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A Conjecture Regarding Quantum Fluctuation of Gravitation and Elementary Particles as Excitons in a Turbulent Gravitational Field

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Based on Wheeler's conjecture that the quantum fluctuations of the metric create a multiple connected foam-like structure of the vacuum with a structure constant of $L^* = (\hbar G/c^3)^{1/2} \approx 10^{-33}$ cm and large virtual energy densities of the order $c^4/GL^{*2} \approx 10^{115}$ erg/cm^3 and that elementary particles are exciton-like weak coherent perturbations in the violent vacuum physics, a model theory is constructed in which real turbulent fluctuations are superimposed on the average metric with the fluctuating metric satisfying the free space Einstein equations.

A stationary turbulent field of "mixing length" L^* represents the vacuum fluctuations, long-range coherent fluctuations the exciton-like particles. Averaging gives the Einstein equations for the average metric which because of the nonlinear terms contains a vacuum term associated with the small-scale fluctuations and a term associated with the coherent part which is interpreted as the energy momentum tensor. Estimate of the leading terms in the energy momentum tensor gives for the size L of the excitons the relation $L \sim L^{*2/3} R_0^{1/3}$ with the average universal curvature radius R_0. For $R_0 \sim 10^{-28}$ cm, L is of the correct order of magnitude 10^{-13} cm. Further relations between microscopic and cosmologic quantities are derived which appear to be Eddington's relations. The vacuum terms give a modification of Einstein's equations which act as mass production terms and are proportional to the average curvature with an estimated rate of 10^{-46} g/cm^3 sec. Possible consequences due to time dependence of R are briefly discussed.

INTRODUCTION

In a series of articles and books J. A. Wheeler has investigated the "issue of the final state of matter" catalyzed to the endpoint of nuclear evolution [1–3]. Assuming that Einstein's theory of general relativity is relevant to the inner structure of physics, Wheeler has focused attention on the fact that many typical solutions of the equations of general relativity in time develop singularities with infinite curvature.

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In recent years the works of Penrose [4], Hawking [5], and Geroch [6] have proven that Wheeler's earlier conjecture is true, that under quite general assumptions all solutions in general relativity eventually develop singularities of infinite curvature.

We are today far from an understanding by what trick nature might prevent these singularities from actually occurring. However, led by past experience one is inclined to believe that nature never really evolves into a singularity. And so we are looking for new avenues, for some extension of existing physical theories which would do for the collapse problem what the uncertainty principle did for atomic physics, viz., overriding the classical prediction that the orbiting electron would radiate and spiral into the nucleus terminating all physics in a singularity.

There is, however, no observational basis which could guide us in inventing such theories. Objects in which gravitational collapse might play a role, the quasars, are still not understood. But in his analysis of the gravitational collapse problem Wheeler has shown [3] that the assumption of baryon annihilation in the final evolutionary states of a collapsing object is inescapable. This leads to the conjecture that the behavior of matter in very intense gravitational fields and the nature of elementary particles might be intimately connected. A crude model theory of this connection would be expected to link gravitation and elementary particles in such a way that it gives some connection between the typical particle size and mass and gravitational quantities and offers a possibility of escape from the collapse problem.

In his quantum geometrodynamics, Wheeler has pointed the direction in which one might look for a solution of these problems [1, 2, 7]. The existence of a fundamental length which can be produced from the light velocity c, the Planck constant $\hbar$, and the gravitational constant G, the Planck length

$$L^* = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}\,\mathrm{cm} \tag{1}$$

suggests that this length might play the dominant role at very intense curvatures and might govern a quantum theory of strong gravitational fields. Indeed this length determines the principal uncertainty of any length measurement. L^* is the smallest length one could measure without having the probing photon itself distort the space curvature significantly [7, 8]. Wheeler has shown [4] that, based on Feynman's pathintegral method of quantum theory, the length L^* is expected to govern the vacuum fluctuation of the gravitational field functions g_{ik}. They would undergo quantum fluctuations of the order

$$\delta g \sim L^*/L. \tag{2}$$

This means that, considering a spatial region of dimension L, those virtual histories of field evolution contribute most to the propagation function in Feynman's pathintegral for which $\delta g \sim L^*/L$. Over the range L the potentials have fluctuations δg around the local average value. Over regions of the order L^* these fluctuations would be as large as the potentials. Using the geometric interpretation of gravitation, we find these fluctuations to be violent fluctuations of the geometry over regions of the order L^* which would create a virtual multiple connected foamlike structure of space [1, 2, 7].

