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The Nature of the General Relativistic Solution of the Charged Line Mass

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The nature of the solution of a line charge is discussed using the Already Unified Field Theory. It is shown that it is impossible to have charge without mass with this symmetry. Approximate solutions valid for either small or large mass per unit length are presented and compared with previous work. We show that, if charge is present, beyond some critical distance from the axis a neutral test body is repelled instead of attracted.

I. INTRODUCTION

Among the earliest investigations with general relativity was the study of the combined Einstein–Maxwell problem [1]. Several authors [2–4] have, in particular, considered the problem of a static charged line mass. These investigations are incomplete both in a physical sense and in the acceptable range of the parameters they use. Although the formalism presented here can only be solved approximately it has several advantages which will be discussed later in this paper. We will consider the earlier results in some detail in the next section.

In our approach we will use the Rainich [5] Already Unified Field Theory. In this approach the Einstein problem with a stress-tenser due to vacuum nonnull electromagnetic field is equivalent to the following set of fourth-order equations [1]:

$$R_\mu{}^\sigma R_\sigma{}^\nu = \delta_\mu{}^\nu R_{\sigma\tau} R^{\sigma\tau}, \tag{1}$$

$$R = 0, \tag{2}$$

$$R_{00} > 0, \tag{3}$$

$$\alpha_{\beta,\gamma} - \alpha_{\gamma,\beta} = 0, \tag{4}$$

where

$$\alpha_\delta \equiv (-g)^{1/2}\, \epsilon_{\delta\lambda\mu\nu} R^{\lambda\gamma;\mu} R_\gamma{}^\nu / (R_{\alpha\beta} R^{\alpha\beta}). \tag{5}$$

(Einstein summation convention is used throughout, the Minkowski metric is $(1, -1, -1, -1)$ and in general, we follow the "timelike conventions" of Misner, Thorne, and Wheeler [6]. We also choose units such that $c_L = 16\pi G = 1$.) Since the problem is static we can choose a diagonal line element so Eqs. (1)–(5) reduce to

$$R_0{}^0 = R_1{}^1, \qquad R_2{}^2 = R_3{}^3, \qquad R_0{}^0 = -R_2{}^2 \tag{6}$$

and

$$\alpha_\sigma \equiv 0, \tag{7}$$

where (7) follows from the symmetry of the problem.

We choose a line element of the general cylindrical form:

$$ds^2 = e^{2\gamma-2\psi}(dt^2 - d\rho^2) + \rho^2 e^{-2\psi}\, d\phi^2 + e^{2\psi+2\mu}\, dz^2$$
$$-\infty < t, z < +\infty, 0 \leqslant \phi < 2\pi, 0 < \rho < \infty. \tag{8}$$

This puts us into the form used by Witten [7] in his Case III of the Static Cylindrically Symmetric Solutions. Equations (6) which he did not solve become (with $'$ denoting differentiation with respect to ρ)

$$2\psi'' + \mu'' + \mu'^2 + 2\psi'\mu' + 2\psi'/\rho = 0, \tag{9}$$

$$\mu'^2 + \mu'' - 2\gamma'\mu' + 4\mu'\psi' + 2\psi'^2 - 2\gamma'/\rho = 0, \tag{10}$$

and

$$2\psi'' - \gamma'' + 2\psi'/\rho - \gamma'/\rho - \mu'/\rho + 2\psi'\mu' - \gamma'\mu' = 0, \tag{11}$$

while the charge density is given by

$$Q = \pm 2\pi (R_{00})^{1/2}\, \rho e^{\mu+\psi-\gamma}, \tag{12}$$

an expression which is independent of ρ by Maxwell's equations. The nonvanishing components of the Maxwell field tenser are

$$f_{01} = \pm (R_{00})^{1/2}\, e^{\gamma-\psi}, \tag{13}$$

and the measured gravitational force for an observer at rest is

$$g = (\psi' - \gamma'). \tag{14}$$

Witten also showed that the problem could be reduced to solving an integrodifferential equation for μ. We will consider an approximate solution of this in Section III.

The line element (8) we have chosen has the advantage of putting the charge effects explicitly into the function μ. If $\mu' \equiv 0$, then the solution of (9–10) reduces to the uncharged line mass metric [7]

$$ds^2 = \rho^{2c^2+2c}(dt^2 - d\rho^2) - \rho^{-2c}e^{2a}\, d\phi^2 - \rho^{2c+2}\, dz^2. \tag{15}$$

Thus, all the physical effects of the charge depends upon $\mu \not\equiv 0$. Other line elements which we consider in the next section do not have this advantage.

