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Geometrical gauge theory of gravity and elementary particle forces

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The basic forces of nature, mediated by spin-one and spin-two particles, are unified within a geometrical gauge theory. Gravity, electromagnetism, and strong forces are described by two nonsymmetric fields $g_{\mu\nu}$ and $s_{\mu\nu}$. Field equations are derived from a variational principle. The field equations possess an exact (classical) nonsingular solution that corresponds to a geometrical object called a gheon (gravitational-hadronic-electromagnetic entity). Space inside a gheon is Euclidean four-dimensional and a gheon can have event horizons. The gheons describe hadrons and at the classical level permanently confine quarks. The event horizons trap normal hadrons and can also radiate them, producing a thermal hadronic spectrum. There exists a limit in which the field equations reduce to Einstein-Maxwell equations. In this limit, the gheons shrink to point singularities. By matching the boundary conditions of the solutions of a scalar field equation on the surface of a gheon, a discrete mass spectrum for a scalar particle is obtained. A hadronic mass formula of the Gell-Mann–Okubo type is derived for an isolated black gheon.

I. INTRODUCTION

Even though gravity is the weakest force in nature it satisfies the principle of equivalence which distinguishes it from all other known forces at the macroscopic level. This principle is contained in the mathematical statement that gravity couples to the energy-momentum tensor. The gravitational forces acting on matter are always positive and for macroscopic regions they add up to dominate all other forces. Another feature of gravity, shared by the electromagnetic field, is that the field quanta (gravitons) are massless; the forces are of infinite range. Part of the strong forces that act on hadronic quark matter are mediated by massless spin-2^+ quanta, which possess the universal property that they couple to the energy-momentum tensor. The spin-2^+ strong forces satisfy a principle of equivalence which is weaker than that applicable to gravity in that it only applies to hadronic matter. This principle could play an important role in our search for a unified description of forces.

The universal property of gravity is mathematically described by a theory based on Riemannian geometry. The strength of gravity is measured by the magnitude of the curvature at a given point in space-time. In analogy to this one might suspect that the spin-2^+ strong interactions will also be described by a geometrical theory, founded on Riemannian geometry; the strength of the strong force would be determined by the magnitude of a curvature tensor. An extended theory of gravitation has been developed, using non-Riemannian geometry, which combines gravity and electromagnetism in a Hermitian nonsymmetric tensor field $g_{\mu\nu}$.[1,2] The basic gauge invariance of the Lagrangian density of the theory leads to the gauge invariances of gravity and electromagnetism.[2]

We shall formulate in the following an extension of the above theory to include strong interactions, mediated by massless spin-2^+ and -1^- gauge fields. The strong spin-2^+ fields couple to the energy-momentum tensor of hadronic matter, directly affecting the metric of space-time at distances of the order of hadronic Compton wavelengths.

Why should we describe the electromagnetic, strong, and weak interactions by something as complicated as a set of differential equations based on non-Riemannian geometry? We recall that to describe gravity successfully, Einstein used ten field components (the Riemannian metric tensor). In view of this, we cannot hope to obtain a comprehensive description of the other forces of nature in terms of a more economical structure. After many years of search for the basic equations of strong interactions, we are still in the dark as to their precise nature. Only by using a structurally complicated nonlinear theory, such as that described in the following, can we hope to attain a more profound microscopic description of matter.

A concentrated region of mass can produce gravitational fields so strong that neither light nor anything else can escape. The matter can undergo gravitational collapse and form a black hole. In general relativity the matter shrinks to a point singularity in finite proper time. In the generalized theory, the presence of a static electric field ensures that a geometrical singularity does not occur in physical four-dimensional space-time.[2] The singularity is not only hidden by possible event horizons but also by a surface that forms a natural boundary in space-time. The theory provides the ultimate "cosmic censorship," for charged matter inside a sphere with a radius of the Planck length $L_P = 1.6 \times 10^{-33}$ cm will repel all test particles and no physical singularity can occur. The breakdown of physics at space-time singularities in the beginning of the universe or in the collapse of burnt-out stars can be avoided.