The virtual vacuum fluctuation of the metric of magnitude $\delta g \sim L^*/L$ would have corresponding fluctuations of the field strength $\delta \Gamma \sim \delta g/L \sim L^*/L^2$ and of the curvatures

$$\delta R \sim \delta g/L^2 \sim L^*/L^3. \tag{3}$$

The enormous fluctuation of the curvature over distances of the order L^* would correspond to a virtual creation and annihilation of masses of the order $m^* \sim c^2 L^*/G \approx 10^{-5}$ g which corresponds to an energy-density in the vacuum fluctuation of

$$\delta W = m^* c^2/L^{*3} \approx 10^{115}\ \mathrm{erg/cm^3} \tag{4}$$

with a life-time of these virtual states of $\delta t \sim \hbar/m^* c^2 \approx 10^{-43}$ sec according to the uncertainty principle. On account of these enormous numbers Wheeler has conjectured [1, 2, 7, 10] that elementary particles with an energy density of 10^{35} erg/cm^3 over a region of $L \approx 10^{-13}$ cm represent only a minor long-range perturbation on the background of the violent vacuum physics and that he has spoken of elementary particles as geometrodynamic excitons, a fantastically weak coherent perturbation in the pattern of the intense vacuum fluctuation. Wheeler has used the picture of the ocean seen by an aviator. From very high above the ocean looks perfectly flat. From a somewhat closer distance larger coherent patterns are visible, and from very close the small-scale ripples, breaking of wave-crests, and formation of multiple connected surfaces become discernable [1, 2].

Recently Wheeler [10], Gerlach [11], and DeWitt [12] have begun, in the theory of superspace, to investigate the nature of these geometry fluctuations and the structure of superspace, in which the points are representatives of possible 3-geometries and in which the propagation of constructively interfering probability waves form a localized wave packet the path of which marks the evolutionary trajectory of the corresponding classical 3-geometry in much the same way as the propagation of a localized solution of Schrödinger's equation marks the path of the classical electron.

The hope is that eventually particles can be described as quantum states of excitation of the geometry in terms of probability amplitudes which are functionals of possible 3-geometries, different functionals for different states with different numbers of particles, and that the typical behavior of probability amplitudes will "spread" the final state of gravitational collapse and avoid the singularity.

It is unlikely that this "ultimate" theory could be formulated and solutions be found in short order. In this situation it could be interesting to see whether, by incorporating the quantum fluctuations through some ad hoc hypothesis into the classical theory, it is possible to produce a modified theory which does offer a connection between gravitation and particles and possible ways of avoiding the collapse problem. Such a hypothetical model will be made here and some of its consequences will be explored.

What if we take Wheeler's vision more literally? What if we forego a correct quantum geometrodynamic description of the virutal metric fluctuation at this point and try a model theory in which we take the fluctuations as real random fluctuations which satisfy the classical equation of motion, i.e., Einstein's equation, just as the turbulent fluid motion satisfies Navier–Stokes equation. The local value of the metric quantities $g_{\mu\nu}$ would then be some average value $\bar{g}_{\mu\nu}$ on which is superimposed a turbulent random field $t_{\mu\nu}$, the average of which is zero. The quantum nature of these turbulent fluctuations would be included by postulating the distribution of these fluctuations to be stationary, homogeneous and isotropic and have, as characteristic "mixing length," the Planck length L^*. Particles or excitons in this picture would be described by a contribution to the random field which has a correlation length L which is long compared with L^* and is inhomogeneous.

In Section I, the assumptions of this model are discussed and the postulates are listed. The autocorrelations of the postulated vacuum fluctuations of characteristic length L^* introduce additional vacuum terms into the Einstein equations for the average metric. The contribution from the long-range correlations are interpreted as the energy momentum tensor. In Section II it is shown that the additional vacuum terms imply nonconservation of energy momentum at large curvatures. The approximate field equation for the excitons is given. In Section III an order of magnitude estimate of the leading terms in the energy momentum tensor is made. Together with the average curvature of the universe this leads to an estimate of the characteristic size L of the excitons which turns out to be of the right order of 10^{-3} cm. Other relations which link cosmological and exciton quantities follow automatically. These relations are identical to the Eddington relations involving the large dimensionless number 10^{40}. In Section IV some cosmological consequences of this model, the effects of a possible time dependence of the exciton quantities and the model's connection with Mach's principle are briefly discussed.

I. The Model

The basic assumption is that the quantum fluctuation of the metric can be incorporated and that it makes its appearance in the classical Einsteinian theory as real random fluctuation of the metric field. This is strictly a hypothetical model, precipitated by "Wheeler's vision." There is no analog to this in classical field theory. Superposition of a random field in a linear theory does not produce any extra terms in the averaged field equation. Only in nonlinear theories does the introduction of turbulence lead to new terms in the equation of motion like the Reynolds stresses in the Navier–Stokes equation of hydromechanics. But we do not know in what sense such "Reynolds stresses" might produce some semi-classical approximation to the correct quantum field theory. It is interesting, however, to note that supersposition of a random field on the action function of classical mechanics does produce the correct field theory for the action function from which the Schrödinger equation is obtained.

