\alpha V^\beta)]dx^\sigma = 0. \tag{2}$$

In Weyl's theory,

$$d(l^2) = l^2(2a \cdot dx) = 2a_\sigma g_{\alpha\beta} V^\alpha V^\beta dx^\sigma. \tag{3}$$

If $(-2a_\sigma + \partial_\sigma) g_{\alpha\beta}$ replaces $\partial_\sigma g_{\alpha\beta}$ in the formula for the Christoffel connections $C^\alpha_{\beta\gamma}$, we have the affine connections $\Gamma^\alpha_{\beta\gamma}$ of the Weyl geometry.

Weyl did not want a conformal transformation of the metric to affect the intrinsic geometry of the manifold. It is clear from the above derivation that the following leaves the affine connections unchanged,

$$\bar{g}_{\alpha\beta} = U g_{\alpha\beta} \quad \text{and} \quad \bar{a}_\sigma = a_\sigma + \tfrac{1}{2}(\partial_\sigma \ln U). \tag{4}$$

That follows because

$$U(-2a_\sigma + \partial_\sigma) g_{\alpha\beta} = (-2\bar{a}_\sigma + \partial_\sigma)\bar{g}_{\alpha\beta}, \tag{5}$$

and the factor U cancels out of (3).

Calculating the Weyl scalar curvature $\bar{W}$, using the affine connections $\Gamma^\alpha_{\beta\gamma}$ with the metric $\bar{g}_{\alpha\beta}$ yields

$$\bar{W} = \bar{R} + 6\bar{a}^2 - 6((-\bar{g})^{1/2}\bar{a}^\sigma)_{,\sigma} / (-\bar{g})^{1/2} \tag{6}$$

where[7] $\bar{g} = \det(\bar{g}_{\alpha\beta})$ and $\bar{R}$ is the Riemannian scalar curvature of $\bar{g}_{\alpha\beta}$. In terms of the metric $g_{\alpha\beta}$ the quantities become

$$\bar{\Gamma}^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma}, \quad \bar{W} = W/U, \quad \bar{a}_\sigma = a_\sigma + \tfrac{1}{2}(\partial_\sigma \ln U)$$

and

$${}_0\bar{f}_{\alpha\beta} = \bar{a}_{\alpha,\beta} - \bar{a}_{\beta,\alpha} = a_{\alpha,\beta} - a_{\beta,\alpha} = {}_0f_{\alpha\beta}.^{8} \tag{7}$$

Combining equations to obtain the conformal scalar curvature, we have

$$\bar{R} = \frac{W}{U} - \frac{6\bar{a}^2}{U} + \frac{6((-\bar{g})^{1/2}\bar{a}^\sigma)_{,\sigma}}{(-\bar{g})^{1/2}}. \tag{8}$$

The last term on the right becomes a divergence in the action principle integral; thus it has no affect on the field equations and is dropped from the following equations.

Assuming the vector a^σ has pure imaginary components, and a^σ has an imaginary part, R is kept real by replacing the following:

$$a^2 \to |a|^2, \quad \bar{a}^2 \to |\bar{a}|^2, \quad \text{and} \quad a^\sigma{}_{;\sigma} = 0. \tag{9}$$

Altogether,

$$\bar{R} = (1/U)(R + 6|a|^2 - 6|\bar{a}|^2), \tag{10}$$

where R is the scalar curvature of the metric $g_{\alpha\beta}$.

The last formula for $\bar{R}$ must be modified to serve as Lagrangian. To do this, use the replacement,

$$\bar{a}_\sigma = a_\sigma + \partial_\sigma \ln|u| \to \bar{a}_\sigma = a_\sigma + \partial_\sigma \ln u, \tag{11}$$

where $|u|^2 = U$.

Therefore, $\bar{R}$ with this change is the scalar curvature when u is real.

