

Relativistic astrometry

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• **Prof. Sergei Kopeikin**, University of Missouri-Columbia

Relativistic astrometry studies the geometric relationships between astronomical objects in warped spacetime of the Universe by making use of high-precision astrometric techniques: optical space interferometry, very-long baseline interferometry (VLBI), pulsar timing, atomic clocks. It is based on the physical principles of Einstein's general theory of relativity and utilizes a powerful mathematical apparatus of differential geometry. Applied relativistic astrometry uses general-relativistic algorithms for processing high-precision astrometric data for building the fundamental catalog of stars and extra-galactic sources - quasars which establishes the inertial reference frame in the sky. Experimental relativistic astrometry explores the existence of presumable deviations from the general theory of relativity predicted by valid alternative theories of gravity.

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Figure 1: Relativistic astrometry uses the optical and radio astronomy techniques to measure accurate positions of astronomical objects on the sky. The most precise positions of quasars, measured by very long baseline interferometry, are superior to those made by optical methods, and they form the International Celestial Reference Frame (ICRF). Picture credit to Dr. Alexander Rodin, Puschino Radio Astronomical Observatory, Russia

Theoretical Foundations

Spacetime manifold

We all live in four-dimensional world called spacetime. The world may have more dimensions at microscopic level as predicted by advanced unified field theories but those dimensions are irrelevant for interpretation of modern astrometric observations and for constructing the fundamental inertial frame in the sky. Hence, we shall ignore the hypothetical but still unobservable,

relativistic effects associated with extra-dimensions. The most primitive physical object can be modeled as a point occupying no volume in spacetime. Such point is called spacetime event. A continuous set of points forms a mathematical structure known as spacetime manifold. The manifold has four dimensions but no rigidly-defined relationships between the points, like the distance. The goal of the differential geometry is to establish the geometric relationships between the points of the manifold. This leads to introduction of more complicated mathematical objects on the manifold - functions, curves, vectors and tensors.

Tensors

The most important tensors of the spacetime manifold are:

1. the metric, $g_{\alpha\beta}$, that establishes the concept of the proper time, the proper distance and the angle between vectors;
2. the curvature (or Riemann) tensor, $R_{\alpha\beta\gamma\delta}$, that characterizes how strong the spacetime manifold is warped;
3. the energy-momentum tensor, $T_{\alpha\beta}$, that describes the matter of the astronomical bodies which are the sources of the gravitational field.

Here the Greek indices $\alpha, \beta, \gamma, \dots$ take the values 0,1,2,3 corresponding to four coordinates among which the index 0 is associated with the time coordinate, and three other indices 1,2,3 do with the spatial ones.

Affine connection

Vectors and tensors can be transported from one point of spacetime to another in accordance to a certain transportation law. Transportation along a curve is associated with a, so-called, parallel transport that introduces a concept of a covariant derivative on the manifold denoted with operator ∇ as contrasted to operator of a partial derivative denoted with ∂ . In case, when a coordinate chart, $x^\alpha = (x^0, x^i)$, is chosen the covariant and partial derivatives along the coordinate axis with index α are denoted as ∇_α and ∂_α correspondingly.

The covariant derivative defines a new geometric object on the manifold called the affine connection. It exists irrespective of the choice of coordinates. However, if coordinates are chosen, the affine connection takes a particular form of the Christoffel symbols $\Gamma_{\beta\gamma}^\alpha$, that is convenient for calculations. If the metric $g_{\alpha\beta}$ is known, the Christoffel symbols are defined by the following formula

$$\Gamma_{\beta\gamma}^\alpha = \frac{1}{2} g^{\alpha\mu} (\partial_\beta g_{\gamma\mu} + \partial_\gamma g_{\beta\mu} - \partial_\mu g_{\beta\gamma}), \quad (1)$$

where here and everywhere else, the repeated Greek indices (in the above equation it is μ) assume the Einstein summation over all values of the index, that is 0,1,2,3, and the co-metric $g^{\alpha\beta}$ is derived from the equation of matrix inversion

$$g^{\alpha\beta} g_{\beta\gamma} = \delta_\gamma^\alpha, \quad (2)$$

with $\delta_\gamma^\alpha = \text{diag}[1, 1, 1, 1]$ being the diagonal unit matrix also known as the Kronecker symbol.

Curvature

The curvature tensor of spacetime is expressed in terms of the Christoffel symbols and their first partial derivatives

$$R_{\alpha\beta\gamma\delta} = g_{\alpha\mu} R^\mu{}_{\beta\gamma\delta}, \quad (3)$$

where

$$R^\mu{}_{\beta\gamma\delta} = \partial_\gamma \Gamma_{\beta\delta}^\mu - \partial_\delta \Gamma_{\beta\gamma}^\mu + \Gamma_{\nu\gamma}^\mu \Gamma_{\beta\delta}^\nu - \Gamma_{\nu\delta}^\mu \Gamma_{\beta\gamma}^\nu. \quad (4)$$

The curvature tensor is also known as the Riemann tensor. It is made of the metric tensor and its first and second partial derivatives. The curvature of the Minkowski spacetime is identically zero, which means that the Minkowski spacetime is flat and can be entirely covered by a single Cartesian coordinate system with the Christoffel symbols vanishing everywhere. In general case of non-vanishing curvature it is impossible to nullify the Christoffel symbols over the entire spacetime.

The Einstein Gravity Field Equations

The metric tensor plays a double role in general relativity. On one hand it defines the metric properties of the spacetime manifold, on the other hand it represents (tensor) potentials of the gravitational field generalizing the scalar gravitational potential U of the Newtonian theory of gravity. The metric tensor is found by solving the Einstein field equations with the appropriate boundary and initial conditions. The Einstein equations connect the curvature with the energy-momentum tensor of matter as follows:

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} T_{\alpha\beta} , \quad (5)$$

where the Ricci tensor $R_{\alpha\beta} = R^\mu{}_{\alpha\mu\beta}$, the Ricci scalar $R = g^{\alpha\beta} R_{\alpha\beta}$, G is the universal gravitational constant, and c is the fundamental speed numerically equal to the speed of weak gravitational waves and electromagnetic waves in vacuum.