The classical Hamilton–Jacobi equation for the action function $S(\mathbf{r}, t)$, i.e.,

$$-\partial S/\partial t = (\nabla S)^2/2m + U(\mathbf{r}) \tag{5}$$

could be interpreted in analogy to fluid dynamics as the equation of motion for a turbulent fluid

$$S = S_0 + S_1 , \tag{6}$$

where S_0 is the average field an S_1 a random field with zero average. Inserting S into the Hamilton–Jacobi equation and taking time averages (which makes terms that are linear in the random field cancel out) one obtains

$$-\partial S_0/\partial t = (\nabla S_0)^2/2m + U(\mathbf{r}) + \overline{(\nabla S_1)^2}/2m. \tag{7}$$

In this equation of motion the last term would be the analog to the Reynolds stresses in the Navier–Stokes equations for turbulent flow. If one postulates that the random field is stationary and that the higher order autocorrelations are small then one can show that the first nonzero term in an expansion for $\overline{(\nabla S_1^2)}$ would be $2\beta\nabla^2 S_0$ where β is a small constant. This gives as the modified Hamilton–Jacobi equation

$$-\partial S_0/\partial t = (\nabla S_0)^2/2m + U(\mathbf{r}) + (\beta/m)\,\nabla^2 S_0 \tag{8}$$

which is identical with the equation for the action function which one obtains from Schrödinger equation with

$$\psi(\mathbf{r}, t) = \exp[(i/\hbar)\, S(\mathbf{r}, t)], \tag{9}$$

where β is taken to be

$$\beta = -i\hbar/2. \tag{10}$$

The corresponding procedure for the Einsteinian theory would be to superimpose a random contribution on the Hamilton–Jacobi function in the Einstein–

Hamilton–Jacobi equation of general relativity [10, 11], solve the turbulence problem under suitable assumptions of stationarity, homogeneity, and neglect of higher moments, and to find the "Reynolds stresses" in terms of the average action function. Repetition of Gerlachs derivation [11] of the classical field equations from the now modified Hamilton–Jacobi equation together with the principle of constructive interference would lead to modified field equations. The connection $\psi = \exp[(i/\hbar) S]$ would lead from the modified Hamilton–Jacobi equation to the Schrödinger equation of the problem.

If it can be established that the connection between the turbulent action field and quantum theory is meaningful, then this might be a way of finding the right Schrödinger equation of geometrodynamics.

Instead of embarking on this ambitious program we consider that the action is a functional of the metric field functions and that a fluctuation of the action has corresponding fluctuations of the field functions. Introducing "Reynolds stresses" due to the fluctuation of the field functions in the nonlinear terms of the Einsteinian field equation will give a different modification of the theory from that one would obtain from the above-mentioned amendment of the Einstein–Hamilton–Jacobi equation. But since the validity of this amendment is not clear one could be satisfied with the less complicated model. The additional terms in this model do originate in the nonlinear terms of the field equations and, therefore, express the geometrodynamic idea that it is through quantum fluctuations of intense self-interacting gravitational fields and the manifestation of small fractions of this self-energy that particles derive their existence.

The model rests on the following assumptions and postulates:

(a) All physics is described by the metric field

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + t_{\mu\nu} ,$$

where $t_{\mu\nu}$ is a random variable field in space and time and $\bar{g}_{\mu\nu}$ is an average field which may be slowly variable. The average of the random field is zero:

$$\langle t_{\mu\nu} \rangle = (\Delta V)^{-4} \int t_{\mu\nu}(x)(-g)^{1/2}\, d^4x = 0. \tag{11}$$

(b) The random field is assumed to have a spectral distribution which allows one to decompose it into two parts:

$$t_{\mu\nu} = f_{\mu\nu} + \varphi_{\mu\nu} . \tag{12}$$

The part $f_{\mu\nu}$ is to be associated with the vacuum fluctuations. It is postulated to be a stationary, homogeneous, isotropic random field of normal distribution with $\langle f_{\mu\nu} \rangle = 0$.

The correlation length is taken to be of the order L^* and the amplitude is taken to be of the order of $\bar{g}_{\mu\nu}$.

The field $\varphi_{\mu\nu}$ also has zero average but it is of long correlation length and is nonhomogeneous. It is zero for the vacuum and, when nonzero represents the excitons in terms of the correlation functions $\langle\varphi_{\mu\nu}(x)\, \varphi_{\delta\sigma}(x')\rangle$ and higher order correlations.

The amplitude of $\varphi_{\mu\nu}$ is taken to be very small compared to $\bar{g}_{\mu\nu}$ and $f_{\mu\nu}$.

