Are Rindler Quanta Real?
Inequivalent Particle Concepts
in
Quantum Field Theory

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Abstract

Philosophical reflection on quantum field theory has tended to focus on how it revises our conception of what a particle is. However, there has been relatively little discussion of the threat to the "reality" of particles posed by the possibility of inequivalent quantizations of a classical field theory, i.e., inequivalent representations of the algebra of observables of the field in terms of operators on a Hilbert space. The threat is that each representation embodies its own distinctive conception of what a particle is, and how a "particle" will respond to a suitably operated detector. Our main goal is to clarify the subtle relationship between inequivalent representations of a field theory and their associated particle concepts. We also have a particular interest in the Minkowski versus Rindler quantizations of a free Boson field, because they respectively entail two radically different descriptions of the particle content of the field in the *very same* region of spacetime. We shall defend the idea that these representations provide *complementary descriptions* of the same state of the field against the claim that they embody completely *incommensurable theories* of the field.

> *Sagredo*: Do we not see here another example of that all-pervading principle of complementarity which excludes the simultaneous applicability of concepts to the real objects of our world?
>
> Is it not so that, rather than being frustrated by this limitation of our conceptual grasp of the reality, we see in this unification of opposites the deepest and most satisfactory result of the dialectical process in our struggle for understanding?
>
> *Are Quanta Real? A Galilean Dialogue* (Jauch [1973], p. 48)

1 Introduction

Philosophical reflection on quantum field theory has tended to focus on how it revises our conception of what a particle is. For instance, though there is a self-adjoint operator in the theory representing the total number of particles of a field, the standard "Fock space" formalism does not individuate particles from one another. Thus, Teller ([1995], Ch. 2) suggests that we speak of quanta that can be "aggregated", instead of (enumerable) particles — which implies that they can be distinguished and labelled. Moreover, because the theory *does* contain a total number of quanta observable (which, therefore, has eigenstates corresponding to different values of this number), a field state can be a nontrivial superposition of number eigenstates that fails to predict any particular number of quanta with certainty. Teller ([1995], pp. 105-6) counsels that we think of these superpositions as not actually containing any quanta, but only propensities to *display* various numbers of quanta when the field interacts with a "particle detector".

The particle concept seems so thoroughly denuded by quantum field theory that is hard to see how it could possibly underwrite the particulate nature of laboratory experience. Those for whom fields are the fundamental objects of the theory are especially aware of this explanatory burden:

> ...quantum field theory is the quantum theory of a field, not a theory of "particles". However, when we consider the manner in which a quantum field interacts with other systems to which it is coupled, an interpretation of the states in [Fock space] in terms of "particles" naturally arises. It is, of course, essential that this be the case if quantum field theory is to describe observed phenomena, since "particle-like" behaviour is commonly observed (Wald [1994], pp. 46-7).

These remarks occur in the context of Wald's discussion of yet another threat to the "reality" of quanta.

The threat arises from the possibility of inequivalent representations of the algebra of observables of a field in terms of operators on a Hilbert space. Inequivalent representations are required in a variety of situations; for example, interacting field theories in which the scattering matrix does not exist ("Haag's theorem"), free fields whose dynamics cannot be unitarily implemented (Arageorgis *et al* [2000]), and states in quantum statistical mechanics corresponding to different temperatures (Emch [1972]). The catch is that each representation carries with it a distinct notion of "particle". Our main goal in this paper is to clarify the subtle relationship between inequivalent representations of a field theory and their associated particle concepts.

Most of our discussion shall apply to any case in which inequivalent representations of a field are available. However, we have a particular interest in the case of the Minkowski versus Rindler representations of a free Boson field. What makes this case intriguing is that it involves two radically different descriptions of the particle content of the field in the *very same* spacetime region. The questions we aim to answer are:

- Are the Minkowski and Rindler descriptions nevertheless, in some sense, *physically* equivalent?
- Or, are they incompatible, even theoretically *incommensurable*?
- Can they be thought of as *complementary* descriptions in the same way that the concepts of position and momentum are?

- Or, can at most one description, the "inertial" story in terms Minkowski quanta, be the *correct* one?

Few discussions of Minkowski versus Rindler quanta broaching these questions can be found in the philosophical literature, and what discussion there is has not been sufficiently grounded in a rigorous mathematical treatment to deliver cogent answers (as we shall see). We do not intend to survey the vast physics literature about Minkowski versus Rindler quanta, nor all physical aspects of the problem. Yet a proper appreciation of what is at stake, and which answers to the above questions are sustainable, requires that we lay out the basics of the relevant formalism. We have strived for a self-contained treatment, in the hopes of opening up the discussion to philosophers of physics already familiar with elementary non-relativistic quantum theory. (We are inclined to agree with Torretti's recent diagnosis that most philosophers of physics tend to neglect quantum field theory because they are "sickened by untidy math" ([1999], p. 397).)

We begin in section **2** with a general introduction to the problem of quantizing a classical field theory. This is followed by a detailed discussion of the conceptual relationship between inequivalent representations in which we reach conclusions at variance with some of the extant literature. In section **3**, we explain how the state of motion of an observer is taken into account when constructing a Fock space representation of a field, and how the Minkowski and Rindler constructions give rise to inequivalent representations. Finally, in section **4**, we examine the subtle relationship between the different particle concepts implied by these representations. In particular, we defend the idea that they supply *complementary* descriptions of the same field against the claim that they embody different, incommensurable *theories*.

A certain number of mathematical results play an important role in our exposition and in our philosophical arguments. The results are stated in the main text as propositions, and the proofs of those that cannot be found in the literature are included in an appendix.

2 Inequivalent Field Quantizations

In section **2.1** we discuss the Weyl algebra, which in the case of infinitely many degrees of freedom circumscribes the basic kinematical structure of a free Boson field. After introducing in section **2.2** some important concepts concerning representations of the Weyl algebra in terms of operators on Hilbert space, we shall be in a position to draw firm conclusions about the conceptual relation between inequivalent representations in section **2.3**.

2.1 The Weyl Algebra

Consider how one constructs the quantum-mechanical analogue of a classical system with a finite number of degrees of freedom, described by a $2n$-dimensional phase space S. Each point of S is determined by a pair of vectors $\vec{a}, \vec{b} \in \mathbb{R}^n$ whose components $\{a_j\}$ and $\{b_j\}$ encode all the position and momentum components of the system

$$x(\vec{a}) = \sum_{j=1}^{n} a_j x_j, \;\; p(\vec{b}) = \sum_{j=1}^{n} b_j p_j. \tag{1}$$

To quantize the system, we impose the *canonical commutation relations* (CCRs)

$$[x(\vec{a}), x(\vec{a}')] = [p(\vec{b}), p(\vec{b}')] = 0, \;\; [x(\vec{a}), p(\vec{b})] = i(\vec{a} \cdot \vec{b})I, \tag{2}$$

and, then, seek a representation of these relations in terms of operators on a Hilbert space $\mathcal{H}$. In the standard *Schrödinger representation*, $\mathcal{H}$ is the space of square-integrable wavefunctions $L^2(\mathbb{R}^n)$, $x(\vec{a})$ becomes the operator that multiplies a wavefunction $\Psi(x_1, \ldots, x_n)$ by $\sum_{j=1}^{n} a_j x_j$, and $p(\vec{b})$ is the partial differential operator $-i \sum_{j=1}^{n} b_j \frac{\partial}{\partial x_j}$.

Note the action of $x(\vec{a})$ is not defined on an element $\Psi \in L^2(\mathbb{R}^n)$ unless $x(\vec{a})\Psi$ is again square-integrable, and $p(\vec{b})$ is not defined on Ψ unless it is suitably differentiable. This is not simply a peculiarity of the Schrödinger representation. Regardless of the Hilbert space on which they act, two self-adjoint operators whose commutator is a nonzero scalar multiple of the identity, as in (2), cannot both be everywhere defined (Kadison & Ringrose (henceforth, KR) [1997], Remark 3.2.9). To avoid the technical inconvenience of dealing with domains of definition, it is standard to reformulate the representation problem in terms of unitary operators which are bounded.

Introducing the two n-parameter families of unitary operators

$$U(\vec{a}) := e^{ix(\vec{a})}, \;\; V(\vec{b}) := e^{ip(\vec{b})}, \qquad \vec{a}, \vec{b} \in \mathbb{R}^n, \tag{3}$$

it can be shown, at least formally, that the CCRs are equivalent to

$$U(\vec{a})U(\vec{a}') = U(\vec{a} + \vec{a}'), \;\; V(\vec{b})V(\vec{b}') = V(\vec{b} + \vec{b}'), \tag{4}$$

$$U(\vec{a})V(\vec{b}) = e^{i(\vec{a}\cdot\vec{b})}V(\vec{b})U(\vec{a}), \tag{5}$$

called the *Weyl form* of the CCRs. This equivalence holds rigorously in the Schrödinger representation, however there are "irregular" representations in which it fails (see Segal [1967], Sec. 1; Summers [1998], Sec. 1). Thus, one reconstrues the goal as that of finding a representation of the Weyl form of the CCRs in terms of two concrete families of unitary operators $\{U(\vec{a}), V(\vec{b}) : \vec{a}, \vec{b} \in \mathbb{R}^n\}$ acting on a Hilbert space $\mathcal{H}$ that *can* be related, via (3), to canonical position and momentum operators on $\mathcal{H}$ satisfying the CCRs. We shall return to this latter "regularity" requirement later in this section.

Though the position and momentum degrees of freedom have so far been treated on a different footing, we can simplify things further by introducing the composite *Weyl operators*

$$W(\vec{a}, \vec{b}) := e^{i(\vec{a}\cdot\vec{b})/2}V(\vec{b})U(\vec{a}), \qquad \vec{a}, \vec{b} \in \mathbb{R}. \tag{6}$$

Combining this definition with Eqns. (4) and (5) yields the multiplication rule

$$W(\vec{a}, \vec{b})W(\vec{a}', \vec{b}') = e^{-i\sigma((\vec{a},\vec{b}),(\vec{a}',\vec{b}'))/2}W(\vec{a}+\vec{a}', \vec{b}+\vec{b}'), \tag{7}$$

where

$$\sigma((\vec{a}, \vec{b}), (\vec{a}', \vec{b}')) := (\vec{a}' \cdot \vec{b}) - (\vec{a} \cdot \vec{b}'). \tag{8}$$

Observe that $\sigma(\cdot, \cdot)$ is an anti-symmetric, bilinear form on S, called a *symplectic* form. (Note, also, that σ is nondegenerate; i.e., if for any $f \in S$, $\sigma(f, g) = 0$ for all $g \in S$, then $f = 0$.) We set

$$W(\vec{a}, \vec{b})^* := e^{-i(\vec{a}\cdot\vec{b})/2}U(-\vec{a})V(-\vec{b}) = W(-\vec{a}, -\vec{b}). \tag{9}$$

Clearly, then, any representation of the Weyl operators $W(\vec{a}, \vec{b})$ on a Hilbert space $\mathcal{H}$ gives rise to a representation of the Weyl form of the CCRs, and vice-versa.

Now, more generally, we allow our classical phase space S to be any infinite-dimensional vector space, possibly constructed out of solutions to some relativistic wave equation. We assume S comes equipped with a (non-degenerate) symplectic form σ, and we say that a family $\{W_\pi(f) : f \in S\}$ of unitary operators acting on some Hilbert space $\mathcal{H}_\pi$ satisfies *the Weyl relations* just in case (cf. (7), (9))

$$W_\pi(f)W_\pi(g) = e^{-i\sigma(f,g)/2}W_\pi(f+g), \qquad f, g \in S, \tag{10}$$

$$W_\pi(f)^* = W_\pi(-f), \qquad f \in S. \tag{11}$$

We may go on to form arbitrary linear combinations of the Weyl operators, and thus obtain (at least some of) the self-adjoint operators that will serve as observables of the system.

Let $\mathcal{F}$ be a family of bounded operators on $\mathcal{H}_\pi$. We say that a bounded operator A on $\mathcal{H}_\pi$ may be *uniformly* approximated by operators in $\mathcal{F}$ just in case for every $\epsilon > 0$, there is an operator $\tilde{A} \in \mathcal{F}$ such that

$$\|(A - \tilde{A})x\| < \epsilon, \text{ for all unit vectors } x \in \mathcal{H}_\pi. \tag{12}$$

Let $\mathcal{W}_\pi$ denote the set of all bounded operators on $\mathcal{H}_\pi$ that can be uniformly approximated by elements in $\mathcal{F}$, where $\mathcal{F}$ is the set of linear combinations of Weyl operators $W_\pi(f)$ acting on $\mathcal{H}_\pi$. $\mathcal{W}_\pi$ is called the C^**-algebra* generated by the Weyl operators $\{W_\pi(f)\}$. In particular, $\mathcal{W}_\pi$ is a subalgebra of the algebra $\mathbf{B}(\mathcal{H}_\pi)$ of *all* bounded operators on $\mathcal{H}_\pi$ that is itself uniformly closed and closed under taking adjoints $A \mapsto A^*$.

Suppose, now, that $\{W_\pi(f)\}$ and $\{W_\phi(f)\}$ are systems of Weyl operators acting, respectively, on Hilbert spaces $\mathcal{H}_\pi, \mathcal{H}_\phi$. Let $\mathcal{W}_\pi, \mathcal{W}_\phi$ denote the corresponding C^*-algebras. A bijective mapping $\alpha : \mathcal{W}_\pi \mapsto \mathcal{W}_\phi$ is called a $*$*-isomorphism* just in case α is linear, multiplicative, and commutes with the adjoint operation. We then have the following uniqueness result for the C^*-algebra generated by Weyl operators (see Bratteli & Robinson (henceforth, BR) [1996], Thm. 5.2.8).

Proposition 1. *There is a $*$-isomorphism α from $\mathcal{W}_\pi$ onto $\mathcal{W}_\phi$ such that $\alpha(W_\pi(f)) = W_\phi(f)$ for all $f \in S$.*

This establishes that the C^*-algebra generated in any representation by Weyl operators satisfying the Weyl relations is, in fact, representation-independent. We shall denote this abstract algebra, called the *Weyl algebra over* (S, σ), by $\mathcal{W}[S, \sigma]$ (and, when no confusion can result, simply say "Weyl algebra" and write $\mathcal{W}$ for $\mathcal{W}[S, \sigma]$). So our problem boils down to choosing a *representation* $(\pi, \mathcal{H}_\pi)$ of the Weyl algebra, given by a mapping $\pi : \mathcal{W}[S, \sigma] \mapsto \mathbf{B}(\mathcal{H}_\pi)$ preserving all algebraic relations. Note, also, that since the image $\pi(\mathcal{W})$ will always be an isomorphic copy of $\mathcal{W}$, π will always be one-to-one, and hence provide a *faithful* representation of $\mathcal{W}$.

With the representation-independent character of the Weyl algebra $\mathcal{W}$, why should we care any longer to choose a representation? After all, there is no technical obstacle to proceeding abstractly. We can take the self-adjoint elements of $\mathcal{W}$ to be the quantum-mechanical observables of our system. A linear functional ω on $\mathcal{W}$ is called a *state* just in case ω is positive (i.e., $\omega(A^*A) \geq 0$) and normalized (i.e., $\omega(I) = 1$). As usual, a state ω is taken to be *pure* (and *mixed* otherwise) just in case it is not a nontrivial convex combination of other states of $\mathcal{W}$. The dynamics of the system can be represented by a one-parameter group α_t of automorphisms of $\mathcal{W}$ (i.e., each α_t is just a map of $\mathcal{W}$ onto itself that preserves all algebraic relations). Hence, if we have some initial state ω_0, the final state will be given by $\omega_t = \omega_0 \circ \alpha_t$. We can even supply definitions for the probability in the state ω_t that a self-adjoint element $A \in \mathcal{W}$ takes a value lying in some Borel subset of its spectrum (Wald [1994], pp. 79-80), and for transition probabilities between, and superpositions of, pure states of $\mathcal{W}$ (Roberts & Roepstorff [1969]). At no stage, it seems, need we ever introduce a Hilbert space as an essential element of the formalism. In fact, Haag and Kastler ([1964], p. 852) and Robinson ([1966], p. 488) maintain that the choice of a representation is largely a matter of analytical convenience without physical implications.

Nonetheless, the abstract Weyl algebra does not contain unbounded operators, many of which are naturally taken as corresponding to important physical quantities. For instance, the total energy of the system, the canonically conjugate position and momentum observables — which in field theory play the role of the local field observables — and the total number of particles. Also, we shall see later that not even any *bounded* function of the total number of particles (apart from zero and the identity) lies in the Weyl algebra. Surprisingly, Irving Segal (founder of the mathematically rigorous approach to quantum field theory) has written that this:

> ... has the simple if quite rough and somewhat oversimplified interpretation that the total number of "bare" particles is devoid of physical meaning (Segal [1963], p. 56; see also his [1959], p. 12).

We shall return to this issue of physical meaning shortly. First, let us see how a representation can be used to expand the observables of a system beyond the abstract Weyl algebra.

Let $\mathcal{F}$ be a family of bounded operators acting on a representation space $\mathcal{H}_\pi$. We say that a bounded operator A on $\mathcal{H}_\pi$ can be *weakly* approximated by elements of $\mathcal{F}$ just in case for any vector $x \in \mathcal{H}$, and any $\epsilon > 0$, there is some $\tilde{A} \in \mathcal{F}$ such that

$$\left| \langle x, Ax \rangle - \langle x, \tilde{A}x \rangle \right| < \epsilon. \tag{13}$$

(Note the important quantifier change between the definitions of uniform and weak approximation, and that weak approximation has no abstract representation-independent counterpart.) Consider the family $\pi(\mathcal{W})^-$ of bounded operators that can be weakly approximated by elements of $\pi(\mathcal{W})$, i.e., $\pi(\mathcal{W})^-$ is the weak closure of $\pi(\mathcal{W})$. By von Neumann's double commutant theorem, $\pi(\mathcal{W})^- = \pi(\mathcal{W})''$, where the prime operation on a family of operators (here applied twice) denotes the set of all bounded operators on $\mathcal{H}_\pi$ commuting with that family. $\pi(\mathcal{W})''$ is called the *von Neumann algebra* generated by $\pi(\mathcal{W})$. Clearly $\pi(\mathcal{W}) \subseteq \pi(\mathcal{W})''$, however we can hardly expect that $\pi(\mathcal{W}) = \pi(\mathcal{W})''$ when $\mathcal{H}_\pi$ is infinite-dimensional (which it *must* be, since there is no finite-dimensional representation of the Weyl algebra for even a single degree of freedom). Nor should we generally expect that $\pi(\mathcal{W})'' = \mathbf{B}(\mathcal{H}_\pi)$, though this does hold in "irreducible" representations, as we explain in the next subsection.