The modification can be described in another way. Notice that $\bar{a}_\sigma$ is the result of a conformal transformation from the vacuum metric to the charge metric. The inverse should give the vector potential that the charge would predict for the vacuum, call it $a_{v\sigma}$,

$$a_{v\sigma} = \bar{a}_\sigma - \partial_\sigma \ln|u| = a_\sigma + \partial_\sigma \ln u - \partial_\sigma \ln|u| = a_\sigma + i\,\mathrm{Im}\,\partial_\sigma \ln u,$$

$$\text{where} \quad i = \sqrt{-1} \text{ and } \mathrm{Im}X = (X - X^*)/2i. \tag{12}$$

If we take $a_{v\sigma}$ as the vector potential of the vacuum, the modification is to replace the a_σ in the term $6a^2/U$ by $(a_{v\sigma} - i\,\mathrm{Im}\,\partial_\sigma \ln u)$, in (10). From this point of view it is clear $\bar{R}$ is no longer the scalar curvature since we use two different gauges for the same potential in the formula for $\bar{R}$. To emphasize the change, we define

$$S = (1/U)(R + 6|a_v|^2 - 6|\bar{a}|^2). \tag{13}$$

Notice that if a Weyl geometry and the Lagrangian S are used for the vacuum, then the constraints on the vacuum metric are unchanged if $u=1$, since then $S=R$. Furthermore, this is a rigorous way of introducing a vacuum vector potential to the geometry. The total vector potential in the vacuum ϕ_σ is assumed to be the sum of the charge geometry potential plus a vacuum geometry contribution γ_σ

$$\phi_\sigma = a_{v\sigma} + \gamma_\sigma. \tag{14}$$

The field equations depend on which form of a_σ is varied in the Lagrangian; we chose $a_{v\sigma}$.

The vector a_σ of Weyl's theory must be allowed only purely imaginary numerical components. To justify this assumption, consider a point charge electron in circular orbit about a point charge proton.[9] Calculate the change in length after one revolution. Select those orbits for which there is no change,

$$\oint dl/l = 2\pi n i, \text{ where } n \text{ is an integer.} \tag{15}$$

For the case considered, a_σ has only a time component, $-ke/r$, where k is constant, e is the proton charge, and r is the distance from the proton. Then, applying classical mechanics to cancel radial forces and using (1), we find

$$r = -n^2/mk^2. \tag{16}$$

If $k=\pm ie/\hbar$,[10] then the radii selected are those of Bohr's model of the hydrogen atom. This is the justification for assuming a_σ is purely imaginary.

III. FIELD EQUATIONS

The field equations result from a variational principle constructed from the scalar curvature R of the Riemannian geometry, the modified form[11] S of the curvatures in the Weyl geometries, and the square of the electromagnetic field. Only the case of one charge is derived; thus two Weyl geometries are needed, labeled by a preindex. The Lagrangian is

$$L = (R + cf^2)(-g)^{1/2} + \sum_{j=1}^{2} b\, {}_jS(-{}_j\bar{g})^{1/2}, \tag{17}$$

where c and b are constants, and $f_{\alpha\beta} = \phi_{\alpha,\beta} - \phi_{\beta,\alpha}$, where ϕ^σ is the total potential in vacuum.

The field equations are covariant, but not gauge invariant. This last property is used in the physical interpretation. Specifically, the equations are written with these substitutions:

$${}_ju = {}_jv \exp(i\, {}_jp \cdot x) \quad\text{and}\quad {}_ja_v = a'_\sigma - i\, {}_jp_\sigma \text{ for } j=1,2, \tag{18}$$

where the vectors ${}_jp$ are constant, and a'_σ is the same for both particle component geometries. A more complete description may require a more general transformation. Each ${}_ju$ may be thought of as an amplitude modulated plane wave. The field equations are labeled by the function varied,

$$\begin{aligned} {}_ju^*\, Eq.:\ \ 0 &= (R/6 + |{}_ja_v|^2)\, {}_jv \\ &\quad + (-g)^{-1/2}(a'_\alpha + \partial_\alpha) g^{\alpha\beta}(-g)^{1/2}(a'_\beta + \partial_\beta)\, {}_jv, \\ a\, Eq.:\ \ j^\alpha &= \frac{1}{4\pi} F^{\alpha\sigma}{}_{;\sigma} = q\sum_{j=1}^{2}[{}_jp^\alpha\, {}_jU + \mathrm{Im}({}_jv^* \partial^\alpha\, {}_jv)], \end{aligned} \tag{19}$$

where $q = 3bi/4\pi ck$, (20)