(c) The function $\varphi_{\mu\nu}(x)$ is taken to represent a measure for the probability to find matter at point x.

(d) The total field $g_{\mu\nu} = \bar{g}_{\mu\nu} + f_{\mu\nu} + \varphi_{\mu\nu}$ satisfies the free space Einsteinian equation

$$E_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = 0. \tag{13}$$

This is an expression of our assumption that matter is not some foreign agent in the arena of geometry but certain coherent wrinkles in the randomly fluctuating geometry.

The Ricci tensor $R_{\mu\nu}$ and the scalar R are functions of the metric field $g_{\mu\nu}$ and decompose into contributions from the average field $\bar{g}_{\mu\nu}$, the fast vacuum fluctuations $f_{\mu\nu}$ and the long-range perturbations $\varphi_{\mu\nu}$ of these.

The gravitational field strengths which are represented by the Christoffel symbols also appear as superposition of an average field and a random field:

$$\Gamma^{\rho}_{\mu\nu} = \bar{\Gamma}^{\rho}_{\mu\nu} + b^{\rho}_{\mu\nu}. \tag{14}$$

In what follows we wish to maintain the geometric point of view of gravitation for the average metric. But we propose to consider the random field as a tensor field superimposed on the average geometry. Any Riemannian geometry which is characterized by $g_{\mu\nu}$ can be expressed as a field over another Riemannian geometry characterized by $\bar{g}_{\mu\nu}$ [13]. Both are understood to be expressed in the same coordinate system with

$$g_{\mu\nu} g^{\nu\lambda} = \bar{g}_{\mu\nu} \bar{g}^{\nu\lambda} = \delta_{\mu}{}^{\lambda}. \tag{15}$$

For the geometric objects in the two spaces, i.e., the tensors of interest, the Ricci tensor $R_{\mu\nu}$ and the scalar R, one finds

$$\begin{aligned} R_{\mu\nu} &= \bar{R}_{\mu\nu} + \delta R_{\mu\nu}, \\ R &= \bar{R} + \delta R. \end{aligned} \tag{16}$$

The variations can be expressed in covariant form

$$\delta R_{\nu\alpha} = \tfrac{1}{2} g^{\alpha\omega} \{ 2 t_{\omega(\sigma;\bar{\nu})\bar{\alpha}} - t_{\alpha\omega;(\bar{\nu}\bar{\sigma})} - t_{\nu\sigma;(\bar{\omega}\bar{\alpha})} + 4 g \kappa_{\tau} b^{\tau}_{\sigma[\omega} b^{\kappa}_{\nu]\alpha} \}, \tag{17}$$

$$\delta R = g^{\mu\beta} t_{\beta\alpha} \bar{R}_{\mu}{}^{\alpha} + g^{\nu\sigma} g^{\omega\rho} \{ t_{\rho\nu;(\bar{\omega}\bar{\sigma})} - t_{\rho\omega;(\bar{\sigma}\bar{\nu})} + 2 g_{\alpha\beta} b^{\beta}_{\sigma[\omega} b^{\alpha}_{\nu]\rho} \}. \tag{18}$$

Here the semicolon represents covariant differentiation, and the subscript $;\bar{\nu}$ means covariant differentiation with respect to the average metric, i.e., when written out, the additional terms contain the Christoffel symbols of the average metric. In the case of flat average metric these covariant differentiations reduce to ordinary differentiation. The parentheses ( ) and brackets [ ], as usual, mean symmetrization and antisymmetrization, resp., in the enclosed indices. The $b^{\alpha}_{\nu\rho}$ are the differences of the two affine connections and can be shown to be tensors

$$b^{\rho}_{\nu\alpha} = \tfrac{1}{2} g^{\rho\sigma}\{t_{\sigma\nu;\bar{\alpha}} + t_{\sigma\alpha;\bar{\nu}} - t_{\nu\alpha;\bar{\sigma}}\}. \tag{19}$$

The Einstein tensors for the two geometries are

$$E_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R \tag{20}$$

and

$$\bar{E}_{\mu\nu} = \bar{R}_{\mu\nu} - \tfrac{1}{2} \bar{g}_{\mu\nu} \bar{R}, \tag{21}$$

and one has

$$E_{\mu\nu} = \bar{E}_{\mu\nu} + \delta E_{\mu\nu} \tag{22}$$

$$\delta E_{\mu\nu} = \delta R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} \delta R - \tfrac{1}{2} t_{\mu\nu} \bar{R}. \tag{23}$$

The vanishing of the Einstein tensor $E_{\mu\nu}$ therefore gives

$$\bar{E}_{\mu\nu} = \tfrac{1}{2} g_{\mu\nu} \delta R - \tfrac{1}{2} t_{\mu\nu} \bar{R} - \delta R_{\mu\nu}\,. \tag{24}$$