II. Some Previous Solutions

The earliest study of the charged line source was done by Mukherji in 1938 [2]. He used a line element of the form,

$$ds^2 = e^{2\nu}\, dt^2 - e^{2\mu}(dr^2 + r^2\, d\phi^2) - e^{2\lambda}\, dz^2. \tag{16}$$

His solution was

$$e^{2\nu} = X[1 - \pi\epsilon^2 X/(4M^2)]^{-2}, \tag{17}$$

$$e^{2\lambda} = X[1 - \pi\epsilon^2 X/(4M^2)]^2, \tag{18}$$

$$e^{2\mu} = X^{-1/2}[1 - \pi\epsilon^2 X/(4M^2)]^2\, (a/r)^2, \tag{19}$$

where

$$X = 1 + 4M\ln(r/a)$$

and the nonvanishing covariant components of the Maxwell Field tensor are

$$f_{01} = -\epsilon e^{\nu-\lambda}/r. \tag{20}$$

Thus, ϵ is a measure of the electric field strength.

However, in his derivation he assumes that as $\epsilon \to 0$ that $g_{00} = 1 - 2\Omega$ where $\Omega = -2M\ln(r/a)$ is the classical gravitational potential. Hence, his solution is at best only valid to first order in his mass parameter M. There are also other difficulties connected with his solution. In the limit $\epsilon \to 0$ $e^{2\mu} \to (a/r)^2 + 0(M)$ so the line element [16] is certainly not in a physically obvious form. The reason for this behavior can be seen if we look at the physically measured radial component of the electric field given by

$$E_r = (-g^{00}g^{11})^{1/2} f_{01} \tag{21}$$

and at the gravitational force given by

$$g = \tfrac{1}{2} d/dr(\ln g_{00}). \tag{22}$$

Neither of these are of a simple form for r large; in fact, the gravitational force changes sign at $r = a \exp[(\pi\epsilon^2 - 4M^2)/(\pi\epsilon^2)]$, unless $4M^2 < \pi\epsilon^2$.

Bonner in 1952 [3] studied the static charged line source using a line element of the form,

$$ds^2 = e^\sigma\, dt^2 - e^\lambda\, d\rho^2 - \rho^2 e^{-\sigma}\, d\phi^2 - e^\lambda\, dz^2. \tag{23}$$

His metric only involves two functions instead of the three we used. The transformation between his and our coordinates is given by

$$d\rho_{\text{ours}}/d\rho_{\text{Bonner}} = \exp(2\psi + \mu - \gamma), \tag{24}$$

and σ is related to $\gamma - \psi$ and λ to $\psi + \mu$. Thus, his field equations have less freedom than ours.

Thorne has shown for a different axial electromagnetic field problem with $R_0{}^0 = -R_1{}^1$ that μ can be chosen zero with no loss of generality [8]. His proof is not valid for the problem we are considering ($R_0{}^0 = R_1{}^1$) as he indicates.

Bonner shows that his solution corresponds to a line charge along the axis of $(\frac{1}{4})\,\pi^{1/2}C$ where C is one of his constants of integration. If we also calculate the gravitational force we find that it is proportional to C. Thus, the solutions he finds are such that the mass along the axis is proportional to the charge. This supports our contention that the line element he has chosen does not reflect the complete freedom possible with axial symmetry.

Finally, we consider the work of Raychaudhuri in 1960 who also uses the already unified field theory approach [4]. He considers the solutions of the line element,

$$ds^2 = D(x)\, dt^2 - A(x)\, dx^2 - B(x)\, dy^2 - C(x)\, dt^2. \tag{25}$$

The solution he finds which corresponds to our problem is of the form (rewritten in new notation),

$$ds^2 = A^{-1}\, dt^2 - A\, d\rho^2 - \rho^{2-\lambda} A\, d\phi^2 - \rho^\lambda A\, dz^2, \tag{26}$$

where

$$A = (C_1\rho^\sigma - C_2\rho^{-\sigma})^2 \tag{27}$$

with

$$\sigma^2 C_1 C_2 > 0 \qquad \text{and} \qquad \sigma^2 = (\lambda/2)(2 - \lambda).$$