$$\begin{aligned} g\, Eq.:\ \ 0 &= \left(1 + \sum_{j=1}^{2} b\, {}_jU\right) G^{\alpha\beta} - 8\pi ck^2 T^{\alpha\beta}_{\mathrm{em}} \\ &\quad + b\sum_{j=1}^{2}\{(g^{\alpha\beta}\Box\, {}_jU - {}_jU^{;\alpha;\beta}) \\ &\quad + 6\, {}_jU({}_jp^\alpha\, {}_jp^\beta + 2ia'^{(\alpha}\mathrm{Im}(\partial^{\beta)} \ln {}_jv) \\ &\quad - \partial^{(\alpha}\ln {}_jv^* \partial^{\beta)}\ln {}_jv) - 3\, {}_jUg^{\alpha\beta} \\ &\quad \times [({}_jp)^2 + 2ia' \cdot \mathrm{Im}\partial \ln {}_jv - |\partial \ln {}_jv|^2]\}, \end{aligned} \tag{21}$$

where

$$\Box U = U^{;\sigma}{}_{;\sigma},\ \ F^{\alpha\beta} = f^{\alpha\beta}/k,\ \ G^{\alpha\beta} = R^{\alpha\beta} - \tfrac{1}{2} R g^{\alpha\beta}, \text{ and}$$

$$T^{\alpha\beta}_{\mathrm{em}} = -(1/4\pi)(F^{\alpha\beta}F^{\beta}{}_{\sigma} - \tfrac{1}{4} g^{\alpha\beta}F^2). \tag{22}$$

We have kept the notation ${}_jU$ because ${}_jU = |{}_jv|^2$. The ${}_ju^*$ equation may be rewritten in a simpler form,

$$0 = [R/6 + ({}_jp)^2 - 2i\, {}_jp \cdot a' + a'^\sigma{}_{;\sigma}]\, {}_jv + 2a' \cdot \partial\, {}_jv + \Box\, {}_jv. \tag{23}$$

The charge density of the source is spread out over a volume of space as a glance at (20) shows. If one section of this density were to be repelled by another section of the charge density, there are no external forces which could hold it together. It must be assumed that the charge reacts to that part of the vector potential which has other charges as its source. Accordingly, a'_σ is assumed to be the external vector potential and γ_σ is assumed to be the vector potential arising from the charge itself. This assumption destroys the time inversion symmetry of the theory. If a charge absorbs energy from the field corresponding to a'_σ, the inverse process is the emission of energy from the charge to the field due to a'_σ. This means γ_σ and a'_σ are the same. But then a'_σ has the charge as part of its source and the charge blows up. Therefore, time inversion symmetry fails.

There are two identities which must be satisfied. Since $F^{\alpha\beta}$ is antisymmetric, the divergence of the current j^σ is identically zero,

$$j^\sigma{}_{;\sigma} = 0 = \sum_{j=1}^{2} \mathrm{Im}({}_ju^* \Box\, {}_ju). \tag{24}$$

Together with the ${}_ju^*$ equation, this implies

$$\sum_{j=1}^{2} ({}_jU\, {}_ja^\sigma_v)_{;\sigma} = 0. \tag{25}$$

Care must be exercised so that ${}_ju$ and ${}_ja_{v\sigma}$ satisfy this equation.

The divergence of $G^{\alpha\beta}$ is identically zero, so

$$\begin{aligned} 0 &= -8\pi ck^2 T^{\alpha\beta}_{\mathrm{em};\beta} + b\sum_{j=1}^{2}[{}_jU_{,\beta}(G^{\alpha\beta} + R^{\alpha\beta} + {}_jp^\alpha\, {}_jp^\beta \\ &\quad - \tfrac{1}{2} g^{\alpha\beta}({}_jp)^2) + 6i\, {}_0f^{\alpha\beta}\,\mathrm{Im}({}_jv^* \partial_\beta\, {}_jv) \\ &\quad - 6i\, {}_jU^{,\alpha} a' \cdot \mathrm{Im}(\partial \ln {}_jv) - 6(\partial^{(\alpha}\, {}_jv^* \partial^{\beta)}\, {}_jv \\ &\quad - \tfrac{1}{2} g^{\alpha\beta} |\partial\, {}_jv|^2)_{;\beta}]. \end{aligned} \tag{26}$$