The rhs represents a random source function in the Einsteinian equation for the average metric. Because of the nonlinear terms, an average of this function does not disappear but can act as the effective energy-momentum tensor

$$\langle S_{\mu\nu}\rangle = \tfrac{1}{2}\langle g_{\mu\nu} \delta R\rangle - \langle \delta R_{\mu\nu}\rangle. \tag{25}$$

In the absence of matter, all $\varphi_{\mu\nu}$ would be zero and $\delta R_{\mu\nu}$ and δR in (23) would be the functions as defined in (17) and (18) with $t_{\mu\nu}$ replaced by $f_{\mu\nu}$ which we will indicate by ${}^{f}\delta R_{\mu\nu}$ and ${}^{f}\delta R$. For pure vacuum we would have then

$$\bar{E}_{\mu\nu} = \tfrac{1}{2}\langle(\bar{g}_{\mu\nu} + f_{\mu\nu})\, {}^{f}\delta R - {}^{f}\delta R_{\mu\nu}\rangle. \tag{26}$$

The ${}^{f}\delta R_{\mu\nu}$ and ${}^{f}\delta R$ also contain nonlinear terms, the averages of which do not vanish. The requirement of homogeneity and isotropy restrict the number of nonzero terms as such requirements restrict the number of nonzero correlation functions in the turbulence theory (see Hinze [14]). Further restriction is obtained by the requirement of normal distribution for the random fields which, in the turbulence theory, causes all odd-order moments to be zero and allows the higher even-order moments to be expressed in terms of second-order moments. After taking the average the rhs represents a vacuum term in the average geometry

$$\tfrac{1}{2}\langle(\bar{g}_{\mu\nu} + f_{\mu\nu})\, {}^{f}\delta R - {}^{f}\delta R_{\mu\nu}\rangle = \langle V_{\mu\nu}\rangle. \tag{27}$$

We will write Einstein's Eq. (24) now in the form

$$\bar{E}_{\mu\nu} = \langle V_{\mu\nu}\rangle + \langle Y_{\mu\nu}\rangle, \tag{28}$$

where

$$\langle Y_{\mu\nu}\rangle = \langle \tfrac{1}{2} g_{\mu\nu}\delta R + \delta R_{\mu\nu} - V_{\mu\nu}\rangle. \tag{29}$$

$Y_{\mu\nu}$ collects all the terms which contain the matter field $\varphi_{\mu\nu}$ and may be interpreted as the effective energy-momentum tensor multiplied with $8\pi G/c^2$

$$\langle Y_{\mu\nu}\rangle = (8\pi G/c^2)\, T_{\mu\nu}, \tag{30}$$

where

$$T_{00} = \rho \tag{31}$$

is the mass density.

II. Conservation of Energy-Momentum and Field Equation of Matter

The Bianchi identity says that

$$E^{\nu}_{\mu;\nu} \equiv 0, \tag{32}$$

where $;\nu$ is the covariant differentiation with respect to the metric $g_{\mu\nu}$. With respect to the metric $\bar{g}_{\mu\nu}$, we have

$$\bar{E}^{\nu}_{\mu;\bar{\nu}} \equiv 0. \tag{33}$$

Therefore, we find that

$$\bar{g}^{\sigma\nu}\bar{E}_{\mu\nu} - \bar{g}^{\sigma\nu}(\langle V_{\mu\nu}\rangle - \langle Y_{\mu\nu}\rangle) = 0 \tag{34}$$

and

$$[\bar{g}^{\sigma\nu}(\langle V_{\mu\nu}\rangle - \langle Y_{\mu\nu}\rangle)]_{;\bar{\sigma}} = 0. \tag{35}$$

It appears therefore that in this conjecture the effective energy-momentum would not always be conserved. The term $(\bar{g}^{\sigma\nu}\langle V_{\mu\nu}\rangle)_{;\bar{\sigma}}$ acts as a source term which depends on the curvature of the average metric through the Christoffel symbols in the covariant derivative

$$(\bar{g}^{\sigma\nu}\langle V_{\mu\nu}\rangle)_{;\bar{\sigma}} = \frac{\partial}{\partial x^{\sigma}}(\bar{g}^{\sigma\nu}\langle V_{\mu\nu}\rangle) + \bar{\Gamma}^{\sigma}_{\alpha\sigma}(\bar{g}^{\alpha\nu}\langle V_{\mu\nu}\rangle) - \bar{\Gamma}^{\alpha}_{\mu\sigma}(\bar{g}^{\sigma\nu}\langle V_{\alpha\nu}\rangle). \tag{36}$$

The derivatives of $\langle V_{\mu\nu}\rangle$ are zero because of our assumption of homogeneity of the vacuum field. The Christoffel symbols measure the gravitational field strenght and are proportional to 1/curvature radius. We would expect, therefore, significant contributions from the source term in the phases of large curvature; i.e., in the contracted phase of the closed universe and in the final phase of the collapse of a stellar object.