Thus, σ is real and $0 \leqslant \sigma \leqslant \frac{1}{2}$ for $0 \leqslant \lambda \leqslant 2$ and $C_1, C_2 > 0$; otherwise, σ is imaginary and no rules are given for making the line element real. The only comment for σ imaginary is that it can be transformed into Bonner's solution [3]. A choice of σ imaginary is thus in an unusable form. The real $0 \leqslant \sigma \leqslant \frac{1}{2}$, which is claimed to be transformable into Mukherji's solution, gives the physically measured electric field as

$$E_r \sim \begin{matrix} 2\sigma\sqrt{C_2}/\rho^{1+4\sigma} & \rho \text{ large} \\ 2\sigma\sqrt{C_2}/\rho & p \text{ small}, \end{matrix} \tag{28}$$

while the radial gravitational force is

$$g = +(\sigma/\rho)(C_1\rho^\sigma + C_2\rho^{-\sigma})(C_1\rho^\sigma - C_2\rho^{-\sigma})^{-5}. \tag{29}$$

Thus, σ is not interpretable simply as the mass parameter, but restricting σ to the range 0 to $\frac{1}{2}$ is the same as restricting the mass parameter. So once again we only have understandable solutions for small masses. Another problem is that all the $g_{\mu\nu}$'s vanish when $\rho = (C_2/C_1)^{1/\sigma}$. This presents a definite difficulty in interpretation since g changes sign at this critical value in addition to its divergence.

In summary then, Mukherji's solution is an approximation only valid for small masses and charge densities not too large. Bonner's solution is one which seems to assume a proportionality between the charge and mass density. Finally, the physical interpretation of Raychaudhuri's result is not clear.

III. The Nature of the Solution

Witten in his study of this problem [7, 9] noted that the substitutions

$$\gamma' = -\mu' + (A/\rho)\, e^{-\mu} \tag{30}$$

and

$$\psi' = -e^{-\mu}/(2\rho) \int_0^\rho d\xi\, \xi(\mu'' + \mu'^2)\, e^\mu + (B/\rho)\, e^{-\mu} \tag{31a}$$

satisfies Eqs. (9) and (11) and that Eq. (10) then becomes a differential-integral equation in μ (Witten's equation (9-6.54)[7]). This was about as far as he proceeded. We note that instead of the relation (31a) we could also express ψ' as

$$\psi' = e^{-\mu}/(2\rho) \int_\mu^\infty d\xi\, \xi(\mu'' + \mu'^2)\, e^\mu + (B/\rho)\, e^{-\mu}. \tag{31b}$$

At least one integral of (31) should be well defined since the space is flat at infinity and since

$$R_{00} = \mu'/\rho + (\mu'^2 + \mu'')/2. \tag{32}$$

If we now make the substitution

$$\mu = \ln \alpha, \tag{33}$$

we obtain

$$\gamma' = -\alpha'/\alpha + A/(\rho\alpha) \tag{34}$$

and

$$\psi' = 1/(2\rho) - \alpha'/(2\alpha) + D/(\rho\alpha), \tag{35}$$

where either

$$2(D - B) = \lim_{\rho\to 0}(\rho\alpha' - \alpha) \tag{36a}$$

or

$$2(D - B) = \lim_{\rho\to\infty}(\rho\alpha' - \alpha), \tag{36b}$$

depending upon whether (31a) or (31b) is used. We want solutions in which if there is no electric field, $\mu = 0$ and, hence, $\alpha = 1$. Then (34) and (35) should give us the metric (15). Thus, we can without loss of generality choose

$$D = c + \tfrac{1}{2} \tag{37a}$$

and

$$A = (c + 1)^2, \tag{37b}$$

where $c(c + 1)$ is the mass parameter of the line element (15). The differential integral equation for μ reduces to

$$\alpha\alpha'' + 3\alpha\alpha'/\rho - (q + 1)(\alpha + \rho\alpha')/\rho^2 + \alpha^2/(2\rho^2) + \alpha'^2/2 + (q + \tfrac{1}{2})/\rho^2 = 0, \tag{38}$$

where we have set $q = 2c(c + 1)$ for simplicity. A final substitution,

$$y = \rho\alpha, \tag{39}$$

reduces (38) to the form,

$$yy'' + y'^2/2 - (q + 1)\,y' + (q + \tfrac{1}{2}) = 0. \tag{40}$$

This is the equation we must solve subject to the requirement of Eq. (3) which becomes

$$R_{00} = y''/(2y) \geqslant 0. \tag{41}$$

An immediate consequence is that we see when the mass parameter vanishes ($q = 0$), the electric field must also vanish. This can be seen by putting $y = \rho + x(\rho)$; then (40) with $q = 0$ gives $yy'' = -(x')^2/2$ which is inconsistant with Eq. (41).