In most practical situations gravitational field is weak, and (5) can be simplified after making use of a linearized approximation around the Minkowski spacetime,

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} . \quad (6)$$

Here, $\eta_{\alpha\beta} = \text{diag}[-1, 1, 1, 1]$ is the diagonal Minkowski metric, and $h_{\alpha\beta} = h_{\alpha\beta}(t, \vec{x})$ is its gravitational perturbation that can be found from the linearized Einstein equations

$$\square_\eta h_{\alpha\beta} = - \frac{16\pi G}{c^4} S_{\alpha\beta} , \quad (7)$$

where the source of the gravitational field is the energy-momentum tensor of matter,

$$S_{\alpha\beta} = T_{\alpha\beta} - \frac{1}{2} \eta_{\alpha\beta} T , \quad T = \eta^{\alpha\beta} T_{\alpha\beta} , \quad (8)$$

the wave operator of the Minkowski spacetime is defined as

$$\square_\eta = - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \Delta , \quad (9)$$

with Δ being the Laplace operator, and c being the fundamental speed of the Minkowski spacetime. Be aware that some researchers, for example [Brumberg](http://www.scholarpedia.org/article/Relativistic_Celestial_Mechanics) (http://www.scholarpedia.org/article/Relativistic_Celestial_Mechanics), take the signature of the Minkowski metric being opposite to the convention of this article. It implicitly changes the sign of the perturbation $h_{\alpha\beta}$. This article adopts the metric signature convention from the textbook by Will (1993).

The most general physical solution of (7) is a weak gravitational perturbation propagating with the speed c from the source of gravity to the photon,

$$h_{\alpha\beta} = \frac{4G}{c^4} \int_V \frac{S_{\alpha\beta} \left(t - \frac{1}{c} |\vec{x} - \vec{x}'|, \vec{x}' \right)}{|\vec{x} - \vec{x}'|} d^3 x' , \quad (10)$$

where the integration is performed over the volume V of all bodies contributing to the generation of the gravitational field. In general relativity both the speed of light and the speed of gravity are numerically equal to the fundamental speed c . It is important to realize, however, that this theoretical postulate was made by Einstein on the premise of the concept of the general covariance. The speed of gravity in general relativity enters time derivatives of the metric tensor both in the gravity field equations and in the equations of motion of test particles and massive bodies and parametrizes the post-Newtonian expansion of these equations in the form of a slow-motion parameter (Kopeikin 2004). Astrometric observations of the bending of light by moving major planets of the solar system provide the experimental access to the numerical value of the slow-motion parameter and, hence, allows us to set a stringent upper limit on the speed of gravity (Fomalont and Kopeikin 2003, Fomalont et al. 2010). Results of this theoretical and experimental research are still under discussion and various scientists have alternative opinions summarized by Will (2006), and by Kopeikin et al. (2011).

In a particular case of a static mass M_a the energy-momentum tensor is given by $T_{\alpha\beta} = M_a c^2 \eta_{\alpha 0} \eta_{\beta 0}$, that is in the rest frame of the mass, the energy-momentum tensor has the following components

$$T_{00} = M_a c^2, \quad T_{0i} = 0, \quad T_{ij} = 0.$$

If the mass is located at a point with coordinates $\vec{x}^a$, equation (10) is simplified to

$$h_{00} = h_{11} = h_{22} = h_{33} = \frac{2G}{c^2} \frac{M_a}{|\vec{x} - \vec{x}^a|}, \quad (11)$$

and all other components of the metric tensor perturbation are equal to zero. In the linearized approximation, the gravitational perturbations obey the principle of linear superposition. Hence, the field of N masses can be found as a simple algebraic sum of the solutions given by (11) for each mass $a = 1, 2, \dots, N$.

Propagation of Light

Astrometry uses electromagnetic radiation to detect and to measure precise positions of astronomical objects on the sky. The most precise astrometric measurements are done with very long baseline interferometry which operates in radio frequencies and measures the phase of radio waves arriving from radio sources (quasars, radio stars, pulsars). Optical techniques utilize the property of a light ray to propagate along a straight line at large astronomical distances. Relativistic astrometry takes into account various aberrations in the propagation of the light ray caused by the relative motion of observer with respect to a reference frame and/or the source of light as well as by the influence of gravitational field acting on the light propagation effectively like a refractive medium (though this analogy is not fully exact since gravitational field is more complex physical entity than any medium).

Equation of light geodesics

The propagation of an electromagnetic signal (photon) in vacuum and in the presence of gravitational field obeys the equation of light geodesics. The photon moves along a light-ray curve $x^\alpha(\lambda)$ where $x^\alpha = (ct, \vec{x})$ are coordinates in spacetime, and λ is an affine parameter along the curve. The tangent vector to the light-ray curve is identified with the wave vector of the propagating photon, $K^\alpha = dx^\alpha/d\lambda$, which is parallel-transported along the curve. The equation of the parallel transport

$$\frac{dK^\alpha}{d\lambda} + \Gamma_{\mu\nu}^\alpha K^\mu K^\nu = 0, \quad (12)$$

where the Christoffel symbols $\Gamma_{\mu\nu}^\alpha$ have been defined in (1). For integration of (12) it is convenient to use the coordinate time t as an independent variable instead of the affine parameter λ . Changing the argument yields the following equation

$$\ddot{x}^i + \Gamma_{00}^i + 2\Gamma_{0j}^i \dot{x}^j + \Gamma_{jp}^i \dot{x}^j \dot{x}^p - \left(\Gamma_{00}^0 + 2\Gamma_{0j}^0 \dot{x}^j + \Gamma_{jp}^0 \dot{x}^j \dot{x}^p \right) \dot{x}^i = 0, \quad (13)$$

where $x^i = x^i(t)$, and the overdot here and everywhere else means a total derivative with respect to time, that is $\dot{x}^i \equiv dx^i(t)/dt$.

This is an ordinary, second-order differential equation which solution is defined by the choice of the initial-boundary conditions. More specifically, these conditions are defined by postulating the value of the coordinate speed of light and the direction $\vec{k}^i$ of its propagation at infinite past ($t \rightarrow -\infty$), and the condition that the light ray is passing at the time of observation, $t = t_1$, through a particular observer located at point $\vec{x}_1$,

$$\dot{\vec{x}}(-\infty) = c\vec{k}, \quad \vec{x}(t_1) = \vec{x}_1, \quad (14)$$

where c is again the fundamental speed of the Minkowski space, and $\vec{k} = (k^i)$ is the unit vector, $\vec{k} \cdot \vec{k} = \delta_{ij} k^i k^j = 1$.