The origin of the confinement of quarks has been the subject of much investigation in recent years. The conventional picture of color confinement is based on the assumption that the strong color gauge group $SU(3)_c$ represents an exact symmetry of nature; the electrically neutral massless gluons produce long-range forces, accompanied by infrared effects. These effects are presumed to be so singular that infinitely rising long-range forces build up, permanently confining quarks and gluons inside color-singlet hadronic states. It has not been proved that this conjecture leads to exact (or partial) confinement within the context of non-Abelian (Yang-Mills) theories. Perturbation calculations reveal that the infrared behavior of the non-Abelian gauge theories is not more singular than that of the familiar Abelian gauge theory of quantum electrodynamics. Thus perturbation theory does not appear to lead to a mechanism of confinement of quarks. Efforts have recently been devoted to deriving confinement from nonperturbative semiclassical methods. So far no conclusive results have been obtained.[3]

In the unified theory formulated here two possible schemes may be considered:

(I) We view the geometry of space-time as an "arena." The quarks are present in the geometrical theory to generate the spectroscopy of particles. Quark confinement would have a geometrical origin.

(II) The structure of particles and particle spectroscopy would originate from a purely geometrical theory of spacetime. The quarks would be replaced by a *geometrical* explanation of particles.

In developing a unified theory we shall have both of these possibilities in mind although we shall concentrate, at present, on the less ambitious approach I.

Isham, Salam, and Strathdee[4,5] have introduced the idea of a "strong-gravity" theory of massive spin-2^+ particles. The event horizons of the strong-gravity black holes trap the color quarks and gluons; a particle that once gets inside the event horizons cannot escape and is confined at the classical level.

In the theory presented here, the spin-1 and spin-2 particles are unified in a geometrical gauge scheme, leading to (classical) rigorous solitonlike solutions which describe particles as extended objects, confining quarks and gluons exactly. The origin in the spherically symmetric solution is enclosed in a sphere S inside which space is Euclidean with signature -4. The generalized curvature tensor and its invariants are finite on the surface of S. The physical energy-momentum tensor is finite. We call these objects *gheons*[6] (gravitational-hadronic-electromagnetic entities) and they are the size of hadrons. At the classical level the quarks residing inside the gheons are permanently confined. Leptons can penetrate the gheons, for they do not interact strongly with hadrons.

The gheons can possess black-hole event horizons which also confine ordinary hadrons, provided certain physical conditions are met. These event horizons may radiate[5,7,8] with a hadronic temperature given by $(4\pi k_B T)^{-1} = R$, where R is the radius of the event horizon and k_B is Boltzmann's constant.

The unified theory also offers the possibility of an explanation of the discreteness of hadronic matter. In conventional quantum field theories, formulated in flat Minkowski space-time, the masses of particles occur as arbitrary parameters; the representations of the Lorentz group describe continuous distributions of matter. In the unified theory a matching of boundary conditions on the surface of gheons leads to a discrete mass spectrum.

The results presented are mainly classical or semiclassical and a quantum picture of gheons remains to be formulated e.g. in terms of the Feynman path-integral method.[9] The possibility of quantizing the theory in a flat space-time leads to a more immediate way of performing calculations. But many of the most important features of the nonlinear theory would be lost. A purely geometrical theory with a curved space-time still seems to be at odds with quantum theory, a problem of concern for modern physics. Since the classical solutions are nonsingular, it is hoped that the quantized version of the theory is already *finite* with a cutoff for gravity $1/\sqrt{G_N} \simeq 10^{19}$ GeV, for weak interactions $1/\sqrt{G_F} \simeq 10^3$ GeV, and for strong interactions $1/\sqrt{G_s} \simeq 1$ GeV. The cutoffs would be dynamical consequences of the theory, arising from the geometrical structure of space-time.

II. THE FIELD EQUATIONS

The field structure which describes the spin-1 and spin-2 particles is based on two Hermitian (complex) nonsymmetric fields $g_{\mu\nu}$ and $s_{\mu\nu}$. We write[10]

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]},$$
$$s_{\mu\nu} = s_{(\mu\nu)} + s_{[\mu\nu]}. \tag{2.1}$$

Associated with the $g_{\mu\nu}$ and $s_{\mu\nu}$ tensors are two (Hermitian) nonsymmetric affine connections