We may now expand our observables to include all self-adjoint operators in $\pi(\mathcal{W})''$. And, although $\pi(\mathcal{W})''$ still contains only bounded operators, it is easy to associate (potentially physically significant) unbounded observables with this algebra as well. We say that a (possibly unbounded) self-adjoint operator A on $\mathcal{H}_\pi$ is *affiliated* with $\pi(\mathcal{W})''$ just in case all A's spectral projections lie in $\pi(\mathcal{W})''$. Of course, we could have adopted the same definition for self-adjoint operators "affiliated to" $\pi(\mathcal{W})$ itself, but C^*-algebras do not generally contain nontrivial projections (or, if they do, will not generally contain even the spectral projections of their self-adjoint members).

As an example, suppose we now demand to have a (so-called) *regular* representation π, in which the mappings $t \in \mathbb{R} \mapsto \pi(W(tf))$, for all $f \in S$, are all weakly continuous. Then Stone's theorem will guarantee the existence of unbounded self-adjoint operators $\{\Phi(f) : f \in S\}$ on $\mathcal{H}_\pi$ satisfying $\pi(W(tf)) = e^{i\Phi(f)t}$, and it can be shown that all these operators are affiliated to $\pi(\mathcal{W})''$ (KR [1997], Ex. 5.7.53(ii)). In this way, we can recover as observables our original canonically conjugate positions and momenta (cf. Eqn. (3)), which the Weyl relations ensure will satisfy the original unbounded form of the CCRs.

It is important to recognize, however, that by enlarging the set of observables to include those affiliated to $\pi(\mathcal{W})''$, we have now left ourselves open to arbitrariness. In contrast to Proposition 1, we now have

Proposition 2. *There are (even regular) representations π, ϕ of $\mathcal{W}[S, \sigma]$ for which there* is no *$*$-isomorphism α from $\pi(\mathcal{W})''$ onto $\phi(\mathcal{W})''$ such that $\alpha(\pi(W(f))) = \phi(W(f))$ for all $f \in S$.*

This occurs when the representations are "disjoint", which we discuss in the next subsection.

Proposition 2 is what motivates Segal to argue that observables affiliated to the weak closure $\pi(\mathcal{W})''$ in a representation of the Weyl algebra are "somewhat unphysical" and "have only analytical significance" ([1963], pp. 11–14, 134).[1] Segal is explicit that by "physical" he means "empirically measurable in principle" ([1963], p. 11). We should not be confused by the fact that he often calls observables that fail this test "conceptual" (suggesting they are more than mere analytical crutches). For in Baez *et al* ([1992], p. 145), Segal gives as an example the bounded self-adjoint operator $\cos p + (1 + x^2)^{-1}$ on $L^2(\mathbb{R})$ "for which no known 'Gedanken experiment' will actually directly determine the spectrum, and so [it] represents an observable in a purely conceptual sense". Thus, the most obvious reading of Segal's position is that he subscribes to an operationalist view about the physical significance of theoretical quantities. Indeed, since good reasons *can* be given for the impossibility of exact ("sharp") measurements of all the observables in a von Neumann algebra generated by a C^*-algebra (see Wald [1994], Halvorson [2000a]), operationalism explains Segal's dismissal of the physical (as opposed to analytical) significance of observables not in the Weyl algebra *per se*. (Also, it is worth recalling that Bridgman himself was similarly unphased by having to relegate much of the mathematical structure of a physical theory to "a ghostly domain with no physical relevance" ([1936], p. 116).)

Of course, insofar as operationalism is philosophically defensible at all, it does not compell assent. And, in this instance, Segal's operationalism has not dissuaded others from taking the more liberal view apparently advocated by Wald:

> ...one should not view [the Weyl algebra] as encompassing *all* observables of the theory; rather, one should view [it] as encompassing a "minimal" collection of observables, which is sufficiently large to enable the theory to be formulated. One may later wish to

[1] Actually, Segal consistently finds it convenient to work with a strictly larger algebra than our (minimal) Weyl algebra, sometimes called the *mode finite* or *tame* Weyl algebra. However, both Proposition 1 (see Baez *et al* [1992], Thm. 5.1) and Proposition 2 continue to hold for the tame Weyl algebra (also cf. Segal [1967], pp. 128-9).

> enlarge [the algebra] and/or further restrict the notion of "state" in order to accommodate the existence of additional observables ([1994], p. 75).

The conservative and liberal views entail quite different commitments about the physical equivalence of representations — or so we shall argue.

2.2 Equivalence and Disjointness of Representations

It is essential that precise mathematical definitions of equivalence be clearly distinguished from the, often dubious, arguments that have been offered for their conceptual significance. We confine this section to discussing the definitions.

Since our ultimate goal is to discuss the Minkowski and Rindler quantizations of the Weyl algebra, we only need to consider the case where one of the two representations at issue, say π, is "irreducible" and the other, ϕ, is "factorial". A representation π of $\mathcal{W}$ is called *irreducible* just in case no non-trivial subspace of the Hilbert space $\mathcal{H}_\pi$ is invariant under the action of all operators in $\pi(\mathcal{W})$. It is not difficult to see that this is equivalent to $\pi(\mathcal{W})'' = \mathbf{B}(\mathcal{H}_\pi)$ (using the fact that an invariant subspace will exist just in case the projection onto it commutes with all of $\pi(\mathcal{W})$). A representation ϕ of $\mathcal{W}$ is called *factorial* whenever the von Neumann algebra $\phi(\mathcal{W})''$ is a *factor*, i.e., it has trivial centre (the only operators in $\phi(\mathcal{W})''$ that commute with all other operators in that set are multiples of the identity). Since $\mathbf{B}(\mathcal{H}_\pi)$ is a factor, it is clear that π's irreducibility entails its factoriality. Thus, the Schrödinger representation of the Weyl algebra is both irreducible and factorial.

The strongest form of equivalence between representations is unitary equivalence: ϕ and π are said to be *unitarily equivalent* just in case there is a unitary operator U mapping $\mathcal{H}_\phi$ isometrically onto $\mathcal{H}_\pi$, and such that

$$U\phi(A)U^{-1} = \pi(A) \qquad \forall A \in \mathcal{W}. \tag{14}$$

There are two other weaker definitions of equivalence.

Given a family π_i of irreducible representations of the Weyl algebra on Hilbert spaces $\mathcal{H}_i$, we can construct another (reducible) representation ϕ of the Weyl algebra on the direct sum Hilbert space $\sum \oplus \mathcal{H}_i$, by setting

$$\phi(A) = \sum_i \oplus\, \pi_i(A), \qquad A \in \mathcal{W}. \tag{15}$$

If each representation $(\pi_i, \mathcal{H}_i)$ is unitarily equivalent to some representation $(\pi, \mathcal{H})$, we say that $\phi = \sum \oplus \pi_i$ is a *multiple* of the representation π. Furthermore, we say that two representations of the Weyl algebra, ϕ (factorial) and π (irreducible), are *quasi-equivalent* just in case ϕ is a multiple of π. It should be obvious from this characterization that quasi-equivalence weakens unitary equivalence.

Another way to see this is to use the fact (KR [1997], Def. 10.3.1, Cor. 10.3.4) that quasi-equivalence of ϕ and π is equivalent to the existence of a $*$-isomorphism α from $\phi(\mathcal{W})''$ onto $\pi(\mathcal{W})''$ such that $\alpha(\phi(A)) = \pi(A)$ for all $A \in \mathcal{W}$. Unitary equivalence is then just the special case where the $*$-isomorphism α can be implemented by a unitary operator.

If ϕ is not even quasi-equivalent to π, then we say that ϕ and π are *disjoint* representations of $\mathcal{W}$.[2] Note, then, that if both π and ϕ are irreducible, they are either unitarily equivalent or disjoint.

We can now state the following pivotal result (von Neumann [1931]).

Stone-von Neumann Uniqueness Theorem. *When S is finite-dimensional, every regular representation of the Weyl algebra $\mathcal{W}[S, \sigma]$ is quasi-equivalent to the Schrödinger representation.*

This theorem is usually interpreted as saying that there is a unique quantum theory corresponding to a classical theory with finitely-many degrees of freedom. The theorem *fails* in field theory — where S is infinite-dimensional — opening the door to disjoint representations and Proposition 2.

There is another way to think of the relations between representations, in terms of states. Recall the abstract definition of a state of a C^*-algebra, as simply a positived normalized linear functional on the algebra. Since, in any representation π, $\pi(\mathcal{W})$ is just a faithful copy of $\mathcal{W}$, π induces a one-to-one correspondence between the abstract states of $\mathcal{W}$ and the abstract states of $\pi(\mathcal{W})$. Note now that *some* of the abstract states on $\pi(\mathcal{W})$ are the garden-variety density operator states that we are familiar with from elementary

[2] In general, disjointness is not defined as the negation of quasi-equivalence, but by the more cumbersome formulation: Two representations π, ϕ are disjoint just in case π has no "subrepresentation" quasi-equivalent to ϕ, and ϕ has no subrepresentation quasi-equivalent to π. Since we are only interested, however, in the special case where π is irreducible (and hence has no non-trivial subrepresentations) and ϕ is "factorial" (and hence is quasi-equivalent to each of its subrepresentations), the cumbersome formulation reduces to our definition.

quantum mechanics. In particular, define ω_D on $\pi(\mathcal{W})$ by setting

$$\omega_D(A) := \mathrm{Tr}(DA), \qquad A \in \pi(\mathcal{W}). \tag{16}$$

In general, however, there will be abstract states of $\pi(\mathcal{W})$ *that are not given by density operators via Eqn. (16).*[3] We say then that an abstract state ω of $\pi(\mathcal{W})$ is *normal* just in case it is given (via Eqn. (16)) by some density operator D on $\mathcal{H}_\pi$. We let $\mathfrak{F}(\pi)$ denote the subset of the abstract state space of $\mathcal{W}$ consisting of those states that correspond to normal states in the representation π, and we call $\mathfrak{F}(\pi)$ the *folium* of the representation π. That is, $\omega \in \mathfrak{F}(\pi)$ just in case there is a density operator D on $\mathcal{H}_\pi$ such that

$$\omega(A) = \mathrm{Tr}(D\pi(A)), \qquad A \in \mathcal{W}. \tag{17}$$

We then have the following equivalences (KR [1997], Prop. 10.3.13):

$$\begin{aligned} \pi \text{ and } \phi \text{ are quasi-equivalent} &\iff \mathfrak{F}(\pi) = \mathfrak{F}(\phi), \\ \pi \text{ and } \phi \text{ are disjoint} &\iff \mathfrak{F}(\pi) \cap \mathfrak{F}(\phi) = \emptyset. \end{aligned}$$

In other words, π and ϕ are quasi-equivalent just in case they share the same normal states. And π and ϕ are disjoint just in case they have *no* normal states in common.

In fact, if π is disjoint from ϕ, then all normal states in the representation π are "orthogonal" to all normal states in the representation ϕ. We may think of this situation intuitively as follows. Define a third representation ψ of $\mathcal{W}$ on $\mathcal{H}_\pi \oplus \mathcal{H}_\phi$ by setting

$$\psi(A) = \pi(A) \oplus \phi(A), \qquad A \in \mathcal{W}. \tag{18}$$

Then, every normal state of the representation π is orthogonal to every normal state of the representation ϕ.[4] This makes sense of the oft-repeated phrase (see, e.g., Gerlach [1989]) that "The Rindler vacuum is orthogonal to all states in the Minkowski vacuum representation".

[3] Gleason's theorem does not rule out these states because it is not part of the definition of an abstract state that it be countably additive over mutually orthogonal projections. Indeed, such additivity does not even make sense abstractly, because an infinite sum of orthogonal projections can never converge uniformly, only weakly (in a representation).

[4] This intuitive picture may be justified by making use of the "universal representation" of $\mathcal{W}$ (KR [1997], Thm. 10.3.5).

While not every abstract state of $\mathcal{W}$ will be in the folium of a given representation, there is always *some* representation of $\mathcal{W}$ in which the state *is* normal, as a consequence of the following (see KR [1997], Thms. 4.5.2 and 10.2.3).

Gelfand-Naimark-Segal Theorem. *Any abstract state ω of a C^*-algebra $\mathcal{A}$ gives rise to a unique (up to unitary equivalence) representation $(\pi_\omega, \mathcal{H}_\omega)$ of $\mathcal{A}$ and vector $\Omega_\omega \in \mathcal{H}_\omega$ such that*

$$\omega(A) = \langle \Omega_\omega, \pi_\omega(A)\Omega_\omega \rangle, \qquad A \in \mathcal{A}, \tag{19}$$

and such that the set $\{\pi_\omega(A)\Omega_\omega : A \in \mathcal{A}\}$ is dense in $\mathcal{H}_\omega$. Moreover, π_ω is irreducible just in case ω is pure.

The triple $(\pi_\omega, \mathcal{H}_\omega, \Omega_\omega)$ is called the *GNS representation* of $\mathcal{A}$ induced by the state ω, and Ω_ω is called a *cyclic* vector for the representation. We shall see in the next main section how the Minkowski and Rindler vacuums induce disjoint GNS representations of the Weyl algebra.

There is a third notion of equivalence of representations, still weaker than quasi-equivalence. Let π be a representation of $\mathcal{W}$, and let $\mathfrak{F}(\pi)$ be the folium of π. We say that an abstract state ω of $\mathcal{W}$ can be *weak* approximated* by states in $\mathfrak{F}(\pi)$ just in case for each $\epsilon > 0$, and for each finite collection $\{A_i : i = 1, \ldots, n\}$ of operators in $\mathcal{W}$, there is a state $\omega' \in \mathfrak{F}(\pi)$ such that

$$|\omega(A_i) - \omega'(A_i)| < \epsilon, \qquad i \in [1, n]. \tag{20}$$

Two representations π, ϕ are then said to be *weakly equivalent* just in case all states in $\mathfrak{F}(\pi)$ may be weak* approximated by states in $\mathfrak{F}(\phi)$ and vice-versa. We then have the following fundamental result (Fell [1960]).

Fell's Theorem. *Let π be a faithful representation of a C^*-algebra $\mathcal{A}$. Then, every abstract state of $\mathcal{A}$ may be weak* approximated by states in $\mathfrak{F}(\pi)$.*

In particular, then, it follows that *all* representations of $\mathcal{W}$ are weakly equivalent.

In summary, we have the following implications for any two representations π, ϕ:

$$\text{Unitarily equivalent} \implies \text{Quasi-equivalent} \implies \text{Weakly equivalent.}$$

If π and ϕ are both irreducible, then the first arrow is reversible.

2.3 Physical Equivalence of Representations

Do disjoint representations yield *physically* inequivalent theories? It depends on what one takes to be the physical content of a theory, and what one means by "equivalent theories" — subjects about which philosophers of science have had plenty to say.

Recall that Reichenbach [1938] deemed two theories "the same" just in case they are empirically equivalent, i.e., they are confirmed equally under all possible evidence. Obviously this criterion, were we to adopt it here, would beg the question against those who (while agreeing that, strictly speaking, only self-adjoint elements of the Weyl algebra can actually be measured) invest physical importance to "global" quantities only definable in a representation, like the total number of particles.

A stronger notion of equivalence, due originally to Glymour [1971] (who proposed it only as a *necessary* condition), is that two theories are equivalent only if they are "intertranslatable". This is often cashed out in logical terms as the possibility of defining the primitives of one theory in terms of those of the other so that the theorems of the first appear as logical consequences of those of the second, and vice-versa. Prima facie, this criterion is ill-suited to the present context, because the different "theories" are not presented to us as syntactic structures or formalized logical systems, but rather two competing algebras of observables whose states represent physical predictions. In addition, intertranslatability *per se* has nothing to say about what portions of the mathematical formalism of the two physical theories being compared ought to be intertranslatable, and what should be regarded as "surplus mathematical structure" not required to be part of the translation.

Nevertheless, we believe the intertranslatability thesis can be naturally expressed in the present context and rendered neutral as between the conservative and liberal approaches to physical observables discussed earlier. Think of the Weyl operators $\{\phi(W(f)) : f \in S\}$ and $\{\pi(W(f)) : f \in S\}$ as the primitives of our two "theories", in analogy with the way the natural numbers can be regarded as the primitives of a "theory" of real numbers. Just as we may define rational numbers as ratios of natural numbers, and then construct real numbers as the limits of Cauchy sequences of rationals, we construct the Weyl algebras $\phi(\mathcal{W})$ and $\pi(\mathcal{W})$ by taking linear combinations of the Weyl operators and then closing in the uniform topology. We then close in the weak topology of the two representations to obtain the von Neumann algebras $\phi(\mathcal{W})''$ and $\pi(\mathcal{W})''$. Whether the observables affiliated with this second closure have physical significance is up for grabs, as is whether we should be conservative and take only normal states in the given representation to be physical, or be more liberal and admit a broader class of algebraic states. The analogue of the "theorems" of the theory are then statements about the expectation values dictated by the physical states for the self-adjoint elements in the physically relevant algebra of the theory.

We therefore propose the following formal rendering of Glymour's intertranslatability thesis adapted to the present context. Representations ϕ and π are *physically equivalent* only if there exists a bijective mapping α from the physical observables of the representation ϕ to the physical observables of the representation π, and another bijective mapping β from the physical states of the representation ϕ to the physical states of the representation π, such that

$$\alpha(\phi(W(f))) = \pi(W(f)),\ \forall f \in S, \tag{21}$$
$$\text{(“primitives”)}$$

$$\beta(\omega)(\alpha(A)) = \omega(A),\ \forall \text{ states } \omega,\ \forall \text{ observables } A. \tag{22}$$
$$\text{(“theorems”)}$$

Of course, the notion of equivalence we obtain depends on how we construe the phrases "physical observables of a representation π" and "physical states of a representation π". According to a conservative rendering of observables, only the self-adjoint elements of the Weyl algebra $\pi(\mathcal{W})$ are genuine physical observables of the representation π. (More generally, an unbounded operator on $\mathcal{H}_\pi$ is a physical observable only if all of its bounded functions lie in $\pi(\mathcal{W})$.) On the other hand, a liberal rendering of observables considers all self-adjoint operators in the weak closure $\pi(\mathcal{W})^-$ of $\pi(\mathcal{W})$ as genuine physical observables. (More generally, those unbounded operators whose bounded functions lie in $\pi(\mathcal{W})^-$ should be considered genuine physical observables.) A conservative with respect to states claims that only those density operator states (i.e., normal states) of the algebra $\pi(\mathcal{W})$ are genuine physical states. On the other hand, a liberal with respect to states claims that all algebraic states of $\pi(\mathcal{W})$ should be thought of as genuine physical states. We thereby obtain *four distinct* necessary conditions for physical equivalence, according to whether one is conservative or liberal about observables, and conservative or liberal about states.

Arageorgis ([1995], p. 302) and Arageorgis *et al* ([2000], p. 3) also take the correct notion of physical equivalence in this context to be intertranslatability. On the basis of informal discussions (with rather less supporting argument than one would have liked), they claim that physical equivalence of representations requires that they be unitarily equivalent. (They do not discuss quasi-equivalence.) We disagree with this conclusion, but there is still substantial overlap between us. For instance, with our precise necessary condition for physical equivalence above, we can now establish the following elementary result.