Light-ray path

The light-ray (13) can not be exactly integrated in the most general case due to its complexity. However, in case of a weak gravitational field (13) can be solved approximately by making use of successive approximations. Switching off the gravitational field yields the unperturbed ("Newtonian") path of the light ray

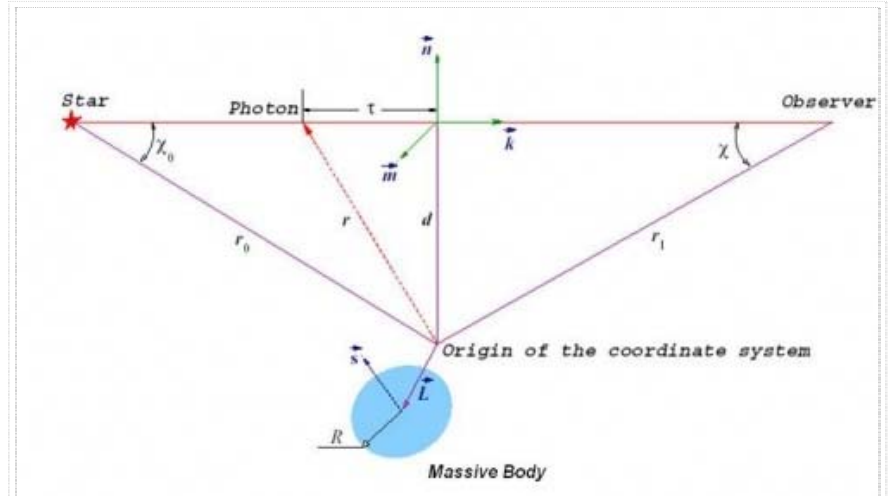


Figure 2: Integration parameters of the unperturbed light-ray path. The massive body deflects light. Position of its center of mass is shown by vector $\vec{L} = \vec{x}_a$. Photon propagates from the source of light (star) towards observer. The source of light has coordinates $\vec{x}_0$ at the time of emission of light t_0 . Observer has coordinates $\vec{x}_1$ at the time of observation t_1 . The light ray passes near the origin of the coordinate system at the distance $d = |\vec{\xi}|$ at the time t^* . The unperturbed current coordinate of the photon is $\vec{x}_N(t) = c\vec{k}\tau + \vec{\xi}$, where $\tau = t - t^*$, and $\vec{\xi} = \vec{k} \times (\vec{x} \times \vec{k}) = \vec{k} \times (\vec{x}_0 \times \vec{k})$. The unit vectors $\vec{n}$ and $\vec{m}$ are mutually orthogonal to each other and to the unit vector $\vec{k}$, and they are lying in the "plane of the sky" that is orthogonal to the line of sight of the observer to the star. Distances $r_0 = |\vec{x}_0|$ and $r_1 = |\vec{x}_1|$, and for any source of light residing outside of the solar system $r_1 \ll r_0$.

$$\vec{x}_N(t) = \vec{x}_0 + c\vec{k}(t - t_0), \quad (15)$$

where $\vec{x}_0$ is the point of emission of the light ray at time t_0 . Post-Newtonian gravitational perturbation of the light-ray propagation is determined as a small correction $\vec{\Xi}$ to the unperturbed trajectory

$$\vec{x} = \vec{x}_N + \vec{\Xi}, \quad \dot{\vec{x}} = c\vec{k} + \dot{\vec{\Xi}}. \quad (16)$$

Substituting (16) to (13) and keeping only linear terms in the gravitational perturbation yields the equation for the light-ray perturbation, which can be integrated. In the particular case of the static and spherically-symmetric gravitational field of a point-like mass given by (11), the light-ray geodesic is

$$\ddot{\vec{\Xi}} = \frac{2GM_a}{c^2} \left[\frac{\vec{k} \times (\vec{r}_{0a} \times \vec{k})}{r_{0a} - \vec{k} \cdot \vec{r}_{0a}} - \frac{\vec{k} \times (\vec{r}_a \times \vec{k})}{r_a - \vec{k} \cdot \vec{r}_a} + \vec{k} \ln \frac{r_a - \vec{k} \cdot \vec{r}_a}{r_{0a} - \vec{k} \cdot \vec{r}_{0a}} \right], \quad (17)$$

$$\frac{\dot{\vec{\Xi}}}{c} = - \frac{2GM_a}{c^2 r_a} \left[\frac{\vec{k} \times (\vec{r}_a \times \vec{k})}{r_a - \vec{k} \cdot \vec{r}_a} + \vec{k} \right], \quad (18)$$

where $\vec{r}_a = \vec{x} - \vec{x}_a$, $\vec{r}_{0a} = \vec{x}_0 - \vec{x}_a$, $r_a = |\vec{r}_a|$, $r_{0a} = |\vec{r}_{0a}|$. Equation (16) predicts two main relativistic effects in propagation of light caused by the presence of gravitational field -- the bending of light and the Shapiro time delay.

Gravitational deflection of light

The gravitational deflection of light is the angle $\vec{\alpha}$ defined as the difference between the initial direction of the light ray and that of the tangent vector to the light-ray path at the point of observation, projected on the plane of the sky,