$\Gamma^{\lambda}_{\mu\nu}(g)$ and $\Gamma^{\lambda}_{\mu\nu}(s)$. The $\Gamma^{\lambda}_{\mu\nu}(g)$ and $\Gamma^{\lambda}_{\mu\nu}(s)$ can be expressed in terms of the connections $W^{\lambda}_{\mu\nu}(g)$ and $W^{\lambda}_{\mu\nu}(s)$ by means of the projective transformations[2]

$$\Gamma^{\lambda}_{\mu\nu}(g)=W^{\lambda}_{\mu\nu}(g)+\tfrac{2}{3}\delta^{\lambda}_{\mu}W_{\nu}(g)\,,\qquad \Gamma^{\lambda}_{\mu\nu}(s)=W^{\lambda}_{\mu\nu}(s)+\tfrac{2}{3}\delta^{\lambda}_{\mu}W_{\nu}(s)\,, \tag{2.2}$$

where $W_{\nu}(g)$ and $W_{\nu}(s)$ are purely imaginary vectors defined by

$$W_{\nu}(g)=\tfrac{1}{2}[W^{\sigma}_{\nu\sigma}(g)-W^{\sigma}_{\sigma\nu}(g)]\,,\qquad W_{\nu}(s)=\tfrac{1}{2}[W^{\sigma}_{\nu\sigma}(s)-W^{\sigma}_{\sigma\nu}(s)]\,. \tag{2.3}$$

It follows from (2.2) that

$$\Gamma_{\mu}(g)\equiv\Gamma^{\sigma}_{[\mu\sigma]}(g)=0\,,\qquad \Gamma_{\mu}(s)\equiv\Gamma^{\sigma}_{[\mu\sigma]}(s)=0\,. \tag{2.4}$$

The contravariant tensors $g^{\mu\nu}$ and $s^{\mu\nu}$ are related to the covariant tensors $g_{\mu\nu}$ and $s_{\mu\nu}$ by

$$g^{\mu\nu}g_{\sigma\nu}=g^{\nu\mu}g_{\nu\sigma}=\delta^{\mu}_{\sigma}\,,\qquad s^{\mu\nu}s_{\sigma\nu}=s^{\nu\mu}s_{\nu\sigma}=\delta^{\mu}_{\sigma}\,. \tag{2.5}$$

The contracted curvature tensors formed from the (non-Hermitian) connections $W^{\lambda}_{\mu\nu}(g)$ and $W^{\lambda}_{\mu\nu}(s)$ are given by

$$\begin{aligned}K_{\mu\nu}(g)&=\partial_{\beta}W^{\beta}_{\mu\nu}(g)-\partial_{\nu}W^{\beta}_{\mu\beta}(g)\\&\quad-W^{\beta}_{\alpha\nu}(g)W^{\alpha}_{\mu\beta}(g)+W^{\beta}_{\alpha\beta}(g)W^{\alpha}_{\mu\nu}(g),\\K_{\mu\nu}(s)&=\partial_{\beta}W^{\beta}_{\mu\nu}(s)-\partial_{\nu}W^{\beta}_{\mu\beta}(s)\\&\quad-W^{\beta}_{\alpha\nu}(s)W^{\alpha}_{\mu\beta}(s)+W^{\beta}_{\alpha\beta}(s)W^{\alpha}_{\mu\nu}(s)\,.\end{aligned} \tag{2.6}$$

There are also contracted curvature tensors composed of the connections $\Gamma^{\lambda}_{\mu\nu}(g)$ and $\Gamma^{\lambda}_{\mu\nu}(s)$:

$$\begin{aligned}R_{\mu\nu}(g)&=\partial_{\beta}\Gamma^{\beta}_{\mu\nu}(g)-\partial_{\nu}\Gamma^{\beta}_{(\mu\beta)}(g)\\&\quad-\Gamma^{\beta}_{\alpha\nu}(g)\Gamma^{\alpha}_{\mu\beta}(g)+\Gamma^{\beta}_{(\alpha\beta)}(g)\Gamma^{\alpha}_{\mu\nu}(g)\,,\\R_{\mu\nu}(s)&=\partial_{\beta}\Gamma^{\beta}_{\mu\nu}(s)-\partial_{\nu}\Gamma^{\beta}_{(\mu\beta)}(s)\\&\quad-\Gamma^{\beta}_{\alpha\nu}(s)\Gamma^{\alpha}_{\mu\beta}(s)+\Gamma^{\beta}_{(\alpha\beta)}(s)\Gamma^{\alpha}_{\mu\nu}(s)\,.\end{aligned} \tag{2.7}$$