IV. MASS AND CHARGE

The mass M and charge $-e$ are defined as the volume integrals of the time components of the particle stress–energy tensor $T^{\alpha\beta}_p$ and the current vector j^σ, respectively:

$$M=\int T_p^{00}d^3x \quad \text{and} \quad -e=\int j^0\, d^3x, \tag{27}$$

where

$$-8\pi T_p^{00}=G^{00}+8\pi T_{\text{em}}^{00}. \tag{28}$$

Since the amount of mass and charge is defined in terms of one component of a tensor and vector, the result depends on the coordinate system of the observer. The only well-defined coordinate system is the rest frame of the charge. Therefore, we assume the expectation value of momentum is zero. However, the kinetic energy of the charge need not be zero, for example, if the charge is the electron in a one-electron atom. The treatment is further simplified by considering the wavefunctions ${}_jv$ to be eigenstates of energy. Equation (25) will then be satisfied if a'_σ has only a time component. The mass is spread out over a volume much larger than the Schwarzschild limit. This is obvious, since nuclei are the tightest binders, and atomic electrons are spread out over volumes of radius many times bigger than the Schwarzschild radius. Thus we assume $g^{\alpha\beta}$ is the flat space metric and

$$\partial_0\, {}_jv=i\, {}_jw\, {}_jv \quad \text{and} \quad |\partial_n\, {}_jv| \ll |{}_jw|. \tag{29}$$

Before evaluating the mass and charge, certain problems with the form of the current must be eliminated. If the particle at rest is to have no current, the following terms of (20) must be zero:

$$0={}_1p\, {}_1U+{}_2p\, {}_2U. \tag{30}$$

Assuming equal normalization of components, we have

$$0={}_1p+{}_2p. \tag{31}$$

By thinking of ${}_ju$ as amplitude modulated plane waves, the carrier waves are complex conjugates of each other. Thus,

$$\int {}_1u^*\, {}_2u\, d^3x=\int {}_1v^*\, {}_2v \exp(2i\, {}_2p\cdot x)\, d^3x. \tag{32}$$

The wavelength of the plane wave will turn out to be roughly $\frac{1}{2}\hbar/m$, which is 10^{-2} Å for electrons. Changes in ${}_1v^*\, {}_2v$ are on the order of angstroms, so the integral is approximately zero. This near orthogonality mimics that of the spin components in elementary quantum mechanics. Here, however, the wave equation (19) does not mix spin components, so there are no spin effects.

Returning to the mass and charge evaluation, using (31), and assuming $b\, {}_jU$ and R negligible, one finds

$$M=(3bV/4\pi)(p^n)^2+(3b/4\pi)\int_j v^*\Delta\, {}_j v\, d^3x. \tag{33}$$

where $\frac{1}{2}V=\int_j U d^3x$, $(p^n)^2=({}_jp^n)^2$, $\Delta v=v^{;n}{}_{;n}$, and the surface of the volume of integration is far from the charge so that divergences in T_p^{00} integrate out. The constant term on the right in (33) is the active rest mass M_0,

$$M_0=(3bV/4\pi)(p^n)^2. \tag{34}$$

Keeping only the lowest order terms in (19) implies

$${}_jw^2={}_jp^2=p^2. \tag{35}$$

The charge is approximately

$$-e=\tfrac{1}{2}V({}_1w+{}_2w). \tag{36}$$

Multiplying by $({}_1w-{}_2w)$ implies ${}_1w={}_2w=w$.

To evaluate the constants of the theory, assume (19) is an approximate form of the Klein–Gordon equation

$$[m^2/\hbar^2+(-ieA_\sigma/\hbar+\partial^\sigma)(-ieA^\sigma/\hbar+\partial^\sigma)]v=0, \tag{37}$$

where A_σ is the electromagnetic vector potential. Therefore,

$$p^2=m^2/\hbar^2 \quad \text{and} \quad k=-ie/\hbar. \tag{38}$$

Note that this value for k agrees with the Bohr atom treatment in Sec. II. Comparison of Eq. (21) with the general relativity result for electromagnetism, shows $ck^2=-1$. Now the constants can be determined:

$$w=\pm m/\hbar,\ bV=\pm 4\pi\hbar^2/3m,\ (p^n)^2=\pm mM_0/\hbar^2,\ ck^2=-1,$$
$$k=-ie/\hbar, \quad \text{and} \quad (p^0)^2=w^2+(p^n)^2=(m^2/\hbar^2)(1\pm M_0/m),$$
$$\text{where } m>0. \tag{39}$$