It is conceivable that the source term at large curvatures could prevent the solutions of Einstein's equations from becoming singular. The global theorems on the singularities by Penrose, Hawking and Geroch [6–8] depend on a strong form of the principle of the conservation of energy-momentum.

It is interesting to note that a quantum theory of gravitation leads to observable vacuum terms as DeWitt has shown [12]. In this covariant quantum theory of gravity he has estimated the vacuum-to-vacuum amplitudes. The contributions which originate in the vacuum polarization which the background geometry induces give rise to unobservable renormalizations as well as physically real radiative corrections. The observable corrections appear as corrections to Einstein's equations. Hill [15] has investigated the effect of such correction on the collapse of a Friedmann universe and has found that such term may prevent the occurrence of the singularity.

Parker has shown recently that the generalization of quantum field theory of massive spin-0 particles to an expanding Friedmann universe gives pair creation; and Sexl and Urbantke have shown that the time-dependent gravitational field stemming from the expansion of the universe will give rise to pair production, which is significant during the first 10^{-20} sec of the universal expansion [16].

It appears then that our model theory contains qualitative features which are obtained in theories more firmly based on conventional theories.

In the limit of negligible source strength; i.e., in a flat average metric, effective energy-momentum is conserved. The conservation equations

$$(\bar{g}^{\sigma\nu}\langle Y_{\mu\nu}\rangle)_{;\bar{\sigma}} = 0 \tag{37}$$

would be the field equations for matter. These are differential equations for the correlation function $\langle \varphi_{\mu\nu}^{(x)} \varphi_{\lambda\sigma}^{(x')} \rangle$ and higher order moments. Starting from these equations a hierarchy of equations could be derived in the fashion of the turbulence theory of fluids. The solutions of these equations would have to represent the nature of the possible excitons which can be described by these field equations. They would have to describe the lifetimes and the decays into other excitons. They would have to explain why some excitons are apparently stable and they would have to describe the interactions of these excitons.

One might envision a formulation of excitons similar to the theory of elementary domains which in recent years was developed by the Yukawa School. The correlation functions here would play a role similar to that of the multilocal fields there [17].

III. Order of Magnitude Estimates

The explicit form of the effective matter tensor $Y_{\mu\nu}$ is involved. All terms are of the form $\varphi^n f^m (\partial\varphi)^q (\partial f)^r$ with $1 \leqslant n \leqslant 4$, $0 \leqslant m \leqslant 3$, $0 \leqslant q \leqslant 2$, $0 \leqslant r \leqslant 2$. In order to estimate the average effective energy-momentum we make use of the difference of frequencies from which the major contributions to the fields f and φ come as postulated in (b).

The fields f have a frequency spectrum centering around $\Omega_f \sim c/L^*$ which is very large compared to $\omega_\varphi \sim c/L$, the characteristic frequency for the φ-fields, and

$$\varphi^n (\partial\varphi)^q f^m (\partial f)^r = H(\omega_\varphi , t)\, F(\Omega_f , t).$$

Because of $\Omega_f \gg \omega_\varphi$, and because of the assumed stationarity of the f-fluctuations we may perform the averages by averaging the functions H and F separately, i.e.,

$$\langle HF \rangle \sim \langle H \rangle \langle F \rangle. \tag{38}$$

We will now use the following orders of magnitude:

(1) $[\varphi] \sim \varphi$, the amplitude of φ is to be found in connection with other significant quantities in the theory.

(2) The extension of the excitons is taken to be characterized by the length L which is to be found and which characterizes the inhomogeneity of the long-wave correlation φ:

$$[\partial\varphi] \sim \varphi/L. \tag{39}$$

(3) The rapid vacuum fluctuations f are taken to have amplitudes comparable to the average metric:

$$[f] \sim 1. \tag{40}$$

(4) The characteristic length of these vacuum fluctuations is taken to be L^*:

$$[\partial f] \sim 1/L^*. \tag{41}$$

Collecting all terms in Y and forming the average according to the above-given rule and considering that terms which are linear in f or φ cancel when averaged, one finds as the leading term:

$$\langle Y \rangle \sim \langle \varphi\varphi\partial f \partial f \rangle \sim \varphi^2/L^{*2}. \tag{42}$$

We can now relate the amplitude φ, the sizes L and L^* by an argument similar to the one used by Rosen [18]. We note that $\langle Y \rangle$ is also proportional to the average deviation of the curvature $\langle \delta R \rangle$ from the average curvature $\bar{R}_{\mu\nu}$:

$$\delta R \sim \varphi^2/L^{*2}. \tag{43}$$

The local variation of the curvature due to the presence of matter described by the field φ brings about a variation of the action,

$$\delta S \sim (c^3/G) \int \langle \delta R \rangle \, d^4x. \tag{44}$$

The integral is to be extended over the four volume characterized by the particle size L. The lower level excitons will be associated with a change of action from the vacuum value of the order $\hbar$. So we have

$$\hbar \sim (c^3/G)(\varphi^2/L^{*2})\, L^4$$

or

$$\varphi \sim (L^*/L)^2. \tag{45}$$

The component Y_{00} of the effective energy-momentum tensor at the location of the exciton would be of the order $G\rho_n/c^2$ with ρ_n the density of nuclear matter.