Proposition 3. *Under the conservative approach to states, ϕ (factorial) and π (irreducible) are physically equivalent representations of $\mathcal{W}$ only if they are quasi-equivalent.*

Proof. Let ω be a normal state of $\phi(\mathcal{W})$. Then, by hypothesis, $\beta(\omega)$ is a normal state of $\pi(\mathcal{W})$. Define a state ρ on $\mathcal{W}$ by

$$\rho(A) = \omega(\phi(A)), \qquad A \in \mathcal{W}. \tag{23}$$

Since ω is normal, $\rho \in \mathfrak{F}(\phi)$. Define a state ρ' on $\mathcal{W}$ by

$$\rho'(A) = \beta(\omega)(\pi(A)), \qquad A \in \mathcal{W}. \tag{24}$$

Since $\beta(\omega)$ is normal, $\rho' \in \mathfrak{F}(\pi)$. Now, conditions (21) and (22) entail that

$$\omega(\phi(A)) = \beta(\omega)(\alpha(\phi(A))) = \beta(\omega)(\pi(A)), \tag{25}$$

for any $A = W(f) \in \mathcal{W}$, and thus $\rho(W(f)) = \rho'(W(f))$ for any $f \in S$. However, a state of the Weyl algebra is uniquely determined (via linearity and uniform continuity) by its action on the generators $\{W(f) : f \in S\}$. Thus, $\rho = \rho'$ and since $\rho \in \mathfrak{F}(\phi) \cap \mathfrak{F}(\pi)$, it follows that ϕ and π are quasi-equivalent. $\square$

With somewhat more work, the following result may also be established.[5]

Proposition 4. *Under the liberal approach to observables, ϕ (factorial) and π (irreducible) are physically equivalent representations of $\mathcal{W}$ only if they are quasi-equivalent.*

[5]Our proof in the appendix makes rigorous Arageorgis' brief (and insufficient) reference to Wigner's symmetry representation theorem in his ([1995], p. 302, footnote).

The above results leave only the position of the "conservative about observables/liberal about states" undecided. However, we claim, *pace* Arageorgis *et al*, that a proponent of this position can satisfy conditions (21),(22) *without* committing himself to quasi-equivalence of the representations. Since he is conservative about observables, Proposition 1 already guarantees the existence of a bijective mapping α — in fact, a *-isomorphism from the whole of $\phi(\mathcal{W})$ to the whole of $\pi(\mathcal{W})$ — satisfying (21). And if he is liberal about states, the state mapping β need not map any normal state of $\phi(\mathcal{W})$ into a normal state of $\pi(\mathcal{W})$, bypassing the argument for Proposition 3. Consider, for example, the liberal who takes *all* algebraic states of $\phi(\mathcal{W})$ and $\pi(\mathcal{W})$ to be physically significant. Then for any algebraic state ω of $\phi(\mathcal{W})$, the bijective mapping β that sends ω to the state $\omega \circ \alpha^{-1}$ on $\pi(\mathcal{W})$ trivially satisfies condition (22) even when ϕ and π are disjoint.

Though we have argued that Segal was conservative about observables, we are not claiming he was a liberal about states. In fact, Segal consistently maintained that only the "regular states" of the Weyl algebra have physical relevance ([1961], p. 7; [1967], pp. 120, 132). A state ω of $\mathcal{W}[S,\sigma]$ is called *regular* just in case the map $f \mapsto \omega(W(f))$ is continuous on all finite-dimensional subspaces of S; or, equivalently, just in case the GNS representation of $\mathcal{W}[S,\sigma]$ determined by ω is regular (Segal [1967], p. 134). However, note that, unlike normality of a state, regularity is representation-*independent*. Taking the set of all regular states of the Weyl algebra to be physical is therefore still liberal enough to permit satisfaction of condition (22). For the mapping β of the previous paragraph trivially preserves regularity, insofar as both ω and $\omega \circ \alpha^{-1}$ induce the same abstract regular state of $\mathcal{W}$.

Our verdict, then, is that Segal is not committed to saying only quasi-equivalent representations can be physically equivalent. And this explains why he sees fit to *define* physical equivalence of representations in such a way that Proposition 1 secures the physical equivalence of all representations (see Segal [1961], Defn. 1(c)). (Indeed, Segal regards Proposition 1 as the appropriate generalization of the Stone-von Neumann uniqueness theorem to infinite-dimensional S.) One might still ask what the point of passing to a concrete Hilbert space representation of $\mathcal{W}$ is if one is going to allow as physically possible regular states not in the folium of the chosen representation. The point, we take it, is that if we are interested in drawing out the predictions of some particular regular state, such as the Minkowski vacuum or the Rindler vacuum, then passing to a particular representation will put at our disposal all the standard analytical techniques of Hilbert space quantum mechanics to facilitate calculations in that particular state.[6]

Haag & Kastler ([1964], p. 852) and Robinson ([1966], p. 488) have argued that *by itself* the *weak* equivalence of all representations of the Weyl algebra entails their physical equivalence.[7] Their argument starts from the fact that, by measuring the expectations of a finite number of observables $\{A_i\}$ in the Weyl algebra, each to a finite degree of accuracy ϵ, we can only determine the state of the system to within a weak* neighborhood. But by Fell's density theorem, states from the folium of *every* representation lie in this neighborhood. So for all practical purposes, we can never determine which representation is the physically "correct" one and they all, in some (as yet, unarticulated!) sense, carry the same physical content. And as a corollary, choosing a representation is simply a matter of convention.

Clearly the necessary condition for physical equivalence we have proposed constitutes a very different notion of equivalence than weak equivalence, so we are not disposed to agree with this argument. Evidently it presupposes that only the observables in the Weyl algebra itself are physically significant, which we have granted *could* be grounded in operationalism. However, there is an additional layer of operationalism that the argument must presuppose: scepticism about the physical meaning of postulating an *absolutely precise* state for the system. If we follow this scepticism to its logical conclusion, we should instead think of physical states of the Weyl algebra as represented by weak* neighborhoods of algebraic states. What it would then mean to falsify a state, so understood, is that some finite number of expectation values measured to within finite accuracy are found to be incompatible with all the algebraic states in some proposed weak* neighborhood. Unfortunately, no particular "state" in this sense can ever be fully empirically adequate, for any hypothesized state (= weak* neighborhood) will be subject to constant revision as the accuracy and number of our experiments increase. We agree

[6]In support of not limiting the physical states of the Weyl algebra to any one representation's folium, one can also cite the cases of non-unitarily implementable dynamics discussed by Arageorgis *et al* ([2000]) in which dynamical evolution occurs between regular states that induce disjoint GNS representations. In such cases, it would hardly be coherent to maintain that regular states *dynamically accessible to one another* are not physically co-possible.

[7]Indeed, the term "physical equivalence" is often used synonomously with weak equivalence; for example, by Emch ([1972], p. 108), who, however, issues the warning that "we should be seriously wary of semantic extrapolations" from this usage. Indeed!

with Summers [1998] that this would do irreparable damage to the predictive power of the theory — damage that can only be avoided by maintaining that there is a correct algebraic state.

We do not, however, agree with Summers' presumption (tacitly endorsed by Arageorgis *et al* [2000]) that we not only need the correct algebraic state, but "...the correct state *in the correct representation*" ([2000], p. 13; italics ours). This added remark of Summers' is directed against the conventionalist corollary to Fell's theorem. Yet we see nothing in the point about predictive power that privileges any particular representation, not even the GNS representation of the predicted state. We might well have good reason to deliberately choose a representation in which the *precise* algebraic state predicted is not normal. (For example, Kay [1985] does exactly this, by "constructing" the Minkowski vacuum as a thermal state in the Rindler quantization.) The role Fell's theorem plays is then, at best, methodological. All it guarantees is that when we calculate with density operators in our chosen represention, we can always get a reasonably good indication of the predictions of *whatever* precise algebraic state we have postulated for the system.

So much for the conservative stance on observables. An interpreter of quantum field theory is not likely to find it attractive, if only because none of the observables that have any chance of underwriting the particle concept lie in the Weyl algebra. But suppose, as interpreters, we adopt the liberal approach to observables. Does the physical inequivalence of disjoint representations entail their incompatibility, or even incommensurability? By this, we do not mean to conjure up Kuhnian thoughts about incommensurable "paradigms", whose proponents share no methods to resolve their disputes. Rather, we are pointing to the (more Feyerabendian?) possibility of an unanalyzable shift in meaning between disjoint representations as a consequence the fact that the concepts (observables and/or states) of one representation are not wholly definable or translatable in terms of those of the other.

One might think of neutralizing this threat by viewing disjoint representations as sub-theories or models of a more general theory built upon the Weyl algebra. Consider the analogy of *two different* classical systems, modelled, say, by phase spaces of different dimension. Though not physically equivalent, these models hardly define incommensurable theories insofar as they share the characteristic kinematical and dynamical features that warrant the term "classical". Surely the same could be said of disjoint representations of the Weyl algebra?

Alas, there is a crucial disanalogy. In the case of the Minkowski and

Rindler representations, physicists freely switch between them to describe the quantum state of the *very same* "system" — in this case, the quantum field in a fixed region of spacetime (see, e.g., Unruh and Wald [1984] and Wald [1994], Sec. 5.1). And, as we shall see later, the weak closures of these representations provide physically inequivalent descriptions of the particle content in the region. So it is tempting to view this switching back and forth between disjoint representations as conceptually incoherent (Arageorgis [1995], p. 268), and to see the particle concepts associated to the representations as not just different, but outright incommensurable (Arageorgis *et al* [2000]).

We shall argue that this view, tempting as it is, goes too far. For suppose we *do* take the view that the observables affiliated to the von Neumann algebras generated by two disjoint representations ϕ and π simply represent different physical aspects of the same physical system. If we are also liberal about states (not restricting ourselves to any one representation's folium), then it is natural to ask what implications a state ω of our system, that happens to be in the folium of ϕ, has for the observables in $\pi(\mathcal{W})''$. In many cases, it is possible to extract a definite answer.

In particular, any abstract state ω of $\mathcal{W}$ gives rise to a state on $\pi(\mathcal{W})$, which may be extended to a state on the weak closure $\pi(\mathcal{W})''$ (KR [1997], Thm. 4.3.13). The only catch is that unless $\omega \in \mathfrak{F}(\pi)$, this extension will not be unique. For, only normal states of $\pi(\mathcal{W})$ possess sufficiently nice continuity properties to ensure that their values on $\pi(\mathcal{W})$ uniquely fix their values on the weak-closure $\pi(\mathcal{W})''$ (see KR [1997], Thm. 7.1.12). *However*, it may happen that all extensions of ω agree on the expectation value they assign to a *particular observable* affiliated to $\pi(\mathcal{W})''$. This is the strategy we shall use to make sense of assertions such as "The Minkowski vacuum in a (Rindler) spacetime wedge is full of Rindler quanta" (cf., e.g., DeWitt [1979a]). The very fact that such assertions can be made sense of *at all* takes the steam out of claims that disjoint representations are incommensurable. Indeed, we shall ultimately argue that this shows disjoint representations should not be treated as *competing* "theories" in the first place.

3 Constructing representations

We now explain how to construct "Fock representations" of the CCRs. In sections **3.1** and **3.2** we show how this construction depends on one's choice of preferred timelike motion in Minkowski spacetime. In section **3.3**, we show that alternative choices of preferred timelike motion can result in unitarily inequivalent — indeed, disjoint — representations.

3.1 First Quantization ("Splitting the Frequencies")

The first step in the quantization scheme consists in turning the classical phase space (S, σ) into a quantum-mechanical "one particle space" — i.e., a Hilbert space. *The non-uniqueness of the quantization scheme comes in at this very first step.*

Depending on our choice of preferred timelike motion, we will have a one-parameter group T_t of linear mappings from S onto S representing the evolution of the classical system in time. The flow $t \mapsto T_t$ should also preserve the symplectic form. A bijective real-linear mapping $T : S \mapsto S$ is called a *symplectomorphism* just in case T preserves the symplectic form; i.e., $\sigma(Tf, Tg) = \sigma(f, g)$ for all $f, g \in S$.

We say that J is a *complex structure* for (S, σ) just in case

1. J is a symplectomorphism,
2. $J^2 = -I$,
3. $\sigma(f, Jf) > 0, \qquad 0 \neq f \in S$.

Relative to a complex structure J, we may extend the scalar multiplication on S to complex numbers; viz., take multiplication by $a + bi$ as given by $a + bi := af + bJf \in S$. We may also define an inner product $(\cdot, \cdot)_J$ on the resulting complex vector space by setting

$$(f, g)_J := \sigma(f, Jg) + i\sigma(f, g), \qquad f, g \in S. \tag{26}$$

We let $\mathcal{S}_J$ denote the Hilbert space that results when we equip (S, σ) with the extended scalar multiplication and inner product $(\cdot, \cdot)_J$.

A symplectomorphism T is (by assumption) a real-linear operator on S. However, it does not automatically follow that T is a *complex*-linear operator on $\mathcal{S}_J$, since $T(if) = i(Tf)$ may fail. If, however, T commutes with J, then T will be a complex-linear operator on $\mathcal{S}_J$, and it is easy to see that $(Tf, Tg)_J = (f, g)_J$ for all $f, g \in \mathcal{S}_J$, so T would in fact be unitary. Accordingly, we say that a group T_t of symplectomorphisms on (S, σ) is *unitarizable* relative to J just in case $[J, T_t] = 0$ for all $t \in \mathbb{R}$.

If T_t is unitarizable and $t \mapsto T_t$ is weakly continuous, so that we have $T_t = e^{itH}$ (by Stone's theorem), we say that T_t has *positive energy* just in case H is a positive operator. In general, we say that $(\mathcal{H}, U_t)$ is a *quantum one particle system* just in case $\mathcal{H}$ is a Hilbert space and U_t is a weakly continuous one-parameter unitary group on $\mathcal{H}$ with positive energy. Kay ([1979]) proved:

Proposition 5. *Let T_t be a one-parameter group of symplectomorphisms of (S, σ). If there is a complex structure J on (S, σ) such that $(\mathcal{S}_J, T_t)$ is a quantum one particle system, then J is unique.*

Physically, the time translation group T_t determines a natural decomposition (or "splitting") of the solutions of the relativistic wave equation we are quantizing into those that oscillate with purely positive and with purely negative frequency with respect to the motion. This has the effect of uniquely fixing a choice of J, and the Hilbert space $\mathcal{S}_J$ then provides a representation of the positive frequency solutions alone.[8]

3.2 Second Quantization (Fock space)

Once we have a fixed complex structure J on (S, σ), the "second quantization" procedure yields a unique representation $(\pi, \mathcal{H}_\pi)$ of the Weyl algebra $\mathcal{W}[S, \sigma]$.

Let $\mathcal{H}^n$ denote the n-fold symmetric tensor product of $\mathcal{S}_J$ with itself. That is, using $\mathcal{S}_J^n$ to denote $\mathcal{S}_J \otimes \cdots \otimes \mathcal{S}_J$ (n times), $\mathcal{H}^n = P_+(\mathcal{S}_J^n)$ where P_+ is the projection onto the symmetric subspace. Then we define a Hilbert space

$$\mathcal{F}(\mathcal{S}_J) := \mathbb{C} \oplus \mathcal{H}^1 \oplus \mathcal{H}^2 \oplus \mathcal{H}^3 \oplus \cdots , \tag{27}$$

called the *bosonic Fock space over $\mathcal{S}_J$*. Let

$$\Omega := 1 \oplus 0 \oplus 0 \oplus \cdots , \tag{28}$$

denote the privileged "Fock vacuum" state in $\mathcal{F}(\mathcal{S}_J)$.

Now, we define creation and annihilation operators on $\mathcal{F}(\mathcal{S}_J)$ in the usual way. For any fixed $f \in S$, we first consider the unique bounded linear

[8] For more physical details, see Fulling ([1972], Secs. VIII.3,4) and Wald ([1994], pp. 41-2, 63, 111).

extensions of the mappings $a_n^*(f) : \mathcal{S}_J^{n-1} \to \mathcal{S}_J^n$ and $a_n(f) : \mathcal{S}_J^n \to \mathcal{S}_J^{n-1}$ defined by the following actions on product vectors

$$a_n^*(f)(f_1 \otimes \cdots \otimes f_{n-1}) = f \otimes f_1 \otimes \cdots \otimes f_{n-1}, \tag{29}$$

$$a_n(f)(f_1 \otimes \cdots \otimes f_n) = (f, f_1)_J \; f_2 \otimes \cdots \otimes f_n. \tag{30}$$

We then define the *unbounded* creation and annihilation operators on $\mathcal{F}(\mathcal{S}_J)$ by

$$a^*(f) := a_1^*(f) \oplus \sqrt{2} P_+ a_2^*(f) \oplus \sqrt{3} P_+ a_3^*(f) \oplus \cdots , \tag{31}$$

$$a(f) := 0 \oplus a_1(f) \oplus \sqrt{2} a_2(f) \oplus \sqrt{3} a_3(f) \oplus \cdots . \tag{32}$$

(Note that the mapping $f \mapsto a^*(f)$ is linear while $f \mapsto a(f)$ is *anti*-linear.)

As the definitions and notation suggest, $a^*(f)$ and $a(f)$ are each other's adjoint, $a^*(f)$ is the creation operator for a particle with wavefunction f, and $a(f)$ the corresponding annihilation operator. The unbounded self-adjoint operator $N(f) = a^*(f)a(f)$ represents the number of particles in the field with wavefunction f (unbounded, because we are describing bosons to which no exclusion principle applies). Summing $N(f)$ over any J-orthonormal basis of wavefunctions in $\mathcal{S}_J$, we obtain the *total* number operator N on $\mathcal{F}(\mathcal{S}_J)$, which has the form

$$N = 0 \oplus 1 \oplus 2 \oplus 3 \oplus \cdots . \tag{33}$$

Next, we define the self-adjoint "field operators"

$$\Phi(f) := 2^{-1/2}(a^*(f) + a(f)), \qquad f \in S. \tag{34}$$

(In heuristic discussions of free quantum field theory, these are normally encountered as "operator-valued solutions" $\Phi(x)$ to a relativistic field equation at some fixed time. However, if we want to associate a properly defined self-adjoint field operator with the spatial point x, we must consider a neighborhood of x, and an operator of form $\Phi(f)$, where the "test-function" $f \in S$ has support in the neighborhood.[9]) Defining the unitary operators

$$\pi(W(tf)) := \exp(it\Phi(f)), \qquad t \in \mathbb{R}, \; f \in S, \tag{35}$$

it can then be verified (though it is not trivial) that the $\pi(W(f))$ satisfy the Weyl form of the CCRs. In fact, the mapping $W(f) \mapsto \pi(W(f))$ gives an irreducible regular representation π of $\mathcal{W}$ on $\mathcal{F}(\mathcal{S}_J)$.