However, it is possible that Eq. (40) with $q = 0$ has solutions which do not satisfy Eq. (41). If $q \neq 0$ Eq. (40) has a first integral

$$Fy^q = (y' - 1)/(y' - 1 - 2q)^{1+2q} \; (F \text{ a constant}),$$

the integral of which has not been found.

Since when $q = 0$ we want the solution $y = \rho$, we look for solutions of the form,

$$y = \rho + qx(\rho), \tag{42}$$

subject to the requirement (41) which becomes

$$R_{00} = qx''/2y^2 = x'(1 - x'/2)/[2(\rho/q + x)^2] \geqslant 0; \tag{43}$$

that is,

$$0 \leqslant x' \leqslant 2, \qquad 0 \leqslant x'', \tag{44}$$

while Eq. (40) becomes

$$(\rho/q + x)\, x'' - x'(1 - x'/2) = 0. \tag{45}$$

Since (41) tells us that both x'' and x' are nonnegative, the solution x as well as x' must be monatonically increasing; that is, if $\rho_1 \geqslant \rho_2$ then $x(\rho_1) \geqslant x(\rho_2)$ and also $x'(\rho_1) \geqslant x'(\rho_2)$. This is consistent with the type of solution which was found using this metrical form for the line mass (15) or for the other possible static cylindrical electromagnetic universes [7, 9]. The preceeding being true we expect that $x, x', x'' \to 0$ as $\rho \to 0$. An exact series solution of the form,

$$\alpha = 1 + q \sum_{n=1} B_n k^n \rho^{nq},$$

exists with k a parameter and

$$B_n = -1/[2(nq + 1)(n - 1)] \sum_{m=1}^{n-1} (mq + 1)((n + m)\, q + 1)\, B_m B_{n-1}\, , \; B_1 = 1.$$

The B_n alternate in sign and became successively larger; and the series does not converge in mean. The range, if any, for which this series converges has not been determined so we are forced to look for approximate solutions.

The search for approximate solutions is made possible by the restriction (44) and by the vanishing of x if the electric field vanishes. If $q \ll \frac{1}{2}$ then clearly $\rho/q \gg x$. This is also true for any q provided ρ is sufficiently small; but, for ρ large and $q \gg \frac{1}{2}$ then $x \gg \rho/q$.

IV. Solution for $q \ll \frac{1}{2}$

When $q \ll \frac{1}{2}$, $\rho/q \gg 2\rho$ while (44) implies that $x \leqslant 2\rho$, hence the first integral of (45) is approximately

$$x' = 2k^2\rho^q/(k^2\rho^q + 2), \tag{46}$$

where k^2 is an integration constant which we will show is associated with the charge per unit length. Equation (46) can be easily integrated by series techniques to give either

$$x = 2(k^2/2)^{-1/q} \sum_{n=0}^{\infty} C_n^{-1-1/q}(-1)^n(1 + q + nq)^{-1}[(1 + 2\rho^{-q}/k^2)^{-1-n-1/q} - 1] \tag{47}$$

or

$$x = 2(k^2/2)^{-1/q} \sum_{n=0}^{\infty} C_n^{1/q}(-1)^n(1 - nq)^{-1}[(1 + k^2\rho^q/2)^{1/q-n} - 1], \tag{48}$$

with

$$C_n{}^m = [m(m - 1)(m - 2) \cdots (m - n + 1)]/n!,$$

where the series in (48) terminates with a $(-1)^N \ln(1 + k^2\rho^q/2)$ if $q = 1/N$. Both of these series converge for all values of k^2 and ρ but (47) is most useful when $\rho < (2/k^2)^{1/q}$ and (48) when $\rho > (2/k^2)^{1/q}$ in which cases they have leading terms

$$x \sim k^2\rho^{1+q}/(1 + q)$$

and $x \sim 2\rho - 4\rho^{1-q}/[k^2q(1 - q)]$, respectively.