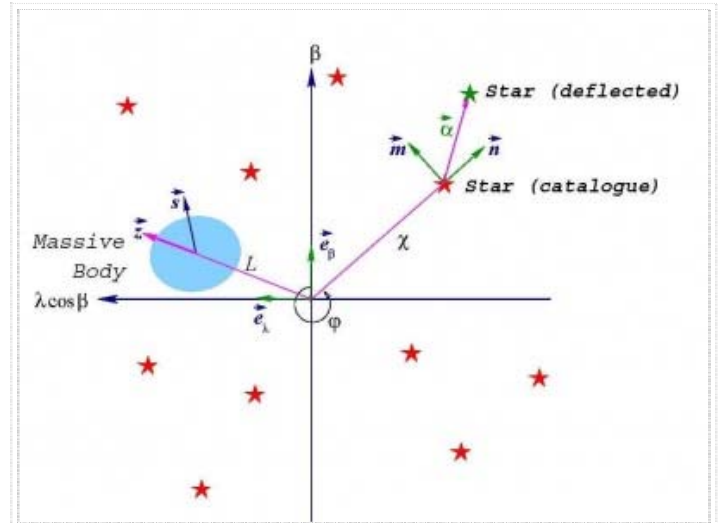


Figure 3: Relativistic deflection of light $\vec{\alpha}$ as seen in the plane of the sky. The bending displaces the undisturbed (catalogue) position of the star at the angle $\vec{\alpha}$ which is decomposed in two components - radial (along the vector $\vec{n}$) and tangential (along the vector $\vec{m}$). Radial deflection is caused by the spherically-symmetric component of the gravitational field of the massive body. The tangential component of the gravitational bending of light is produced by high-order gravitational multipoles (Kopeikin and Makarov 2007) and by the gravitomagnetic dragging of light due to the motion of the massive body with respect to observer (Fomalont and Kopeikin 2003)

$$\vec{\alpha} = \vec{k} - \frac{1}{c} \vec{k} \times (\dot{\vec{x}} \times \vec{k}), \quad (19)$$

Making use of (18) yields

$$\vec{\alpha} = (1 + \cos \chi_a) \frac{2GM_a}{c^2} \frac{\vec{\xi}_a}{d_a^2}, \quad (20)$$

where $\vec{\xi}_a = \vec{k} \times (\vec{r}_a \times \vec{k})$, $d_a = |\vec{\xi}_a|$ is the impact parameter of the unperturbed light-ray with respect to the massive body, χ_a is the angle between the light ray and the vector $\vec{r}_a = \vec{x} - \vec{x}_a$ pointing out of the massive body to the observer. Visualization of the bending angle $\vec{\alpha}$ is given in Figure 3.

In case of observer at infinity, one has $\chi_a \rightarrow 1$ and the light bending is given by the famous Einstein's equation

$$\vec{\alpha} = \frac{4GM_a}{c^2} \frac{\vec{\xi}_a}{d_a^2}, \quad (21)$$

which gives the angle of the gravitational light bending two times larger than the Newtonian theory. Einstein's result has been tested many times with gradually improving accuracy of measurements from 30% to 0.03%. The most precise VLBI measurement of $\vec{\alpha}$ was achieved by [Edward Fomalont](http://en.wikipedia.org/wiki/Edward_Fomalont) (http://en.wikipedia.org/wiki/Edward_Fomalont) and colleagues in 2005, reaching a precision of one part in 3000 (see Figure 4).

Gravitational time delay

The gravitational time delay results from the change in the coordinate velocity of light $\dot{\vec{x}}$, along the unperturbed light-ray path as the photon propagates through the gravitational field. Equation (18) is instrumental for calculation of the time delay. Indeed, this equation yields the relationship between the overall travel time of photon and the coordinates of the points of its emission and observation

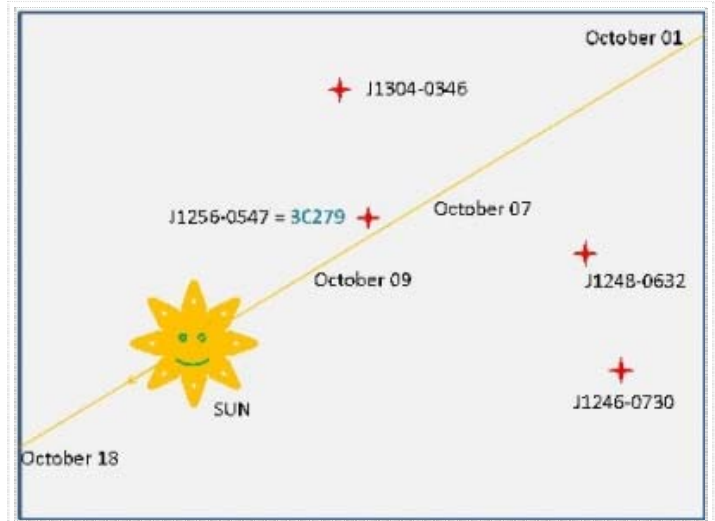


Figure 4: Fomalont et al. (2005) used the VLBA (<http://www.nrao.edu/index.php/about/facilities/vlba>), a continent-wide American system of radio telescopes ranging from Hawaii to the Virgin Islands to make the most precise test of the bending of light. The VLBA offers the power to make the most accurate position measurements in the sky and the most detailed images of radio sources. The observations were made as the Sun passed nearly in front of four distant quasars - faraway galaxies with supermassive black holes at their cores - in October of 2005. The Sun's gravity causes slight changes in the apparent positions of the quasars because it deflects the radio waves coming from them. It is these changes which had been precisely measured and compared with Einstein's prediction in (21).

$$t - t_0 = \frac{1}{c} |\vec{x} - \vec{x}_0| - \frac{2GM_a}{c^3} \ln \frac{r_a - \vec{k} \cdot \vec{r}_a}{r_{0a} - \vec{k} \cdot \vec{r}_{0a}} . \quad (22)$$

Making simple algebraic transformations of vectors, (22) can be transformed to another useful form

$$t - t_0 = \frac{r}{c} + \frac{2GM_a}{c^3} \ln \frac{r_a + r_{0a} + r}{r_a + r_{0a} - r} , \quad (23)$$

where the distance $r = |\vec{x} - \vec{x}_0|$. This equation clearly reveals that in the presence of gravitational field the light takes longer to propagate from one point to another, since the logarithmic function in (23) is always positive for any configuration of the points of emission and observation with respect to the massive body. Equations (22) and (23) are mathematically equivalent. However, for some good reasons (22) is more preferable for analyzing radio-interferometric observations, while (23) fits better for processing radio and laser ranging observations of planets and spacecrafts in the solar system.

The gravitational time delay of light propagating through a static and spherically-symmetric gravitational field was discovered in 1964 by Irwin Shapiro (http://en.wikipedia.org/wiki/Irwin_I._Shapiro). Since then, the measurement of the Shapiro delay has been performed in many experiments with the Sun as a source of gravity that includes radio ranging of planets and spacecrafts in the solar system, and VLBI radio microwave observations of quasars. The most precise measurement of the Shapiro delay (a few parts in 10,000) was achieved by the Doppler tracking technique in the Cassini experiment (Bertotti et al. 2003, Anderson et al. 2004).