[9] The picture of a quantum field as an operator-valued *field* — or, as Teller ([1995], Ch. 5) aptly puts it, a field of "determinables" — unfortunately, has no mathematically rigorous foundation.

We also have

$$\langle \Omega, \pi(W(f))\Omega \rangle = e^{-(f,f)_J/4}, \qquad f \in S. \tag{36}$$

(We shall always distinguish the inner product of $\mathcal{F}(\mathcal{S}_J)$ from that of $\mathcal{S}_J$ by using angle brackets.) The vacuum vector $\Omega \in \mathcal{F}(\mathcal{S}_J)$ defines an abstract regular state ω_J of $\mathcal{W}$ via $\omega_J(A) := \langle \Omega, \pi(A)\Omega \rangle$ for all $A \in \mathcal{W}$. Since the action of $\pi(\mathcal{W})$ on $\mathcal{F}(\mathcal{S}_J)$ is irreducible, $\{\pi(A)\Omega : A \in \mathcal{W}\}$ is dense in $\mathcal{F}(\mathcal{S}_J)$ (else its closure would be a non-trivial subspace invariant under all operators in $\pi(\mathcal{W})$). Thus, the Fock representation of $\mathcal{W}$ on $\mathcal{F}(\mathcal{S}_J)$ is unitarily equivalent to the GNS representation of $\mathcal{W}$ determined by the pure state ω_J.

In sum, a complex structure J on (S, σ) gives rise to an abstract vacuum state ω_J on $\mathcal{W}[S, \sigma]$ whose GNS representation $(\pi_{\omega_J}, \mathcal{H}_{\omega_J}, \Omega_{\omega_J})$ is just the standard Fock vacuum representation $(\pi, \mathcal{F}(\mathcal{S}_J), \Omega)$. Note also that inverting Eqn. (34) yields

$$a^*(f) = 2^{-1/2}(\Phi(f) - i\Phi(if)),\ a(f) = 2^{-1/2}(\Phi(f) + i\Phi(if)),\ f \in S. \tag{37}$$

Thus, we could just as well have arrived at the Fock representation of $\mathcal{W}$ "abstractly" by *starting* with the pure regular state ω_J on $\mathcal{W}[S, \sigma]$ as our proposed vacuum, exploiting its regularity to guarantee the existence of field operators $\{\Phi(f) : f \in S\}$ acting on $\mathcal{H}_{\omega_J}$, and then using Eqns. (37) to *define* $a^*(f)$ and $a(f)$ (and, from thence, the number operators $N(f)$ and N).

There is a natural way to construct operators on $\mathcal{F}(\mathcal{S}_J)$ out of operators on the one-particle space $\mathcal{S}_J$, using the *second quantization map* Γ and its "derivative" $d\Gamma$. Unlike the representation map π, the operators on $\mathcal{F}(\mathcal{S}_J)$ in the range of Γ and $d\Gamma$ do not "come from" $\mathcal{W}[S, \sigma]$, but rather $\mathbf{B}(\mathcal{S}_J)$. Since the latter depends on how S was complexified, we cannot expect second quantized observables to be representation-independent.

To define $d\Gamma$, first let H be a self-adjoint (possibly unbounded) operator on $\mathcal{S}_J$. We define H_n on $\mathcal{H}^n$ by setting $H_0 = 0$ and

$$H_n(P_+(f_1 \otimes \cdots \otimes f_n)) = P_+ \left(\sum_{i=1}^{n} f_1 \otimes f_2 \otimes \cdots \otimes Hf_i \otimes \cdots \otimes f_n \right), \tag{38}$$

for all f_i in the domain of H, and then extending by continuity. It then follows that $\oplus_{n \geq 0} H_n$ is an "essentially selfadjoint" operator on $\mathcal{F}(\mathcal{S}_J)$ (see BR [1996], p. 8). We let

$$d\Gamma(H) := \overline{\bigoplus_{n \geq 0} H_n}, \tag{39}$$

denote the resulting (closed) self-adjoint operator. The simplest example occurs when we take $H = I$, in which case it is easy to see that $d\Gamma(H) = N$. In stark contrast to this, we have the following.[10]

Proposition 6. *When S is infinite-dimensional, $\pi(\mathcal{W}[S,\sigma])$ contains no non-trivial bounded functions of the total number operator on $\mathcal{F}(\mathcal{S}_J)$.*

In particular, $\pi(\mathcal{W})$ does not contain any of the spectral projections of N. Thus, while the *conservative* about observables is free to refer to the abstract state ω_J of $\mathcal{W}$ as a "vacuum" state, he cannot use that language to underwrite the claim that ω_J is a state of "no particles"!

To define Γ, let U be a unitary operator on $\mathcal{S}_J$. Then $U_n = P_+(U\otimes\cdots\otimes U)$ is a unitary operator on $\mathcal{H}^n$. We define the unitary operator $\Gamma(U)$ on $\mathcal{F}(\mathcal{S}_J)$ by

$$\Gamma(U) := \bigoplus_{n\geq 0} U_n. \tag{40}$$

If $U_t = e^{itH}$ is a weakly continuous unitary group on $\mathcal{S}_J$, then $\Gamma(U_t)$ is a weakly continuous group on $\mathcal{F}(\mathcal{S}_J)$, and we have

$$\Gamma(U_t) = e^{itd\Gamma(H)}. \tag{41}$$

In particular, the one-particle evolution $T_t = e^{itH}$ that was used to fix J "lifts" to a field evolution given by $\Gamma(T_t)$, where $d\Gamma(H)$ represents the energy of the field and has the vacuum Ω as a ground state.

It can be shown that the representation and second quantization maps interact as follows:

$$\pi(W(Uf)) = \Gamma(U)^*\pi(W(f))\Gamma(U), \qquad f \in S, \tag{42}$$

for any unitary operator U on $\mathcal{S}_J$. Taking the phase transformation $U = e^{it}I$, it follows that

$$\pi(W(e^{it}f)) = e^{-itN}\pi(W(f))e^{itN}, \qquad f \in S,\ t \in \mathbb{R}. \tag{43}$$

Using Eqn. (36), it also follows that

$$\langle\Gamma(U)\Omega, \pi(W(f))\Gamma(U)\Omega\rangle = \langle\Omega, \pi(W(Uf))\Omega\rangle = \langle\Omega, \pi(W(f))\Omega\rangle. \tag{44}$$

[10] Our proof in the appendix reconstructs the argument briefly sketched in Segal ([1959], p. 12).

Since the states induced by the vectors Ω and $\Gamma(U)\Omega$ are both normal in π and agree on $\pi(\mathcal{W})$, they determine the same state of $\pi(\mathcal{W})'' = \mathbf{B}(\mathcal{F}(\mathcal{S}_J))$. Thus Ω must be an eigenvector of $\Gamma(U)$ for any unitary operator U on $\mathcal{S}_J$. In particular, the vacuum is invariant under the group $\Gamma(T_t)$, and is therefore time-translation invariant.

3.3 Disjointness of the Minkowski and Rindler representations

We omit the details of the construction of the classical phase space (S, σ), since they are largely irrelevant to our concerns. The only information we need is that the space S may be taken (roughly) to be solutions to some relativistic wave equation, such as the Klein-Gordon equation. More particularly, S may be taken to consist of pairs of smooth, compactly supported functions on $\mathbb{R}^3$: one function specifies the values of the field at each point in space at some initial time (say $t = 0$), and the other function is the time-derivative of the field (evaluated at $t = 0$). If we then choose a "timelike flow" in Minkowski spacetime, we will get a corresponding flow in the solution space S; and, in particular, this flow will be given by a one-parameter group T_t of symplectomorphisms on (S, σ).

First, consider the group T_t of symplectomorphisms of (S, σ) induced by the standard inertial timelike flow. (See Figure 1, which suppresses two spatial dimensions. Note that it is irrelevant which inertial frame's flow we pick, since they all determine the same representation of $\mathcal{W}[S, \sigma]$ up to unitary equivalence; see Wald [1994], p. 106.) It is well-known that there is a complex structure M on (S, σ) such that $(\mathcal{S}_M, T_t)$ is a quantum one-particle system (see Kay [1985]; Horuzhy [1988], Ch. 4). We call the associated pure regular state ω_M of $\mathcal{W}[S, \sigma]$ the *Minkowski vacuum state*. As we have seen, it gives rise via the GNS construction to a unique Fock vacuum representation π_{ω_M} on the Hilbert space $\mathcal{H}_{\omega_M} = \mathcal{F}(\mathcal{S}_M)$.

Next, consider the group of Lorentz boosts about a given centre point O in spacetime. This also gives rise to a one-parameter group T_s of symplectomorphisms of (S, σ) (cf. Figure 1). Let $S(\triangleleft)$ be the subspace of S consisting of Cauchy data with support in the right Rindler wedge ($x_1 > 0$); i.e., at $s = 0$, both the field and its first derivative vanish when $x_1 \leq 0$. Let $\mathcal{W}_\triangleleft := \mathcal{W}[S(\triangleleft), \sigma]$ be the Weyl algebra over the symplectic space $(S(\triangleleft), \sigma)$. Then, T_s leaves $S(\triangleleft)$ invariant, and hence gives rise to a one-parameter group

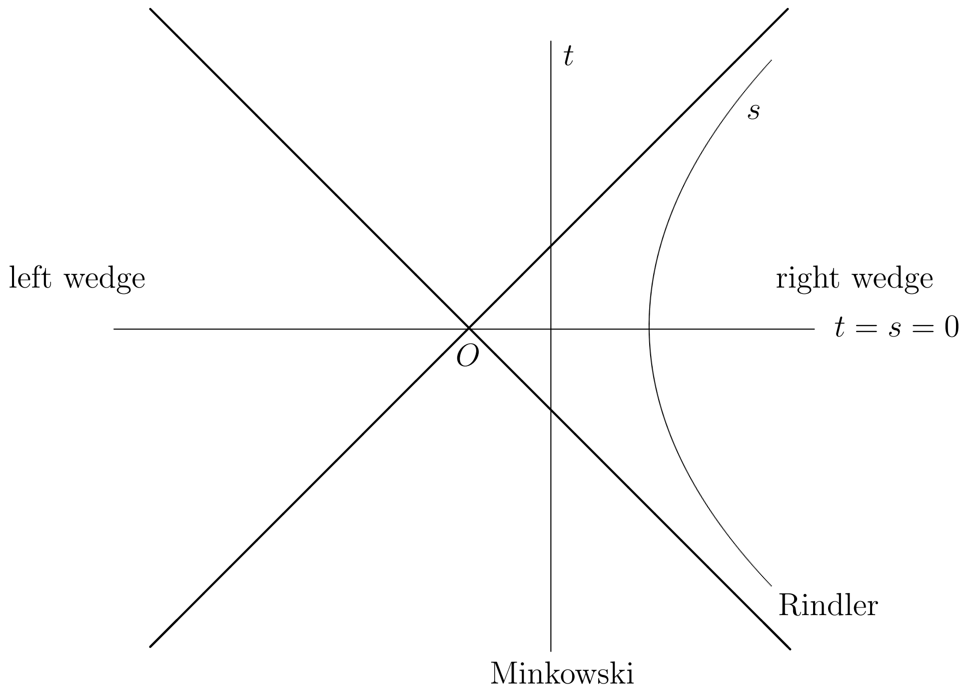

Figure 1: Minkowski and Rindler Motions.

of symplectomorphisms of $(S(\triangleleft), \sigma)$. Kay ([1985]) has shown rigorously that there is indeed a complex structure R on $(S(\triangleleft), \sigma)$ such that $(\mathcal{S}(\triangleleft)_R, T_s)$ is a quantum one particle system. We call the resulting state $\omega_R^{\triangleleft}$ of $\mathcal{W}_{\triangleleft}$ the *(right) Rindler vacuum state*. It gives rise to a unique GNS-Fock representation $\pi_{\omega_R^{\triangleleft}}$ of $\mathcal{W}_{\triangleleft}$ on $\mathcal{H}_{\omega_R^{\triangleleft}} = \mathcal{F}(\mathcal{S}(\triangleleft)_R)$ and, hence, a quantum field theory for the spacetime consisting of the right wedge *alone*.

The Minkowski vacuum state ω_M of $\mathcal{W}$ also determines a state $\omega_M^{\triangleleft}$ of $\mathcal{W}_{\triangleleft}$, by restriction (i.e., $\omega_M^{\triangleleft} := \omega_M|_{\mathcal{W}_{\triangleleft}}$). Thus, we may apply the GNS construction to obtain the Minkowski representation $(\pi_{\omega_M^{\triangleleft}}, \mathcal{H}_{\omega_M^{\triangleleft}})$ of $\mathcal{W}_{\triangleleft}$. It can be shown (using the "Reeh-Schlieder theorem" — see Clifton and Halvorson [2000]) that $\omega_M^{\triangleleft}$ is a highly mixed state (unlike $\omega_R^{\triangleleft}$). Therefore, $\pi_{\omega_M^{\triangleleft}}$ is reducible.

To obtain a concrete picture of this representation, note that (again, as a consequence of the "Reeh-Schlieder theorem") Ω_{ω_M} is a cyclic vector for the subalgebra $\pi_{\omega_M}(\mathcal{W}_{\triangleleft})$ acting on the "global" Fock space $\mathcal{F}(\mathcal{S}_M)$. Thus, by the uniqueness of the GNS representation $(\pi_{\omega_M^{\triangleleft}}, \mathcal{H}_{\omega_M^{\triangleleft}})$, it is unitarily equivalent to the representation $(\pi_{\omega_M}|_{\mathcal{W}_{\triangleleft}}, \mathcal{F}(\mathcal{S}_M))$. It can be shown that $\pi_{\omega_M}(\mathcal{W}_{\triangleleft})''$ is a factor (Horuzhy ([1988], Thm. 3.3.4). Thus, while reducible, $\pi_{\omega_M^{\triangleleft}}$ is still factorial.

Under the liberal approach to observables, the representations $\pi_{\omega_M^{\triangleleft}}$ (factorial) and $\pi_{\omega_R^{\triangleleft}}$ (irreducible) provide physically inequivalent descriptions of the physics in the right wedge.

Proposition 7. *The Minkowski and Rindler representations of $\mathcal{W}_{\triangleleft}$ are disjoint.*

Now let $\triangleright$ denote the left Rindler wedge, and define the subspace $S(\triangleright)$ of S as $S(\triangleleft)$ was defined above. (Of course, by symmetry, Proposition 7 holds for $\mathcal{W}_{\triangleright}$ as well.) Let $\mathcal{W}_{\bowtie} := \mathcal{W}[S(\triangleright) \oplus S(\triangleleft), \sigma]$ denote the Weyl algebra over the symplectic space $(S(\triangleright) \oplus S(\triangleleft), \sigma)$. Then $\mathcal{W}_{\bowtie} = \mathcal{W}_{\triangleright} \otimes \mathcal{W}_{\triangleleft}$, and $\omega_M^{\bowtie} := \omega_M|_{\mathcal{W}_{\bowtie}}$ is *pure* (Kay [1985], Defn., Thm. 1.3(iii)).[11] The GNS representation $\omega_M^{\bowtie}$ induces is therefore irreducible, and (again invoking the uniqueness of the GNS representation) it is equivalent to $(\pi_{\omega_M}|_{\mathcal{W}_{\bowtie}}, \mathcal{F}(\mathcal{S}_M))$ (since $\Omega_{\omega_M} \in \mathcal{F}(\mathcal{S}_M)$ is a cyclic vector for the subalgebra $\pi_{\omega_M}(\mathcal{W}_{\bowtie})$ as well).

The tensor product of the pure left and right Rindler vacua $\omega_R^{\bowtie} := \omega_R^{\triangleright} \otimes \omega_R^{\triangleleft}$ is of course also a pure state of $\mathcal{W}_{\bowtie}$.[12] It will induce a GNS representation of the latter on the Hilbert space $\mathcal{H}_{\omega_R^{\bowtie}}$ given by $\mathcal{F}(\mathcal{S}_R) \equiv \mathcal{F}(\mathcal{S}(\triangleright)_R) \otimes \mathcal{F}(\mathcal{S}(\triangleleft)_R)$. It is not difficult to show that $\omega_R^{\bowtie}$ and $\omega_M^{\bowtie}$, both now irreducible, are also disjoint.

Proposition 8. *The Minkowski and Rindler representations of $\mathcal{W}_{\bowtie}$ are disjoint.*

In our final main section we shall discuss the conceptually problematic implications that the *M-vacuum* states $\omega_M^{\bowtie}$ and $\omega_M^{\triangleleft}$ have for the *presence*

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[11]The restriction of ω_M to $\mathcal{W}_{\bowtie}$ is a pure "quasifree" state. Thus, there is a complex structure M' on $S(\triangleright) \oplus S(\triangleleft)$ such that

$$\omega_M(W(f)) = \exp(-\sigma(f, M'f)/4) = \exp(-\sigma(f, Mf)/4), \tag{45}$$

for all $f \in S(\triangleright) \oplus S(\triangleleft)$ (Petz [1990], Prop. 3.9). It is not difficult to see then that $M|_{S(\triangleright)\oplus S(\triangleleft)} = M'$ and therefore that M leaves $S(\triangleright) \oplus S(\triangleleft)$ invariant. Hereafter, we will use M to denote the complex structure on S as well as its restriction to $S(\triangleright) \oplus S(\triangleleft)$.

[12]More precisely, $\omega_R^{\triangleleft}$ arises from a complex structure $R_{\triangleleft}$ on $S(\triangleleft)$, $\omega_R^{\triangleright}$ arises from a complex structure $R_{\triangleright}$ on $S(\triangleright)$, and $\omega_R^{\bowtie}$ arises from the complex structure $R_{\triangleright} \oplus R_{\triangleleft}$ of $S(\triangleright) \oplus S(\triangleleft)$. When no confusion can result, we will use R to denote the complex structure on $S(\triangleright) \oplus S(\triangleleft)$ and its restriction to $S(\triangleleft)$.

of R-quanta in the double and right wedge spacetime regions. However, we note here an important difference between Rindler and Minkowski observers.

The total number of R-quanta, according to a Rindler observer confined to the left (resp., right) wedge, is represented by the number operator $N_{\triangleright}$ (resp., $N_{\triangleleft}$) on $\mathcal{F}(\mathcal{S}(\triangleright)_R)$ (resp., $\mathcal{F}(\mathcal{S}(\triangleleft)_R)$). However, because of the spacelike separation of the wedges, no single Rindler observer has access, even in principle, to the expectation value of the "overall" total Rindler number operator $N_R = N_{\triangleright} \otimes I + I \otimes N_{\triangleleft}$ acting on $\mathcal{F}(\mathcal{S}(\triangleright)_R) \otimes \mathcal{F}(\mathcal{S}(\triangleleft)_R)$.