Notice that negative w implies negative rest mass M_0, since $(p^n)^2>0$. Antiparticles are well known to exist, and can be interpreted as negative energy states. To include such species, we need only expand the two Weyl geometry theory to four, two having positive w and two negative. Wave components with opposite w's can be considered orthogonal in many cases.

$$\int_1 v^*\, {}_3v\, d^3x=I\exp(2i\, {}_3wt), \tag{40}$$

where I is a function of position and time, t. If the function I does not change appreciably in the span of time on the order of $1/w=10^{-20}$s, then, averaged over a few such time spans, (40) shows the effective orthogonality of components with opposite w's.

A four component wavefunction is most natural to describe charge. The need for a two-component theory can now be seen to arise because the constant vector p^σ is needed so that the rest mass of the charge is nonzero, (34), but yet the electromagnetic source cannot have a constant part essentially independent of the wavefunction, (20). The mass depends on the square of p^σ, and the constant part of the current is linear in p^σ. Thus, having two components with vectors ${}_jp^\sigma$ of equal magnitudes, but opposite directions, cancels the unwanted terms in the current, but retains nonzero rest mass. Two more components are needed to include antiparticles in the description.

Two rest masses, m and M_0, appear in the equations; one is the source of a gravitational field, the other measures the inertial resistance to applied force in the wave equation. The difference in these two masses has nothing to do with the Eötvös–Dicke experiment and the equivalence principle. In the absence of electromagnetic and gravitational forces, it is evident from (19) that charges follow straight line paths. Covariance of the equations requires that in the absence of electromagnetic forces, charges follow geodesics. Thus the path of a charge unaffected by electromagnetism is independent of any intrinsic characteristic of the charge. These predictions are the content of the equivalence principle supported by experiment. Thus, first, the theory obeys the equivalence principle. Secondly, the "gravitational mass" m_g arises from Newton's equation

$$m_i a=m_g K/r^2, \tag{41}$$

where K is constant and r is the distance from the gravitational source. It may be argued that m_i, the inertial mass, is the mass m. But the active mass M is a factor in K, and is definitely not m_i.

For a bound charge, (33) can be put in the form

$$M = M_0 - 2\langle \text{k.e.}\rangle,$$

$$\text{where } \langle \text{k.e.}\rangle = \frac{1}{V}\sum_{j=1}^{2}\int_j v^*\left(-\frac{\hbar^2}{2m}\Delta\right)_j v\, d^3x. \tag{42}$$

For a one electron atom, the vector potential may be assumed to have only a time component, which is inversely proportional to the distance from the nucleus. Application of the virial theorem implies

$$M = M_0 - 2E, \quad \text{where } E \text{ is the binding energy.} \tag{43}$$

Discussion of this effect will be delayed until radiation has been treated.

The discussion of mass would be incomplete without noticing that chargeless particles are essentially massless in this theory. This follows because charge null implies k null implies ${}_j a_{v\sigma}$ null, so

$$m = \hbar(R/6)^{1/2}. \tag{44}$$

Mesons are "strongly" charged even if neutral electromagnetically. A discussion of strong charges lies beyond the scope of this report, so (44) does not contradict observation.

V. RADIATION

The conservation of energy equation is (26) with $\alpha = 0$, which we use in the nonrelativistic limit, (29). By using (20), (22), and (30), two of the terms combine,

$$8\pi T^{0\beta}_{\text{em};\beta} + \sum_{j=1}^{2} 6bi\,{}_0 f^{0\beta}\,\text{Im}({}_j v^*\partial_\beta\, {}_j v) = (8\pi/k) j_\beta (f^{0\beta} - {}_0 f^{0\beta})$$

$$= (8\pi/k) j_\beta\, {}_\gamma f^{0\beta} = 8\pi\, {}_\gamma T^{0\beta}_{\text{em};\beta}, \tag{45}$$