The Shapiro time delay plays an important role in the theory and observations of gravitational lenses (Perlick 2004) as well as in testing the general theory of relativity in the strong gravitational field regime of binary/double pulsars (Lorimer and Kramer 2004).

Standard IAU Reference Systems

Over the last few decades, various groups within the International Astronomical Union (IAU) have been active in exploring the application of the general theory of relativity to the modeling and interpretation of high-accuracy astrometric observations in the solar system and beyond. A Working Group on Relativity in Celestial Mechanics and Astrometry was formed in 1994 to define and implement a relativistic theory of reference frames and time scales. This task was successfully completed with the adoption of a series of resolutions on astronomical reference systems, time scales, and Earth rotation models by 24th General Assembly of the IAU, held in Manchester, UK, in 2000. The IAU resolutions are based on the first post-Newtonian approximation of general relativity which is a conceptual basis of the fundamental astronomy in the solar system (Soffel et al. 2003). The IAU resolutions define two basic reference (coordinate) systems.

Barycentric Celestial Reference System

Barycentric Celestial Reference System (BCRS), $x^\alpha = (ct, \vec{x})$, is defined in terms of a metric tensor $g_{\alpha\beta}$ with components

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + O(c^{-5}) , \quad (24)$$

$$g_{0i} = -\frac{4w^i}{c^3} + O(c^{-5}) , \quad (25)$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2w}{c^2} \right) + O(c^{-4}) . \quad (26)$$

Here, the post-Newtonian gravitational potentials w and w^i are defined by solving the gravity field equations

$$\square w = -4\pi G \sigma , \quad (27)$$

$$\square w^i = -4\pi G \sigma^i , \quad (28)$$

where $\square \equiv -c^{-2} \partial^2 / \partial t^2 + \nabla^2$ is the wave operator, $\sigma = c^{-2} (T^{00} + T^{ss})$, $\sigma^i = c^{-1} T^{0i}$, and $T^{\mu\nu}$ are the components of the stress-energy tensor of the solar system bodies, $T^{ss} = T^{11} + T^{22} + T^{33}$.

Equations (27), (28) are solved by iterations

$$w(t, \vec{x}) = G \int \frac{\sigma(t, \vec{x}') d^3 x'}{|\vec{x} - \vec{x}'|} + \frac{G}{2c^2} \frac{\partial^2}{\partial t^2} \int d^3 x' \sigma(t, \vec{x}') |\vec{x} - \vec{x}'| + O(c^{-4}) , \quad (29)$$

$$w^i(t, \vec{x}) = G \int \frac{\sigma^i(t, \vec{x}') d^3 x'}{|\vec{x} - \vec{x}'|} + O(c^{-2}) , \quad (30)$$

which are to be substituted to the metric tensor (24)–(26). Each of the potentials, w and w^i , can be linearly decomposed in two pieces

$$w = w_E + \bar{w} , \quad (31)$$

$$w^i = w_E^i + \bar{w}^i , \quad (32)$$

where w_E and w_E^i are BCRS potentials depending on the distribution of mass and current only inside the Earth, and $\bar{w}$ and $\bar{w}^i$ are gravitational potentials of external bodies.

The IAU 2000 resolutions define the coordinate time t as the Barycentric Coordinate Time (TCB). The corresponding spatial coordinates $(x^i) = \vec{x}$ are called the TCB coordinates. TCB is the relativistic generalization of the Newtonian concept of the Ephemeris Time (ET) which was defined as a dynamic time argument derived from the equation of motion of the Earth around the Sun in correspondence with a numerical expression of the geometric mean longitude of the Sun given by Newcomb and adopted by the IAU in 1952 (Guinot (1989)).

Geocentric Celestial Reference System

Geocentric Celestial Reference System (GCRS) is denoted $X^\alpha = (cT, \vec{X})$. It has the metric tensor $G_{\alpha\beta}$ with components

$$G_{00} = -1 + \frac{2W}{c^2} - \frac{2W^2}{c^4} + O(c^{-5}) , \quad (33)$$

$$G_{0i} = -\frac{4W^i}{c^3} + O(c^{-5}) , \quad (34)$$

$$G_{ij} = \delta_{ij} \left(1 + \frac{2W}{c^2} \right) + O(c^{-4}) . \quad (35)$$

Here $W = W(T, \vec{X})$ is the post-Newtonian gravitational potential and $W^i(T, \vec{X})$ is a vector-potential both expressed in the geocentric coordinates. They satisfy to the same type of the wave equations (27), (28). Planetocentric metric for any planet can be introduced in the same way as the GCRS.

The geocentric potentials, W_E and W_E^i , are split into three parts

$$W(T, \vec{X}) = W_E(T, \vec{X}) + W_{\text{kin}}(T, \vec{X}) + W_{\text{dyn}}(T, \vec{X}) , \quad (36)$$

$$W^i(T, \vec{X}) = W_E^i(T, \vec{X}) + W_{\text{kin}}^i(T, \vec{X}) + W_{\text{dyn}}^i(T, \vec{X}) . \quad (37)$$

associated respectively with the gravitational field of the Earth, external tidal field and kinematic inertial force. IAU resolutions imply that the external and kinematic parts must vanish at the geocenter and admit an expansion in powers of $\vec{X}$. Geopotentials W_E and W_E^i are defined in the same way as w_E and w_E^i in equations (29)-(30) but with σ and σ^i calculated in the GCRS. They are related to the barycentric gravitational potentials w_E and w_E^i by the post-Newtonian transformations.