The reverse is true for a Minkowski observer. While she has access, at least in principle, to the total number of M-quanta operator N_M acting on $\mathcal{F}(\mathcal{S}_M)$, N_M is a purely global observable that does not split into the sum of two separate number operators associated with the left and right wedges (as a general consequence of the "Reeh-Schlieder theorem" — see Redhead [1995]). In fact, since the Minkowski complex structure M is an "anti-local" operator (Segal and Goodman [1965]), it fails to leave either of the subspaces $S(\triangleright)$ or $S(\triangleleft)$ invariant, and it follows that no M-quanta number operator is affiliated with $\pi_{\omega_M^{\triangleleft}}(\mathcal{W}_{\triangleleft})''$.[13] Thus, even a liberal about observables must say that a Minkowski observer with access only to the right wedge does not have the capability of counting M-quanta.

So, while it might be sensible to ask for the probability in state $\omega_M^{\triangleleft}$ that a Rindler observer detects particles in the right wedge, it is *not* sensible to ask, conversely, for the probability in state $\omega_R^{\triangleleft}$ that a Minkowski observer will detect particles in the right wedge. Note also that since N_M is a purely global observable (i.e., there is no sense to be made of "the number of Minkowski quanta in a bounded spatial or spacetime region"), what a Minkowski observer might *locally* detect with a "particle detector" (over an extended, but finite, interval of time) can at best give an approximate indication of the global Minkowski particle content of the field.

4 Minkowski versus Rindler Quanta

We have seen that a Rindler observer will construct "his quantum field theory" of the right wedge spacetime region differently from a Minkowski observer. He will use the complex structure R picked out uniquely by the boost group about O, and build up a representation of $\mathcal{W}_{\triangleleft}$ on the Fock space

[13] See Halvorson [2000b] for further details and a critical analysis of different approaches to the problem of particle localization in quantum field theory.

$\mathcal{F}(\mathcal{S}(\triangleleft)_R)$. However, suppose that the state of $\mathcal{W}_\triangleleft$ is the state $\omega_M^\triangleleft$ of *no* particles (globally!) according to a Minkowski observer. What, if anything, will our Rindler observer say about the particle content in the right wedge? And does *this* question even make sense?

We shall argue that it does, notwithstanding the disjointness of the Minkowski and Rindler representations. And the answer is surprising. Not only does a Rindler observer have a nonzero chance of detecting the presence of R-quanta. In section **4.2** we shall show that if our Rindler observer were able to build a detector sensitive to the *total* number of R-quanta in the right wedge, he would always find that the probability of an *infinite* total number is *one*!

We begin in section **4.1** by discussing the paradox of observer-dependence of particles to which such results lead. In particular, we shall criticize Teller's ([1995,1996]) resolution of this paradox. Later, in section **4.3**, we shall also criticize the arguments of Arageorgis [1995] and Arageorgis et al [2000] for the incommensurability of inequivalent particle concepts, and argue, instead, for their complementarity (in *support* of Teller).

4.1 The Paradox of the Observer-Dependence of Particles

Not surprisingly, physicists initially found a Rindler observer's ability to detect particles in the Minkowski vacuum paradoxical (see Rüger [1989], p. 571; Teller [1995], p. 110). After all, particles are the sorts of things that are either there or not there, so how could their presence depend on an observer's state of motion?

One way to resist this paradox is to reject from the outset the physicality of the Rindler representation, thereby withholding bona fide particle status from Rindler quanta. For instance, one could be bothered by the fact the Rindler representation cannot be globally defined over the whole of Minkowski spacetime, or that the one-particle Rindler Hamiltonian lacks a mass gap, allowing an arbitrarily large number of R-quanta to have a fixed finite amount of energy ("infrared divergence"). Arageorgis ([1995], Ch. 6) gives a thorough discussion of these and other "pathologies" of the Rindler representation.[14] In consequence, he argues that the phenomenology associated with a Rindler observer's "particle detections" in the Minkowski vac-

[14]See also, more recently, Belinskiǐ [1997], Fedotov *et al.* [1999], and Nikolić [2000].

uum ought to be explained entirely in terms of observables affiliated to the Minkowski representation (such as garden-variety Minkowski vacuum fluctuations of the local field observables).

This is not the usual response to the paradox of observer-dependence. Rüger [1989] has characterized the majority of physicists' responses in terms of the *field approach* and the *detector approach*. Proponents of the field approach emphasize the need to forfeit particle talk at the fundamental level, and to focus the discussion on measurement of local field quantities. Those of the detector approach emphasize the need to relativize particle talk to the behaviour of concrete detectors following specified world-lines. Despite their differing emphases, and the technical difficulties in unifying these programs (well-documented by Arageorgis [1995]), neither eschews the Rindler representation as unphysical, presumably because of its deep connections with quantum statistical mechanics and blackhole thermodynamics (Sciama *et al* [1981]). Moreover, pathological or not, it remains of philosophical interest to examine the consequences of taking the Rindler representation seriously — just as the possibility of time travel in general relativity admitted by certain "pathological" solutions to Einstein's field equations is of interest. And it is remarkable that there should be *any* region of Minkowski spacetime that admits two physically inequivalent quantum field descriptions.

Teller ([1995,1996]) has recently offered his own resolution of the paradox. We reproduce below the relevant portions of his discussion in Teller ([1995], p. 111). However, note that he does not distinguish between left and right Rindler observers, $|0; M\rangle$ refers, in our notation, to the Minkowski vacuum vector $\Omega_{\omega_M} \in \mathcal{F}(\mathcal{S}_M)$, and $|1, 0, 0, \ldots\rangle_M$ (resp., $|1, 0, 0, \ldots\rangle_R$) is a one-particle state $0 \oplus f \oplus 0 \oplus 0 \oplus \cdots \in \mathcal{F}(\mathcal{S}_M)$ (resp., $\in \mathcal{F}(\mathcal{S}_R)$).

> . . . Rindler raising and lowering operators are expressible as superpositions of the Minkowski raising and lowering operators, and states with a definite number of Minkowski quanta are superpositions of states with different numbers of Rindler quanta. In particular, $|0; M\rangle$ is a superposition of Rindler quanta states, including states for arbitrarily large numbers of Rindler quanta. In other words, $|0; M\rangle$ has an exact value of zero for the Minkowski number operator, and is simultaneously highly indefinite for the Rindler number operator.
>
> ...In $|0; M\rangle$ there is *no* definite number of Rindler quanta. There is only a propensity for detection of one or another number of

> Rindler quanta by an accelerating detector. A state in which a quantity has no exact value is one in which no values for that quantity are definitely, and so actually, exemplified. Thus in $|0;M\rangle$ no Rindler quanta actually occur, so the status of $|0;M\rangle$ as a state completely devoid of quanta is not impugned.
>
> To be sure, this interpretive state of affairs is surprising. To spell it out one step further, in $|1,0,0,\ldots\rangle_M$ there is one actual Minkowski quantum, no actual Rindler quanta, and all sorts of propensities for manifestation of Rindler quanta, among other things. In $|1,0,0,\ldots\rangle_R$ the same comment applies with the role of Minkowski and Rindler reversed. It turns out that there are various kinds of quanta, and a state in which one kind of quanta actually occurs is a state in which there are only propensities for complementary kinds of quanta. Surprising, but perfectly consistent and coherent.

Teller's point is that R-quanta only exist (so to speak) potentially in the M-vacuum, not actually. Thus it is still an invariant observer-independent fact that there are no *actual quanta* in the field, and the paradox evaporates. Similarly for Minkowski states of one or more particles as seen by Rindler observers. There is the same definite number of *actual quanta* for all observers. Thus, since actual particles are the "real stuff", the real stuff *is* invariant!

Notice, however, that there is something self-defeating in Teller's final concession, urged by advocates of the field and detector approaches, that different kinds of quanta need to be distinguished. For if we do draw the distinction sharply, it is no longer clear why even the actual presence of R-quanta in the M-vacuum should bother us. Teller seems to want to have it both ways: while there are different kinds of quanta, there is still only one kind of *actual* quanta, and it better be invariant.

Does this invariance really hold? In one sense, Yes. Disjointness does not prevent us from building Rindler creation and annihilation operators on the Minkowski representation space $\mathcal{F}(\mathcal{S}_M)$. We simply need to define Rindler analogues, $a^*_R(f)$ and $a_R(f)$, of the Minkowski creation and annihilation operators via Eqns. (37) with $\Phi(Rf)$ in place of $\Phi(if)$ $(= \Phi(Mf))$ (noting that $f \mapsto a_R(f)$ will now be anti-linear with respect to the *Rindler* conjugation R). It is then easy to see, using (34), that

$$a_R(f) = 2^{-1}[a^*_M((I+MR)f) + a_M((I-MR)f)]. \tag{46}$$

This linear combination would be trivial if $R = \pm M$. However, we know $R \neq M$, and $R = -M$ is ruled out because it is inconsistent with both complex structures being positive definite. Consequently, $\Omega_{\omega_M^{\bowtie}}$ must be a nontrivial superposition of eigenstates of the Rindler number operator $N_R(f) := a_R^*(f)a_R(f)$; for an easy calculation, using (46), reveals that

$$N_R(f)\Omega_{\omega_M^{\bowtie}} = 2^{-2}[\Omega_{\omega_M^{\bowtie}} + a_M^*((I - MR)f)a_M^*((I + MR)f)\Omega_{\omega_M^{\bowtie}}], \qquad (47)$$

which (the presence of the nonzero second term guarantees) is not a simply a multiple of $\Omega_{\omega_M^{\bowtie}}$. Thus, Teller would be correct to conclude that the Minkowski vacuum implies dispersion in the number operator $N_R(f)$. And the same conclusion would follow if, instead, we considered the Minkowski creation and annihilation operators as acting on the Rindler representation space $\mathcal{F}(\mathcal{S}_R)$. Since only finitely many degrees of freedom are involved, this is guaranteed by the Stone-von Neumann theorem.

However, therein lies the rub. $N_R(f)$ merely represents the number of R-quanta with a specified wavefunction f. What about the *total* number of R-quanta in the M-vacuum (which involves *all* degrees of freedom)? If Teller cannot assure us that this too has dispersion, his case for the invariance of "actual quanta" is left in tatters. In his discussion, Teller fails to distinguish $N_R(f)$ from the total number operator N_R, but the distinction is crucial. It is a well-known consequence of the disjointness of $\pi_{\omega_R^{\bowtie}}$ and $\pi_{\omega_M^{\bowtie}}$ that neither representation's total number operator is definable on the Hilbert space of the other (BR [1996], Thm. 5.2.14). Therefore, it is literally *nonsense* to speak of $\Omega_{\omega_M^{\bowtie}}$ as a superposition of eigenstates of N_R![15] If $x_n, x_m \in \mathcal{F}(\mathcal{S}_R)$ are eigenstates of N_R with eigenvalues n, m respectively, then $x_n + x_m$ again lies in $\mathcal{F}(\mathcal{S}_R)$, and so is "orthogonal" to all eigenstates of the Minkowski number operator N_M acting on $\mathcal{F}(\mathcal{S}_M)$. And, indeed, taking infinite sums of Rindler number eigenstates will again leave us in the folium of the Rindler representation. As Arageorgis ([1995], p. 303) has also noted: "The Minkowski

[15] In their review of Teller's [1995] book, Huggett and Weingard [1996] question whether Teller's "quanta interpretation" of quantum field theory can be implemented in the context of inequivalent representations. However, when they discuss Teller's resolution of the observer-dependence paradox, in terms of mere *propensities to display* R-quanta in the M-vacuum, they write "This seems all well and good" ([1996], p. 309)! Their only criticism is the obvious one: legitimizing such propensity talk ultimately requires a solution to the measurement problem. Teller's response to their review is equally unsatisfactory. Though he pays lip-service to the possibility of inequivalent representations ([1998], pp. 156-7), he fails to notice how inequivalence undercuts his discussion of the paradox.

vacuum state is not a superposition of Rindler quanta states, despite 'appearances' ".[16]

Yet this point, by itself, does not tell us that Teller's discussion cannot be salvaged. Recall that a state ρ is *dispersion-free* on a (bounded) observable X just in case $\rho(X^2) = \rho(X)^2$. Suppose, now, that Y is a possibly *un*bounded observable that is definable in some representation π of $\mathcal{W}$. We can then rightly say that an algebraic state ρ of $\mathcal{W}$ *predicts dispersion in* Y just in case, for *every* extension $\hat{\rho}$ of ρ to $\pi(\mathcal{W})''$, $\hat{\rho}$ is *not* dispersion-free on all bounded functions of Y. We then have the following result.

Proposition 9. *If J_1, J_2 are distinct complex structures on (S, σ), then ω_{J_1} (resp., ω_{J_2}) predicts dispersion in N_{J_2} (resp., N_{J_1}).*

As a consequence, the Minkowski vacuum $\omega_M^{\bowtie}$ indeed predicts dispersion in the Rindler total number operator N_R (and in both $N_{\triangleright} \otimes I$ and $I \otimes N_{\triangleleft}$, invoking the symmetry between the wedges).

Teller also writes of the Minkowski vacuum as being a superposition of eigenstates of the Rindler number operator with *arbitrary large* eigenvalues. Eschewing the language of superposition, the idea that there is no finite number of R-quanta to which the M-vacuum assigns probability one can also be rendered sensible. The relevant result was first proved by Fulling ([1972], Appendix F; [1989], p. 145):

Fulling's "Theorem". *Two Fock vacuum representations $(\pi, \mathcal{F}(\mathcal{H}), \Omega)$ and $(\pi', \mathcal{F}(\mathcal{H}'), \Omega')$ of $\mathcal{W}$ are unitarily equivalent if and only if $\langle \Omega, N'\Omega \rangle < \infty$ (or, equivalently, $\langle \Omega', N\Omega' \rangle < \infty$).*

As stated, this "theorem" also fails to make sense, because it is only in the case where the representations are *already* equivalent that the primed

[16] Arageorgis presumes Teller's discussion is based upon the appearance of the following purely formal (i.e., non-normalizable) expression for $\Omega_{\omega_M^{\bowtie}}$ as a superposition in $\mathcal{F}(\mathcal{S}_R) \equiv \mathcal{F}(\mathcal{S}(\triangleright)_R) \otimes \mathcal{F}(\mathcal{S}(\triangleleft)_R)$ over left ("I") and right ("II") Rindler modes (Wald [1994], Eqn. (5.1.27)):

$$\prod_i \left\{ \sum_{n=0}^{\infty} \exp(-n\pi\omega_i/a)\, |n_{iI}\rangle \otimes |n_{iII}\rangle \right\} . \tag{48}$$

However, it bears mentioning that, as this expression suggests: (a) the restriction of $\omega_M^{\bowtie}$ to either $\mathcal{W}_{\triangleright}$ or $\mathcal{W}_{\triangleleft}$ is indeed mixed; (b) $\omega_M^{\bowtie}$ can be shown rigorously to be an entangled state of $\mathcal{W}_{\triangleright} \otimes \mathcal{W}_{\triangleleft}$ (Clifton and Halvorson [2000]); and (c) the thermal properties of the "reduced density matrix" for either wedge obtained from this formal expression can be derived rigorously (Kay [1985]). In addition, see Propositions 9 and 10 below!

total number operator is definable on the unprimed representation space and an expression like "$\langle\Omega, N'\Omega\rangle$" is well-defined. (We say more about why this is so in the next section.) However, there *is* a way to understand the expression "$\langle\Omega, N'\Omega\rangle < \infty$" (resp., "$\langle\Omega, N'\Omega\rangle = \infty$") in a rigorous, non-question-begging way. We can take it to be the claim that all extensions $\hat{\rho}$ of the abstract unprimed vacuum state of $\mathcal{W}$ to $\mathbf{B}(\mathcal{F}(\mathcal{H}'))$ assign (resp., do *not* assign) N' a finite value; i.e., for any such extension, $\sum_{n'=1}^{\infty} \hat{\rho}(P_{n'})n'$ converges (resp., does not converge), where $\{P_{n'}\}$ are the spectral projections of N'. With this understanding, the following rigorization of Fulling's "theorem" can then be proved.

Proposition 10. *A pair of Fock representations $\pi_{\omega_{J_1}}$, $\pi_{\omega_{J_2}}$ are unitarily equivalent if and only if ω_{J_1} assigns N_{J_2} a finite value (equivalently, ω_{J_2} assigns N_{J_1} a finite value).*

It follows that $\omega_M^{\bowtie}$ cannot assign probability one to any finite number of R-quanta (and vice-versa, with $R \leftrightarrow M$).

Unfortunately, neither Proposition 9 or 10 is sufficient to rescue Teller's "actual quanta" invariance argument, for these propositions give no further information about the shape of the probability distribution that $\omega_M^{\bowtie}$ prescribes for N_R's eigenvalues. In particular, both propositions are compatible with there being a probability of *one* that *at least* $n > 0$ R-quanta obtain in the M-vacuum, for any $n \in \mathbb{N}$. If that were the case, Teller would then be forced to withdraw and concede that at least *some*, and perhaps many, Rindler quanta *actually* occur in a state with no actual Minkowski quanta. In the next section, we shall show that this — Teller's worst nightmare — is in fact the case.

4.2 Minkowski Probabilities for Rindler Number Operators

We now defend the claim that a Rindler observer will say that there are actually *infinitely many* quanta while the field is in the Minkowski vacuum state (or, indeed, in any other state of the Minkowski folium).[17] This result

[17]In fact, this was first proved, in effect, by Chaiken [1967]. However his lengthy analysis focussed on comparing Fock with non-Fock (so-called "strange") representations of the Weyl algebra, and the implications of his result for disjoint Fock representations based on inequivalent one-particle structures seem not to have been carried down into the textbook

applies more generally to any pair of disjoint regular representations, at least one of which is the GNS representation of an abstract Fock vacuum state. We shall specialize back down to the Minkowski/Rindler case later on.

Let ρ be a regular state of $\mathcal{W}$ inducing the GNS representation $(\pi_\rho, \mathcal{H}_\rho)$, and let ω_J be the abstract vacuum state determined by a complex structure J on (S, σ). The case we are interested in is, of course, when π_ρ, π_{ω_J} are disjoint. We first want to show how to define representation-independent probabilities in the state ρ for any J-quanta number operator that "counts" the number of quanta with wavefunctions in a fixed *finite*-dimensional subspace $F \subseteq \mathcal{S}_J$. (Parts of our exposition below follow BR ([1996], pp. 26-30), which may be consulted for further details.)