where ${}_\gamma f^{\alpha\beta} = \gamma^{\alpha,\beta} - \gamma^{\beta,\alpha}$, and ${}_\gamma T^{\alpha\beta}_{\text{em}}$ is the electromagnetic stress—energy tensor (22) calculated with ${}_\gamma F^{\alpha\beta} = (1/k)_\gamma f^{\alpha\beta}$. In the derivation of (45), we used the result that a'_σ is sourceless, as previously discussed. The cancellation involves the rate of doing work on the charge by the field due to a'_σ, and the rate that energy is lost by the field. To see this notice that the term ${}_0 f^{0\beta}{}_{j\beta}$ is the Lorentz force. The energy balance between charge and field due to a'_σ occurs in detail.

Of more interest is the energy emitted from the region of space containing the charge. So we integrate over space, assuming the volume of integration large enough so that certain surface integrals vanish. The expression for the rate of change of energy in the field due to γ_σ, in the volume of integration becomes

$${}_\gamma \dot{P}^{0\prime} = -\frac{b}{8\pi}\sum_{j=1}^{2}\frac{d}{dt}\int [{}_j U(G^{00} + R^{00} + 6({}_j p^0)^2 - 3g^{00}p^2$$

$$- 6ig^{00}a'\cdot \text{Im}(\partial \ln {}_j v)) - 6\partial^0\, {}_j v^* \partial^0\, {}_j v + 3g^{00}|\partial\, {}_j v|^2]d^3x. \tag{46}$$

To simplify this, use (23) multiplied by $-3\,{}_j v^*$, (30), assume ${}_j v$ is negligible on the surface of the volume of integration, and

$${}_j v^{,0} \approx iw\, {}_j v, \quad \text{where } w = m/\hbar > 0. \tag{47}$$

All this implies

$${}_\gamma \dot{P}^0 = -\frac{b}{8\pi}\sum_{j=1}^{2}\frac{d}{dt}\int [6\, {}_j U(({p^0})^2 - w^2)]d^3x$$

$$= -\frac{M_0}{V}\sum_{j=1}^{2}\frac{d}{dt}\int_j U d^3x. \tag{48}$$

If ${}_j v$ satisfy (19), then $d/dt(\int_j U d^3x) = 0$, implying no radiation. This argument requires more faith in the wave equation than is justified. Terms for the spin must be added before the equation can be regarded as exact. We do not consider it accurate enough to deny the delicate process of radiation to proceed. With the assumption that the coefficients in an eigenfunction expansion of the wavefunction depend on time, we explore the consequences of the source equation (20).

Assume a two-level system,

$${}_j v = [c_1 y_1 \exp(i\epsilon_1 t) + c_2 y_2 \exp(i\epsilon_2 t)]\exp(iwt), \tag{49}$$

where c_n are real and y_n are orthogonal energy eigenstates of (19) normalized to $V/2$. Then $c_1^2 + c_2^2 = 1$, so define θ such that

$$c_1 = \sin\theta, \quad c_2 = \cos\theta, \quad \dot{c}_1 = \dot{\theta} c_2, \quad \text{and } \dot{c}_2 = -\dot{\theta} c_1. \tag{50}$$

Even when radiating, the charge of the electron in an atomic system should have a constant part,

$$-\dot{e} = 0 = \frac{d}{dt}\langle \int j^0 d^3x\rangle_t$$

$$= q\frac{d}{dt}(w\int U d^3x + \epsilon_1 c_1^2 V + \epsilon_2 c_2^2 V). \tag{51}$$

Therefore,

$$\frac{d}{dt}\int U d^3x = -\frac{V}{w} 2c_1 c_2 \dot{\theta}(\epsilon_1 - \epsilon_2) \tag{52}$$

and

$${}_\gamma \dot{P}^0 = \frac{M_0}{m} 2c_1 c_2 \dot{\theta}\hbar(\epsilon_1 - \epsilon_2). \tag{53}$$

Integrating over all time, the energy liberated is $(M_0/m)\hbar(\epsilon_1 - \epsilon_2)$. In order that the atom not be a source or sink of energy, this value must agree with (43). Therefore, noticing that V changes in (34),

$$M_0 = m. \tag{54}$$

An evaluation of the current shows the emitted radiation has frequency $|\epsilon_1 - \epsilon_2|$. It is well known that absorption of energy from radiation occurs at the same frequency. The total energy absorbed and the total emitted in radiative transitions divided by the frequency of the radiation is constant, h. Electromagnetic radiation from atomic electrons appears in quantized energy packets, in agreement with this result.