The kinematic contributions are linear in the GCRS spatial coordinates $\vec{X}$

$$W_{\text{kin}} = Q_i X^i , \quad W_{\text{kin}}^i = \frac{1}{4} c^2 \varepsilon_{ipq} (\Omega^p - \Omega_{\text{prec}}^p) X^q , \quad (38)$$

where Q_i characterizes a deviation of the actual world line of the geocenter from a fiducial world line of a hypothetical spherically-symmetric Earth

$$Q_i = \partial_i \bar{w}(\vec{x}_E) - a_E^i + O(c^{-2}) . \quad (39)$$

Here $a_E^i = dv_E^i/dt$ is the barycentric acceleration of the geocenter. Function Ω_{prec}^a describes the relativistic precession of dynamically non-rotating spatial axes of GCRS with respect to reference quasars

$$\Omega_{\text{prec}}^i = \frac{1}{c^2} \varepsilon_{ijk} \left(-\frac{3}{2} v_E^j \partial_k \bar{w}(\vec{x}_E) + 2 \partial_k \bar{w}^j(\vec{x}_E) - \frac{1}{2} v_E^j Q^k \right) . \quad (40)$$

The three terms on the right-hand side of this equation represent the geodetic, Lense-Thirring, and Thomas precessions, respectively. Dynamic potentials W_{dyn} and W_{dyn}^i are generalizations of the Newtonian tidal potential in the form of a polynomial starting from the quadratic with respect to $\vec{X}$ terms.

Finally, we notice that the IAU 2000 resolutions define the coordinate time T as the Geocentric Coordinate Time (TCG). The corresponding spatial coordinates $(X^i) = \vec{x}$ are called the TCG coordinates. TCG differs in rate from the Terrestrial Time (TT) scale, which has the same rate as clocks placed on geoid (Fukushima 1989, see section "The scaling rules" below).

Coordinate Transformations Between the Reference Systems

General-relativistic transformations

The spacetime coordinate transformations between the BCRS and GCRS are found by matching the BCRS and GCRS metric tensors in the vicinity of the world line of the Earth by making use of their tensor transformation properties. The transformations are written as (Kopeikin 1988, Soffel et al. 2003)

$$T = t - \frac{1}{c^2} [A + \vec{v}_E \cdot \vec{r}_E] + \frac{1}{c^4} [B + B^i r_E^i + B^{ij} r_E^i r_E^j], \quad (41)$$

$$X^i = r_E^i + \frac{1}{c^2} \left[\frac{1}{2} v_E^i \vec{v}_E \cdot \vec{r}_E + \bar{w}(\vec{x}_E) r_E^i + r_E^i \vec{a}_E \cdot \vec{r}_E - \frac{1}{2} a_E^i r_E^2 \right], \quad (42)$$

where $\vec{r}_E = \vec{x} - \vec{x}_E$, functions A, B, B^i, B^{ij} obey equations

$$\frac{dA}{dt} = \frac{1}{2} v_E^2 + \bar{w}(\vec{x}_E), \quad (43)$$

$$\frac{dB}{dt} = -\frac{1}{8} v_E^4 - \frac{3}{2} v_E^2 \bar{w}(\vec{x}_E) + 4 v_E^i \bar{w}^i + \frac{1}{2} \bar{w}^2(\vec{x}_E), \quad (44)$$

$$B^i = -\frac{1}{2} v_E^2 v_E^i + 4 \bar{w}^i(\vec{x}_E) - 3 v_E^i \bar{w}(\vec{x}_E), \quad (45)$$

$$B^{ij} = -v_E^i Q_j + 2 \partial_j \bar{w}^i(\vec{x}_E) - v_E^i \partial_j \bar{w}(\vec{x}_E) + \frac{1}{2} \delta^{ij} \dot{\bar{w}}(\vec{x}_E), \quad (46)$$

where x_E^i, v_E^i , and a_E^i are the BCRS position, velocity and acceleration vectors of the Earth, the overdot stands for the total derivative with respect to t , and one has neglected all terms of the order $O(r_E^3)$.

The scaling rules

The Earth's orbit in BCRS is almost circular. Making use of the orbital parameters allows us to represent the right side of (43) in the form

$$\frac{1}{2} v_E^2 + \bar{w}(\vec{x}_E) = c^2 L_C + (\text{periodic terms}), \quad (47)$$

where the constant

$$L_C = 1.48082686741 \times 10^{-8} \pm 2 \times 10^{-17}, \quad (48)$$

and the periodic terms have been calculated with a great precision by Irwin and Fukushima (2005). Equations (41), (43) and (47) indicate that TCG gradually diverges from TCB if one uses the same unit of time for measuring both TCB and TCG. The secular divergence between the two time scales is eliminated by selecting other coordinate times, TDB for the BCRS and TT for the GCRS, chosen to have the same rate as atomic clocks on the geoid.

The new barycentric time scale is called TDB, and it is defined by equation

$$TDB = t(1 - L_B) , \quad (49)$$

and the new geocentric time scale TT is defined by equation

$$TT = T(1 - L_G) , \quad (50)$$

where the constants L_B and L_G are now defining constants,

$$L_G = 6.969290134 \times 10^{-10} , \quad L_B = 1.55051976772 \times 10^{-8} . \quad (51)$$

Because in the first post-Newtonian approximation the constant L_B obeys the following equation (Brumberg and Kopeikin 1991, Irwin and Fukushima 2005)

$$L_B = L_C + L_G - L_C L_G , \quad (52)$$

the calibration procedure effectively rescales TCB (that is, t) and TCG (that is, T) so that the constant L_C is removed from equation (47).

The scaled coordinates TT and TDB have been used before the IAU adopts in 2000 a consistent relativistic framework (Soffel et al. 2003), and are still in use for continuity though IAU resolutions in general, and IAU 2000 resolutions in particular, do not recommend scaled coordinates, they just acknowledge their use. We notice that the re-scaling of time entails re-scaling of spatial coordinates and also changes the equations of motion of planets, their satellites and light propagation (Brumberg and Kopeikin 1991). In order to keep the equations of motion invariant one has also to re-scale the masses of the solar system bodies. These scaling transformations are included to IAU 2000 resolutions (Kopeikin et al. 2011) and IERS standards (McCarthy and Petit 2003).