We know that, for any $f \in S$, there exists a self-adjoint operator $\Phi_\rho(f)$ on $\mathcal{H}_\rho$ such that

$$\pi_\rho(W(tf)) = \exp\left(it\Phi_\rho(f)\right), \qquad t \in \mathbb{R}. \tag{49}$$

We can also define unbounded annihilation and creation operators on $\mathcal{H}_\rho$ for J-quanta by

$$a_\rho(f) := 2^{-1/2}(\Phi_\rho(f) + i\Phi_\rho(Jf)),\ a^*_\rho(f) := 2^{-1/2}(\Phi_\rho(f) - i\Phi_\rho(Jf)). \tag{50}$$

Earlier, we denoted these operators by $a_J(f)$ and $a^*_J(f)$. However, we now want to emphasize the representation space upon which they act; and only the single complex structure J shall concern us in our general discussion, so there is no possibility of confusion with others.

Next, define a "quadratic form" $n_\rho(F) : \mathcal{H}_\rho \mapsto \mathbb{R}^+$. The domain of $n_\rho(F)$ is

$$D(n_\rho(F)) := \bigcap_{f \in F} D(a_\rho(f)), \tag{51}$$

where $D(a_\rho(f))$ is the domain of $a_\rho(f)$. Now let $\{f_k : k = 1, \ldots, m\}$ be some J-orthonormal basis for F, and define

$$[n_\rho(F)](\psi) := \sum_{k=1}^{m} \|a_\rho(f_k)\psi\|^2, \tag{52}$$

tradition of the subject. (The closest result we have found is BR ([1996], Thm. 5.2.14) which we are able to employ as a lemma to recover Chaiken's result for disjoint Fock representations — see the appendix.)

for any $\psi \in D(n_\rho(F))$. It can be shown that the sum in (52) is independent of the chosen orthonormal basis for F, and that $D(n_\rho(F))$ lies dense in $\mathcal{H}_\rho$. Given any densely defined, positive, closed quadratic form t on $\mathcal{H}_\rho$, there exists a unique positive self-adjoint operator T on $\mathcal{H}_\rho$ such that $D(t) = D(T^{1/2})$ and

$$t(\psi) = \langle T^{1/2}\psi, T^{1/2}\psi\rangle, \qquad \psi \in D(t). \tag{53}$$

We let $N_\rho(F)$ denote the finite-subspace J-quanta number operator on $\mathcal{H}_\rho$ arising from the quadratic form $n_\rho(F)$.

We seek a representation-*independent* value for "$\mathrm{Prob}^\rho(N(F) \in \Delta)$", where $\Delta \subseteq \mathbb{N}$. So let τ be *any* regular state of $\mathcal{W}$, and let $N_\tau(F)$ be the corresponding number operator on $\mathcal{H}_\tau$. Let $\mathcal{W}_F$ be the Weyl algebra over $(F, \sigma|_F)$, and let $E_\tau(F)$ denote the spectral measure for $N_\tau(F)$ acting on $\mathcal{H}_\tau$. Then, $[E_\tau(F)](\Delta)$ (the spectral projection representing the proposition "$N_\tau(F) \in \Delta$") is in the weak closure of $\pi_\tau(\mathcal{W}_F)$, by the Stone-von Neumann uniqueness theorem. In particular, there is a net $\{A_i\} \subseteq \mathcal{W}_F$ such that $\pi_\tau(A_i)$ converges weakly to $[E_\tau(F)](\Delta)$. Now, the Stone-von Neumann uniqueness theorem also entails that there is a density operator D_ρ on $\mathcal{H}_\tau$ such that

$$\rho(A) = \mathrm{Tr}(D_\rho \pi_\tau(A)), \qquad A \in \mathcal{W}_F. \tag{54}$$

We therefore define

$$\begin{aligned} \mathrm{Prob}^\rho(N(F) \in \Delta) &:= \lim_i \rho(A_i) && (55)\\ &= \lim_i \mathrm{Tr}(D_\rho \pi_\tau(A_i)) && (56)\\ &= \mathrm{Tr}(D_\rho [E_\tau(F)](\Delta)). && (57) \end{aligned}$$

The final equality displays that this definition is independent of the chosen approximating net $\{\pi_\tau(A_i)\}$, and the penultimate equality displays that this definition is independent of the (regular) representation π_τ. In particular, since we may take $\tau = \rho$, it follows that

$$\mathrm{Prob}^\rho(N(F) \in \Delta) = \langle \Omega_\rho, [E_\rho(F)](\Delta)\Omega_\rho\rangle, \tag{58}$$

exactly as expected.

We can also define a positive, closed quadratic form on $\mathcal{H}_\rho$ corresponding to the *total* J-quanta number operator by:

$$n_\rho(\psi) = \sup_{F \in \mathbb{F}} [n_\rho(F)](\psi)\,, \tag{59}$$

$$D(n_\rho) = \left\{ \psi \in \mathcal{H}_\rho : \psi \in \bigcap_{f \in S} D(a_\rho(f)),\, n_\rho(\psi) < \infty \right\}, \tag{60}$$

where $\mathbb{F}$ denotes the collection of all finite-dimensional subspaces of $\mathcal{S}_J$. If $D(n_\rho)$ is dense in $\mathcal{H}_\rho$, then it makes sense to say that the total J-quanta number operator N_ρ exists on the Hilbert space $\mathcal{H}_\rho$. In general, however, $D(n_\rho)$ will not be dense, and may contain only the 0 vector. Accordingly, we cannot use a direct analogue to Eqn. (57) to define the probability, in the state ρ, that there are, say, n or fewer J-quanta.

However, we can still proceed as follows. Fix $n \in \mathbb{N}$, and suppose $F \subseteq F'$ with both $F, F' \in \mathbb{F}$. Since any state with n or fewer J-quanta with wavefunctions in F' cannot have *more* than n J-quanta with wavefunctions in the (smaller) subspace F,

$$\mathrm{Prob}^\rho(N(F) \in [0,n]) \;\geq\; \mathrm{Prob}^\rho(N(F') \in [0,n]). \tag{61}$$

Thus, whatever value we obtain for "$\mathrm{Prob}^\rho(N \in [0,n])$", it should satisfy the inequality

$$\mathrm{Prob}^\rho(N(F) \in [0,n]) \;\geq\; \mathrm{Prob}^\rho(N \in [0,n]), \tag{62}$$

for any finite-dimensional subspace $F \subseteq \mathcal{S}_J$. However, the following result holds.

Proposition 11. *If ρ is a regular state of $\mathcal{W}$ disjoint from the Fock state ω_J, then $\inf_{F \in \mathbb{F}} \left\{ \mathrm{Prob}^{\rho}(N_F \in [0,n]) \right\} = 0$ for every $n \in \mathbb{N}$.*

Thus ρ must assign every finite number of J-quanta probability zero; i.e., ρ predicts an infinite number of J-quanta with probability 1!

Let us tighten this up some more. Suppose that we are in any regular representation $(\pi_\omega, \mathcal{H}_\omega)$ in which the total J-quanta number operator N_ω exists and is affiliated to $\pi_\omega(\mathcal{W})''$. (For example, we may take the Fock representation where $\omega = \omega_J$.) Let E_ω denote the spectral measure of N_ω on $\mathcal{H}_\omega$. Considering ρ as a state of $\pi_\omega(\mathcal{W})$, it is then reasonable to define

$$\mathrm{Prob}^\rho(N \in [0,n]) := \hat{\rho}(E_\omega([0,n])), \tag{63}$$

where $\hat{\rho}$ is any extension of ρ to $\pi_\omega(\mathcal{W})''$, provided the right-hand side takes the same value for all extensions. (And, of course, it will when $\rho \in \mathfrak{F}(\pi_\omega)$, where (63) reduces to the standard definition.) Now clearly

$$[E_\omega(F)]([0,n]) \;\geq\; E_\omega([0,n]),\;\; F \in \mathbb{F}. \tag{64}$$

("If there are at most n J-quanta in total, then there are at most n J-quanta whose wavefunctions lie in any finite-dimensional subspace of $\mathcal{S}_J$".) Since states preserve order relations between projections, every extension $\hat{\rho}$ must therefore satisfy

$$\mathrm{Prob}^{\rho}(N(F) \in [0,n]) \;=\; \hat{\rho}([E_\omega(F)]([0,n])) \;\geq\; \hat{\rho}(E_\omega([0,n])). \tag{65}$$

Thus, if ρ is disjoint from ω, Proposition 11 entails that $\mathrm{Prob}^{\rho}(N \in [0,n]) = 0$ for all finite n.[18]

As an immediate consequence of this and the disjointness of the Minkowski and Rindler representations, we have (reverting back to our earlier number operator notation):

$$\mathrm{Prob}^{\omega^{\bowtie}_M}(N_R \in [0,n]) = 0 = \mathrm{Prob}^{\omega^{\bowtie}_R}(N_M \in [0,n]),\;\text{for all } n \in \mathbb{N}, \tag{66}$$

$$\mathrm{Prob}^{\omega^{\triangleright}_M}(N_\triangleright \in [0,n]) = 0 = \mathrm{Prob}^{\omega^{\triangleleft}_M}(N_\triangleleft \in [0,n]),\;\text{for all } n \in \mathbb{N}. \tag{67}$$

The same probabilities obtain when the Minkowski vacuum is replaced with any other state normal in the Minkowski representation.[19] So it could not be farther from the truth to say that there is merely the potential for Rindler quanta in the Minkowski vacuum, or any other eigenstate of N_M.

One must be careful, however, with an informal statement like "The M-vacuum contains infinitely many R-quanta with probability 1". Since Rindler wedges are unbounded, there is nothing unphysical, or otherwise metaphysically incoherent, about thinking of wedges as containing an infinite number of

[18]Notice that such a prediction could never be made by a state in the folium of π_ω, since normal states are countably additive (see note 3).

[19]This underscores the utter bankruptcy, from the standpoint of the liberal about observables, in taking the weak equivalence of the Minkowski and Rindler representations to be sufficient for their physical equivalence. Yes, every Rindler state of the Weyl algebra is a weak* limit of Minkowski states. But the former all predict a finite number of Rindler quanta with probability 1, while the latter all predict an *infinite* number with probability 1! (Wald ([1994], pp. 82-3) makes the exact same point with respect to states that do and do not satisfy the "Hadamard" property.)

Rindler quanta. But we must not equate this with the quite different *empirical* claim "A Rindler observer's particle detector has the sure-fire disposition to register the value '∞' ". There is no such value! Rather, the empirical content of equations (66) and (67) is simply that an idealized "two-state" measuring apparatus designed to register whether there are $> n$ Rindler quanta in the Minkowski vacuum will always return the answer 'Yes'. This is a perfectly sensible physical disposition for a measuring device to have. Of course, we are not pretending to have in hand a specification of the physical details of such a device. Indeed, when physicists model particle detectors, these are usually assumed to couple to specific "modes" of the field, represented by finite-subspace, not total, number operators (cf., e.g., Wald [1994], Sec. 3.3). But this is really beside the point, since Teller advertises his resolution of the paradox as a way to *avoid* a "retreat to instrumentalism" about the particle concept ([1995], p. 110).

On Teller's behalf, one might object that there are still no grounds for saying any R-quanta obtain in the M-vacuum, since for any particular number n of R-quanta you care to name, equations (66) and (67) entail that n is *not* the number of R-quanta in the M-vacuum. But remember that the same is true for $n = 0$, and that, therefore, $n \geq 1$ R-quanta has probability 1! A further tack might be to deny that probability 0 for $n = 0$, or any other n, entails impossibility or non-actuality of that number of R-quanta. This would be similar to a common move made in response to the lottery paradox, in the hypothetical case where there are an infinite number of ticket holders. Since *someone* has to win, each ticket holder must still have the potential to win, even though his or her probability of winning is zero. The difficulty with this response is that in the Rindler case, we have no independent reason to think that some particular finite number of R-quanta *has* to be detected at all. Moreover, if we were to go soft on taking probability 0 to be sufficient for "not actual", we should equally deny that probability 1 is sufficient for "actual", and by Teller's lights the paradox would go away at a stroke (because there could never be actual Rindler *or* Minkowski quanta in *any* field state).

We conclude that Teller's resolution of the paradox of observer-dependence of particles fails. And so be it, since it was ill-motivated in the first place. We already indicated in the previous subsection that it should be enough of a resolution to recognize that there are different kinds of quanta. We believe the physicists of the field and detector approaches are correct to bite the bullet hard on this, even though it means abandoning naïve realism about particles (though not, of course, about detection events). We turn, next, to arguing that a coherent story can still be told about the relationship between the different kinds of particle talk used by different observers.

4.3 Incommensurable or Complementary?

At the beginning of this paper, we reproduced a passage from Jauch's amusing Galilean dialogue on the question "Are Quanta Real?". In that passage, Sagredo is glorying in the prospect that complementarity may be applicable even in classical physics; and, more generally, to solving the philosophical problem of the specificity of individual events versus the generality of scientific description. It is well-known that Bohr himself sought to extend the idea of complementarity to all different walks of life, beyond its originally intended application in quantum theory. And even within the confines of quantum theory, it is often the case that when the going gets tough, tough quantum theorists cloak themselves in the mystical profundity of complementarity, sometimes just to get philosophers off their backs.

So it seems with the following notorious comments of a well-known advocate of the detector approach that have received a predictably cool reception from philosophers:

> Bohr taught us that quantum mechanics is an algorithm for computing the results of measurements. Any discussion about what is a "real, physical vacuum", must therefore be related to the behaviour of real, physical measuring devices, in this case particle-number detectors. Armed with such heuristic devices, we may then assert the following. There are quantum states and there are particle detectors. Quantum field theory enables us to predict probabilistically how a particular detector will respond to that state. That is all. That is all there can ever be in physics, because physics is about the observations and measurements that we can make in the world. We can't talk meaningfully about whether such-and-such a state contains particles except in the context of a specified particle detector measurement. To claim (as some authors occasionally do!) that when a detector responds (registers particles) in somebody's cherished vacuum state that the particles concerned are "fictitious" or "quasi-particles", or

> that the detector is being "misled" or "distorted", is an empty statement (Davies [1984], p. 69).

We shall argue that, cleansed of Davies' purely operationalist reading of Bohr, complementarity *does*, after all, shed light on the relation between inequivalent particle concepts.

Rüger [1989] balks at this idea. He writes:

> The "real problem" — how to understand how there might be particles for one observer, but none at all for another observer in a different state of motion — is not readily solved by an appeal to Copenhagenism... Though quantum mechanics can tell us that the *properties* of micro-objects (like momentum or energy) depend in a sense on observers measuring them, the standard interpretation of the theory still does not tell us that whether there is a micro-object or not depends on observers. At least the common form of this interpretation is not of immediate help here (Rüger [1989], pp. 575-6).

Well, let us consider the "common form" of the Copenhagen interpretation. Whatever one's preferred embellishment of the interpretation, it must at least imply that observables represented by noncommuting "complementary" self-adjoint operators cannot have simultaneously determinate values in all states. Since field quantizations are built upon an abstract noncommutative algebra, the Weyl algebra, complementarity retains its application to quantum field theory. In particular, in any *single* Fock space representation — setting aside inequivalent representations for the moment — there will be a total number operator and nontrivial superpositions of its eigenstates. In these superpositions, which are eigenstates of observables failing to commute with the number operator, it is therefore perfectly in line with complementarity that we say they contain no actual particles in any substantive sense.[20] In addition, there will be different number operators on Fock space that count the number of quanta with wavefunctions lying in different subspaces of the one-particle space, and they will only commute if the corresponding subspaces are compatible. So even before we consider inequivalent particle concepts,

[20] As Rüger notes earlier ([1989], p. 571), in ordinary non-field-theoretic quantum theory, complementarity only undermined a naïve substance-properties ontology. However, this was only because there was no "number of quanta" observable in the theory!

we must already accept that there are different *complementary* "kinds" of quanta, according to what their wavefunctions are.

Does complementarity extend to the particle concepts associated with inequivalent Fock representations? *Contra* Rüger [1989], we claim that it does. We saw earlier that one can build finite-subspace J-quanta number operators in *any* regular representation of $\mathcal{W}[S,\sigma]$, provided only that J defines a proper complex structure on S that leaves it invariant. In particular, using the canonical commutation relation $[\Phi(f),\Phi(g)] = i\sigma(f,g)I$, a tedious but elementary calculation reveals that, for any $f,g \in S$,

$$\begin{aligned} & [N_{J_1}(f), N_{J_2}(g)] \\ = \quad & i/2\{\sigma(f,g)[\Phi(f),\Phi(g)]_+ + \sigma(f,J_2g)[\Phi(f),\Phi(J_2g)]_+ \\ & +\sigma(J_1f,g)[\Phi(J_1f),\Phi(g)]_+ + \sigma(J_1f,J_2g)[\Phi(J_1f),\Phi(J_2g)]_+\}, \end{aligned} \tag{68}$$

in any regular representation.[21] Thus, there are well-defined and, in general, *nontrivial* commutation relations between finite-subspace number operators, even when the associated particle concepts are inequivalent. We also saw in Eqn. (47) that when $J_2 \neq J_1$, no $N_{J_2}(f)$, for any $f \in \mathcal{S}_{J_2}$, will leave the zero-particle subspace of N_{J_1} invariant. Since it is a necessary condition that this nondegenerate eigenspace be left invariant by any self-adjoint operator commuting with N_{J_1}, it follows that $[N_{J_2}(f), N_{J_1}] \neq 0$ for all $f \in \mathcal{S}_{J_2}$. Thus finite-subspace number operators for one kind of quanta are complementary to the total number operators of inequivalent kinds of quanta.

Of course, we cannot give the same argument for complementarity between the *total* number operators N_{J_1} and N_{J_2} pertaining to inequivalent kinds of quanta, because, as we know, they cannot even be defined as operators on the same Hilbert space. However, we disagree with Arageorgis ([1995], pp. 303-4) that this means Teller's "complementarity talk" in relation to the Minkowski and Rindler total number operators is wholly inapplicable. We have two reasons for the disagreement.

First, since it is a necessary condition that a (possibly unbounded) self-adjoint observable Y on $\mathcal{H}_{\omega_{J_1}}$ commuting with N_{J_1} have $\Omega_{\omega_{J_1}}$ as an eigenvector, it is also necessary that the abstract vacuum state ω_{J_1} be dispersion-free on Y. But this latter condition is purely algebraic and makes sense even

[21] As a check on expression (68), note that it is invariant under the one-particle space phase transformations $f \to (\cos t + J_1 \sin t)f$ and $g \to (\cos t + J_2 \sin t)g$, and when $J_1 = J_2 = J$, reduces to zero just in case the rays generated by f and g are compatible subspaces of $\mathcal{S}_J$.

when Y does *not* act on $\mathcal{H}_{\omega_{J_1}}$. Moreover, as Proposition 9 shows, this condition fails when Y is taken to be the total number operator of any Fock representation inequivalent to $\pi_{\omega_{J_1}}$. So it is entirely natural to treat Proposition 9 as a vindication of the idea that inequivalent pairs of total number operators are complementary.

Secondly, we have seen that any state in the folium of a representation associated with one kind of quanta assigns probability zero to any finite number of an inequivalent kind of quanta. This has a direct analogue in the most famous instance of complementarity: that which obtains between the concepts of position and momentum.