The cosmological mass density ρ_c is the nuclear mass density multiplied by the probability to have an exciton at the location around x. This probability is, according to our postulate (C), proportional to the amplitude φ:

$$(G/c^2)\, \rho_c \sim (G/c^2)\, \rho_n \varphi. \tag{46}$$

Most cosmological models based on Einstein's theory relate the scale parameter of the metric or the universal radius R_0 with the mass density:

$$(G/c^2)\, \rho_c \sim R_0^{-2}. \tag{47}$$

The present mass density or the corresponding universal radius must be considered as fact, reflecting a sort of initial condition which is to be entered into the theory.

We have, therefore,

$$R_0^{-2} \sim (G/c^2)\, \rho_n \varphi \sim \langle Y \rangle\, \varphi \sim \varphi^3/L^{*2}, \tag{48}$$

or, because of (45),

$$R_0/L^* \sim (L/L^*)^3 \tag{49}$$

and

$$R_0/L \sim (L/L^*)^2 \sim \varphi^{-1}. \tag{50}$$

If one takes for R_0 the frequently quoted value of $R_0 \approx 10^{28}$ cm and $L^* = 10^{-33}$ cm, then one finds

$$L/L^* \approx 10^{20}, \qquad L \approx 10^{-13}\ \text{cm}, \tag{51}$$

and (50) becomes

$$R_0/L \sim \varphi^{-1} \sim (L/L^*)^2 \sim 10^{40} = N^{1/2}. \tag{52}$$

This is one of Eddington's relations [19]. The large number $N = 10^{80}$ in Eddington's book represents the number of particles in the universe. Some of the other Eddington relations follow. The characteristic nuclear density becomes

$$(G/c^2)\,\rho_n \sim (\varphi/L^*)^2 \sim (L^*)^{-2}\,N^{-1}, \tag{53}$$

and with $G/c^2 = 0.7 \times 10^{28}\ \mathrm{g}^{-1}\,\mathrm{cm}$ one obtains $\rho_n \sim 4 \times 10^{14}\ \mathrm{g\,cm^{-3}}$.

The mass of a typical exciton would be

$$m_n \sim L^3\rho_n \sim (c^2/G)\,L^*N^{-1/4} \sim (c^2/G)\,LN^{-1/2} \sim \hbar/cL \approx 0.3 \times 10^{-24}\ \mathrm{g}. \tag{54}$$

This is Eq. (40.7) in Eddington's book.

The total mass in the universe would be given by $M \sim$ number of possible sites $L^3 \times$ mass of exciton $\times$ probability that site is occupied

$$M \sim (R_0^3/L^3) \times m_n \times \varphi \sim Nm_n\,. \tag{55}$$

We can also find the mass M directly from

$$M \sim \rho_c R_0^3, \tag{56}$$

and from Eqs. (47), (49), and (54), which give

$$M \sim (c^2/G)\,R_0 = (\hbar/c)(c^3/G\hbar)\,R_0 = (\hbar/c)(L^*)^{-2}\,L^*N^{3/4} = Nm_n\,. \tag{57}$$

These are the Eqs. (5.3) and (5.41) in Eddington's book.

We wish to emphasize that Eddington's relations are here arrived at by considerations which are quite different from Eddington's approach. Eddington arrives at these relations from consideration of the usual Bernoulli fluctuation of physical origin, in a large number N of particles. The original distribution function of the fluctuation has a standard deviation σ which puts a scale to all physics. The combination of the original fluctuation σ and the uncertainty in the volume occupied by the N particles and the subsequent uncertainty σ_E in distance measurement which is $\sigma_E \sim N^{-1/2}$ (according to Eddington!) makes the metric of space appear like that of a spherical space of radius $R_0 = \sigma/\sigma_E$ which leads to $\sigma = R_0N^{-1/2}$. The scale parameter σ is identified by Eddington with the range of the nuclear force field, i.e., with the size of the hadrons. Here we have arrived at these relations from a dynamical model.