The quality of our approximation can be estimated by computing the term xx'' that we have neglected in the differential equation and comparing it with the dominate term that we kept. That is when $\rho < (2/k^2)^{1/q}$, $x' \sim 2k^2\rho^q$ while

$$xx'' < qk^4\rho^{2q}/[2(q + 1)]$$

and when $\rho > (2/k^2)^{1/q}$, $(\rho/q)\, x'' \sim 4\rho^{-q}/k^2$ while $xx'' < 2q\rho^{-q}/k^2$. In both cases, these ratios are less than q. Better approximations to the solution can be found by successive approximations, however, we will not proceed with this since we are already in a position to extract the physical information about charge and gravitational force.

We find from (43) that

$$R_{00} = (q^2k^2/2)\, \rho^{-2-q}(k^2/2 + \rho^{-q})^{-2}\, (1 + qx/\rho)^{-2}, \tag{49}$$

while

$$\psi' - \gamma' = -(q/2)(1 - x')\, \rho^{-1}(1 + qx/\rho)^{-1}. \tag{50}$$

Hence, the measured radial gravitational force is

$$g = -(q/2)\, \rho^{-1}[1 - 2k^2/(k^2 + 2\rho^{-q})](1 + qx/\rho)^{-1}. \tag{51}$$

Since $qx/\rho \ll 1$ for $q \ll \frac{1}{2}$, we have approximately

$$\psi - \gamma = \ln[\rho^{q/2}(k^2/2 + \rho^{-q})];$$

hence, from Eq. (12)

$$Q = \pm\sqrt{2}\, \pi q k. \tag{52}$$

The physically measured component of the radial electric field which is

$$E_r = f_{01}/(-g_{00}g_{11})^{1/2} = (R_{00})^{1/2}\, e^{\psi-\gamma} \tag{53}$$

becomes

$$E_r = \pm(qk/2)\, \rho^{-1}(1 + qx/\rho)^{-1}, \tag{54}$$

where the choice of $\pm$ is made in accordance with the choice in (52).

Since $q \ll \frac{1}{2}$, Eq. (52) enables us to interpret k as the parameter describing the strength of the electric source. As expected when $q \to 0$ there can be no electric charge or field. The apparent problem is with the radial gravitational force which changes sign at $\rho = (2/k)^{1/q}$. This force is defined in the sense that it is equal to d^2 (proper radial distance)/d(proper time)^2 for a test mass, provided the proper velocity of the test object is small. Our calculations indicate that the presence of charge distorts space in the opposite sense as the effect of matter.

If we calculate the condition for circular orbits by integrating the geodesic equations, we obtain (requiring that ρ and z be constants)

$$-g_{11}(dt/ds) = E,$$
$$-g_{22}(d\phi/ds) = J,$$

and

$$(\gamma' - \psi')(E/J)^2 = (g_{11}/g_{22})^2\, \rho^2(1/\rho - \psi')\, e^{-2\gamma},$$

where E and J can be interpreted as the energy, and angular momentum constants respectively. For $\rho > (2/k)^{1/q}$, $(E/J)^2 < 0$ and there can be no timelike circular orbits for a neutral test particle. Equation (51) implies that this is occuring because the effective gravitational force is repulsive. A somewhat similar but opposite effect occurs in Melvin's Universe [10]. A similar change in sign of the ratio $(E/J)^2$ and in the gravitational force occurs in Raychaudhuri's solution near $\rho_{\mathrm{Ray}} = (C_2/C_1)^{1/2\sigma}$; however, since his parameters, σ, C_1 and C_2, do not have straight forward mass and charge interpretation, what is occuring for his solution is not as clear. In any event the effect of the charge is to reduce the gravitational force. This effect falls off more slowly than the effect of the mass so it finally overpowers and reverses the measured gravitational force no matter how small the charge.

Inserting dimensions into Eq. (52), we obtain

$$Q = \pm\pi 2^{-1/2} G^{-1/2} C_L{}^2 qk = \pm 2\sqrt{2}\,\pi G^{1/2} C_L M k = \pm 2.2 \times 10^4 Mk, \tag{55}$$

where

$$q = 4MG/C_L = 9.1 \times 10^{-19} M, \tag{56}$$

where G is Newton's Gravitational constant, C_L the speed of light, M the mass per unit length in kilogram/meter and Q is the charge in Coulombs/meter. The distance at which the gravitational force changes from attractive to repulsive is (in meters if Q is in C and M in kg/m) $(4.4 \times 10^4 M/Q)^{(1.1\times 10^{18} M)}$ which is extremely large for any experimentally reasonable values of Q and M. The restriction of $q \ll \frac{1}{2}$ means that $M \ll 5.5 \times 10^{17}$ kg/m, and hence probably includes all approximations to real physical systems. The matching of external solutions of cylindrical systems has been discussed elsewhere [8, 11–13] and presents no problem. We expect that the charge could either reside on the surface or be distributed within the interior of the source.