Consider the unbounded position and momentum operators, x and p ($= -i\frac{\partial}{\partial x}$), acting on $L^2(\mathbb{R})$. Let E_x and E_p be their spectral measures. We say that a state ρ of $\mathbf{B}(L^2(\mathbb{R}))$ assigns x a *finite* dispersion-free value just in case ρ is dispersion-free on x and there is a $\lambda \in \mathbb{R}$ such that $\rho(E_x((a,b))) = 1$ if and only if $\lambda \in (a,b)$. (Similarly, for p.) Then the following is a direct consequence of the canonical commutation relation $[x,p] = iI$ (see Halvorson and Clifton [1999], Prop. 3.7).

Proposition 12. *If ρ is a state of $\mathbf{B}(L^2(\mathbb{R}))$ that assigns x (resp., p) a finite dispersion-free value, then $\rho(E_p((a,b))) = 0$ (resp., $\rho(E_x((a,b))) = 0$) for any $a,b \in \mathbb{R}$.*

This result makes rigorous the fact, suggested by Fourier analysis, that if either of x or p has a sharp finite value in any state, the other is "maximally indeterminate". But the same goes for pairs of inequivalent number operators (N_{J_1}, N_{J_2}): if a regular state ρ assigns N_{J_1} a finite dispersion-free value, then $\rho \in \mathfrak{F}(\pi_{\omega_{J_1}})$ which, in turn, entails that ρ assigns probability zero to any finite set of eigenvalues for N_{J_2}. Thus, (N_{J_1}, N_{J_2}) are, in a natural sense, *maximally* complementary, despite the fact that they have no well-defined commutator.

One might object that our analogy is only skin deep; after all, x and p still act on the *same* Hilbert space, $L^2(\mathbb{R})$! So let us deepen the analogy. Let $\mathcal{W}$ be the Weyl algebra for one degree of freedom, and let $U(a) \equiv W(a,0)$ and $V(b) \equiv W(0,b)$ be the unitary operators corresponding, respectively, to position and momentum. Now, if we think of position as analogous to the Minkowski number operator and momentum as analogous to the Rindler number operator, the standard Schrödinger representation is not the analogue of the Minkowski vacuum representation — since the Minkowski vacuum representation is constructed so as to have eigenvectors for N_M, whereas the Schrödinger representation obviously does not have eigenvectors for x. Thus, to find a representation analogous to the Minkowski vacuum representation, first choose a state ρ of $\mathcal{W}$ that is dispersion-free on all elements $\{U(a) : a \in \mathbb{R}\}$. In particular, we may choose ρ such that $\rho(U(a)) = e^{ia\lambda}$ for all $a \in \mathbb{R}$. If we then let $(\pi_\rho, \mathcal{H}_\rho, \Omega_\rho)$ denote the GNS representation of $\mathcal{W}$ induced by ρ, it follows that we may construct an unbounded position operator x on $\mathcal{H}_\rho$ which has Ω_ρ as an eigenvector with eigenvalue λ. But, lo and behold, it is not possible to define a momentum operator p on the Hilbert space $\mathcal{H}_\rho$.

Indeed, since ρ is dispersion-free on $U(a)$, it is multiplicative for the product of $U(a)$ with any other element of $\mathcal{W}$ (KR [1997], Ex. 4.6.16). In particular,

$$\rho(U(a))\rho(V(b)) = e^{iab}\rho(V(b))\rho(U(a)), \qquad a, b \in \mathbb{R}. \tag{69}$$

Since $\rho(U(a)) = e^{ia\lambda} \neq 0$, this implies

$$\rho(V(b)) = e^{iab}\rho(V(b)), \qquad a, b \in \mathbb{R}. \tag{70}$$

However, when $a \neq 0$, (70) cannot hold for all $b \neq 0$ unless $\rho(V(b)) = 0$. Thus,

$$\langle \Omega_\rho, \pi_\rho(V(b))\Omega_\rho \rangle = 0, \qquad \forall b \neq 0. \tag{71}$$

On the other hand,

$$\langle \Omega_\rho, \pi_\rho(V(0))\Omega_\rho \rangle = \langle \Omega_\rho, I\Omega_\rho \rangle = 1. \tag{72}$$

Thus, $\pi_\rho(V(b))$ is not weakly continuous in b, and there can be no self-adjoint operator p on $\mathcal{H}_\rho$ such that $V(b) = e^{ibp}$. On the other hand, since $a \in \mathbb{R} \mapsto \rho(U(a)) = e^{ia\lambda}$ is continuous, and hence π_ρ is regular with respect to the subgroup of unitary operators $\{U(a) : a \in \mathbb{R}\}$, there *is* a position operator on $\mathcal{H}_\rho$.

Similarly, if ω is a state of $\mathcal{W}$ that is dispersion-free on the momentum unitary operators $\{V(b) : b \in \mathbb{R}\}$, then it is not possible to define a position operator on the Hilbert space $\mathcal{H}_\omega$. Moreover, the GNS representations π_ρ and π_ω are disjoint — precisely as in the case of the GNS representations induced by the Minkowski and Rindler vacuum states. Indeed, suppose for reductio that there is a unitary operator T from $\mathcal{H}_\omega$ to $\mathcal{H}_\rho$ such that $T^{-1}\pi_\rho(A)T = \pi_\omega(A)$ for all $A \in \mathcal{W}$. Then, it would follow that $\pi_\omega(U(a)) = T^{-1}\pi_\rho(U(a))T$ is weakly continuous in a, in contradiction to the fact that x cannot be defined on $\mathcal{H}_\omega$.

So we maintain that there are compelling formal reasons for thinking of Minkowski and Rindler quanta as complementary. What's more, when a Minkowski observer sets out to detect particles, her state of motion determines that her detector will be sensitive to the presence of Minkowski quanta. Similarly for a Rindler observer and his detector. This is borne out by the analysis of Unruh and Wald [1984] in which they show how his detector will *itself* "define" (in a "nonstandard" way) what solutions of the relativistic wave equation are counted as having positive frequency, via the way the detector couples to the field. So we may think of the choice of an observer to follow an inertial or Rindler trajectory through spacetime as analogous to the choice between measuring the position or momentum of a particle. Each choice requires a distinct kind of coupling to the system, and both measurements cannot be executed on the field simultaneously and with infinite precision.[22] Moreover, execution of one type of measurement precludes meaningful discourse about the values of the observable that the observer did not choose to measure. All this is the essence of "Copenhagenism".

And it should *not* be equated with operationalism! The goal of the detector approach to the paradox of observer-dependence was to achieve clarity on the problem by reverting back to operational definitions of the word "particle" with respect to the concrete behaviour of particular kinds of detectors (cf., e.g., DeWitt [1979b], p. 692). But, as with early days of special relativity and quantum theory, operationalism can serve its purpose and then be jettisoned. Rindler quanta get their status as such not because they are, *by definition*, the sort of thing that accelerated detectors detect. This gets things backwards. Rindler detectors display Rindler quanta in the Minkowski vac-

[22]Why can't *both* a Minkowski and a Rindler observer set off in different spacetime directions and *simultaneously* measure their respective (finite-subspace or total) number operators? Would it not, then, be a violation of microcausality when the Minkowski observer's measurement disturbs the statistics of the Rindler observer's measurement outcomes? No. We must remember that the Minkowski particle concept is global, so our Minkowski observer cannot make a precise measurement of any of her number operators unless it is executed throughout the whole of spacetime, which would necessarily destroy her spacelike separation from the Rindler observer. On the other hand, if she is content with only an approximate measurement of one of her number operators in a bounded spacetime region, it is well-known that simultaneous, nondisturbing "unsharp" measurements of incompatible observables *are* possible. For an analysis of the case of simultaneous measurements of unsharp position and momentum, see Busch *et al* [1995].

uum *because* they couple to *Rindler* observables of the field that are distinct from, and indeed complementary to, Minkowski observables.

Arageorgis [1995] himself, together with his collaborators (Arageorgis *et al* [1995]), prefer to characterize inequivalent particle concepts, not as complementary, but *incommensurable*. At first glance, this looks like a trivial semantic dispute between us. For instance Glymour, in a recent introductory text on the philosophy of science, summarizes complementarity using the language of incommensurability:

> Changing the experiments we conduct is like changing conceptual schemes or paradigms: we experience a different world. Just as no world of experience combines different conceptual schemes, no reality we can experience (even indirectly through our experiments) combines precise position and precise momentum (Salmon *et al* [1992], p. 128).

However, philosophers of science usually think of incommensurability as a relation between theories *in toto*, not different parts of the same physical theory. Arageorgis *et al* maintain that inequivalent quantizations define incommensurable *theories*.

Arageorgis [1995] makes the claim that "the degrees of freedom of the field in the Rindler model *simply cannot be described* in terms of the ground state and the elementary excitations of the degrees of freedom of the field in the Minkowski model" ([1995], p. 268; our italics). Yet so much of our earlier discussion proves the contrary. Disjoint representations *are* commensurable, via the abstract Weyl algebra they share. The result is that the ground state of one Fock representation makes definite, if sometimes counterintuitive, predictions for the "differently complexified" degrees of freedom of other Fock representations.

Arageorgis *et al* [2000] offer an *argument* for incommensurability — based on Fulling's "theorem". They begin by discussing the case where the primed and unprimed representations are unitarily equivalent. (Notice that they speak of two different "theorists", rather than two different observers.)

> ...while *different*, these particle concepts can nevertheless be deemed to be *commensurable*. The two theorists are just labelling the particle states in different ways, since each defines particles of a given type by mixing the creation and annihilation operators

> of the other theorist. Insofar as the primed and unprimed theorists disagree, they disagree over which of two inter-translatable descriptions of the same physical situation to use.
>
> The gulf of disagreement between two theorists using unitarily inequivalent Fock space representations is much deeper. If in this case the primed-particle theorist can speak sensibly of the unprimed-particle theorist's vacuum at all, he will say that its primed-particle content is infinite (or more properly, undefined), and the unprimed-theorist will say the same of the unprimed-particle content of the primed vacuum. Such disagreement is profound enough that we deem the particle concepts affiliated with unitarily inequivalent Fock representations *incommensurable* ([2000], p. 26).

The logic of this argument is curious. In order to make Fulling's "theorem" do the work for incommensurability that Arageorgis *et al* want it to, one must first have in hand a rigorous version of the theorem (otherwise their argument would be built on sand). But any rigorous version, like our Proposition 10, has to presuppose that there is sense to be made of using a vector state from one Fock representation to generate a prediction for the expectation value of the total number operator in another inequivalent representation. Thus, one cannot even *entertain* the philosophical implications of Fulling's result if one has not first granted a certain level of commensurability between inequivalent representations.

Moreover, while it may be tempting to *define* what one means by "incommensurable representations" in terms of Fulling's characterization of inequivalent representations, it is difficult to see the exact motivation for such a definition. Even vector states *in the folium* of the unprimed "theorist's" Fock representation can fail to assign his total number operator a finite expectation value (just consider any vector not in the operator's domain). Yet it would be alarmist to claim that, were the field in such a state, the unprimed "theorist" would lose his conceptual grasp on, or his ability to talk about, his *own* unprimed kind of quanta! So long as a state prescribes a well-defined probability measure over the spectral projections of the unprimed "theorist's" total number operator — and all states in his *and* the folium of any primed theorist's representation *will* — we fail to see the difficulty.

5 Conclusion

Let us return to answer the questions we raised in our introduction.

We have argued that a conservative operationalist about physical observables is not committed to the physical inequivalence of disjoint representations, so long as he has no particular attachment to states in a particular folium being the only physical ones. On the other hand, a liberal about physical observables, no matter what his view on states, *must* say that disjoint representations yield physically inequivalent descriptions of a field. However, we steadfastly resisted the idea that this means an interpreter of quantum field theory must say disjoint representations are incommensurable, or even different, *theories*.

Distinguishing "potential" from "actual" quanta won't do to resolve the paradox of observer-dependence. Rather, the paradox forces us to thoroughly abandon the idea that Minkowski and Rindler observers moving through the same field are both trying to detect the presence of particles *simpliciter*. Their motions cause their detectors to couple to *different* incompatible particle observables of the field, making their perspectives on the field necessarily complementary. Furthermore, taking this complementary seriously means saying that neither the Minkowski nor Rindler perspective yields the uniquely "correct" story about the particle content of the field, and that *both* are necessary to provide a complete picture.

So "Are Rindler Quanta Real?" This is a loaded question that can be understood in two different ways.

First, we could be asking "Are *Any* Quanta Real?" without regard to inequivalent notions of quanta. Certainly particle detection events, modulo a resolution of the measurement problem, are real. But it should be obvious by now that detection events do not generally license naïve talk of individuatable, localizable, particles that come in determinate numbers in the *absence* of being detected.

A fuller response would be that quantum field theory is "fundamentally" a theory of a field, not particles. This is a reasonable response given that: (i) the field operators $\{\Phi(f) : f \in S\}$ exist in every regular representation; (ii) they can be used to construct creation, annihilation, and number operators; and (iii) their expectation values evolve in significant respects like the values of the counterpart classical field, modulo non-local Bell-type correlations. This "field approach" response might seem to leave the ontology of the theory somewhat opaque. The field operators, being subject to the canonical commutation relations, do not all commute; so we cannot speak sensibly of them all simultaneously having determinate values! However, the right way to think of the field approach, compatible with complementary, is to see it as viewing a quantum field as a collection of correlated "objective propensities" to display values of the field operators in more or less localized regions of spacetime, relative to various measurement contexts. This view makes room for the reality of quanta, but only as a kind of epiphenomenon of the field associated with certain functions of the field operators.

Second, we could be specifically interested in knowing whether it is sensible to say that *Rindler*, as opposed to just Minkowski, quanta are real. An uninteresting answer would be 'No' — on the grounds that quantum field theory on flat spacetime is not a serious candidate for describing our actual universe, or that the Rindler representation is too "pathological". But, as philosophers, we are content to leave to the physicists the task of deciding the question "Are Rindler Quanta *Empirically Verified*?". All we have tried to determine (to echo words of van Fraassen) is how the world *could possibly be* if both the Rindler and Minkowski representations were "true". We have argued that the antecedent of this counterfactual makes perfect sense, and that it forces us to view Rindler and Minkowski quanta as complementary. Thus, Rindler and Minkowski would be equally amenable to achieving "reality status" provided the appropriate measurement context is in place. As Wald has put it:

> Rindler particles are "real" to accelerating observers! This shows that different notions of "particle" are useful for different purposes ([1994], p. 116).

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Appendix

Proposition 4. *Under the liberal approach to observables, ϕ (factorial) and π (irreducible) are physically equivalent representations of $\mathcal{W}$ only if they are quasi-equivalent.*

Proof. By hypothesis, the bijective mapping α must map the self-adjoint part of $\phi(\mathcal{W})''$ onto that of $\pi(\mathcal{W})''$. Extend α to *all* of $\phi(\mathcal{W})''$ by defining

$$\alpha(X) := \alpha(\mathrm{Re}(X)) + i\alpha(\mathrm{Im}(X)), \qquad X \in \phi(\mathcal{W})''. \tag{73}$$

Clearly, then, α preserves adjoints.

Recall that a family of states S_0 on a C^*-algebra is called *full* just in case S_0 is convex, and for any $A \in \mathcal{A}$, $\rho(A) \geq 0$ for all $\rho \in S_0$ only if $A \geq 0$. By hypothesis, there is a bijective mapping β from the "physical" states of $\phi(\mathcal{W})''$ onto the "physical" states of $\pi(\mathcal{W})''$. According to both the conservative and liberal construals of physical states, the set of physical states includes normal states. Since the normal states are full, the domain and range of β contain full sets of states of the respective C^*-algebras.

By condition (22) and the fact that the domain and range of β are full sets of states, α arises from a *symmetry* between the C^*-algebras $\phi(\mathcal{W})''$ and $\pi(\mathcal{W})''$ in the sense of Roberts & Roepstorff ([1969], Sec. 3).[23] Their Propositions 3.1 and 6.3 then apply to guarantee that α must be linear and preserve Jordan structure (i.e., anti-commutator brackets). Thus α is a Jordan $*$-isomorphism.

Now both $\phi(\mathcal{W})''$ and $\pi(\mathcal{W})'' = \mathbf{B}(\mathcal{H}_\pi)$ are von Neumann algebras, and the latter has a trivial commutant. Thus KR ([1997], Ex. 10.5.26) applies, and α is either a $*$-isomorphism or a $*$-anti-isomorphism, that reverses the order of products. However, such reversal is ruled out, otherwise we would have, using the Weyl relations (10),

$$\alpha(\phi(W(f))\phi(W(g))) = e^{-i\sigma(f,g)/2}\alpha(\phi(W(f+g))), \tag{74}$$

$$\Rightarrow \quad \alpha(\phi(W(g)))\alpha(\phi(W(f))) = e^{-i\sigma(f,g)/2}\alpha(\phi(W(f+g))), \tag{75}$$

$$\Rightarrow \quad \pi(W(g))\pi(W(f)) = e^{-i\sigma(f,g)/2}\pi(W(f+g)), \tag{76}$$

$$\Rightarrow \quad e^{i\sigma(f,g)/2}\pi(W(f+g)) = e^{-i\sigma(f,g)/2}\pi(W(f+g)), \tag{77}$$

[23] Actually, they consider only symmetries of a C^*-algebra onto *itself*, but their results remain valid for our case.

for all $f, g \in S$. This entails that the value of σ on any pair of vectors is always is a multiple of 2π which, since σ is bilinear, cannot happen unless $\sigma = 0$ identically (and hence $S = \{0\}$). It follows that α is in fact a $*$-isomorphism. And, by condition (21), α must map $\phi(A)$ to $\pi(A)$ for all $A \in \mathcal{W}$. Thus ϕ is quasi-equivalent to π. $\square$

Proposition 6. *When S is infinite-dimensional, $\pi(\mathcal{W}[S, \sigma])$ contains no non-trivial bounded functions of the total number operator on $\mathcal{F}(\mathcal{S}_J)$.*

Proof. For clarity, we suppress the representation map π. Suppose that $F : \mathbb{N} \mapsto \mathbb{C}$ is a bounded function. We show that if $F(N) \in \mathcal{W}$, then $F(n) = F(n+1)$ for all $n \in \mathbb{N}$.