The scaling of time and spatial coordinates can be interpreted from the point of view of transformation of the GCRS metric tensor only, without apparent re-scaling of time and space coordinates. Indeed, (38) is a solution of the Laplace equation which is defined up to an arbitrary function of time $Q = Q(t)$ that can be naturally incorporated to the time-time component G_{00} of the GCRS metric via the potential

$$W_{\text{kin}} = Q + Q_i X^i . \quad (53)$$

Matching of the so-defined GCRS metric tensor with the BCRS metric tensor reveals that (43) is to be replaced with

$$\frac{dA}{dt} = \frac{1}{2} v_E^2 + \bar{w}(\vec{x}_E) - Q . \quad (54)$$

It is clear that if one chooses $Q = c^2 L_C$, it eliminates the secular drift between times T and t without explicit re-scaling (49) of t . It turns out that the relativistic definition of mass (Kopeikin and Vlasov 2004) depends on function Q and is re-scaled automatically in such a way that the Newtonian equations of motion remain invariant (Xie and Kopeikin 2010). Introduction of Q to the potential W_{kin} appropriately transforms the space-space component G_{ij} of the GCRS metric tensor that is formally equivalent to the IAU 2000 re-scaling of the GCRS spatial coordinates. One concludes that introducing the function Q to the GCRS metric tensor without apparent re-scaling of coordinates and masses is practically equivalent to the scaling laws of time, space, and masses. It does not change the status of the defining constants L_B and L_G . Similar procedure of the transformation of the topocentric metric tensor can be applied to take into account the linear drift existing between time T and the Terrestrial Time.

The parametrized coordinate transformations

The parametrized post-Newtonian (PPN) formalism (Will 2006) is not consistent with the IAU resolutions. It limits applicability of the IAU resolutions in testing gravity theories. PPN equations of motion depend on two parameters, β and γ and they are presently compatible with the IAU resolutions only in the case of $\beta = \gamma = 1$. Rapidly growing precision of astrometric observations as well as advent of gravitational-wave detectors urgently demand a PPN theory of relativistic transformations between the local and global coordinate systems. This theory has been developed in papers (Kopeikin and Vlasov, Xie and

Kopeikin 2010).

PPN parameters β and γ are characteristics of a scalar field which makes the metric tensor different from general relativity. In order to extend the IAU 2000 resolutions to PPN formalism one used a general class of Brans-Dicke theories based on the metric tensor $g_{\alpha\beta}$ and a scalar field ϕ that couples with the metric tensor via function $\theta(\phi)$. Both ϕ and $\theta(\phi)$ are analytic functions which can be expanded in a Taylor series about their background values $\bar{\phi}$ and $\bar{\theta}$.

The parametrized theory of relativistic reference frames in the solar system is built in accordance to the same rules as used in the IAU resolutions. The PPN transformations between BCRS and GCRS are found by matching the BCRS and GCRS metric tensors and the scalar field in the vicinity of the world line of the Earth. They have the following form

$$T = t - \frac{1}{c^2} [A + \vec{v}_E \cdot \vec{r}_E] + \frac{1}{c^4} [B + B^i r_E^i + B^{ij} r_E^i r_E^j], \quad (55)$$

$$X^i = r_E^i + \frac{1}{c^2} \left[\frac{1}{2} v_E^i v_E^j r_E^j + \gamma Q r_E^i + \gamma \bar{w}(\vec{x}_E) r_E^i + r_E^i a_E^j r_E^j - \frac{1}{2} a_E^i r_E^2 \right] \quad (56)$$

where $\vec{r}_E = \vec{x} - \vec{x}_E$, and functions $A(t), B(t), B^i(t), B^{ij}(t)$ obey

$$\frac{dA}{dt} = \frac{1}{2} v_E^2 + \bar{w} - Q(\vec{x}_E), \quad (57)$$

$$\frac{dB}{dt} = -\frac{1}{8} v_E^4 - \left(\gamma + \frac{1}{2} \right) v_E^2 \bar{w}(\vec{x}_E) + 2(1 + \gamma) v_E^i \bar{w}^i + \left(\beta - \frac{1}{2} \right) \bar{w}^2(\vec{x}_E), \quad (58)$$

$$B^i = -\frac{1}{2} v_E^2 v_E^i + 2(1 + \gamma) \bar{w}^i(\vec{x}_E) - (1 + 2\gamma) v_E^i \bar{w}(\vec{x}_E), \quad (59)$$

$$B^{ij} = -v_E^i Q_j + (1 + \gamma) \partial_j \bar{w}^i(\vec{x}_E) - \gamma v_E^i \partial_j \bar{w}(\vec{x}_E) + \frac{1}{2} \delta^{ij} \dot{\bar{w}}(\vec{x}_E). \quad (60)$$

These transformations depends explicitly on the PPN parameters β and γ and the scaling function Q , and should be compared with those (41)-(46) currently adopted in the IAU resolutions.

PPN parameters β and γ have a fundamental physical meaning in the scalar-tensor theory of gravity along with the universal gravitational constant G and the fundamental speed c . It means that if the parameterized transformations (55)-(60) are adopted by the IAU, the parameters β and γ are to be considered as new astronomical constants which values have to be determined experimentally.

Cosmological Reference System for Astrometry

BCRS assumes that the solar system is isolated and space-time is asymptotically flat. This idealization will not work at some level of accuracy of astronomical observations because the universe is expanding and its space-time is described by the Friedman-Robertson-Walker (FRW) metric tensor having non-zero Riemannian curvature (Anderson and Nieto 2010). It may turn out that some, yet unexplained anomalies in the orbital motion of the solar system bodies are indeed associated with the cosmological expansion. Moreover, radioastronomical observations of cosmic microwave background radiation and other large-scale cosmological effects require clear understanding of how the solar system is embedded to the cosmological model. Therefore, it seems reasonable to match the cosmological description of space-time with the IAU 2000 theory of reference frames in the solar system (Kopeikin 2009).

Because the universe is not asymptotically-flat the gravitational field of the solar system can not vanish at infinity. Instead, it must match with the cosmological metric tensor. This imposes the cosmological boundary condition. The cosmological model is not unique has a number of free parameters depending on the amount of visible and dark matter, and on the presence of dark

energy. We considered a FRW universe driven by a scalar field imitating the dark energy ϕ and having a spatial curvature equal to zero. The universe is perturbed by a localized distribution of matter of the solar system. In this model the perturbed metric tensor reads

$$g_{\alpha\beta} = a^2(\eta)f_{\alpha\beta} , \quad (61)$$

$$f_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} ,$$

where $h_{\alpha\beta}$ is the perturbation of the background FRW metric tensor $\bar{g}_{\alpha\beta} = a^2\eta_{\alpha\beta}$ caused by the presence of the solar system, $a(\eta)$ is a scale factor of the universe depending on the, so-called, conformal time η related to coordinate time t by simple differential equation

$$dt = a(\eta)d\eta .$$

In what follows, a linear combination of the metric perturbations

$$\gamma^{\alpha\beta} = h^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} h , \quad h = \eta^{\mu\nu} h_{\mu\nu} , \quad (62)$$

is more convenient for calculations.