The field equations are derived from the Lagrangian density

$$\mathfrak{L}=\mathfrak{L}_g+\mathfrak{L}_s\,, \tag{2.8}$$

where

$$\mathfrak{L}_g=\frac{1}{16\pi G_N}\sqrt{-g}\left(g^{\mu\nu}K_{\mu\nu}(g)+\frac{4\pi G_N}{k_G^2}g^{[\mu\nu]}g_{[\nu\mu]}\right), \tag{2.9}$$

$$\mathfrak{L}_s=\frac{1}{16\pi G_s}\sqrt{-s}\left(s^{\mu\nu}K_{\mu\nu}(s)+\frac{4\pi G_s}{k_s^2}s^{[\mu\nu]}s_{[\nu\mu]}\right). \tag{2.10}$$

Here G_N is the Newtonian gravitational constant $G_N=6.6\times10^{-39}$ GeV^{-2}, and G_s is the strong coupling constant for the spin-2^+ gluons interacting with quarks where $G_s\simeq1$ GeV^{-2}. The purely imaginary constants k_G and k_s are denoted by $k_G=i\kappa_G$ and $k_s=i\kappa_s$ where $\kappa_G=G_N/e$ and $\kappa_s=G_s/g$; the constants e and g are the electric charge and the gluon coupling constant, respectively.

We have not taken the weak interactions explicitly into account in the Lagrangian density. The formalism for doing this will be presented elsewhere.[11] The fundamental length associated with the weak interactions is $\sqrt{G_F}=L_W\simeq6\times10^{-17}$ cm. For gravity it is the Planck length $\sqrt{G_N}=L_P=1.6\times10^{-33}$ cm, while for strong spin-2^+ forces it is $\sqrt{G_s}=L_s\simeq2\times10^{-14}$ cm.

The Lagrangian densities (2.9) and (2.10) will yield free-field equations. They can readily be extended to include the energy-momentum tensors $T_{\mu\nu}$ (hadrons) and $T_{\mu\nu}$ (leptons).

The field equations obtained from the independent variations of $g_{\mu\nu}$, $s_{\mu\nu}$, $W^{\lambda}_{\mu\nu}(g)$, and $W^{\lambda}_{\mu\nu}(s)$ are[2]

$$\partial_{\lambda}(\sqrt{-g}g^{\mu\nu})+\sqrt{-g}g^{\sigma\nu}\Gamma^{\mu}_{\sigma\lambda}(g)+\sqrt{-g}g^{\mu\sigma}\Gamma^{\nu}_{\lambda\sigma}(g)-\sqrt{-g}g^{\mu\nu}\Gamma^{\sigma}_{(\lambda\sigma)}(g)=0\,, \tag{2.11}$$

$$\partial_{\lambda}(\sqrt{-s}s^{\mu\nu})+\sqrt{-s}s^{\sigma\nu}\Gamma^{\mu}_{\sigma\lambda}(s)+\sqrt{-s}s^{\mu\sigma}\Gamma^{\nu}_{\lambda\sigma}(s)-\sqrt{-s}s^{\mu\nu}\Gamma^{\sigma}_{(\lambda\sigma)}(s)=0\,, \tag{2.12}$$

$$\partial_{\nu}(\sqrt{-g}g^{[\mu\nu]})=0\,, \tag{2.13}$$

$$\partial_{\nu}(\sqrt{-s}s^{[\mu\nu]})=0\,, \tag{2.14}$$

$$P_{\mu\nu}(g)=0\,, \tag{2.15}$$

$$P_{\mu\nu}(s)=0\,. \tag{2.16}$$

The Hermitian tensor $P_{\mu\nu}$ is given by

$$P_{\mu\nu}={}^*R_{\mu\nu}+\tfrac{2}{3}(\partial_{\mu}W_{\nu}-\partial_{\nu}W_{\mu})\,, \tag{2.17}$$

where

$${}^*R_{\mu\nu}(g)=R_{\mu\nu}(g)+\frac{4\pi G_N}{k_G^2}I_{\mu\nu}(g)\,, \tag{2.18}$$

$${}^*R_{\mu\nu}(s)=R_{\mu\nu}(s)+\frac{4\pi G_s}{k_s^2}I_{\mu\nu}(s)\,, \tag{2.19}$$

and

$$\begin{aligned}I_{\mu\nu}(g)&=-(g_{\mu\sigma}g^{[\sigma\rho]}g_{\rho\nu}+\tfrac{1}{2}g_{\mu\nu}g_{\sigma\rho}g^{[\sigma\rho]}+g_{[\mu\nu]})\,,\\I_{\mu\nu}(s)&=-(s_{\mu\sigma}s^{[\sigma\rho]}s_{\rho\nu}+\tfrac{1}{2}s_{\mu\nu}s_{\sigma\rho}s^{[\sigma\rho]}+s_{[\mu\nu]})\,.\end{aligned} \tag{2.20}$$