The Weyl operators on $\mathcal{F}(\mathcal{S}_J)$ satisfy the commutation relation (BR [1996], Prop. 5.2.4(1,2)):

$$W(g)\Phi(f)W(g)^* = \Phi(f) - \sigma(g, f)I. \tag{78}$$

Using Eqns. (26) and (37), we find

$$W(g)a^*(f)W(g)^* = a^*(f) + 2^{-1/2}i(g, f)_J I, \tag{79}$$

and from this, $[W(g), a^*(f)] = 2^{-1/2}i(g, f)_J W(g)$. Now let $\psi \in \mathcal{F}(\mathcal{S}_J)$ be in the domain of $a^*(f)$. Then a straightforward calculation shows that

$$\begin{aligned} &\langle a^*(f)\psi, W(g)a^*(f)\psi\rangle \\ &\quad = \; 2^{-1/2}i(g, f)_J\langle a^*(f)\psi, W(g)\psi\rangle + \langle a(f)a^*(f)\psi, W(g)\psi\rangle. \end{aligned} \tag{80}$$

Let $\{f_k\}$ be an infinite orthonormal basis for $\mathcal{S}_J$, and let $\psi \in \mathcal{F}(\mathcal{S}_J)$ be the vector whose n-th component is $P_+(f_1 \otimes \cdots \otimes f_n)$ and whose other components are zero. Now, for any $k > n$, we have $a(f_k)a^*(f_k)\psi = (n+1)\psi$. Thus, Eqn. (80) gives

$$\begin{aligned} &\langle a^*(f_k)\psi, W(g)a^*(f_k)\psi\rangle \\ &\quad = \; 2^{-1/2}i(g, f_k)_J\langle a^*(f_k)\psi, W(g)\psi\rangle + (n+1)\langle \psi, W(g)\psi\rangle. \end{aligned} \tag{81}$$

Hence,

$$\lim_{k\to\infty} \langle a^*(f_k)\psi, W(g)a^*(f_k)\psi\rangle = (n+1)\langle \psi, W(g)\psi\rangle. \tag{82}$$

Since $\mathcal{W}$ is generated by the $W(g)$, Eqn. (82) holds when $W(g)$ is replaced with any element in $\mathcal{W}$. On the other hand, ψ is an eigenvector with eigenvalue n for N while $a^*(f_k)\psi$ is an eigenvector with eigenvalue $n+1$ for N. Thus, $\langle\psi, F(N)\psi\rangle = F(n)\|\psi\|^2$ while

$$
\begin{aligned}
\langle a^*(f_k)\psi, F(N)a^*(f_k)\psi\rangle &= F(n+1)\,\|a^*(f_k)\psi\|^2 &\quad (83)\\
&= (n+1)F(n+1)\,\|\psi\|^2, &\quad (84)
\end{aligned}
$$

for all $k > n$. Thus, the assumption that $F(N)$ is in $\mathcal{W}$ (and hence satisfies (82)) entails that $F(n+1) = F(n)$. $\square$

Proposition 7. *The Minkowski and Rindler representations of $\mathcal{W}_\triangleleft$ are disjoint.*

Proof. By Horuzhy ([1988], Thm. 3.3.4), $\pi_{\omega_M^\triangleleft}(\mathcal{W}_\triangleleft)''$ is a "type III" von Neumann algebra which, in particular, contains no atomic projections. Since $\pi_{\omega_R^\triangleleft}$ is irreducible and $\pi_{\omega_M^\triangleleft}$ factorial, either $\pi_{\omega_R^\triangleleft}$ and $\pi_{\omega_M^\triangleleft}$ are disjoint, or they are quasi-equivalent. However, since $\pi_{\omega_R^\triangleleft}(\mathcal{W}_\triangleleft)'' = \mathbf{B}(\mathcal{F}(\mathcal{S}(\triangleleft)_R))$, the weak closure of the Rindler representation clearly contains atomic projections. Moreover, $*$-isomorphisms preserve the ordering of projection operators. Thus there can be no $*$-isomorphism of $\pi_{\omega_M^\triangleleft}(\mathcal{W}_\triangleleft)''$ onto $\pi_{\omega_R^\triangleleft}(\mathcal{W}_\triangleleft)''$, and the Minkowski and Rindler representations of $\mathcal{W}_\triangleleft$ are disjoint. $\square$

Proposition 8. *The Minkowski and Rindler representations of $\mathcal{W}_{\bowtie}$ are disjoint.*

Proof. Again, we use the fact that $\pi_{\omega_M^{\bowtie}}(\mathcal{W}_\triangleleft)''$ ($\equiv \pi_{\omega_M^\triangleleft}(\mathcal{W}_\triangleleft)''$) does not contain atomic projections, whereas $\pi_{\omega_R^{\bowtie}}(\mathcal{W}_\triangleleft)''$ ($\equiv \pi_{\omega_R^\triangleleft}(\mathcal{W}_\triangleleft)''$) does. Suppose, for reductio ad absurdum, that $\omega_R^{\bowtie}$ and $\omega_M^{\bowtie}$ are not disjoint. Since both these states are pure, they induce irreducible representations, which therefore must be unitarily equivalent. Thus, there is a *weakly continuous* $*$-isomorphism α from $\pi_{\omega_M^{\bowtie}}(\mathcal{W}_{\bowtie})''$ onto $\pi_{\omega_R^{\bowtie}}(\mathcal{W}_{\bowtie})''$ such that $\alpha(\pi_{\omega_M^{\bowtie}}(A)) = \pi_{\omega_R^{\bowtie}}(A)$ for each $A \in \mathcal{W}_{\bowtie}$. In particular, α maps $\pi_{\omega_M^{\bowtie}}(\mathcal{W}_\triangleleft)$ onto $\pi_{\omega_R^{\bowtie}}(\mathcal{W}_\triangleleft)$; and, since α is weakly continuous, it maps $\pi_{\omega_M^{\bowtie}}(\mathcal{W}_\triangleleft)''$ onto $\pi_{\omega_R^{\bowtie}}(\mathcal{W}_\triangleleft)''$. Consequently, $\pi_{\omega_M^{\bowtie}}(\mathcal{W}_\triangleleft)''$ contains an atomic projection — contradiction. $\square$

Proposition 9. *If J_1, J_2 are distinct complex structures on (S, σ), then ω_{J_1} (resp., ω_{J_2}) predicts dispersion in N_{J_2} (resp., N_{J_1}).*

Proof. We shall prove the contrapositive. Suppose, then, that there is some extension $\hat{\omega}_{J_1}$ of ω_{J_1} to $\mathbf{B}(\mathcal{F}(\mathcal{S}_{J_2}))$ that is dispersion-free on all bounded functions of N_{J_2}. Then $\hat{\omega}_{J_1}$ is multiplicative for the product of the bounded operator $e^{\pm itN_{J_2}}$ with any other element of $\mathbf{B}(\mathcal{F}(\mathcal{S}_{J_2}))$ (KR [1997], Ex. 4.6.16). Hence, by Eqn. (43),

$$\begin{aligned}
\omega_{J_1}(W(\cos t + \sin t J_2 f)) &= \hat{\omega}_{J_1}\left(e^{-itN_{J_2}}\pi_{\omega_{J_2}}(W(f))e^{itN_{J_2}}\right) && (85)\\
&= \hat{\omega}_{J_1}(e^{-itN_{J_2}})\,\omega_{J_1}(W(f))\,\hat{\omega}_{J_1}(e^{itN_{J_2}}) && (86)\\
&= \omega_{J_1}(W(f)), && (87)
\end{aligned}$$

for all $f \in S$ and $t \in \mathbb{R}$. In particular, we may set $t = \pi/2$, and it follows that $\omega_{J_1}(W(J_2 f)) = \omega_{J_1}(W(f))$ for all $f \in S$. Since e^{-x} is a one-to-one function of $x \in \mathbb{R}$, it follows from (36) that

$$(f, f)_{J_1} = (J_2 f, J_2 f)_{J_1}, \qquad f \in S, \tag{88}$$

and J_2 is a real-linear isometry of the Hilbert space $\mathcal{S}_{J_1}$. We next show that J_2 is in fact a unitary operator on $\mathcal{S}_{J_1}$.

Since J_2 is a symplectomorphism, $\mathrm{Im}(J_2 f, J_2 g)_{J_1} = \mathrm{Im}(f, g)_{J_1}$ for any two elements $f, g \in S$. We also have

$$|f + g|^2_{J_1} = |f|^2_{J_1} + |g|^2_{J_1} + 2\mathrm{Re}(f, g)_{J_1}, \tag{89}$$

$$\begin{aligned}
|J_2 f + J_2 g|^2_{J_1} &= |J_2 f|^2_{J_1} + |J_2 g|^2_{J_1} + 2\mathrm{Re}(J_2 f, J_2 g)_{J_1} && (90)\\
&= |f|^2_{J_1} + |g|^2_{J_1} + 2\mathrm{Re}(J_2 f, J_2 g)_{J_1}, && (91)
\end{aligned}$$

using the fact that J_2 is isometric. But $J_2(f + g) = J_2 f + J_2 g$, since J_2 is real-linear. Thus,

$$|J_2 f + J_2 g|^2_{J_1} = |J_2(f + g)|^2_{J_1} = |f + g|^2_{J_1}, \tag{92}$$

using again the fact that J_2 is isometric. Cancellation with Eqns. (89) and (91) then gives $\mathrm{Re}(f, g)_{J_1} = \mathrm{Re}(J_2 f, J_2 g)_{J_1}$. Thus, J_2 preserves the inner product between any two vectors in $\mathcal{S}_{J_1}$. All that remains to show is that J_2 is complex-linear. So let $f \in \mathcal{S}_{J_1}$. Then,

$$(J_2(if), J_2 g)_{J_1} = (if, g)_{J_1} = -i(f, g)_{J_1} = -i(J_2 f, J_2 g)_{J_1} = (iJ_2 f, J_2 g)_{J_1}, \tag{93}$$

for all $g \in \mathcal{H}$. Since J_2 is onto, it follows that $(J_2(if), g)_{J_1} = (iJ_2 f, g)_{J_1}$ for all $g \in \mathcal{H}$ and therefore $J_2(if) = iJ_2 f$.

Finally, since J_2 is unitary and $J_2^2 = -I$, it follows that $J_2 = \pm iI = \pm J_1$. However, if $J_2 = -J_1$, then

$$-\sigma(f, J_1 f) = \sigma(f, J_2 f) \geq 0, \qquad f \in S, \tag{94}$$

since J_2 is a complex structure. Since J_1 is also a complex structure, it follows that $\sigma(f, J_1 f) = 0$ for all $f \in S$ and $S = \{0\}$. Therefore, $J_2 = J_1$. □

Proposition 10. *A pair of Fock representations $\pi_{\omega_{J_1}}$, $\pi_{\omega_{J_2}}$ are unitarily equivalent if and only if ω_{J_1} assigns N_{J_2} a finite value (equivalently, ω_{J_2} assigns N_{J_1} a finite value).*

Proof. S may be thought of as a *real* Hilbert space relative to either of the inner products μ_1, μ_2 defined by

$$\mu_{1,2}(\cdot,\cdot) := \mathrm{Re}(\cdot,\cdot)_{J_{1,2}} = \sigma(\cdot, J_{1,2}\cdot). \tag{95}$$

We shall use Van Daele and Verbeure's [1971] Theorem 2: $\pi_{\omega_{J_1}}, \pi_{\omega_{J_2}}$ are unitarily equivalent if and only if the positive operator $-[J_1, J_2]_+ - 2I$ on S is a trace-class relative to μ_2. (Since unitary equivalence is symmetric, the same "if and only if" holds with $1 \leftrightarrow 2$.)

As we know, we can build any number operator $N_{J_2}(f)$ ($f \in S$) on $\mathcal{H}_{\omega_{J_1}}$ by using the complex structure J_2 in Eqns. (37). In terms of field operators, the result is

$$N_{J_2}(f) = 2^{-1}(\Phi(f)^2 + \Phi(J_2 f)^2 + i[\Phi(f), \Phi(J_2 f)]). \tag{96}$$

Observe that $N_{J_2}(J_2 f) = N_{J_2}(f)$, which had better be the case, since $N_{J_2}(f)$ represents the number of J_2-quanta with wavefunction in the *subspace* of $\mathcal{S}_{J_2}$ generated by f. The expectation value of an arbitrary "two-point function" in J_1-vacuum is given by

$$\begin{aligned}
& \langle \Omega_{\omega_{J_1}}, \phi(f_1)\phi(f_2)\Omega_{\omega_{J_1}} \rangle && (97)\\
= \;& (-i)^2 \tfrac{\partial^2}{\partial t_1 \partial t_2} \omega_{J_1}(W(t_1 f_1) W(t_2 f_2))|_{t_1=t_2=0} && (98)\\
= \;& -\tfrac{\partial^2}{\partial t_1 \partial t_2} \exp(-\tfrac{1}{2} t_1 t_2 (f_1, f_2)_{J_1} - \tfrac{1}{4} t_1^2 (f_1, f_1)_{J_1} - \tfrac{1}{4} t_2^2 (f_2, f_2)_{J_1})|_{t_1=t_2=0} && (99)\\
= \;& \tfrac{1}{2}(f_1, f_2)_{J_1}, && (100)
\end{aligned}$$

invoking (35) in the first equality, and the Weyl relations (10) together with Eqns. (26), (36) to obtain the second. Plugging Eqn. (100) back into (96) and using (95) eventually yields

$$\langle \Omega_{\omega_{J_1}}, N_{J_2}(f)\Omega_{\omega_{J_1}} \rangle = 2^{-2}\mu_2(f, (-[J_1, J_2]_+ - 2I)f). \tag{101}$$

Next, recall that on the Hilbert space $\mathcal{H}_{\omega_{J_2}}$, $N_{J_2} = \sum_{k=1}^{\infty} N_{J_2}(f_k)$, where $\{f_k\} \subseteq \mathcal{S}_{J_2}$ is any orthonormal basis. Let $\widehat{\omega}_{J_1}$ be any extension of ω_{J_1} to $\mathbf{B}(\mathcal{H}_{\omega_{J_2}})$. The calculation that resulted in expression (101) was done in $\mathcal{H}_{\omega_{J_1}}$, however, only finitely many-degrees of freedom were involved. Thus the Stone-von Neumann uniqueness theorem ensures that (101) gives the value of each individual $\widehat{\omega}_{J_1}(N_{J_2}(f_k))$. Since for any finite m, $\sum_{k=1}^{m} N_{J_2}(f_k) \leq N_{J_2}$ as positive operators, we must also have

$$\sum_{k=1}^{m} \widehat{\omega}_{J_1}(N_{J_2}(f_k)) = \widehat{\omega}_{J_1}\left(\sum_{k=1}^{m} N_{J_2}(f_k)\right) \leq \widehat{\omega}_{J_1}(N_{J_2}). \tag{102}$$

Thus, $\widehat{\omega}_{J_1}(N_{J_2})$ will be defined just in case the sum

$$\sum_{k=1}^{\infty} \widehat{\omega}_{J_1}(N_{J_2}(f_k)) = \sum_{k=1}^{\infty} \widehat{\omega}_{J_1}(N_{J_2}(J_2 f_k)) \tag{103}$$

converges. Using (101), this is, in turn, equivalent to

$$\sum_{k=1}^{\infty} \mu_2(f_k, (-[J_1, J_2]_+ - 2I)f_k) + \sum_{k=1}^{\infty} \mu_2(J_2 f_k, (-[J_1, J_2]_+ - 2I)J_2 f_k) < \infty. \tag{104}$$

However, it is easy to see that $\{f_k\}$ is a J_2-orthonormal basis just in case $\{f_k, J_2 f_k\}$ forms an orthonormal basis in S relative to the inner product μ_2. Thus, Eqn. (104) is none other than the statement that the operator $-[J_1, J_2]_+ - 2I$ on S is trace-class relative to μ_2, which is equivalent to the unitary equivalence of $\pi_{\omega_{J_1}}, \pi_{\omega_{J_2}}$. (The same argument, of course, applies with $1 \leftrightarrow 2$ throughout.) $\square$

Proposition 11. *If ρ is a regular state of $\mathcal{W}$ disjoint from the Fock state ω_J, then* $\inf_{F \in \mathbb{F}} \left\{ \mathrm{Prob}^{\,\rho}(N_F \in [0, n]) \right\} = 0$ *for every* $n \in \mathbb{N}$.

Proof. Suppose that ω_J and ρ are disjoint; i.e., $\mathfrak{F}(\omega_J) \cap \mathfrak{F}(\rho) = \emptyset$. First, we show that $D(n_\rho) = \{0\}$, where n_ρ is the quadratic form on $\mathcal{H}_\rho$ which, if densely defined, would correspond to the total J-quanta number operator.

Suppose, for reductio ad absurdum, that $D(n_\rho)$ contains some unit vector ψ. Let ω be the state of $\mathcal{W}$ defined by

$$\omega(A) = \langle \psi, \pi_\rho(A)\psi \rangle\,, \qquad A \in \mathcal{W}. \tag{105}$$

Since $\omega \in \mathfrak{F}(\rho)$, it follows that ω is a regular state of $\mathcal{W}$ (since ρ itself is regular), and that $\omega \notin \mathfrak{F}(\omega_J)$. Let P be the projection onto the closed subspace in $\mathcal{H}_\rho$ generated by the set $\pi_\rho(\mathcal{W})\psi$. If we let $P\pi_\rho$ denote the subrepresentation of π_ρ on $P\mathcal{H}_\rho$, then $(P\pi_\rho, P\mathcal{H}_\rho)$ is a representation of $\mathcal{W}$ with cyclic vector ψ. By the uniqueness of the GNS representation, it follows that $(P\pi_\rho, P\mathcal{H}_\rho)$ is unitarily equivalent to $(\pi_\omega, \mathcal{H}_\omega)$. In particular, since Ω_ω is the image in $\mathcal{H}_\omega$ of $\psi \in P\mathcal{H}_\rho$, $D(n_\omega)$ contains a vector cyclic for $\pi_\omega(\mathcal{W})$ in $\mathcal{H}_\omega$. However, by BR ([1996], Thm. 4.2.14, (3) $\Rightarrow$ (1)), this implies that $\omega \in \mathfrak{F}(\omega_J)$ — a contradiction. Therefore, $D(n_\rho) = \{0\}$.

Now suppose, again for reductio ad absurdum, that

$$\inf_{F \in \mathbb{F}} \Big\{ \mathrm{Prob}^{\,\rho}(N_F \in [0,n]) \Big\} \neq 0. \tag{106}$$

Let $E_F := [E_\rho(F)]([0,n])$ and let $E := \bigwedge_{F\in\mathbb{F}} E_F$. Since the family $\{E_F\}$ of projections is downward directed (i.e., $F \subseteq F'$ implies $E_F \geq E_{F'}$), we have

$$0 \;\neq\; \inf_{F\in\mathbb{F}} \{\langle \Omega_\rho, E_F \Omega_\rho \rangle\} \;=\; \langle \Omega_\rho, E\Omega_\rho \rangle \;=\; \|E\Omega_\rho\|^2. \tag{107}$$

Now since $E_F E\Omega_\rho = E\Omega_\rho$, it follows that

$$[n_\rho(F)](E\Omega_\rho) \leq n, \tag{108}$$

for all $F \in \mathbb{F}$. Thus, $E\Omega_\rho \in D(n_\rho)$ and $D(n_\rho) \neq \{0\}$ — contradicting the conclusion of the previous paragraph. $\square$

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