VI. AN ENERGY LEVEL SHIFT

The effect of the extra mass terms in (19), $p\cdot a'$ and a'^2, may be treated as a small perturbation. Their effect is greatest when a'_σ is the largest. Therefore, assume a one-electron atom. To first order the energy change will be

$$\Delta E = -\frac{h^2}{2mV}\sum_{j=1}^{2}\int_j U a'^2 d^3x. \tag{55}$$

The term linear in p^σ is zero by (30). If a'_σ has only a time component, $-ke/r$, then

$$\Delta E=\frac{e^4Z^2}{2m}\left\langle\frac{1}{r^2}\right\rangle,\quad \text{where}\left\langle\frac{1}{r^2}\right\rangle=\frac{1}{V}\sum_{j=1}^{2}\int\frac{{}_jU}{r^2}d^3x. \tag{56}$$

Assuming both wave components are the eigenfunction belonging to the same level in hydrogen yields a difference in ΔE for the $2S-2P$ levels

$$\Delta E_S-\Delta E_p=mZ^4\alpha^4/12,\quad \text{where } \alpha=e^2/\hbar. \tag{57}$$

The equivalent frequency shift is 3×10^4 Mhz. A shift of this magnitude is not observed. We conclude that the theory does not represent the details of atomic spectroscopy accurately. This is not such a bad failure for a spinless geometric theory of charge.

VII. CONCLUSION

This paper has discussed a specific geometric theory of charge, conceived from the viewpoint that geometry itself most simply describes physical events. The simplicity of the geometry is compromised by the necessary modification of the Weyl scalar curvatures. The physics lacks spin and related concepts. Yet the theory describes, with some accuracy, phenomena ranging from the astronomical, with general relativity as a limit, to the minute with a treatment of electromagneitc radiation.

[1]H. Weyl, *Space, Time, Matter*, transl. by H.L. Brose (Methuen, London, 1922); A.S. Eddington, *The Mathematical Theory of Relativity* (Cambridge U.P., London, 1960), 2nd ed., Chap. VII; R. Adler, *et al.*, *Introduction to General Relativity* (McGraw-Hill, New York, 1965), 1st ed., Chap. 13, p. 401.

[2]H. Bondi, "Negative mass in general relativity," Rev. Mod. Phys. **29**, 423 (1957).

[3]J.A. Wheeler, "Geons," Phys. Rev. **97**, 511 (1955); C.W. Misner and J.A. Wheeler, "Classical physics as geometry: Gravitation, electromagnetism, unquantized charge, and mass as properties of curved empty space," Ann. Phys. (N.Y.) **2**, 525 (1957).

[4]H. Weyl, Ref. 1, Sec. 36, p. 295; A.S. Eddington, Ref. 1, Sec. 90, p. 209.

[5]H.T. Flint and J.W. Fisher, "The fundamental equations of wave mechanics and the metrics of space," Proc. Roy. Soc. (London) A **117**, 625, 630 (1928).

[6]P.A.M. Dirac, "Long range forces and broken symmetries," Proc. Roy. Soc. (London) A **333**, 403 (1973); "Cosmological models and the Large Numbers hypothesis," Proc. Roy. Soc. (London) A **338**, 439 (1974).

[7]Commas denote partial derivatives, semicolons for covariant derivatives. All covariant derivatives are respect to $C^\alpha_{\beta\gamma}$.

[8]The invariance of ${}_0f^{\alpha\beta}$ under conformal transformations is the origin of the term "gauge invariance" of electrodynamics.

[9]R. Adler, *et al.*, Ref. 1, p. 416.

[10]Units throughout have $G=c=1$, where G is the gravitational constant and c is the speed of light.

[11]The term S is not gauge invariant. The use of a gauge noninvariant Lagrangian is discussed by P.A.M. Dirac, "A new classical theory of electrons," Proc. Roy. Soc. (London) A **209**, 293 (1951).