The field equations (2.11) and (2.12) can be reduced to the alternative forms

$$\partial_{\lambda}g_{\mu\nu}-g_{\sigma\nu}\Gamma^{\sigma}_{\mu\lambda}(g)-g_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu}(g)=0\,, \tag{2.21}$$

$$\partial_{\lambda}s_{\mu\nu}-s_{\sigma\nu}\Gamma^{\sigma}_{\mu\lambda}(s)-s_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu}(s)=0\,. \tag{2.22}$$

When half-integer-spin fields are present in the Lagrangian density, we must rewrite the equations using a (complex) vierbein formalism.[12]

The Lagrangian density $\mathcal{L}$ is invariant under groups of local gauge transformations. When $\mathcal{L}$ is reformulated in terms of vierbein fields, it can be shown to be invariant under the group of unitary (local) gauge transformations of U(3, 1), which contains $U(1)\otimes O(3,1)$.[12] The Lagrangian density $\mathcal{L}$ will be invariant under the transformations of $U(3,1)_G \otimes U(3,1)_s$. The theory can be extended to an SU(n) scheme[13, 14] and the Lagrangian density made invariant under $U(1)\otimes SU(2)$ or $U(1)\otimes SU(2) \otimes O(3,1)$ non-Abelian gauge transformations. This opens the door to including weak interactions, mediated by the $W^{\pm}$ and the Z^0 partner of the photon which would acquire masses through a Higgs-type spontaneous-symmetry-breaking term in the Lagrangian.

III. THE LIMIT $k_G = k_s = 0$ AND THE GEOMETRICAL INTERPRETATION OF STRONG INTERACTIONS

We shall adopt the physical identifications

$$g_{[\mu\nu]} = k_G F_{\mu\nu} \tag{3.1}$$

and

$$s_{[\mu\nu]} = k_s f_{\mu\nu}, \tag{3.2}$$

where $F_{\mu\nu}$ and $f_{\mu\nu}$ characterize the electromagnetic and gluon spin-1^- gauge fields, respectively. The tensors $g_{(\mu\nu)}$ and $s_{(\mu\nu)}$ describe the gravitational and strong spin-2^+ fields, respectively.

In the limit $k_G = 0$, we get from (2.13), (2.15), and (2.21) the set of Einstein-Maxwell equations[15]

$$\partial_\lambda g_{(\mu\nu)} - g_{(\sigma\nu)}\left\{{\sigma \atop \mu\lambda}\right\}_g - g_{(\mu\sigma)}\left\{{\sigma \atop \lambda\nu}\right\}_g = 0, \tag{3.3}$$

$$\partial_\nu(\sqrt{-g}\, F^{\mu\nu}) = 0, \tag{3.4}$$

$$G_{\mu\nu}(g) = 8\pi G_N T_{\mu\nu}(F), \tag{3.5}$$

$$\partial_{[\sigma} F_{\mu\nu]} = 0. \tag{3.6}$$

Here

$$\left\{{\lambda \atop \mu\nu}\right\}_g = \tfrac{1}{2} g^{(\lambda\sigma)}(\partial_\nu g_{(\mu\sigma)} + \partial_\mu g_{(\nu\sigma)} - \partial_\sigma g_{(\mu\nu)}) \tag{3.7}$$

and

$$G_{\mu\nu}(g) = \partial_\alpha\left\{{\alpha \atop \mu\nu}\right\}_g - \partial_\nu\left\{{\alpha \atop \mu\alpha}\right\}_g - \left\{{\alpha \atop \mu\sigma}\right\}_g\left\{{\sigma \atop \alpha\nu}\right\}_g + \left\{{\sigma \atop \alpha\sigma}\right\}_g\left\{{\alpha \atop \mu\nu}\right\}_g. \tag{3.8}$$

Moreover, $T_{\mu\nu}(F)$ is the Maxwell stress-energy-momentum tensor

$$T_{\mu\nu}(F) = -F^{\alpha}{}_{\mu}F_{\alpha\nu} + \tfrac{1}{4} g_{(\mu\nu)} F_{\alpha\beta}F^{\alpha\beta}. \tag{3.9}$$