V. Solution for $q \gg \frac{1}{2}$

For $q \gg \frac{1}{2}$ the approximations to Eq. (45) must be made differently depending upon rather $\rho \ll ((q+1)/(qk^2))^{1/q}$ or $\gg ((q+1)/(qk^2))^{1/q}$. In the former case assuming $\rho/q \gg x$ leads to a solution for x' as (46) and hence on x of the form (47). The ratio $x/(\rho/q) \sim qk^{2q}/(q+1)$, which is small when

$$\rho \ll ((q+1)/(qk^2))^{1/q} \sim (1/k^2)^{1/q}.$$

On the otherhand, when ρ is large we cannot neglect x as compared to ρ/q. If we neglect ρ/q compared with x and assume $k \neq 0$ so the range of interest for ρ is non-trivial, we obtain

$$x' = 2(1 - k^2 x^{1/2}), \tag{57}$$

and, hence,

$$x + 2x^{1/2}k^{-4} + 2k^{-4}\ln(\sqrt{x} - k^{-4}) = 2\rho + \text{const}, \tag{58}$$

where the constant cannot be determined since (58) is valid only when $\rho \gg (1/k^2)^{1/q}$ and the choice of k has been made such that $Q \alpha k$. Equation (58) gives us approximately

$$x \sim 2\rho - 2k^{-4}((2\rho)^{1/2} + \tfrac{1}{2}\ln(2\rho)) + 0(k^{-8}), \tag{59}$$

where we have made the expansion in k^{-4} since we are most interested in k large.

In this approximation (43) becomes

$$R_{00} = q^2 k^2 (2\rho)^{-1/2} \rho^{-2} (1 + (qx/\rho))^{-2}, \tag{60}$$

while $\psi' - \gamma'$ given by (50) becomes

$$\psi' - \gamma' \sim (q/2)(1 + 2q)^{-1} \rho^{-1} \sim 1/(4\rho).$$

Thus the measured radial gravitational force for a test particle at rest is

$$g = (q/(2 + 4q))\, \rho^{-1}. \tag{61}$$

Again from (12) we obtain

$$Q = \pm 2^{5/4} \pi q k, \tag{62}$$

while the physically measured radial component of the electric field given by (53) becomes

$$\begin{aligned} E_r &= \pm(q/(1 + 2q))\, k 2^{-1/4}/\rho \\ &\sim \pm 2^{-5/4} k/\rho, \end{aligned} \tag{63}$$

where the choice of $\pm$ is made in accordance with the choice of $\pm$ in (62).

We see again that for large q the gravitational field reverses direction for large ρ becoming repulsive; however, since our approximation in this case is not good near the change over point in the direction of g, we cannot explicitly indicate the behavior of this point upon the mass parameter q but it is clear that it continues to move closer to $\rho = 0$ as q increases.

VI. Conclusions

We have shown explicitly that we must have some mass per unit length to have charge which has been shown previously [2]. Our proof is exact and shows that it is the positive energy density ($R_{00} > 0$) requirement of the Already Unified Theory formulation which assumes this.

We also show that a neutral test body is repelled if it is located at a distance greater than some critical radius (which decreases as the charge per unit length increases). This is just the opposite of the effect present for a charged point mass where a neutral body is repelled if it is too close to the charge. Murkherji's approximation [2] is sufficiently poor that he obtains (but did not note) this result only for small values of the charge per unit length. Raychaudhuri's solution [4] although exact for some small mass parameters has vanishing metric and diverging gravitational field at a critical distance; however, he did not discuss this effect which we have done by presenting Eq. (29). Since Raychaudhuri's metric vanishes and since the gravitational force diverges at the critical distance, the nature of the space is not obvious. Our approximate solutions show that all that is happening at this distance is a change from gravitational attraction to gravitational repulsion. Both charged and uncharged particles are able to cross from the region of gravitational attraction to that of gravitational repulsion and conversely provided the initial conditions are appropriate. Charged particles initially at rest will be either or repelled in a simple fashion easily calculated from the modified geodesic equation,

$$m(d^2\rho/ds^2) = \psi' - \gamma' - ef_0^{\ 1},$$

where m is the mass and e the charge of the test body.

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