IV. Discussion of Some Consequences of the Model

The relations (45) and (48) express the connection of the submicroscopic length L^* with the universal radius R_0 which produces a length L characteristic of elementary particle sizes. These relations would indicate that φ should be positive definite. However, the functions φ were introduced as random functions with zero average and long-range correlation. What about this apparent contradiction? The particle-like excitons in this model would be solutions of the field Eqs. (37) in the form of two-point correlation functions $\langle \varphi_{\lambda\nu}(x)\, \varphi_{\mu\sigma}(x') \rangle$. These two-point functions should decrease toward zero for distances $(x - x')$ of the order of the exciton size, but there is no *a priori* reason that this must be a monotonic decrease and not some kind of oscillation. We have associated φ^2 with mass density because the leading term in the expression which was interpreted as energy-momentum tensor is proportional to φ^2. We have postulated $\varphi(x)$ to be representative of the probability to find an exciton around x. Whether such association can be consistently made must remain open at this point. The association of field functions with probability is a quantum mechanical concept. The model theory is a kind of semiclassical theory. The underlying quantum field theory for the creation and annihilation operators which would be related to the φ is not known. This model theory is probably not even the semiclassical limit of a "true" quantum geometrodynamics (cf., Section I.) In this model, then, we will assume that in a "true" theory the association can be consistently made, and in the relations (45) and (48) we will interpret φ to mean something positive.

The appearance of the universal radius R_0 and the dependence of the exciton size and mass on this radius raises the question of a possible time dependence of the fundamental particles. One possibility is to interpret R_0 as the radius of the universe at the phase of maximum expansion: an "initial value datum," but also a constant. Another possibility is to identify the length R_0 with the present average curvature of the universe. In this case the existence of the Hubble effect would force one to accept time dependence of the fundamental quantities in this model.

The excitons in this model are expected to derive from the field Eq. (37). The only length that appears explicitly in these equations by postulating the stationarity of the submicroscopic vacuum fluctuation is the length L^*. The radius R_0 which must enter in order to produce a length $L \sim 10^{-13}$ cm could enter in two ways: either through the average metric $\bar{g}_{\mu\nu}$ and the Christoffel symbols $\bar{\Gamma}^{\lambda}_{\mu\nu}$ which appear explicitly in the field equations or through boundary and initial conditions.

If the exciton properties were determined by the average metric, then they would have to be functions of the location because of the varying curvature in the vicinity of big masses. According to Eqs. (54) and (49), the exciton masses would be proportional to $R_0^{-1/3}$. With R_0 interpreted as the local curvature radius, a drastic difference of the exciton masses at the surface of the sun and the earth would result. Our exciton theory would then have to explain why such difference can not be observed or why it is canceled by other changes also due to the change of local curvature.

If R_0 determines the present properties of the excitons as a boundary condition representing the radius of a closed universe, then one would expect the exciton properties to be time-dependent together with R_0. If one assumes for R_0 the time dependence of a cosmological model with zero cosmological constant and acceleration parameter $q_0 = 1/2$, then the Eqs. (49), (50), (54), and (55) give $L \propto t^{2/9}$; $\varphi \sim N^{-1/2} \propto t^{-4/9}$; $m_n \propto t^{-2/9}$; $M \propto t^{2/3}$; $\rho_c \propto t^{-4/3}$. The time variability of the total exciton number N and the total mass M of the universe correspond to a mass production rate of 10^{-46}g/cm^3 sec which is similar to the rate in the C-field theory of Hoyle and Narlikar [20].

The time dependence of the exciton mass would imply a time dependence of the stellar masses and, therefore, of the stellar luminosities. Estimates show that the luminosity for a main sequence star would depend on time like $\mathscr{L} \sim t^{-3}$ which is much weaker than the t^{-7} dependence in theories with variable gravitation constant. Corrections of the evolutionary age of astrophysical objects due to the time dependence of the masses are similar to those given by Dicke [21] on the basis of the Brans–Dicke cosmology.

A way to circumvent the possibility that the masses become time-dependent would be to interpret R_0 as the mass constant in the Friedmann equation for the evolution of a closed universe:

$$\dot{R}^2/R^2 + 1/R^2 = R_0/R^3.$$

The value R_0 is the radius of maximum expansion and is equal to the Schwarzschild radius of the total universal mass. The question then is: How is this universal initial condition imprinted on the excitons? It is quite clear that here the conjectured model is far from the status of a theory. But it seems that this model contains an element of unification regarding the problem of matter and the cosmological singularity. Particles which in this model are postulated to be coherent geometrodynamic low quantum excitons on the background of a turbulent vacuum fluctuation of the metric appear to have sizes L related to the universal radius R_0 as $L \sim R_0^{1/3} L^{*2/3}$ with the correct order of magnitude of 10^{-13} cm. The Eddington relation that the ratio $(R_0/L)^2$ is equal to the number of particles in the closed Einsteinian universe follows from the model, giving meaning to the numerical coincidence.

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