In the limit $k_s = 0$, the equations (2.14), (2.16), and (2.22) become

$$\partial_\lambda s_{(\mu\nu)} - s_{(\sigma\nu)}\left\{{\sigma \atop \mu\lambda}\right\}_s - s_{(\mu\sigma)}\left\{{\sigma \atop \lambda\nu}\right\}_s = 0, \tag{3.10}$$

$$\partial_\nu(\sqrt{-s}\, f^{\mu\nu}) = 0, \tag{3.11}$$

$$G_{\mu\nu}(s) = 8\pi G_s T_{\mu\nu}(f), \tag{3.12}$$

$$\partial_{[\sigma} f_{\mu\nu]} = 0. \tag{3.13}$$

The definitions of $\left\{{\lambda \atop \mu\nu}\right\}_s$ and $G_{\mu\nu}(s)$ follow from replacing $g_{(\mu\nu)}$ in (3.7) by $s_{(\mu\nu)}$ and $\left\{{\lambda \atop \mu\nu}\right\}_g$ in (3.8) by $\left\{{\lambda \atop \mu\nu}\right\}_s$, respectively. Moreover, $T_{\mu\nu}(f)$ is now the Maxwell stress tensor with $F_{\mu\nu}$ replaced by $f_{\mu\nu}$. In terms of the identifications[2]

$$W_\mu(g) = \frac{12\pi G_N}{k_N} A_\mu, \qquad W_\mu(s) = \frac{12\pi G_s}{k_s} B_\mu, \tag{3.14}$$

Eqs. (3.6) and (3.13) correspond to the equations

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad f_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{3.15}$$

When the weak interactions are explicitly included, it would be advantageous to introduce Higgs-type spontaneous-symmetry-breaking terms. This would allow us to preserve the geometrical interpretation of the theory for nonzero massive quanta.

The physical content of our geometrical realization of the theory is this: Gravity couples universally to all matter with the coupling strength G_N, while the massless spin-2^+ strong gluon forces couple universally to all *hadronic* matter with the coupling strength G_s. Hadrons do not couple to leptons via strong forces, but the leptons do experience gravitational and electromagnetic interactions. The forces manifest themselves through the curvature of space-time. The gravitational influence on geometry is universal: All matter is affected by the gravitational metric tensor $g_{(\mu\nu)}$. Lepton test particles are not affected by the "hadronic" metric tensor $s_{(\mu\nu)}$. This kind of "geometrical selectivity" will be a consequence of certain symmetries satisfied by the geometrical structure of space-time. The distortion of flat space-time due to gravity will be dominant at macroscopic distances and contribute significantly at microscopic distances of the order of the Planck length $L_P = 1.6\times10^{-33}$ cm. At distances of the order of hadronic Compton wavelengths $L_s \simeq 10^{-14}$ cm, $g_{(\mu\nu)} \simeq \eta_{\mu\nu}$ where $\eta_{\mu\nu}$ is the Minkowski metric tensor $\eta_{\mu\nu} = \mathrm{diag}\,(-1,-1,-1,1)$. In this domain, the metric of space-time is given by

$$ds^2 = s_{\mu\nu}dx^\mu dx^\nu. \tag{3.16}$$

IV. EXACT CLASSICAL SOLUTIONS AND THE STRUCTURE OF GHEONS

We shall now consider the rigorous solutions that can be obtained from the field equations. The exact spherically symmetric solution of the field equations has been studied in detail.[1,2,16]

Let us first treat the limiting case $G_N \simeq 0$ and $e \simeq 0$ when $g_{(\mu\nu)} \simeq \eta_{\mu\nu}$ and $g_{[\mu\nu]} \simeq 0$. Then the solution of the field equations (2.14), (2.16), and (2.22) gives

$$ds^2 = \left(1 - \frac{2G_s m}{r} + \frac{4\pi G_s e_1^2}{r^2}\right)\left(1 - \frac{\kappa_s^2 e_1^2}{r^4}\right) dt^2 - \left(1 - \frac{2G_s m}{r} + \frac{4\pi G_s e_1^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2 , \tag{4.1}$$

where e_1 is the total strong gluon charge and

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2 . \tag{4.2}$$

The only nonvanishing components of $f_{\mu\nu}$ are

$$f_{14} = \frac{e_1}{r^2} . \tag{4.3}$$

As expected, the Abelian solution is of long-range form. We anticipate that even in the nonclassical limit, when quantum effects are taken into account, the basic features of the solution will be preserved.