There exist a cosmological gauge, which has a number of remarkable properties. In case of the background FRW universe with dust equation of state (that is, the background pressure of matter is zero) this gauge is given by

$$\gamma^{\alpha\beta}{}_{|\beta} = 2H\varphi\delta_0^\alpha , \quad (63)$$

where bar denotes a covariant derivative with respect to the background metric $\bar{g}_{\alpha\beta}$, $\varphi = \phi/a^2$, $H = \dot{a}/a$ is the Hubble parameter in terms of the conformal time η , and the overdot denotes a time derivative with respect to time η . The gauge (63) generalizes the harmonic gauge of asymptotically-flat space-time for the case of the expanding non-flat background universe, drastically simplifies and decouples the linearized Einstein equations in space-time domain without resorting to the Fourier decomposition of the perturbation, the method used in majority of papers on cosmological perturbations. Introducing notations $\gamma_{00} \equiv 4w/c^2$, $\gamma_{0i} \equiv -4w^i/c^3$, $\gamma_{ij} \equiv 4w^{ij}/c^4$, and splitting Einstein's equations in components, yield

$$\square\chi - 2H\partial_\eta\chi + \frac{5}{2}H^2\chi = -4\pi G\sigma , \quad (64)$$

$$\square w - 2H\partial_\eta w = -4\pi G\sigma - 4H^2\chi , \quad (65)$$

$$\square w^i - 2H\partial_\eta w^i + H^2 w^i = -4\pi G\sigma^i , \quad (66)$$

$$\square w^{ij} - 2H\partial_\eta w^{ij} = -4\pi GT^{ij} , \quad (67)$$

where $\partial_\eta \equiv \partial/\partial\eta$, $\square \equiv -c^{-2}\partial_\eta^2 + \vec{\nabla}^2$, $\chi \equiv w - \varphi/2$, the Hubble parameter $H = \dot{a}/a = 2/\eta$ expressed in terms of the coordinate time t , the densities $\sigma = c^{-2}(T^{00} + T^{ss})$, $\sigma^i = c^{-1}T^{0i}$ with $T^{\alpha\beta}$ being the tensor of energy-momentum of matter of the solar system. These equations extend the domain of applicability of (27) and (28) of the IAU standard framework to the case of expanding universe.

Equation (64) describes evolution of the scalar field while (65) describes evolution of the scalar perturbation of the metric tensor. Equation (66) yields evolution of vector perturbations of the metric tensor, and (67) describes generation and propagation of gravitational waves by the isolated N-body system. (64) – (67) contain all relativistic corrections depending on the Hubble parameter and can be solved analytically in terms of the generalized retarded functions of the Bessel types. There exists the exact Green functions for these equations `kram,ramk,pois1`. They revealed that the gravitational perturbations of the isolated

system on expanding background depend not only on the value of the source taken on the past null cone but also on the value of the gravitational field inside the past null cone.

Existence of extra terms in the solutions of (64) – (67) depending on the Hubble parameter brings about cosmological corrections to the Newtonian law of gravity. For example, the post-Newtonian solution of (65) and (66) with a linear correction due to the Hubble expansion are

$$w(\eta, \vec{x}) = G \int \frac{\sigma(\eta, \vec{x}') d^3 x'}{|\vec{x} - \vec{x}'|} + \frac{G}{2c^2} \frac{\partial^2}{\partial \eta^2} \int d^3 x' \sigma(\eta, \vec{x}') |\vec{x} - \vec{x}'| - GH \int d^3 x' \sigma(\eta, \vec{x}') + O(c^{-4}) + O(H^2), \quad (68)$$

$$w^i(\eta, \vec{x}) = G \int \frac{\sigma^i(\eta, \vec{x}') d^3 x'}{|\vec{x} - \vec{x}'|} + O(c^{-2}) + O(H^2). \quad (69)$$

Matching these solutions with those defined in the BCRS of the IAU 2000 framework in (29) and (Figure 1) is achieved after expanding all quantities depending on the conformal time η in the neighborhood of the present epoch in powers of the Hubble parameter.

Current IAU 2000 paradigm assumes that the asymptotically-flat metric, $f_{\alpha\beta}$, is used for calculation of light propagation and ephemerides of the solar system bodies. It means that the conformal time η is implicitly interpreted as TCB in equations of motion of light and planets. However, the physical metric $g_{\alpha\beta}$ differs from $f_{\alpha\beta}$ by the scale factor $a^2(\eta)$, and the time η relates to TCB as a Taylor series that can be obtained after expanding $a(\eta) = a_0 + \dot{a}\eta + \dots$, in polynomial around the initial epoch η_0 and defining TCB at the epoch as $\text{TCB} = a_0 \eta$. Integrating equation $dt = a(\eta) d\eta$ where t is the coordinate time, yields

$$t = \text{TCB} + \frac{1}{2} \mathcal{H} \cdot \text{TCB}^2 + \dots, \quad (70)$$

where $\mathcal{H} = H/a_0$ and ellipses denote terms of higher order in the Hubble constant $\mathcal{H}$. The coordinate time t relates to the atomic time TAI (the proper time of observer) by equation, which does not involve the scale factor $a(\eta)$ at the main approximation. It means that in order to incorporate the cosmological expansion to the equations of motion, one must replace TCB to the quadratic form

$$\text{TCB} \longrightarrow \text{TCB} + \frac{1}{2} \mathcal{H} \cdot \text{TCB}^2. \quad (71)$$

Distances in the solar system are measured by radio ranging spacecrafts and planets. Equations of light propagation preserve their form if one keeps the speed of light constant and replace coordinates $(\eta, \vec{x})$ to $(t, \vec{\Xi})$, where the spatial coordinates $\vec{\Xi}$ relate to coordinates $\vec{x}$ by equation $d\vec{\Xi} = a(\eta) d\vec{x}$. Because one uses TAI for measuring time, the values of the spatial coordinates in the range measurements are given in terms of the capitalized coordinates $\vec{\Xi}$. Therefore, the ranging measurements are not affected by the time transformation given by (70) in contrast to the measurement of the Doppler shift which deals with the time only.

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