The metric (4.1) corresponds to a spinless particle. Solutions of the Kerr-Newman type for spinning particles could be obtained from the field equations and attempts in this direction have already been made.[17]

When $L_s < r$ the quantity $-(ds^2) > 0$. Inside a sphere S_H of radius $r = L_s$ space is Euclidean four-dimensional. This idealized spherical entity describes a hadron. *Quarks are permanently confined inside* S_H. The group of local coordinate transformations within S_H is O(4) isomorphic to SU(2) $\otimes$ SU(2). Only spacelike (classical) orbits occur inside S_H.

A lepton is a *test particle* that can penetrate the surface of S_H, because it does not "see" this *hadronic* surface (parton model). If we ignore at very small distances of order $L_P = 1.6 \times 10^{-33}$ cm the hadronic forces and set $s_{(\mu\nu)} \simeq \eta_{\mu\nu}$ and $s_{[\mu\nu]} \simeq 0$, then from the field equations (2.13), (2.15), and (2.21) we get the metric[1,2]

$$ds^2 = \left(1 - \frac{2G_N M}{r} + \frac{4\pi G_N e^2}{r^2}\right)\left(1 - \frac{\kappa_G^2 e^2}{r^4}\right) dt^2 - \left(1 - \frac{2G_N M}{r} + \frac{4\pi G_N e^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2 . \tag{4.4}$$

Moreover,

$$F_{14} = e/r^2 . \tag{4.5}$$

For $r < L_P$ the quantity $-(ds^2) > 0$ and inside a small sphere S_G with radius $r = L_P$ space is Euclidean four-dimensional. These tiny "space holes" correspond *to fractionally charged quarks* and they repel all leptonic test particles (gravitational repulsion). The quarks owe their existence to the geometrical structure of the exact solution.

We call the complete geometrical object a *gheon*.[6] They are nonsingular solutions of the field equations. When the constants k_G and k_s tend to zero, *the gheons shrink to point singularities*. Two gheons can collide and coalesce, representing e.g. πN or NN scattering.

The exact solution for a gheon also possesses black-hole event horizons. The hadronic solution falls into three classes:

(1) $4\pi G_s e_1^2 < G_s^2 m^2$. The event horizons occur at

$$R_\pm = G_s m \pm (G_s^2 m^2 - 4\pi G_s e_1^2)^{1/2} . \tag{4.6}$$

(2) $4\pi G_s e_1^2 = G_s^2 m^2$. There is only one event horizon occurring at

$$R = G_s m . \tag{4.7}$$

(3) $4\pi G_s e_1^2 > G_s^2 m^2$. In this case there are no event horizons.

The maximal analytic extension of the solution to Kruskal-type coordinates has been found.[16] Nonsingular physical space-time is represented by two submanifolds which are time-reversed images of one another, joined together by a branch point at $r = L_s$. There is a *natural boundary* at $r = L_s$ which forms a surface of confinement for the quarks.

In solution (1) above the temperature is given by[8]

$$2\pi k_B T = 4\pi (R - G_s m) A_s^{-1} , \tag{4.8}$$

where

$$A_s = 4\pi [G_s m + (G_s^2 m^2 - G_s e_1^2)^{1/2}]^2 \tag{4.9}$$

is the area of the black gheon. Case (2) has a zero temperature and there is no thermal radiation of hadrons. Case (3) describes a "naked" gheon, while case (1) can describe a fireball or cluster of hadrons, produced in π-N or N-N collisions, which radiates normal hadrons with a thermal spectrum.[5]

It can be shown that

$$4\pi k_B T = \frac{R_+ - R_-}{R_+^2}\left(1 - \frac{G_S^2}{R_+^4}\right) . \tag{4.10}$$

When $r = \sqrt{G_S}$ the hadronic temperature $T = 0$, leading to a stable hadron such as the proton.

An important difference between the solitonlike gheons described here and the solutions occurring in pure strong-gravity theories[5] is that light quarks cannot be radiated by event horizons, for they are permanently trapped (in the classical theory) inside the gheon spheres S_H. This is a difference in principle between the two possible theories. If quarks are *heavy*, then they will not in any case appear in the hadronic thermal spectrum of pure strong-gravity theories.

V. SOLUTIONS DESCRIBING DISCRETE HADRONIC MATTER

The gheon as a geometrical stable particle could yield an understanding of the nature of the discreteness of hadronic matter and the hadronic mass spectrum. To illustrate this, let us for simplicity restrict ourselves to a calculation in a locally flat space. The problem reduces to matching boundary conditions on the surface of S_H.

The Klein-Gordon equation for a scalar field ϕ with rest mass μ in Minkowski space (with signature -2) is given by

$$(\Box^2+\mu^2)\phi=0\ , \tag{5.1}$$

where

$$\Box^2=\frac{\partial^2}{\partial t^2}-\nabla^2\ . \tag{5.2}$$

This equation is locally valid in the gheon solution for $r>L_s$. For $r<L_s$ we have

$$(\tilde{\Box}^2+\mu^2)\Psi=0\ , \tag{5.3}$$

where

$$\tilde{\Box}^2=\frac{\partial^2}{\partial t^2}+\nabla^2\ . \tag{5.4}$$

The static spherically symmetric solutions of (5.1) and (5.3) are

$$\phi=\frac{\alpha e^{-\mu r}+\beta e^{\mu r}}{r}\ ,\qquad \Psi=\frac{\gamma\sin\mu r+\delta\cos\mu r}{r}\ , \tag{5.5}$$

where α, β, γ, and δ are arbitrary constants. We set $\beta=\delta=0$ to exclude the singularities at $r=0$ and $r=\infty$. By matching the boundary conditions at $r=L_s$ for ϕ and Ψ and $d\phi/dr$, $d\Psi/dr$ we find

$$\tan\mu L_s=-1\ . \tag{5.6}$$

The solutions of (5.6) give

$$\mu L_s=\tfrac{3}{4}\pi+n\pi\ , \tag{5.7}$$

where $n=0, 1, 2, \ldots$ and they lead to a discrete mass spectrum.[18]

These results cannot of course be expected to generate an empirical mass spectrum, but it is hoped that a matching of the solutions of the fully covariant equations for a gheon could lead to an experimentally meaningful answer.

We can consider the problem of the hadronic mass spectrum from a different point of view by studying further the black gheons. The total area of the gheon can be related to its irreducible (bare) mass m_0 by the equation

$$m_0=(A_s/16\pi)^{1/2}\ , \tag{5.8}$$

where A_s is given by Eq. (4.9). m_0 is the mass obtained after all the spin-1 gluon strong charge (and in general angular momentum) has been extracted from the black gheon. The physical mass m of the black gheon is found by combining (4.9) and (5.8) to give[19]

$$m=m_0(1+\tfrac{1}{4}e_1^2)\ , \tag{5.9}$$

where we have set $\sqrt{G_s}=1/m_0$. We now make the identification of the total gluon charge, assuming an SU(3) octet of 1^- gluons,

$$e_1^2=g^2[I(I+1)-\tfrac{1}{4}Y^2]\ , \tag{5.10}$$

in which I and Y are the SU(3) isospin and hypercharge quantum numbers, respectively, and g^2 is the square of the spin-1 Yang-Mills gluon coupling constant. Then using (5.10) in (5.9) we get a Gell-Mann–Okubo type[20] meson mass formula for black gheons:

$$m=m_0+\tfrac{1}{4}g^2m_0[I(I+1)-\tfrac{1}{4}Y^2]\ . \tag{5.11}$$

For a rotating black gheon there would be an additional term in (5.11) depending on the total angular momentum J. The mass formula can also be extended to SU(4), SU(5), etc. by using a more general Casimir operator formula applicable to these higher internal-symmetry groups.[21]

VI. CONCLUSIONS

By adopting a Hermitian nonsymmetric two-tensor field structure, we have succeeded in developing a theory that combines spin-1 and spin-2 gauge fields within a comprehensive mathematical framework. The exact solitonlike solutions of the field equations for the massless gauge fields are nonsingular solutions (gheons) characterized by two fundamental lengths $L_s\sim 10^{-14}$ cm and $L_P\sim 10^{-33}$ cm. This brings the original generalized theory of gravitation[2] into the realm of experimental particle physics. The problem of quark confinement is elegantly solved by using the structure of the gheons, determined by the field equations in the classical limit.

The laws governing the collapse of a massive star and the structure of a hadron are described in the geometrical gauge theory by the same field equations. The unified theory attempts to relate macroscopic catastrophic phenomena such as the collapse of "cold" stars with the microscopic structure of matter and space-time. It is hoped that with this kind of theory we can ultimately achieve a purely geometrical description of the small-scale as well as the large-scale structure of the universe.

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